Adaptive Output Servocontroller for MIMO System with Input Delay*

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Abstract—The paper deals with the problem of simultaneous adaptive compensation of external disturbances and asymptotic reference tracking in linear time-invariant multi-input multi-output square linear time-invariant (LTI) system with input delay and unmeasurable state. The plant to be controlled can be unstable. The external signals — disturbance to be compensated and reference to be tracked — are modeled as multi-harmonic signals with unknown frequencies, phases and amplitudes. The solution is based on suitable external signals parameterization and prediction as well as on special tracking error augmentation allowing one to overcome the problem of input delay in adaptation algorithm. Two adaptation algorithms using special augmented error are proposed: gradient-based algorithm and an algorithm with improved parametric convergence based on application of Kreisselmeier’s scheme. No a priori assumptions about knowledge of external signal parameters as well as about persistent excitation (PE) condition are needed for asymptotic reference tracking and disturbance rejection in the closed-loop system.

I. INTRODUCTION

The paper concerns the problem of output adaptive tracking multi-harmonic reference signal in multi-input multi-output (MIMO) square linear time-invariant (LTI) system with input delay in presence of external disturbance.

For the last several decades the problems of disturbance compensation and/or reference signal tracking have been intensively investigated. A lot of researches are focused on Internal Model Principle (IMP), according to which the external multi-harmonic disturbance and/or reference signal are modeled as the outputs of autonomous dynamic system (exosystem) exited by nonzero initial conditions. The exosystem parameters are connected with the disturbance and reference signal parameters (amplitudes, frequencies, and phases). Therefore, appropriate incorporation of the exosystem model into the structure of the closed-loop system permits to completely compensate for the disturbance and/or asymptotically track the reference signal. Sometimes controller designed on the basis of the IMP is called servocontroller.

The first works [1],[2],[3] were devoted to the classical IMP where exosystem had known parameters, and the disturbance or reference signal were a priori known. Later for the case of unknown exosystem (when external signals have a priori unknown frequencies, amplitudes and phases) adaptive implementation of IMP was developed for linear [4],[5],[6] and nonlinear systems [7],[8], plants with unknown parameters [9],[10],[11] and was extended to multivariable systems [12],[13].

During last several years the problem of adaptive servocontroller design for plants with input delay draws significant attention. A number of proposed solutions are based on adaptive identification of the signal parameters (frequencies, amplitudes, and phases). Identification approach was successfully implemented for classes of linear [14],[15] and nonlinear [16] systems. One of the recent results [17] is devoted to adaptive control problem for MIMO systems. Identification approach allows for flexible selection of the controller structure, but at the same time this method depends on the PE condition [18] and requires a priori knowledge about exact number of harmonics in the external signals and about boundaries on the signal frequencies.

Another area of interest is the improvement of parametric convergence. In general case the standard gradient-based adaptation algorithms can have arbitrarily bad transient performance despite the fact of guaranteed control objective. Fast parametric convergence can be ensured by dynamic regressor extension (DRE) [27] and mixing (DREM) [28] or with the use of special linear operators with “memory” [29].

The case of state-feedback adaptive servocompensation of external disturbances affecting unstable SISO delayed plant was considered in [22], while the dual problem of adaptive servotracking for the same class of linear plants was solved in [24]. The problems of state-feedback adaptive servocompensation and servotracking for MIMO stable delayed plants were solved separately in [23] and [30], respectively. However, it was recognized that the results of [23],[30] can not be directly extended for the case of simultaneous servotracking and servocompensation of external signals when delayed plant is unstable. Additional structural obstacle arise if the state is not measurable and the output-feedback case is considered.

Thus, in comparison with the previous works the present paper provides the main contribution: the problems of output-feedback adaptive servotracking and servocom-
penetration of external signals are solved simultaneously for a class of MIMO unstable plants with input delay and unmeasurable state. Moreover, in spite of the input delay, the proposed solution allows us to avoid any restrictions on the adaptation gain and/or input delay margin and to improve parametric convergence with the use of a special adaptation algorithm being a modification of Kreisselmeier’s scheme.

The remaining of the paper is organized as follows. In Section II the problem is formulated. In Section III the disturbance and reference signals are parameterized for the control design purposes. In Section IV the physically implementable disturbance observer is designed. In Section V the standard and modified adaptation algorithms are synthesized. Stabilizing component of control law is described in Section VI. Simulation results are demonstrated in Section VIII.

