# Efficient Hierarchical Robust Predictive Control Strategy for Motion Control of Four In-Wheel Motor Actuated Vehicles

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Abstract— The application of in-wheel motors makes the motion control of electric vehicles more flexible and enables the possibility of vehicle attitude control without active suspension systems. However, the obvious symmetry of the model of four in-wheel motor actuated vehicles (4IWMAVs) leads to the issue of homogeneous cost, resulting in oscillations in control output. Moreover, the substantial computational burden due to the high-dimensional vehicle dynamics system, along with the uncertainties stemming from road conditions and actuator dynamics, also deteriorates the performance of vehicle motion control. To solve the above problems, an efficient hierarchical robust predictive control strategy is proposed for 4IWMAVs. The hierarchical structure is adopted to reduce the dimensions of the individual solving problems. At the upper layer, a robust predictive control method based on the invariant set theory is applied to generate the generalized force and torque for motion control. The maximum robust positive invariant set of predictive errors is calculated, and the feedback compensating law is design to reduce uncertainty influence. Then, at the lower layer, a dimensionality-reduction-aware torque allocation algorithm is developed, leveraged the model symmetry fully. The devised torque allocation algorithm achieves dimension-reduced solution under homogeneous cost condition by the self-adjusting mapping matrix, solving the control oscillation caused by multiple optimal solutions. Besides, the algorithm considers the generalized force and torque constraints while minimizing overall energy consumption and ensuring the reasonable vehicle attitude. To sum up, the proposed robust hierarchical strategy can achieve balance motion control demand and energy consumption for 4IWMAVs. The performance of the proposed strategy is verified to meet the requirements of the benchmark problem for motion control of 4IWMAVs through the Matlab/Simulink-Modelica co-simulation.

# I. INTRODUCTION

In-wheel motors have excellent prospects in the application of electric vehicles due to their advantages in spatial arrangement and drive efficiency. The four in-wheel motor actuated vehicles (4IWMAVs) can adjust the drive torque of each wheel as needed, thereby achieving more flexible motion control and the ability to adjust the vehicle attitude without the active suspension systems [1]. However,

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Yuhang Zhang, Chao Yang, Weida Wang, and Tianqi Qi are also with the Chongqing Innovation Center, Beijing Institute of Technology, Chongqing 401120, China. (Corresponding author: Yuhang Zhang). the increase in the number of actuating components makes the motion control strategy for 4IWMAVs more complex than that of normal vehicles. For normal vehicles, the focus of motion control considerations lies in uncertainties in the road conditions and actuator dynamics. But for 4IWMAVs, the increased model dimensionality also presents challenges in control. Furthermore, due to the uniformity of in-wheel motors, the system model exhibits significant symmetry. Thus, there will be multiple sets of equally cost-effective torque distribution results, leading to the output fluctuations, i.e., the issue of homogenous cost.

To address the benchmark problems for motion control of 4IWMAVs considering the aforementioned issues, an efficient hierarchical robust predictive control strategy is proposed. A hierarchical structure is adopted to decompose the high-dimensional overall control problem into two lower-dimensional problem: the generalized force control problem for target velocity or trajectory tracking and the torque allocation problem considering vehicle attitude constraints and energy consumption optimization. The upper layer is designed by the tube-based model predictive control method, which is based on the invariant set theory and has excellent robustness against uncertainties. At the lower layer, a dimensionality-reduction-aware torque allocation algorithm is proposed. Leveraging model symmetry, the mapping matrix that can be adjusted in real-time based on the vehicle's state is designed to represent the relationship between symmetric control variables. The reduces the number of control variables that need to be optimized under cost-equivalent conditions. enabling optimization solution efficient. The schematic diagram of the proposed control strategy is shown in Fig. 1. The effectiveness of this strategy is verified through the Matlab/Simulink-Modelica co-simulation.

### II. MODELLING AND PROBLEM STATEMENT

In this section, the system model for motion control of 4WIMAVs is established and the optimal problems are shown.

