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Model Predictive Planing and Control for the Benchmark Problem of Four in-Wheel Motor Actuated Vehicles

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***Abstract*— The benchmark problem requires controlling the four in-wheel motor actuated vehicle by steering and wheel torques to minimize tracking error and energy consumption with given constraints. To this end, we first design a motion planner to generate total torque commands and achievable steering trajectories based on the slow mechanical dynamics of the 6-DOF vehicle body. Then, we distribute the torque to each wheel minimizing the tire slip energy loss by quadratic programming, and track the reference steering by model predictive control. Finally, the simulation results on the benchmark vehicle model validate the safety, tracking performance, and energy efficiency of the proposed method.**

A. *The Benchmark Problem*

The benchmark problem provides a simulator written by Modelica of a four in-wheel motor actuated vehicle, and the challengers are required to design a controller to minimize tracking error (1) and energy consumption (2), as well as satisfy constraints on vertical acceleration, pitch angle, and roll angle (3).

$$J_1 = \int_0^T (v(t) - v_r(t))^2 \, \mathrm{d}t, \tag{1a}$$

$$J_2 = \int_0^T (x(t) - x_r(t))^2 + (y(t) - y_r(t))^2 \, \mathrm{d}t, \tag{1b}$$

$$E = \int_0^T \sum_{i=1}^{4} V_i(t) I_i(t) \, \mathrm{d}t, \tag{2}$$

$$-C_{\max} \le [a_z(t), \varphi_p(t), \varphi_q(t)] \le C_{\max}. \tag{3}$$

where the tracking objectives for task 1 and task 2 are denoted as J_1 and J_2 respectively. The variables $v(t)$, $x(t)$, and $y(t)$ represent the longitudinal speed, position, and lateral position of the vehicle. Similarly, $v_r(t)$, $x_r(t)$, and $y_r(t)$ are the reference signals. The battery voltage and current of the i-th motor are denoted as V_i and I_i respectively. Additionally, $a_z(t)$, $\varphi_p(t)$, and $\varphi_q(t)$ represent the vertical acceleration, pitch angle, and roll angle of the vehicle body.

To solve this problem, we first design a motion planner to generate total torque commands and steering trajectories that match the achievable motion trajectory of the vehicle body. This is necessary because the target vehicle trajectory may contain discontinuities, such as sudden turns. Due to the difference between the commanded steering angle and the actual steering angle, we track the steering trajectories using model predictive control, where the model is obtained via system identification. Finally, we distribute the total torque across the four wheels and adjust each motor's torque based on longitudinal force and slip rate to minimize total slip energy loss. The overall control framework is illustrated in Fig. 1.

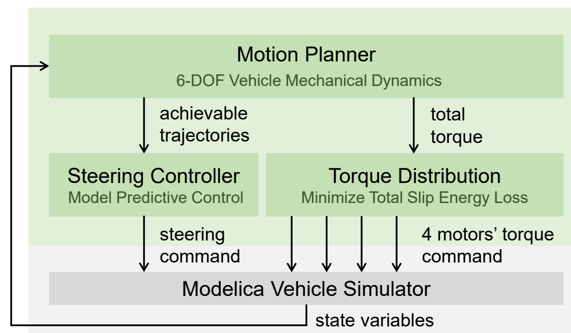

Fig. 1. Overall control framework.

In the following part of this paper, we will introduce the design of motion planner, steering controller, and torque distribution. Finally, we present the simulation results.

B. *Motion Planner Design*

The 6-DOF vehicle mechanical dynamics are

$$\begin{aligned} m(\dot{v}_x - v_y\dot{\varphi}_p + v_z\dot{\varphi}_q) &= F_x, \\ m(\dot{v}_y - v_z\dot{\varphi}_r + v_x\dot{\varphi}_p) &= F_y, \\ m_s(\dot{v}_z - v_x\dot{\varphi}_q + v_y\dot{\varphi}_p) &= F_z, \\ I_x\ddot{\varphi}_p &= M_x, \\ I_y\ddot{\varphi}_r &= M_y, \\ I_z\ddot{\varphi}_q &= M_z. \end{aligned}$$

where m, m_s represent the vehicle and sprung mass, respectively. v_x, v_y, v_z denote the body longitudinal, lateral, and vertical speed. F_x, F_y, F_z are the body longitudinal, lateral, and vertical resultant forces. $\varphi_p, \varphi_q, \varphi_r$ represent the body roll, pitch, and yaw angles. M_x, M_y, M_z denote the body roll, yaw, and pitch moments.