II. PROBLEM STATEMENT

Given the LTI MIMO plant of the form

$$\begin{align*}
\dot{x} &= Ax + Bu(t - \tau) + \delta, \\
y &= Cx,
\end{align*}$$

(1)

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^q$ is the vector of control signals, $y \in \mathbb{R}^p$ is the vector of output variables, $n \geq q$, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times q}$, $C \in \mathbb{R}^{p \times n}$ are the known matrices, $\tau$ is the known delay, $\delta \in \mathbb{R}^q$ is the vector of unmeasurable external disturbances. We consider the problem of designing an output-feedback control providing the boundedness of all the closed-loop signals and achieving the control objective

$$\lim_{t \to +\infty} \|g(t) - y(t)\| = 0,$$

(2)

where $g \in \mathbb{R}^q$ is the vector of reference signals\(^1\). The following assumptions are accepted.

**Assumption 1.** Triple of matrices $(A, B, C)$ is controllable and observable, transfer matrix $W(s) = C(sI - A)^{-1}B$ is invertible and its transmission zeros are asymptotically stable ($s = d/dt$ is the differential operator).

**Assumption 2.** Disturbance $\delta$ and reference signal $g$ can be described as the outputs of corresponding exosystems:

$$\begin{align*}
\dot{\xi}_1 &= \Gamma_1 z_1, \\
\dot{\delta} &= H_1 z_1, \\
\dot{\xi}_2 &= \Gamma_2 z_2, \\
g &= H_2 z_2,
\end{align*}$$

(3) (4)

where $z_1 \in \mathbb{R}^{m_1}$, $z_2 \in \mathbb{R}^{m_2}$ are unmeasurable state vectors with unknown initial conditions, $\Gamma_1 \in \mathbb{R}^{m_1 \times m_1}$, $\Gamma_2 \in \mathbb{R}^{m_2 \times m_2}$ are constant matrices with simple eigenvalues on the imaginary axis, $H_1 \in \mathbb{R}^{q \times m_1}$, $H_2 \in \mathbb{R}^{q \times m_2}$ are constant matrices. Without loss of generality the pairs $(\Gamma_1, H_1)$ and $(\Gamma_2, H_2)$ are assumed to be observable.

**Assumption 3.** The parameters of matrices $\Gamma_1$, $H_1$, $\Gamma_2$, $H_2$ are unknown, but dimensions $m_1$ and $m_2$ are known. The signal $g$ is measurable.

\(^1\)In this paper all the signals (control, reference, disturbance etc.) are vectors. Therefore the term “vector” will be omitted.

\*[Remark 1. In accordance with the problem statement, restrictive requirement to the Hurwitz property of matrix $A$ is not used. Thus, the controller is designed not only to solve the servoprocess, but also to stabilize the plant.

**Remark 2.** In some papers [3], [31], [6] instead of two different exosystems (3) and (4) it is considered a common one with two different outputs

$$\begin{align*}
\dot{x} &= \Gamma z, \\
\delta &= H_1 z, \\
g &= H_2 z.
\end{align*}$$

(5)

However, in the authors’ opinion the case of two exosystems is more general and, at the same time, gives more flexibility for design. All results obtained in the paper can be directly applied to the case of one common exosystem.

**Remark 3.** It is worth noting that Assumption 3 requires a priori knowledge on the upper bound of the number of unknown frequencies in the external signals only, but not on their exact number.

III. DISTURBANCE AND REFERENCE SIGNAL PARAMETERIZATIONS

As the first step of design, we present the disturbance and the reference signals in the forms of linear regressions. These forms are established by the following lemma [4],[7].

**Lemma 1:** The outputs $\delta$ and $g$ of the exosystems (3) and (4) can be represented in the forms:

$$\begin{align*}
\delta &= \Theta_1^\top \xi_1 + \xi_1, \\
g &= \Theta_2^\top \xi_2 + \xi_2,
\end{align*}$$

(6) (7)

where $\Theta_1 \in \mathbb{R}^{n \times m_1}$ and $\Theta_2 \in \mathbb{R}^{n \times m_2}$ are unknown constant matrices, regressors $\xi_1 \in \mathbb{R}^{m_1}$ and $\xi_2 \in \mathbb{R}^{m_2}$ are the state vectors of the filters

$$\begin{align*}
\dot{\xi}_1 &= G_1 \xi_1 + L_1 \delta, \\
\dot{\xi}_2 &= G_2 \xi_2 + L_2 g
\end{align*}$$

