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Dissipativity-Based Decentralized Control and Topology Co-Design for Vehicular Platoons With Disturbance String Stability

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Abstract— **Merging and splitting of vehicles in a platoon is a basic maneuvering that makes the platoons more scalable and flexible. The main challenges lie in simultaneously ensuring the compositionality of the distributed controllers and the string stability of the platoon. To handle this problem, we propose a control and topology co-design method for vehicular platoons, which enables seamless merging and splitting of vehicular platoons. In particular, we first present a centralized linear matrix inequality (LMI)-based control and topology co-design optimization for vehicular platoons with formal (centralized) disturbance string stability (DSS) guarantee. Then, these centralized DSS constraints are made decentralized by developing an alternative set of sufficient conditions. Using these decentralized DSS constraints and Sylvester's criterion-based techniques, the said centralized LMI problem is decomposed into a set of smaller decentralized LMI problems that can be solved at each vehicle in a compositional manner, enabling seamless vehicular merging/splitting. Finally, simulation examples are provided to validate the proposed co-design method through a specifically developed simulator.**

I. Introduction

With the prospect of increasing highway congestion and fuel consumption, vehicular platoons have been proposed as a promising solution for future transportation development [1]. Apart from normal platooning coordination, vehicles inevitably join or leave platoons due to different destinations or schedules of passengers. Even though a variety of platooning control methods have been presented in recent years, e.g., linear (PID, LQR/LQG, H_∞ control) and nonlinear (model predictive control (MPC), sliding mode control, backstepping, intelligent control) methods [2], only a few attention has been paid on merging and splitting maneuvers for vehicular platoons. Existing literature has mainly studied several heuristic or planning-based methods for vehicular merging and splitting, such as PID and MPC [3] without the assuring of the string stability. More importantly, a complete controller re-design is generally needed for the entire platoon after any vehicles join or leave. In other words, these methods are not compositional and lack a formal string stability guarantee.

Another concern is the communication topology synthesis for merging and splitting, as the topology is usually assumed to be fixed in most of the existing works. However, the topology is always expected to be dynamic to enable vehicles to join or leave a platoon at any time. Such dynamic changes frequently occur when ramp vehicles merge into mainlines, long platoons pass intersections, and vehicles exit a platoon or change lanes [4]. This makes the topology synthesis more difficult since the platoons' compositionality and string stability are required to hold after vehicles join or leave the platoons.

Based on the above discussion, in this paper, we propose a dissipativity-based control and topology co-design method with a string stability guarantee for vehicular merging and splitting in a platoon. The main contributions of this paper are summarized as follows:

1) A centralized LMI-based co-design strategy is proposed that enforces a strong notion of string stability (in particular, the so-called disturbance string stability (DSS) [5]);
2) Through identifying an equivalent sequence of compositional LMI conditions along with an alternative sequence of compositional DSS constraints, we decentralize the proposed centralized LMI-based co-design strategy;
3) The compositionality of the proposed decentralized co-design strategy enables seamless merging and splitting for vehicular platoons;
4) Using a specifically developed simulator, the effectiveness of the proposed co-design strategies is explored;
5) The proposed co-design strategy reveals crucial communication links (neighbor information) to preserve string stability in platoons.

This paper is an extension of our preliminary work [6] with the DSS requirements. Due to space constraints, proofs are omitted here but will be available in our journal version.

This paper is organized as follows. Section II summarizes the notations and preliminary concepts. The problem formulation is given in Section III, followed by our main results in Section IV. In Section V, the simulation results are presented. Finally, a concluding remark is given in Section VI.

II. Preliminaries

Notations*:* The real and natural numbers sets are denoted by $\mathbb{R}$ and $\mathbb{N}$, respectively. We define index sets $\mathcal{I}_N := \{1,2,\ldots,N\}$ and $\mathcal{I}_N^0 := \mathcal{I}_N \cup \{0\}$, where $N \in \mathbb{N}$. An $n \times m$ block matrix A can be represented as $A := [A_{ij}]_{i\in\mathcal{I}_n, j\in\mathcal{I}_m}$, where A_{ij} is the $(i,j)^{\text{th}}$ block of A (for indexing purposes, either subscripts or superscripts may be used, i.e., $A_{ij} \equiv A^{ij}$). $[A_{ij}]_{j\in\mathcal{I}_m}$ and $\text{diag}([A_{ii}]_{i\in\mathcal{I}_n})$ represent a block row matrix and a block diagonal matrix, respectively. We define $\{A^i\} := \{A_{ii}\} \cup \{A_{ij}, j \in \mathcal{I}_{i-1}\} \cup \{A_{ji} : j \in \mathcal{I}_i\}$. The zero

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and identity matrices are denoted by $\mathbf{0}$ and $\mathbf{I}$, respectively (dimensions will be obvious from the context). A positive definite (semi-definite) matrix $A \in \mathbb{R}^{n\times n}$ is represented as $A = A^\top > 0$ ($A = A^\top \geq 0$). The sum of a matrix A and its transpose is defined as $\mathcal{S}(A) := A + A^\top$. $\mathbf{1}_{\{\cdot\}}$ is the indicator function and $\mathrm{e}_{ij} := \mathbf{1}_{\{i=j\}}$. We use $\mathcal{K}$, $\mathcal{K}_\infty$ and $\mathcal{KL}$ to denote different classes of comparison functions (e.g., see [7]). For a vector $x \in \mathbb{R}^n$, its Euclidean norm is given by $|x|_2 := |x| := \sqrt{x^\top x}$. For a time-dependent vector $x(t) \in \mathbb{R}^n$, its $\mathcal{L}_2$ and $\mathcal{L}_\infty$ norms are given by $\|x(\cdot)\| := \sqrt{\int_0^\infty |x(\tau)|^2 d\tau}$ and $\|x(\cdot)\|_\infty := \sup_{t\geq 0} \; |x(t)|$, respectively. For a function of time t, we may omit the notation (t) for simplicity.