Let $\boldsymbol{x} = [x, y, z, \varphi_p, \varphi_q, \varphi_r]$, state vector $\boldsymbol{w} = [\boldsymbol{x}, \dot{\boldsymbol{x}}]^\top$, and $\boldsymbol{u} = [F_x, F_y, F_z, M_x, M_y, M_z]^\top$, the state space model can be written as

$$\dot{\boldsymbol{w}} = \boldsymbol{f}(\boldsymbol{w}, \boldsymbol{u}).$$

We discretize and linearize the state space model and implement motion planning by solving the following opti-

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mization problem

$$\begin{aligned}\min_{\boldsymbol{u_0},\boldsymbol{u_1},\cdots,\boldsymbol{u_{N-1}}} \quad & \sum_{k=0}^{N} \|\boldsymbol{w}_k - \boldsymbol{w}_r\|_{\boldsymbol{Q}}^2, \\ \text{s.t.} \quad & \boldsymbol{w}_{k+1} = \boldsymbol{A}(t)\boldsymbol{w}_k + \boldsymbol{B}(t)\boldsymbol{u}_k + \boldsymbol{G}(t), \\ & \boldsymbol{u}_{\min} \leq \boldsymbol{u}_k \leq \boldsymbol{u}_{\max},\ k = 0, 1, \cdots, N-1, \\ & \boldsymbol{w}_0 = \boldsymbol{w}(t).\end{aligned}$$

where $\|\boldsymbol{w}\|_{\boldsymbol{Q}}^2 = x^\top \boldsymbol{Q} w$, N is the prediction horizon, $\boldsymbol{w}_r$ is the reference state, and $\boldsymbol{A}(t), \boldsymbol{B}(t), \boldsymbol{G}(t)$ are the linearized state matrices at time t. $\boldsymbol{Q} \geq 0$ is the weight matrix, and $\boldsymbol{u}_{\min}, \boldsymbol{u}_{\max}$ are the lower and upper bounds of the control input, which play an important role in the continuity of the predicted motion.

C. Steering Controller Design

The reference steering angle trajectory is calculated using the state trajectory generated by the motion planner, which is given by

$$\{\varphi_{\text{ref},k}\}_{k=0}^{N} := \left\{\arcsin\left(L\dot{\varphi}_{r,k}/\sqrt{v_x^2 + v_y^2}\right)\right\}_{k=0}^{N},$$

where L is the distance between the front and rear wheels.

Let $\boldsymbol{\varphi}_k = [\varphi_k, \dot{\varphi}_k]^\top$ denote the augmented state vector, $\boldsymbol{\varphi}_{\text{cmd},k} = [\varphi_{\text{cmd},k}, \dot{\varphi}_{\text{cmd},k}]^\top$ denote the augmented control input, and the augmented state space model obtained through system identification from the commanded steering angle to the actual steering angle can be written as

$$\boldsymbol{\varphi}_{k+1} = \boldsymbol{A}_\varphi \boldsymbol{\varphi}_k + \boldsymbol{B}_\varphi \boldsymbol{\varphi}_{\text{cmd},k}.$$

Then we design a model predictive controller to track the reference steering angle trajectory by solving the following optimization problem

$$\begin{aligned}\min_{\{\boldsymbol{\varphi}_{\text{cmd},k}\}_{k=0}^{N}} \quad & \sum_{k=0}^{N} \|\boldsymbol{\varphi}_k - \boldsymbol{\varphi}_{\text{ref},k}\|_{\boldsymbol{Q}_\varphi}^2 + \|\boldsymbol{\varphi}_{\text{cmd,k}} - \boldsymbol{\varphi}_{\text{ref},k}\|_{\boldsymbol{R}_\varphi}^2, \\ \text{s.t.} \quad & \boldsymbol{\varphi}_{k+1} = \boldsymbol{A}_\varphi \boldsymbol{\varphi}_k + \boldsymbol{B}_\varphi \boldsymbol{\varphi}_{\text{cmd},k}, \\ & \boldsymbol{\varphi}_{\min} \leq \boldsymbol{\varphi}_{\text{cmd,k}} \leq \boldsymbol{\varphi}_{\max},\ k = 0, 1, \cdots, N-1, \\ & \boldsymbol{\varphi}_0 = \boldsymbol{\varphi}(t).\end{aligned}$$

where $\boldsymbol{Q}_\varphi \geq 0$ and $\boldsymbol{R}_\varphi \geq 0$ are the weight matrices.