(8) (9)

with arbitrary Hurwitz matrices $G_1 \in \mathbb{R}^{m_1 \times m_1}$ and $G_2 \in \mathbb{R}^{m_2 \times m_2}$, and constant matrices $L_1 \in \mathbb{R}^{m_1 \times q}$ and $L_2 \in \mathbb{R}^{m_2 \times q}$ to be chosen such that the pairs $(G_1, L_1)$ and $(G_2, L_2)$ are controllable. The signals $\xi_1$ and $\xi_2$ are caused by nonzero initial conditions and exponentially decay\(^2\).

Substituting (6) into (8) and (7) into (9) we obtain canonical forms of the exosystems

$$\begin{align*}
\dot{\xi}_i &= (G_i + L_i \Theta_i^\top) \xi_i, \\
i &= 1, 2.
\end{align*}$$

(10)

Canonical form (10) allows one to immediately obtain the following corollaries which will be used in adaptive control design [32].

**Corollary 1:** The predicted values of the regressors and the disturbance $\delta$ can be calculated as

$$\begin{align*}
\hat{\xi}_i(t + \tau) &= \exp((G_i(t) + L_i \Theta_i^\top) \tau) \xi_i(t), \\
\hat{\delta}(t + \tau) &= \Theta_i^\top \xi_i(t + \tau) = \Psi_i^\top \xi_i(t),
\end{align*}$$

(11) (12)

\(^2\)Since the signals $\xi_1$ and $\xi_2$ are exponentially decay, they do not influence on stability of the closed-loop system and will be omitted.
where \( \Psi_i^\top = \Theta_i^\top \exp((G_i + L_i \Theta_i^\top) \tau), (i = 1, 2) \) are the matrices of unknown parameters.

**Corollary 2:** The \( i \)th time derivative of the disturbance \( \delta \) can be calculated as

\[
\delta^{(i)} = \Psi_i^\top \xi_i,
\]

where \( \Psi_i^\top = \Theta_i^\top (G_i + L_i \Theta_i^\top)^i \).

**Corollary 3:** For any \( q \times q \) matrix of polynomials \( \Omega(s) \) we have

\[
\Omega(s)[\delta] = \tilde{\delta}^\top \xi_i,
\]

where \( \tilde{\delta} \in \mathbb{R}^{m_1 \times q} \) is an unknown matrix.

**Corollary 4:** For filtered disturbance \( \tilde{\delta} = W(s)[\delta] \), where \( W(s) = q \times q \) asymptotically stable transfer matrix, we have

\[
\delta = \Omega_1^\top \xi_i,
\]

where \( \xi_i \in \mathbb{R}^{m_1} \) is generated by filter

\[
\hat{\xi}_i = G_1 \xi_i + L_1 \hat{\delta}.
\]

The above corollaries will be used in adaptive servocontroller design.

**IV. DISTURBANCE OBSERVER**

However, filter (8) is not physically implementable, since it involves unmeasurable input \( \delta \). Therefore we introduce special disturbance observer

\[
\begin{align*}
\dot{\hat{x}} &= A\hat{x} + L(y - \hat{y}) + Bu(t - \tau), \\
\hat{y} &= C\hat{x},
\end{align*}
\]

where \( L \) is \( n \times q \) matrix so that \( A_L = A - LC \) is Hurwitz. It is easy to see that \( y = \hat{y} + \tilde{\delta} \), where \( \tilde{\delta} = W(s)[\delta] \) is the filtered disturbance and \( W_L = C(sI - A_L)^{-1}B \) is asymptotically stable transfer matrix. Therefore in view of Corollary 4 we can present \( \tilde{\delta} \) in form of (15), where \( \hat{\xi}_i \) is the state vector of the physically implementable filter

\[
\hat{\xi}_i = G_1 \hat{\xi}_i + L_1 (y - \hat{y}).
\]

From Corollary 1 we obtain

\[
\delta(t) = \Psi_1^\top \hat{\xi}_i(t - \tau).
\]