*1) **Dissipativity Theory**:* Consider a networked system Σ comprised of N subsystems $\{\Sigma_i : i \in \mathcal{I}_N\}$, where the dynamics of each subsystem $\Sigma_i, i \in \mathcal{I}_N$ are given by

$$\Sigma_i : \begin{cases} \dot{x}_i = f_i(x_i, \{x_j\}_{j\in\mathcal{E}_i}, u_i), \\ y_i = h_i(x_i, u_i), \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$ and $u_i \in \mathbb{R}^{q_i}$ are the subsystem's state and input, respectively. $\{x_j : j \in \mathcal{E}_i\}$ are the states of the neighboring subsystems of the subsystem Σ_i. Consequently, the dynamics of the networked system Σ can be written as

$$\Sigma : \begin{cases} \dot{x} = f(x, u), \\ y = h(x, u), \end{cases} \tag{2}$$

where $x := [x_i^\top]_{i\in\mathcal{I}_N}^\top$, $u := [u_i^\top]_{i\in\mathcal{I}_N}^\top$, $f := [f_i^\top]_{i\in\mathcal{I}_N}^\top : \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}^n$, $n := \sum_{i\in\mathcal{I}_N} n_i$, $q := \sum_{i\in\mathcal{I}_N} q_i$, and $h := [h_i^\top]_{i\in\mathcal{I}_N}^\top : \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}^m$. The function f is assumed to be locally Lipschitz continuous around each equilibrium point $x^* \in \mathcal{X}$ with $f(x^*, u^*) = \mathbf{0}, \forall x^* \in \mathcal{X} \subset \mathbb{R}^n$ ($\mathcal{X}$ denotes a set of equilibrium states, u^* is the input at this equilibrium).

We assume that the equilibrium points of (2) are such that there exists a set $\mathcal{X} \subset \mathbb{R}^n$ where for every $x^* \in \mathcal{X}$, there is a unique $u^* \in \mathbb{R}^q$ that satisfies $f(x^*, u^*) = \mathbf{0}$ while both u^* and $y^* := h(x^*, u^*)$ being implicit functions of x^*. For the dissipativity analysis of (2) without the explicit knowledge of its equilibrium points, the *X-equilibrium-independent dissipativity* (X-EID) property [8] is introduced next.

Definition 1. *(X-EID [8]) The system* (2) *is X-EID under supply rate $s : \mathbb{R}^q \times \mathbb{R}^m \to \mathbb{R}$ if there exists a continuously differentiable storage function $V : \mathbb{R}^n \times \mathcal{X} \to \mathbb{R}$ satisfying: $V(x, x^*) > 0$ with $x \neq x^*$, $V(x^*, x^*) = 0$, and*

$$\dot{V}(x, x^*) = \nabla_x V(x, x^*) f(x, u) \leq s(u - u^*, y - y^*),$$

for all $(x, x^*, u) \in \mathbb{R}^n \times \mathcal{X} \times \mathbb{R}^q$, where the supply rate s is of the quadratic form characterized by a symmetric coefficient matrix $X := [X^{kl}]_{k,l\in\mathcal{I}_2} \in \mathbb{R}^{q+m}$, i.e.,

$$s(u - u^*, y - y^*) := \begin{bmatrix} u - u^* \\ y - y^* \end{bmatrix}^\top \begin{bmatrix} X^{11} & X^{12} \\ X^{21} & X^{22} \end{bmatrix} \begin{bmatrix} u - u^* \\ y - y^* \end{bmatrix}.$$

Note that this notion also includes the conventional dissipativity property [9], particularly when $\mathcal{X} = \{\mathbf{0}\}$ and for $\mathcal{X} \ni x^* = \mathbf{0}$, the corresponding $u^* = \mathbf{0}$ and $y^* = \mathbf{0}$.

Remark 1. *As mentioned in our previous work [10], by different choices of X in Def. 1, the system* (2) *can characterize different input-output behaviors, such as (strict) passive and l_2 stability. Specifically, if* (2) *is strictly passive with indices ν and ρ, i.e., $X = \begin{bmatrix} -\nu\mathbf{I} & \frac{1}{2}\mathbf{I} \\ \frac{1}{2}\mathbf{I} & -\rho\mathbf{I} \end{bmatrix}$, we denote this as* (2) *being IF-OFP(ν,ρ) (input feedforward-output feedback passive). Besides, if* (2) *is l_2-stable with gain γ, then $X = \begin{bmatrix} \gamma^2\mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{bmatrix}$.*

