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Stochastic Linear-Quadratic Optimal Control Problems — Some Recent Results

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***Abstract*—For linear-quadratic (LQ, for short) optimal control problems, there are several notions closely related: Existence of optimal control, two-point boundary value problem (optimality system), and (differential/algebraic) Riccati equation. An impression that people have is that these three notions are roughly equivalent. However, it is too vague.**

The purpose of this talk is to clarify the relation among the above notions for stochastic LQ problems with deterministic coefficients. We introduced open-loop and closed-loop solvability of LQ problems. The following results will be presented: (i) The open-loop solvability is equivalent to the solvability of the optimality system (which is now a forward-backward stochastic differential equation) plus the convexity of the cost functional; (ii) The closed-loop solvability is equivalent to the solvability of the Riccati equation; (iii) Open-loop and closed-loop solvability are not equivalent, in general, and they are equivalent for the infinite horizon LQ problems; (iv) There are corresponding results for two-person differential games of LQ setting in terms of open-loop and closed-loop saddle point; (v) If the LQ optimal control problem has a closed-loop optimal strategy, and the problem also has an open-loop optimal control which admits a closed-loop representation, then the open-loop optimal control coincides with the outcome of the closed-loop optimal strategy; (vi) The conclusion of (v) continue to hold for open-loop and closed-loop saddle points of two-person zero-sum LQ differential games; However, it is not true, in general, for Nash equilibria of non-zero-sum differential games.

This talk is divided into two parts: Part I will cover (i)–(iii), and Part II will cover (iv)–(vi).

This talk is based on the work of the speaker joint with Jingrui Sun of Southern University of Science and Technology, China.

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