Further, define \( \det W_L(s) = \beta(s)/\alpha(s) \), where \( \beta(s) \) and \( \alpha(s) \) are the polynomials of appropriate orders, and \( \beta(s) \) is Hurwitz due to Assumption 1. Then

\[
W_L^{-1} = \frac{\alpha(s)}{\beta(s)} \text{adj} W_L(s) = \frac{1}{\beta(s)} \Omega(s),
\]

where \( \Omega(s) \) is \( q \times q \) polynomial matrix. Therefore, from Corollary 3 we obtain

\[
\delta(t) = W_L^{-1}(s)[\tilde{\delta}(t)] = \frac{1}{\beta(s)} \Omega(s)[\tilde{\delta}(t)] = \frac{1}{\beta(s)} [\Psi_1^\top \hat{\xi}_i(t - \tau)] = \Psi_1^\top \hat{\xi}_i(t - \tau),
\]

where \( \Psi_1 \) is \( m_1 \times q \) unknown matrix, while

\[
\hat{\xi}_i = \Psi_1^\top \hat{\xi}_i \quad \text{and} \quad \hat{\xi}_i = \Psi_1^\top \hat{\xi}_i,
\]

is filtered regressor \( \hat{\xi}_i \).

Thus, the disturbance \( \delta \) can be presented in the form of linear regression

\[
\delta(t) = \Psi_1^\top \xi_\beta(t - \tau),
\]

where regressor \( \xi_\beta \) is generated by physically implementable filters (17), (18) and (20).

**V. ERROR MODEL AND CONTROL LAW DESIGN**

Introduce state and output tracking errors

\[
e = M\hat{\xi}_2 - x, \quad \xi = g - y,
\]

where \( n \times m_2 \) matrix \( M \) is so that \( CM = \Theta_1^\top \). Then differentiating \( e \) in view of (1), (4) and (21) after some calculations we obtain

\[
\dot{e} = Ae + \left( M(G_2 + L_2 \Theta_2^\top) - AM \right) \hat{\xi}_2 - B(u(t - \tau) + \Psi_1^\top \xi_\beta(t - \tau)).
\]

It is known [3],[33] that under Assumption 1 there exist matrices \( M \) and \( \Theta_2 \) satisfying equations

\[
M(G_2 + L_2 \Theta_2^\top) - AM = B\tilde{\Theta}_1^\top, \quad CM = \Theta_1^\top.
\]

Applying these equalities to (24) we rewrite the tracking error model as

\[
\begin{align*}
\dot{\xi} &= Ae + B(\Psi_1^\top \hat{\xi}_i(t - \tau) - u(t - \tau)), \\
\dot{\xi} &= Ce,
\end{align*}
\]

where the extended regressor \( \xi \in \mathbb{R}^{m_1 + m_2} \) and the extended matrix of unknown parameters \( \Psi \in \mathbb{R}^{(m_1 + m_2) \times q} \) are given by

\[
\Psi^\top = [\Psi_1^\top, \Theta_2^\top \exp((G_2 + L_2 \Theta_2) \tau)].
\]

Analysis of the last equations motivates the following structure of the control law

\[
u = \Psi^\top \xi + u,
\]

where \( \Psi \) is the matrix of adjustable parameters, while \( u \) is the stabilizing component.

**VI. ADAPTATION ALGORITHM DESIGN**

Substituting (26) in (25) we obtain

\[
\begin{align*}
\dot{\xi} &= Ae + B(\Psi^\top \xi_\beta(t - \tau) - u(t - \tau)), \\
\dot{\xi} &= Ce,
\end{align*}
\]

where \( \Psi = \Psi - \hat{\Psi} \) is the matrix of parametric errors. Note that this error model can not be used for design of an adaptation algorithm because matrix \( A \) can be unstable, and the model contains delayed values of adjustable parameters \( \Psi(t - \tau) \). To overcome these problems we use special form of augmented error. Idea proposed in [22],[24] permits to use a scheme of augmentation to remove not only destructive dynamic, but also delay in adjustable parameters. In this case such augmentation is defined by the following lemma.
Lemma 2: Introduce the following filter
\[
\dot{\hat{e}} = A \hat{e} + L_e (\varepsilon + C \hat{e}) + B (\Psi^\top (t - \tau) \xi(t - \tau) + u_s(t - \tau)), \\
\dot{\hat{e}} = \varepsilon + C \hat{e} - \Xi(t - \tau) \Psi(t),
\]