*2) **Network Modeling and Topology Synthesis**:*

***Configuration**:* To facilitate our co-design method, we reformulate the networked system Σ as the configuration in Fig. 1a, where each subsystem Σ_i is decoupled as

$$\Sigma_i : \begin{cases} \dot{x}_i = f_i(x_i, u_i), \\ y_i = h_i(x_i, u_i), \end{cases} \tag{4}$$

but they are interconnected (and also controlled) via the static interconnection matrix M with the following relationship:

$$\begin{bmatrix} u \\ z \end{bmatrix} = \begin{bmatrix} M_{uy} & M_{uw} \\ M_{zy} & M_{zw} \end{bmatrix} \begin{bmatrix} y \\ w \end{bmatrix} \equiv M \begin{bmatrix} y \\ w \end{bmatrix}, \tag{5}$$

where $z := [z_i^\top]_{i\in\mathcal{I}_N}^\top$ with each $z_i \in \mathbb{R}^{l_i}$. In analogous to (2), each subsystem $\Sigma_i, i \in \mathcal{I}_N$ in (4) is assumed to have a set $\mathcal{X}_i \subset \mathbb{R}^{n_i}$, where for every $x_i^* \in \mathcal{X}_i$, there is a unique $u_i^* \in \mathbb{R}^{q_i}$ that satisfies $f_i(x_i^*, u_i^*) = 0$ while both u_i^* and $y_i^* := h_i(x_i^*, u_i^*)$ being implicit functions of x_i^*. Moreover, each subsystem $\Sigma_i, i \in \mathcal{I}_N$ is also assumed to be X_i-EID, where $X_i = X_i^\top := [X_i^{kl}]_{k,l\in\mathcal{I}_2}$ (see Def. 1).

In this way, the resulting dynamics of each subsystem is still of the form as (1), but with the input being disturbances w_i. This setup defines how subsystems, exogenous input signal $w \in \mathbb{R}^r$ (e.g., disturbance) and interested output signal $z \in \mathbb{R}^l$ (e.g., performance) are interconnected with each other.

***Dissipativity-Based Topology Synthesis**:* Taking advantage of the configuration as in Fig. 1a, we can synthesize the interconnection matrix by solving an LMI problem, where extra specifications like $\mathbf{Y}$-EID (e.g., $\mathbf{Y} = \begin{bmatrix} \gamma^2\mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{bmatrix}$ to enforce l_2-stability, as in Rmk. 1) can be involved. However, to synthesize M and enforce $\mathbf{Y}$-EID, as in [8], we first need to make an assumption, which leads to a proposition.

Assumption 1. *For the networked system Σ, each subsystem Σ_i is X_i-EID with $X_i^{11} > 0$, $\forall i \in \mathcal{I}_N$, and the given $\mathbf{Y}$-EID specification for the networked system Σ is such that $\mathbf{Y}^{22} < 0$.*

Remark 2. *As we have mentioned in [10], Asm. 1 is mild, since it is always desirable to make the networked system Σ either l_2-stable ($X_i^{11} = \gamma_i^2\mathbf{I} > 0$) or passive $\nu_i > 0$.*

$$\begin{bmatrix} \mathbf{X}_p^{11} & \mathbf{0} & L_{uy} & L_{uw} \\ \mathbf{0} & -\mathbf{Y}^{22} & -\mathbf{Y}^{22} M_{zy} & -\mathbf{Y}^{22} M_{zw} \\ L_{uy}^\top & -M_{zy}^\top \mathbf{Y}^{22} & -L_{uy}^\top \mathbf{X}^{12} - \mathbf{X}^{21} L_{uy} - \mathbf{X}_p^{22} & -\mathbf{X}^{21} L_{uw} + M_{zy}^\top \mathbf{Y}^{21} \\ L_{uw}^\top & -M_{zw}^\top \mathbf{Y}^{22} & -L_{uw}^\top \mathbf{X}^{12} + \mathbf{Y}^{12} M_{zy} & M_{zw}^\top \mathbf{Y}^{21} + \mathbf{Y}^{12} M_{zw} + \mathbf{Y}^{11} \end{bmatrix} > 0 \tag{3}$$

Proposition 1. *[10] Under Asm. 1, the networked system Σ can be made $\boldsymbol{Y}$-EID by solving the following LMI problem to get the interconnection matrix M in* (5)*:*

$$\begin{aligned}&\text{Find: } L_{uy}, L_{uw}, M_{zy}, M_{zw}, \{p_i : i \in \mathcal{I}_N\}\\ &\quad \text{s.t. } p_i > 0, \ \forall i \in \mathcal{I}_N, \ \text{and } (3),\end{aligned} \tag{6}$$

where $\boldsymbol{X}^{12} := \text{diag}((X_i^{11})^{-1}X_i^{12} : i \in \mathcal{I}_N)$, $\boldsymbol{X}^{21} := (\boldsymbol{X}^{12})^\top$ with $M_{uy} := (\boldsymbol{X}_p^{11})^{-1}L_{uy}$ and $M_{uw} := (\boldsymbol{X}_p^{11})^{-1}L_{uw}$.

To evaluate the LMI-based topology synthesis (6) in a decentralized manner, here we recall the concept of "network matrices" and a Sylvester's criterion [11] inspired decentralization technique from [12] (and its extension [13]), i.e., a compositional verification of the positive definiteness of a symmetric block network matrix.