D. Torque Distribution with Energy Saving

The total torque is calculated by

$$T = R\sqrt{F_x^2 + F_y^2}.$$

where R is the wheel radius.

In each wheel of the vehicle, the tire force can be calculated by linear approximation of the magic formula and tire vertical dynamics. The longitudinal force F_i and slip rate k_i are approximated by

$$\begin{aligned}& F_i \approx c_i k_i, \\ & k_i \approx \gamma T_i, \quad i = \text{fl}, \text{fr}, \text{rl}, \text{rr}.\end{aligned}$$

where T_i is the torque of the i-th wheel, and c_i, γ are constants.

The slip power loss is

$$P_{\text{loss}} \approx \sum_{i\in\{\text{fl},\text{fr},\text{rl},\text{rr}\}} F_i k_i v_x, \quad \approx v_x \gamma^2 \sum_{i\in\{\text{fl},\text{fr},\text{rl},\text{rr}\}} c_i T_i^2.$$

Then we distribute the total torque to each wheel minimizing the tire slip energy loss by quadratic programming as follows

$$\begin{aligned}\min_{T_{\text{fl}}, T_{\text{fr}}, T_{\text{rl}}, T_{\text{rr}}} \quad & \sum_{i\in\{\text{fl},\text{fr},\text{rl},\text{rr}\}} c_i T_i^2, \\ \text{s.t.} \quad & T_{\text{fl}} + T_{\text{fr}} + T_{\text{rl}} + T_{\text{rr}} = T, \\ & T_i \leq T_{\max},\ i = \text{fl}, \text{fr}, \text{rl}, \text{rr}.\end{aligned}$$

E. Simulation Results

The simulation results of task 1 are shown in Fig. 2. From top to bottom, the figures show the curves including the total torque of four wheels, the body vertical acceleration, the body roll/pitch angle, body/target longitudinal speed, speed tracking error, energy consumption.

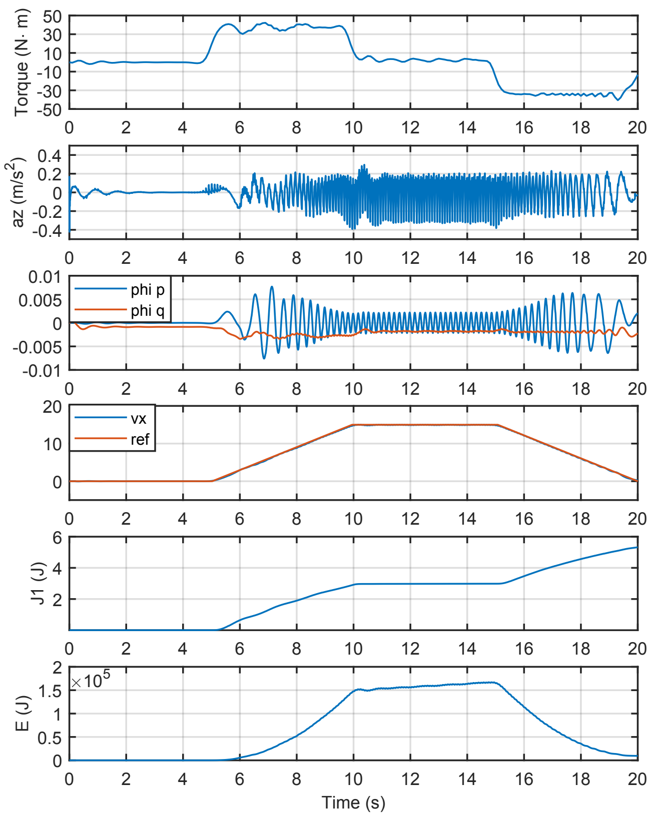

Fig. 2. Simulation results for task 1.

The simulation results of task 2 are presented in the following table only due to the space limitation.

task	max $\lvert a_z \rvert$	max $\lvert \varphi_p \rvert$	max $\lvert \varphi_q \rvert$	J_1 or J_2	E
1	0.3921	0.0077	0.0034	5.3256	9543.7
2	2.3874	0.0239	0.0041	0.5901	3406.4

TABLE I

SIMULATION RESULTS FOR TASK 1 AND TASK 2.