where \( L_e \in \mathbb{R}^{n \times q} \) is such that the matrix \( A + L_e C \) is Hurwitz, \( \Psi = \text{col}(\Psi_1, \ldots, \Psi_q) \in \mathbb{R}^{m_2 + m_2} \) is the vector of adjustable parameters, \( \Psi_i \) are the rows of the matrix \( \Psi, \Xi(t - \tau) \in \mathbb{R}^{q \times (m_1 + m_2)} \) is the matrix of the regressor matrix given by
\[
\Xi(t - \tau) = \begin{bmatrix}
W_{c11}(s) \left[ \tilde{\xi}(t - \tau) \right] & \cdots & W_{c1q}(s) \left[ \tilde{\xi}(t - \tau) \right] \\
\vdots & \ddots & \vdots \\
W_{cql}(s) \left[ \tilde{\xi}(t - \tau) \right] & \cdots & W_{cqq}(s) \left[ \tilde{\xi}(t - \tau) \right]
\end{bmatrix},
\]
and \( W_c(s) = C(sI - A_c)^{-1}B \). Then for the augmented error \( \hat{e} \) we have:
\[
\dot{\hat{e}} = \Xi(t - \tau) \Psi(t),
\]
where \( \Psi = \psi - \tilde{\psi} \) is the vector of parametric errors.

**Proof:** Taking into account the structures of the models (27) and (28), it can be shown that
\[
\dot{\hat{e}} = \varepsilon + C \hat{e} - \Xi(t - \tau) \Psi(t) = W_c(s) \left[ \Psi^\top(t - \tau) \right] - \Xi(t - \tau) \Psi(t).
\] (30)

Then, by direct calculation we can show that
\[
W_c(s) \left[ \Psi^\top(t - \tau) \right] = \Xi(t - \tau) \Psi.
\]
Finally, substituting the last equality in (30) we obtain (29).

A. Standard adaptation algorithm design

Error model (29) is well known in identification and adaptive control theory and allows one to use different standard algorithms of adaptation. In particular case gradient-based one can be used:
\[
\dot{\tilde{\psi}} = \gamma \Xi^\top(t - \tau) \tilde{\psi}(t),
\] (31)
where \( \gamma > 0 \) is the adaptation gain.

Algorithm of adaptation (31) has the following properties.

**Lemma 3:** Under Assumptions 1–3 algorithm of adaptation (31) together with the observer of the disturbance defined by (17), (18) and (20), reference observer (9), control law (26), and scheme of augmentation (28) being applied to plant (1) provides:
1) boundedness of \( \|\hat{e}\|, \|\hat{\psi}\|, \|\tilde{\psi}\|; \\
2) asymptotic convergence \( \|\Psi^\top(t) \tilde{\xi}(t)\| \rightarrow 0 \) and \( \|\Psi^\top(t - \tau) \tilde{\xi}(t - \tau)\| \rightarrow 0 \) for \( t \rightarrow \infty \).

**Proof:** Consider Lyapunov function candidate
\[
V = \frac{1}{2\gamma} \tilde{\psi}^\top \tilde{\psi},
\] (32)
time derivative of which in view of (31) takes the form
\[
\dot{V} = -\tilde{e}^\top \dot{\tilde{e}} \leq 0.
\]
From the latter inequality it directly follows property (3.1) of Lemma 3, and asymptotic convergence \( \|\hat{e}\| \rightarrow 0 \) for \( t \rightarrow \infty \) [18]. Since regressor \( \Xi \) is bounded, then:

a) from (29) it follows \( \|\Xi(t - \tau) \Psi(t)\| \rightarrow 0 \) for \( t \rightarrow \infty \); b) from (31) it follows \( \|\tilde{\psi}\| \rightarrow 0 \) for \( t \rightarrow \infty \). From the latter we obtain that \( \|\tilde{\psi}\| \rightarrow 0 \), and tacking into account the swapping lemma [34] we have \( \|\Xi(t - \tau) \Psi(t)\| \rightarrow 0 \) for \( t \rightarrow \infty \). As a result \( \|\Psi^\top(t - \tau) \tilde{\xi}(t - \tau)\| \rightarrow 0 \) and \( \|\Psi^\top(t) \tilde{\xi}(t)\| \rightarrow 0 \). Proof is complete.

**Remark 4.** Due to Assumption 2 and Lemma 1 matrix regressor \( \Xi(t - \tau) \) is bounded a priori. Therefore, in spite of the fact that the adaptation algorithm (31) involves the augmented error \( \hat{e} \), its normalization is not needed.