Definition 2. *[12] For a network topology $\mathcal{G}_n = (\mathcal{V}, \mathcal{E})$, any $n \times n$ block matrix $\Theta = \left[\Theta_{ij}\right]_{i,j \in \mathcal{I}_n}$ is a* network matrix *if: (1) Θ_{ij} consists of information specific only to the subsystems Σ_i and Σ_j, and (2) $(\Sigma_i, \Sigma_j) \notin \mathcal{E}$ and $(\Sigma_j, \Sigma_i) \notin \mathcal{E}$ implies $\Theta_{ij} = \Theta_{ji} = \mathbf{0}$, for all $i, j \in \mathcal{I}_n$.*

Proposition 2. *[12] A symmetric $N \times N$ block matrix $W = [W_{ij}]_{i,j \in \mathcal{I}_N} > 0$ iff*

$$\tilde{W}_{ii} := W_{ii} - \tilde{W}_i \mathcal{D}_i \tilde{W}_i^\top > 0, \qquad \forall i \in \mathcal{I}_N, \tag{7}$$

where $\tilde{W}_i := [\tilde{W}_{ij}]_{j \in \mathcal{I}_{i-1}} := W_i(\mathcal{D}_i\mathcal{A}_i^\top)^{-1}$, $W_i := [W_{ij}]_{j \in \mathcal{I}_{i-1}}$, $\mathcal{D}_i := \text{diag}(\tilde{W}_{jj}^{-1} : j \in \mathcal{I}_{i-1})$, and $\mathcal{A}_i$ is the block lower-triangular matrix created from $[\tilde{W}_{kl}]_{k,l \in \mathcal{I}_{i-1}}$.

*3) **String Stability**:* To capture the disturbances propagation over the network, we recall the *String Stability* concepts. Consider the network (2), but with the input u being external disturbances $w := [w_i^\top]_{i \in \mathcal{I}_N}^\top \in \mathbb{R}^r$, where each $w_i \in \mathbb{R}^{r_i}$ and $r := \sum_{i \in \mathcal{I}_N} r_i$, after substituting some designed controller $u := u(x)$ into (2). The notion of string stability we use here is the *disturbance string stability (DSS)* initially proposed in [14], which is the dominant time domain string stability notion (for more details, see our review of different string stability notions in [10]).

Definition 3. *(DSS [5]) The networked system* (2) *(but with disturbances w as input) around the equilibrium point $x^* \in \mathcal{X}$ is* disturbance string stable *(DSS), if there exist functions $\beta \in \mathcal{KL}$, $\sigma \in \mathcal{K}_\infty$, and constants $c_x, c_w > 0$, such that for any initial condition $x_i(0)$ and disturbance w_i, $i \in \mathcal{I}_N$ satisfying*

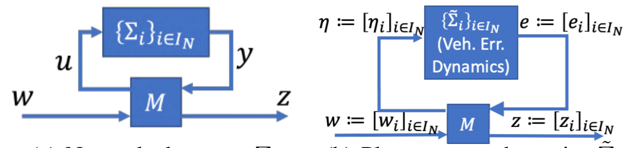
$$\sup_{i \in \mathcal{I}_N} |x_i(0) - x_i^*| < c_x, \quad \text{and} \quad \sup_{i \in \mathcal{I}_N} \|w_i\|_\infty < c_w, \tag{8}$$

respectively, the solution $x_i(t)$, $i \in \mathcal{I}_N$ of (1) *exists for all $t \geq 0$ and satisfies*

$$\sup_{i \in \mathcal{I}_N} |x_i - x_i^*| \leq \beta(\sup_{i \in \mathcal{I}_N} |x_i(0) - x_i^*|, t) + \sigma(\sup_{i \in \mathcal{I}_N} \|w_i\|_\infty), \tag{9}$$

for all $t \geq 0$ and any $N \in \mathbb{N}$.

Remark 3. *The concept of DSS in Def. 3 implies the uniform boundedness of the tracking errors $|x_i - x_i^*|$ for all subsystems as they propagate over the network. More importantly, note that this bound is also independent of N.*



(a) Networked system Σ (b) Platoon error dynamics $\tilde{\Sigma}$

Fig. 1: Network configuration: (a) A generic networked system Σ; (b) Error dynamics as a networked system $\tilde{\Sigma}$.

Before we introduce the condition to guarantee the DSS for the general networked system (2), we first recall the definition of the so-called *Input-to-State Stability (ISS)*. When the subsystems (1) are ISS, the follow-up proposition provides a set of sufficient conditions to guarantee the DSS of the networked system (2).

Definition 4. *(ISS [15]) The subsystem* (1) *(but with disturbances w_i as input) of the networked system* (2) *is* input-to-state stable *(ISS) if there exist functions $\beta_i \in \mathcal{KL}$, and σ_{xi}, $\sigma_{wi} \in \mathcal{K}_\infty$ such that*

$$\begin{aligned}|x_i - x_i^*| \leq &\beta_i(|x_i(0) - x_i^*|, t) + \sigma_{xi}(\max_{j \in \mathcal{E}_i} \|x_j\|_\infty) + \\ &\sigma_{wi}(\|w_i\|_\infty)\end{aligned}$$

is satisfied for all $t \geq 0$.

Proposition 3. *[15] Suppose that each subsystem* (1) *of the networked system* (2) *is ISS and the conditions* (8) *hold for all $i \in \mathcal{I}_N$. Then, the networked system* (2) *is DSS if there exist scalars $\bar{\sigma}_{xi} \in (0, 1)$ such that*

$$\sigma_{xi}(s) \leq \bar{\sigma}_{xi} s \tag{10}$$

holds for all $s \in \mathbb{R}_{\geq 0}$ and $i \in \mathcal{I}_N$.