# A. System Modelling

According to the vehicle dynamics analysis, the system model is as follows[1]:



Figure 1. Schematic diagram of the proposed control strategy

$$\begin{cases} m\dot{v}_{z} = v_{y}\dot{\phi} + \frac{1}{m}\sum\mathcal{F}_{z} - F_{vind} & (a) \\ m\dot{v}_{y} = m_{r}h_{r}\ddot{\theta} + \sum\mathcal{F}_{z} + v_{z}\dot{\phi} & (f) \\ \ddot{\phi} = \frac{1}{I_{zz}}[I_{f}F_{y,\beta}\cos\delta + I_{f}F_{y,\beta}\cos\delta - I_{r}(F_{y,r} + F_{y,r}) + \Delta M_{z} + I_{zz}\ddot{\theta}] \\ \begin{cases} m\ddot{z} = F_{d}\left[-p\tan\theta_{f} + (1-p)\tan\theta_{r}\right] - K_{z}z - C_{z}\dot{z} + \Delta\mathcal{F}_{z} \\ I_{x}\ddot{\theta} = I_{zx}\ddot{\phi} + m_{s}h_{s}\left(\dot{v}_{y} + \dot{\phi}\right) - (K_{\theta} + C)\theta - C_{\theta}\dot{\theta} + \Delta M_{x} & (b) \\ I_{y}\ddot{\psi} = F_{d}\left[pI_{f}\tan\theta_{f} + (1-p)I_{t}\tan\theta_{r} - h\right] - K_{\psi}\Psi - C_{\psi}\dot{\psi} + \Delta M_{y} \end{cases}$$

#### B. Problem statement

Through linearization and discretization, (1a) can be transformed into state-space form. Thus, at the upper layer in the proposed structure, the control problem can be built as follows for MPC method.

$$\min J_{up} = \sum_{k=0}^{n_p} (\left\| \mathcal{X}_{t+k,t} - \mathcal{X}_{ref} \right\|_{\mathcal{Q}_{up}}^2 + \left\| U_{t+k,t} \right\|_{R_{up}}^2 + \xi \varepsilon^2)$$
(2)

For the lower layer, the torque allocation problem based on (1b) can be transmitted into an optimization problem.

$$\min J_{low} = \sum_{i=0}^{\infty} (J_{attitude,i}(\tau_{n,i}) + J_{s,i}(\tau_{n,i}) + J_{e,i}(\tau_{n,i}))$$

$$J_{attitude,i} = f_a(z_i(\tau_{n,i}), \theta_i(\tau_{n,i}), \psi_i(\tau_{n,i})), J_s = f_s(\tau_{n,i}), J_e = f_e(\mathbf{\Omega}, \tau_{n,i}, \eta_i)$$

$$st. \quad \tau_{n,i} \in \Pi \quad n = fl, fr, rl, rr \quad \mathcal{F}_{s,k} = f(\tau_n) \qquad \Delta M_{z,k} = g(\tau_n)$$

$$(3)$$

## III. HIERARCHICAL CONTROL STRATEGY

In this section, the method adopted in the proposed hierarchical control strategy is introduced briefly.

# A. Tube-based MPC method for the upper layer

To deal with the uncertainties from road adhesion and steering dynamics, the tube-based MPC method is applied in the upper layer. The system is divided into two parts, i.e., the nominal system model and the error dynamics model.

The basic idea of the tube-based MPC is to obtain the feedforward control input and the state feedback control input, respectively. [2] The final output is as follows:

$$\boldsymbol{U}_{k} = \boldsymbol{\bar{U}}_{k} + \boldsymbol{\tilde{U}}_{k} = \boldsymbol{\bar{U}}_{k} + K\boldsymbol{e}_{k}$$
(4)

where the nominal output can be obtained by the MPC method based on the nominal system. The feedback gain is determined by the maximum robust positively invariant set of the error dynamics system.

## B. Torque allocation algorithm for the lower layer

Inspired by [3], there exists a mapping function that holds the following equation in the case of homogeneous cost.

$$J_{low}(\tau_n) = J_{low}'(\zeta(\tau_n)) = J_{low}'(\overline{\tau}_n) \quad \overline{\tau}_n^* = \min_{\overline{\tau}_n \in \Pi} J_{low}'(\overline{\tau}_n) \quad \tau_n^* = \zeta^{-1}(\overline{\tau}_n^*)$$
(5)

In this work, the mapping function can be simplified into a mapping matrix, which will be adjusted by the current vehicle state.

#### IV. CONCLUSION

The proposed hierarchical control strategy is to solve the benchmark problems for motion control of 4WIMAVs. At the upper level, motion control considering uncertainties is achieved by tube-based MPC. At the lower level, the dimensionality-reduction-aware torque allocation algorithm can balance vehicle attitude control, energy consumption, and stability requirements.

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