B. Adaptation algorithm design with improved convergence

As well known, the gradient-based algorithm of adaptation (31) can demonstrate arbitrarily bad performance of convergence. Improvement of transient performance of convergence can be ensured by adaptation algorithm with “memory” regressor extension using not only instant but also past values of the regressor [29]. In present work we extend this result to the problem of adaptive output regulation in multivariable systems with input delay.

Consider the following algorithm of adaptation:
\[
\dot{\tilde{\psi}} = \gamma \Xi^\top(t - \tau) \tilde{\psi}(t) + \tilde{\gamma}(H(s) \left[ \Xi^\top(t - \tau) \tilde{\psi}(t) \right] + H(s) \left[ \Xi^\top(t - \tau) \tilde{\psi}(t) \right] \Psi(t) - 0,
\] (33)
where \( \tilde{\gamma} > 0 \) and \( H(s) \) is a linear operator with “memory” with the following properties: \( H(s) : L_\infty \rightarrow L_\infty, H(s) \Xi \Xi^\top \succeq 0, \forall \gamma > 0 \).

Then, using the properties of linear operators and taking into account that \( \tilde{\psi} = \psi - \tilde{\psi} \) and \( \Psi = -\tilde{\psi} \), we can show that in this case the parametric error takes the form
\[
\tilde{\psi} = -\gamma \Xi^\top(t - \tau) \Xi(t - \tau) \Psi - \tilde{\gamma}(H(s) \left[ \Xi^\top(t - \tau) \Xi(t - \tau) \right] \Psi.
\] (34)

**Lemma 4:** Under Assumptions 1–3 algorithm of adaptation (33) together with the observer of the disturbance defined by (17), (18) and (20), reference observer (9), control law (26), and scheme of augmentation (28) being applied to plant (1) provides:
1) boundedness of \( \|\hat{e}\|, \|\hat{\psi}\|, \|\tilde{\psi}\|; \\
2) asymptotic convergence \( \|\Psi^\top(t) \tilde{\xi}(t)\| \rightarrow 0 \) and \( \|\Psi^\top(t - \tau) \tilde{\xi}(t - \tau)\| \rightarrow 0 \) for \( t \rightarrow \infty \).

**Proof:** In view of (34) time derivative of Lyapunov function candidate (32) is as follows:
\[
\dot{V} = -\tilde{e}^\top \tilde{e} - \tilde{\gamma} \tilde{\psi}^\top H(s) \left[ \Xi^\top(t - \tau) \Xi(t - \tau) \right] \Psi
\]
with positive constant \( \tilde{\gamma} \) verifying equality \( \tilde{\gamma} = \gamma \).

623
If the condition (35) is met, Lyapunov function derivative satisfies the following inequality
\[ \dot{V} \leq -2 \bar{\gamma} p_0(t)V. \]
Thus, the norm of \( \dot{V} \) tends to zero asymptotically, the rate of convergence can be increased by increasing gain \( \bar{\gamma} \). Proof is complete.

Remark 5. The adaptation algorithm performance can be degraded by exponentially decaying terms \( \bar{z}_1 \) and \( \bar{z}_2 \) in parameterizations (6), (7). In this regard the eigenvalues of matrices \( G_1 \) and \( G_2 \) have to be chosen sufficiently far from the imaginary axis.

VII. STABILIZING COMPONENT DESIGN

Now we use the error model (27) to design stabilizing component \( u_s \). Since the tracking error \( e \) is not measurable we construct an observer in the form
\[ \dot{\tilde{e}} = A\tilde{e} + L_o(e - C\tilde{e}) - Bu_s(t - \tau), \]
(36)
where \( \tilde{e} \) is the estimate of the vector \( e \), and matrix \( L_o \) is such that the matrix \( \tilde{A} = A - L_oC \) is Hurwitz. Introducing the estimation error \( \tilde{e} = e - \tilde{e} \) and taking its time derivative in view of (27) and (36) we obtain
\[ \dot{\tilde{e}} = \tilde{A}\tilde{e} + B\tilde{\Psi}^T(t - \tau)\tilde{\xi}(t - \tau). \]
(37)

Tacking into account item (3.1) of Lemma 3 we get \( \|\tilde{e}(t)\| \rightarrow 0 \) for \( t \rightarrow \infty \).