III. Problem Formulation

*1) **System Dynamics**:* We consider the longitudinal dynamics of the i^{th} vehicle $\Sigma_i, i \in \mathcal{I}_N^0$ in the platoon as [2]:

$$\Sigma_i : \begin{cases} \dot{x}_i(t) = v_i(t) + d_{xi}(t), \\ \dot{v}_i(t) = a_i(t) + d_{vi}(t), \\ \dot{a}_i(t) = f_i(v_i(t), a_i(t)) + \frac{1}{m_i\tau_i}u_i(t) + d_{ai}(t), \end{cases} \tag{11}$$

where $f_i := f_i(v_i(t), a_i(t)) := -\frac{1}{\tau_i}\big(a_i + \frac{A_{f,i}\rho C_{d,i}v_i^2}{2m_i} + C_{r,i}\big) - \frac{A_{f,i}\rho C_{d,i}v_i a_i}{m_i}$ is the nonlinear aerodynamics of the vehicle. In (11), we use Σ_0 to represent the leading vehicle (virtual and given) and $\{\Sigma_i : i \in \mathcal{I}_N\}$ to represent the following N vehicles (controllable) in the platoon. We lumped all the uncertainties in position x_i, velocity v_i, and acceleration a_i channels together with the external disturbances into the corresponding disturbances d_{xi}, d_{vi}, and d_{ai}, respectively, where x_i, v_i, $a_i \in \mathbb{R}$, and the disturbances d_{xi}, d_{vi}, $d_{ai} \in \mathbb{R}$ are assumed to be bounded and second order differentiable; m_i is the mass; $A_{f,i}$ is the effective frontal area; ρ is the air density; $C_{d,i}$ is the coefficient of the aerodynamic drag; $C_{r,i}$ is the coefficient of the rolling resistance; τ_i is the engine time constant; and $u_i \in \mathbb{R}$ is the control input (to be designed) of Σ_i, $i \in \mathcal{I}_N$.

*2) **Feedback Control Design**:* A basic configuration of a vehicular platoon is shown in Fig. 2. We define the *position tracking error* of the vehicle $\Sigma_i, i \in \mathcal{I}_N$ as $\tilde{x}_i(t) := x_i(t) - (x_0(t) - d_{i0})$, where d_{i0} is the desired separation between vehicles Σ_i and Σ_0. Correspondingly, we also write the velocity and acceleration tracking errors as $\tilde{v}_i(t) := v_i(t) - v_0(t) - d_{x0}(t)$ and $\tilde{a}_i(t) := a_i(t) - a_0(t) - d_{v0}(t) - \dot{d}_{x0}(t)$, respectively. Using these defined errors, we obtain the following tracking error dynamics of the vehicle $\Sigma_i, i \in \mathcal{I}_N$:

$$\tilde{\Sigma}_i : \begin{cases} \dot{\tilde{x}}_i = \tilde{v}_i + d_{xi}, \\ \dot{\tilde{v}}_i = \tilde{a}_i + d_{vi}, \\ \dot{\tilde{a}}_i = f_i + \frac{1}{m_i \tau_i} u_i + d_{ai} - \tilde{u}_0, \end{cases} \tag{12}$$

where $\tilde{u}_0 := f_0 + \frac{1}{m_0 \tau_0} u_0 + d_{a0} + \dot{d}_{v0} + \ddot{d}_{x0}$. In (12), we assume the leader's information (x_0, v_0, a_0) is known to all the followers, and if the leader is noiseless with $\dot{a}_0 \equiv 0$, then $\tilde{u}_0 = 0, \forall t \geq 0$.

Now, to control the error dynamics (15), a state feedback controller $u_i(t)$ can be designed as:

$$u_i = m_i \tau_i \Big(- f_i + (\bar{L}_{ii} + L_{ii}) e_i + \sum_{j \in \mathcal{I}_N \backslash \{i\}} L_{ij}(e_i - e_j) \Big), \tag{13}$$

where we stack the tracking errors as $e_i := \begin{bmatrix} \tilde{x}_i & \tilde{v}_i & \tilde{a}_i \end{bmatrix}^\top$, a **local controller gain** $\bar{L}_{ii} \in \mathbb{R}^{1 \times 3}$ is added to tune the passivity properties of each vehicle Σ_i, and $L_{ij} := \begin{bmatrix} l_{ij}^x & l_{ij}^v & l_{ij}^a \end{bmatrix} \in \mathbb{R}^{1 \times 3}, \forall j \in \mathcal{I}_N$ are **global controller gains**.

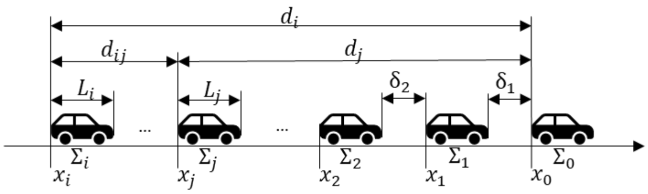


Fig. 2: Vehicle placements in a platoon.

*3) **Modeled as a Networked System**:* For ease of expression, we restate the controller (13) as:

$$u_i = m_i \tau_i \Big(- f_i + \bar{L}_{ii} e_i + \sum_{j \in \mathcal{I}_N} \bar{K}_{ij} e_j \Big) \tag{14}$$

where $\bar{K}_{ij} := -L_{ij}, \forall j \neq i$, and $\bar{K}_{ii} := L_{ii} + \sum_{j \in \mathcal{I}_N \backslash \{i\}} L_{ij}$. By defining $\begin{bmatrix} k_{ij}^x & k_{ij}^v & k_{ij}^a \end{bmatrix} := \bar{K}_{ij}, \forall j \in \mathcal{I}_N$ and the external disturbances $w_i := \begin{bmatrix} d_{xi} & d_{vi} & d_{ai} - \tilde{u}_0 \end{bmatrix}^\top$, the closed-loop error dynamics of the vehicle $\Sigma_i, i \in \mathcal{I}_N$ as:

$$\tilde{\Sigma}_i : \quad \dot{e}_i = (A + B\bar{L}_{ii}) e_i + \eta_i, \tag{15}$$

where $A := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $B := \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, and the input is:

$$\eta_i := \sum_{j \in \mathcal{I}_N} K_{ij} e_j + w_i, \text{ with } K_{ij} := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k_{ij}^x & k_{ij}^v & k_{ij}^a \end{bmatrix}. \tag{16}$$

Note that the error dynamics (15) are of the same form as (4), where each subsystem is interconnected with others through the control η_i. If we collect subsystem inputs as $\eta := [\eta_i^\top]_{i \in \mathcal{I}_N}$ and subsystem outputs as $e := [e_i^\top]_{i \in \mathcal{I}_N}$, each subsystem is interconnected via the similar relationship like (5), but with $u = \eta$ and $y = e$, and thus, $M_{\eta e} := [K_{ij}]_{i,j \in \mathcal{I}_N}$, $M_{\eta w} := \mathbf{I}$, $M_{ze} := \mathbf{I}$, and $M_{zw} := \mathbf{0}$ (see Fig. 1b).

Based on this network configuration, synthesizing $M_{\eta e}$ will reveal the desired individual controllers and a preferable communication topology for the platoon. Hence, our goal in this paper is to propose a co-design framework for the control and topology of a platoon, such that the DSS and the compositionality of the platoon are both ensured.

IV. Main Results

In this section, we present our main results. Note that, to execute the co-design process, we first require the closed-loop error dynamics of each vehicle $\tilde{\Sigma}_i, i \in \mathcal{I}_N$, to be IF-OFP($\nu_i, \rho_i$) (see Rmk. 1). To this end, as shown in [10], at each vehicle $\Sigma_i, i \in \mathcal{I}_N$, we need to find its local controller $\bar{L}_{ii}$, a feasible storage function matrix R_i and passivity indices ν_i, ρ_i by solving the following LMI problem:

$$\begin{aligned} \text{Find: } & \tilde{L}_{ii},\ P_i,\ \nu_i,\ \tilde{\rho}_i,\ \tilde{\gamma}_i, \\ \text{s.t. } & P_i > 0, \\ & \begin{bmatrix} \tilde{\rho}_i \mathbf{I} & P_i & \mathbf{0} \\ P_i & -\mathcal{S}(AP_i + B\tilde{L}_{ii}) & -I + \frac{1}{2}P_i \\ \mathbf{0} & -I + \frac{1}{2}P_i & -\nu_i \mathbf{I} \end{bmatrix} > 0, \\ & -\frac{\tilde{\gamma}_i}{p_i} < \nu_i < 0, \quad 0 < \tilde{\rho}_i < \min\left\{ p_i, \frac{4\tilde{\gamma}_i}{p_i} \right\}, \end{aligned} \tag{17}$$

where $p_i > 0$ is some pre-specified scalar parameter, $\bar{L}_{ii} := \tilde{L}_{ii} P_i^{-1}$, $R_i := P_i^{-1}$, and $\rho_i := \tilde{\rho}_i^{-1}$. As detailed in [10, Thm. 2], the first two constraints in (17) enforce the IF-OFP(ν_i,ρ_i) property of (15) while the latter two constraints in (17) support the feasibility of the co-design process given in the sequel.

*1) **Centralized Co-Design With DSS Guarantee**:* With the obtained local control parameters, we present a centralized LMI-based co-design method with DSS guarantee as follows.

Theorem 1. *The closed-loop platooning system* $\{\tilde{\Sigma}_i\}_{i \in \mathcal{I}_N}$ (15)-(16) *(also shown in Fig. 1b) is both finite-gain l_2-stable with some l_2-gain γ (where $\tilde{\gamma} := \gamma^2 < \bar{\gamma}$) from disturbance input w to performance output z, and DSS with respect to disturbance w and initial error* $e(0)$, *if for some pre-specified ϵ_i, the interconnection matrix block $M_{\eta e} = [K_{ij}]_{i,j \in \mathcal{I}_N}$ (as in Fig. 1b) is synthesized using the centralized LMI problem:*

$$\min_{Q, \gamma, \{p_i : i \in \mathcal{I}_N\}} \sum_{i,j \in \mathcal{I}_N} c_{ij} \|Q_{ij}\|_1 + c_0 \tilde{\gamma},$$

$$\textit{s.t.} \quad p_i > 0,\ \forall i \in \mathcal{I}_N,\ 0 < \tilde{\gamma} < \bar{\gamma},$$

$$\begin{bmatrix} \mathbf{X}_p^{11} & \mathbf{0} & Q & \mathbf{X}_p^{11} \\ \mathbf{0} & \mathbf{I} & \mathbf{I} & \mathbf{0} \\ Q^\top & \mathbf{I} & -Q^\top \mathbf{X}^{12} - \mathbf{X}^{21} Q - \mathbf{X}_p^{22} & -\mathbf{X}^{21} \mathbf{X}_p^{11} \\ \mathbf{X}_p^{11} & \mathbf{0} & -\mathbf{X}_p^{11} \mathbf{X}^{12} & \tilde{\gamma} \mathbf{I} \end{bmatrix} > 0, \tag{18a}$$