Based on the known scheme [35],[36],[37] we design stabilizing component in the form
\[ u_s = -K\left[ \exp(At)\tilde{e}(t) - \int_{t-\tau}^t \exp(A(t - \mu))Bu_s(\mu)d\mu \right], \]
(38)
where matrix \( K \in \mathbb{R}^{n \times q} \) is such that matrix \( A_s = A + BK \) is Hurwitz, \( \tilde{e} \) is the state vector of observer (36).

Then the properties of the closed-loop system are defined by the following statement.

Theorem 1: Under Assumptions 1–3 control law (26) together with the observer of the disturbance defined by (17), (18) and (20), reference observer (9), scheme of augmentation (28), algorithm of adaptation (31) or (33), observer of tracking error (36), and stabilizing component (38) being applied to plant (1) provides the following properties of the closed-loop system: 1) all signals are bounded; 2) control objective (2) is achieved.

Proof: By calculating in view of (36) predicted value of \( \tilde{e} \) we obtain:
\[ \tilde{e}(t + \tau) = \exp(At)\tilde{e}(t) + \int_0^t \exp(A(t - \mu))(L_oC\tilde{e}(\tau + \mu) - Bu_s(\mu))d\mu. \]
Then control law (38) can be presented in the form
\[ u_s = -K(e(t + \tau) - \tilde{e}(t + \tau) - \Delta_e(t + \tau)), \]
where
\[ \Delta_e(t + \tau) = \int_{t-\tau}^t \exp(A(t - \mu))L_oC\tilde{e}(\tau + \mu)d\mu. \]

Substitution of (39) in (27) gives
\[ \dot{e} = A_s e + B\tilde{\Psi}^T(t - \tau)\tilde{\xi}(t - \tau) - K\tilde{e} - K\Delta_e(t - \tau). \]
Due to Lemma 3 \( \|\tilde{\Psi}^T(t - \tau)\tilde{\xi}(t - \tau)\| \rightarrow 0 \), and \( \|\tilde{\xi}\| \rightarrow 0 \).

Therefore: \( \|\Delta_e\| \rightarrow 0 \), \( e \) is bounded, \( \|\tilde{e}\| \rightarrow 0 \), objective (2) is achieved, while \( u_s \) is bounded. In view of the boundedness of \( e \) and \( \tilde{e} \) we have boundedness of \( x \), in view of boundedness of \( u_s \) from (28) we have boundedness of \( \tilde{e} \), and from (36) we have boundedness of \( \tilde{e} \). Proof is complete.

VIII. SIMULATION RESULTS

Consider the second order plant (1) with
\[ A = \begin{bmatrix} 0 & 1 \\ 0.1 & -1.5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad D = 3, \]
input delay \( \tau = 3[s] \), disturbance \( \delta = \text{col}(2\cos(3t), 3\sin(4t)) \), and reference signal \( g = \text{col}((\cos(5t), 4\sin(7t)) \) with \( a \) priori unknown amplitudes, frequencies and phases.

Vector \( \tilde{e} \) is generated by observers (9), (18) with matrices
\[ G_1 = G_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -36 & -60 & -37 & -10 \end{bmatrix}, \quad L_1 = L_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}. \]

For observer (17), augmented error model (28), state error observer (36), and stabilizing component (38) we select the following matrices:
\[ L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad L_o = \begin{bmatrix} 0 & 0 \\ -1.1 & -1/6 \\ 1/1.1 & 0.2 \end{bmatrix}, \quad K = \begin{bmatrix} -1 & -0.5 \\ 0 & 0 \end{bmatrix}. \]

Parameters of the adaptation algorithm with linear operator with “memory” (33)
\[ H(s) = \frac{1}{s + 0.1} \]
are given by
\[ \gamma = 10, \quad \tau = 1000. \]

The simulation results for the control system closed by the algorithm with linear operator with “memory” are presented in Fig. 1 and demonstrate achievement of the control objective as well as the boundedness of the adjustable parameters.

IX. CONCLUSIONS

Servocontrollers with two adaptation algorithms for the problem of adaptive output multi-harmonic tracking for the class of LTI unstable systems with delayed input in presence of multi-harmonic external disturbance are presented. Both algorithms provide the closed-loop system stability for arbitrary adaptation gains and input delays and do not require identification of disturbance parameters. The first algorithm is based on the standard gradient algorithm and has bounded parametric convergence rate. The second algorithm is modified by integral cost function that allows to improve transient performance of the servocontroller.

Our further research is related to the extensions of adaptive servocontroller problem for plants with unknown input delay and parametric uncertainties.
REFERENCES