$$R_i Q_{ii} + Q_{ii}^\top R_i \leq p_i \nu_i \epsilon_i \mathbf{I}, \quad \forall i \in \mathcal{I}_N, \tag{18b}$$

$$\sum_{j \in \mathcal{I}_N \backslash \{i\}} |R_i Q_{ij}| < -p_i \nu_i \delta_i, \quad \forall i \in \mathcal{I}_N, \tag{18c}$$

where $c_0 > 0$ is a pre-specified constant, $\delta_i := \sqrt{\mu_i \lambda_{\min}(R_i)}$ with $\mu_i := \frac{(\rho_i+\epsilon_i-1)}{\lambda_{\max}(R_i)}$, and $0 < \delta_i < 1$, $Q := [Q_{ij}]_{i,j\in\mathcal{I}_N}$ shares the same structure as $M_{\eta e}$, $\boldsymbol{X}^{12} :=$ diag$(-\frac{1}{2\nu_i}\mathbf{I} : i \in \mathcal{I}_N)$, $\boldsymbol{X}^{21} := (\boldsymbol{X}^{12})^\top$, $\boldsymbol{X}_p^{11} :=$ diag$(-p_i\nu_i\mathbf{I} : i \in \mathcal{I}_N)$, $\boldsymbol{X}_p^{22} :=$ diag$(-p_i\rho_i\mathbf{I} : i \in \mathcal{I}_N)$, and $M_{\eta e} := (\boldsymbol{X}_p^{11})^{-1}Q$.

Remark 4. *Here, we provide the direct relationship between the synthesized interconnection matrix block $[K_{ij}]_{i,j\in\mathcal{I}_N}$ in Thm. 1 and the individual vehicle (global) controller gains required in* (16)*. In particular, for the error dynamics* (15)*, the off-diagonal elements of $[K_{ij}]_{i,j\in\mathcal{I}_N}$ are $K_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -l_{ij}^x & -l_{ij}^v & -l_{ij}^a \end{bmatrix}$, for all $i \in \mathcal{I}_N, j \in \mathcal{I}_N \backslash \{i\}$, while the diagonal elements are*

$$K_{ii} = K_{i0} - \sum_{j\in\mathcal{I}_N\backslash\{i\}} K_{ij}, \tag{19}$$

for all $i \in \mathcal{I}_N$, where each $K_{i0} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ l_{ii}^x & l_{ii}^v & l_{ii}^a \end{bmatrix}$.

Remark 5. *The main steps for the implementation of local controller design and centralized global co-design are:*
Step 1: *Select some scalar parameters: $p_i > 0, \forall i \in \mathcal{I}_N$;*
Step 2: *Synthesize local controllers via* (17)*;*
Step 3: *If* (17) *is infeasible, return to **Step 1**;*
Step 4: *Syntesize global co-design using Thm. 1.*

More details of the selection of scalar parameters p_i in Step 1 can be found in [10]. Note that, a similar four-step process can be applied in a decentralized fashion if Step 4 (i.e., global co-design) can be made decentralized. This is introduced next.

2) ***Decentralized Co-design for Merging/Splitting****:* To enable seamless vehicular merging/splitting, the co-design method in (18) is required to be compositional in a decentralized manner. In other words, upon adding or removing a vehicle to or from the platoon, one should not have to re-design the control (and topology) for the entire platoon. Still using the local control parameters by (17), we next show how the centralized co-design (18) can be made decentralized and compositional to synthesize the controllers and the topology for the platoon with DSS guarantee. Also note that the DSS condition (18b) can be made decentralized. However, the decentralization of (18c) is not straightforward. To handle this, we simultaneously propose a sufficient alternative of (18c). More details of how this method enables vehicular merging/splitting can be found in our previous work [10].

Theorem 2. *The closed-loop platooning system $\{\tilde{\Sigma}_i\}_{i\in\mathcal{I}_N}$ can be made both finite-gain l_2-stable with some l_2-gain γ (where $\tilde{\gamma} := \gamma^2 < \bar{\gamma}$, similar as in Thm. 1), and decentralized DSS, if at each vehicle $\Sigma_i, i \in \mathcal{I}_N$: (1) the interconnection matrix blocks $\{K^i\}$ are designed via the decentralized LMI problem:*

$$\min_{\{Q^i\},\hat{\gamma}_i,p_i} \sum_{j\in\mathcal{I}_{i-1}} c_{ij}\|Q_{ij}\|_1 + c_{ji}\|Q_{ji}\|_1 + c_{0i}\hat{\gamma}_i + c_i|\hat{\gamma}_i - \tilde{\gamma}_i|$$
$$\text{s.t.} \quad p_i > 0,\ \hat{\gamma}_i < \bar{\gamma},\ \tilde{W}_{ii} > 0,\ \text{(18b)}, \tag{20a}$$

$$\frac{1}{\delta_i}|R_iQ_{ij}| \leq -\frac{p_i\nu_i}{2^j}, \quad \forall j \in \mathcal{I}_{i-1} \tag{20b}$$

where $\tilde{\gamma}_i$ is from (17) *(obtained in Step 2), and $\tilde{W}_{ii}$ is from* (7) *when enforcing $W = [W_{ij}]_{i,j\in\mathcal{I}_N} > 0$ with*

$$W_{ij} := \begin{bmatrix} \mathsf{e}_{ij}V_p^{ii} & \mathbf{0} & Q_{ij} & \mathsf{e}_{ij}V_p^{ii} \\ \mathbf{0} & \mathsf{e}_{ij}\mathbf{I} & \mathsf{e}_{ij}\mathbf{I} & \mathbf{0} \\ Q_{ji}^\top & \mathsf{e}_{ij}\mathbf{I} & -Q_{ji}^\top S_{jj} - S_{ii}Q_{ij} - \mathsf{e}_{ij}R_p^{ii} & -\mathsf{e}_{ij}S_{ii}V_p^{ii} \\ \mathsf{e}_{ij}V_p^{ii} & \mathbf{0} & -\mathsf{e}_{ij}V_p^{ii}S_{jj} & \hat{\gamma}_i\mathsf{e}_{ij}\mathbf{I} \end{bmatrix},$$

$V_p^{ii} := -p_i\nu_i\mathbf{I}$, $R_p^{ii} := -p_i\rho_i\mathbf{I}$, $S_{ii} := -\frac{1}{2\nu_i}\mathbf{I}$ and blocks $\{K^i\}$ are determined by $K_{ij} = (V_p^{ii})^{-1}Q_{ij}$, and (2) the update:

$$K_{j0}^{New} := K_{j0}^{Old} + K_{ji} \tag{21}$$

is requested at each prior and neighboring vehicle.

V. Simulation Examples

In this section, we provide simulation examples for a platoon's merging/splitting control to verify the effectiveness of our proposed co-design framework. The simulation considers a platoon with ten homogeneous vehicles, and the first vehicle is selected as the leader and the remaining ones are the followers. Due to space constraints, here we only show the results for merging vehicles sequentially into a platoon using Thm. 2 (the used exact system and controller parameters can be found in [10]). Simulation results are generated by a simulator developed in MATLAB[1].

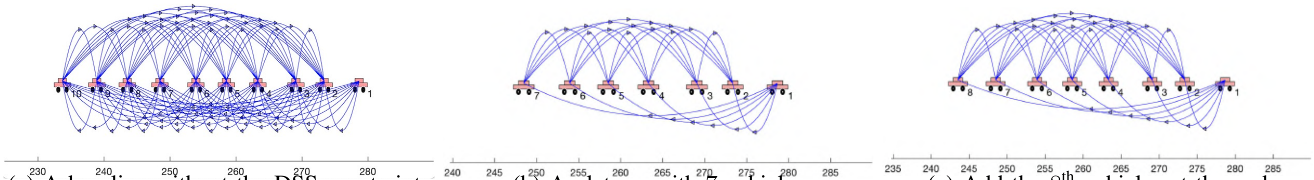
For the considered platoon, the decentralized topology is shown in Fig. 3a- 4a. For comparison, Fig. 3a is provided here as a baseline that uses the decentralized co-design without DSS constraints (18b) and (18c). Compared to Fig. 3a, the main differences of Fig. 4a obtained with DSS constraints lie in the cancellation of the front-to-back links. This indicates that the back-to-front information is more critical in enhancing a platoon's string stability, as seen in Fig. 3b-4a. Also, we observed that this cancellation of links results in an improvement of the l_2-gain from disturbance input to tracking performance output, as $\gamma = 2.5154$ for Fig. 3a while $\gamma = 2.0491$ for Fig. 4a.

For Fig. 3b-4a, it is worth noting that the follower vehicles are sequentially added, i.e., the optimization (20) in Thm. 2 is incrementally/iteratively solved. Specifically, the resulting controller gains $\{K^i\} \cup \{K_{i0}\}$, $i \in \mathcal{I}_N$, are evaluated in a compositional manner.

Remark 6. *To address vehicles splitting from a platoon, the remaining vehicles can directly use their K_{j0}^{Old} values without any elaborate re-designs, and other controller gains remain unchanged, according to Thm. 2. In essence, splitting can be viewed as the inverse process of merging.*

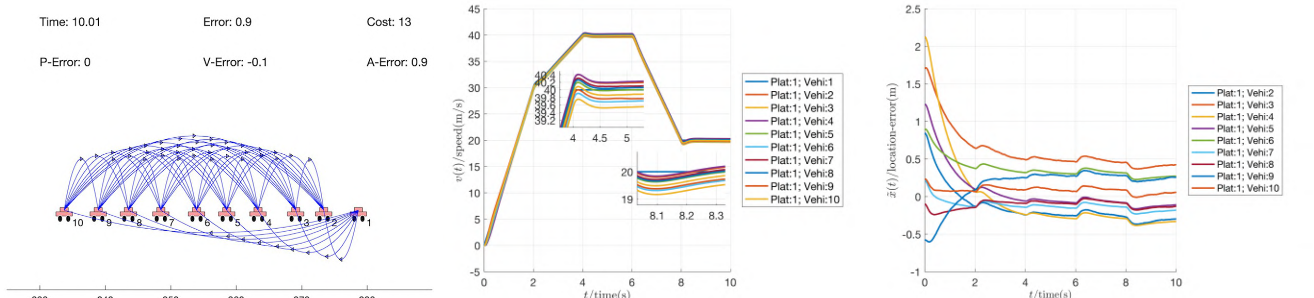
Besides, a measure of DSS of the platoon can be obtained by computing (at each vehicle) the DSS estimate $J := \max_i \frac{1}{\delta_i}\sum_{j\in\mathcal{I}_{10}\backslash\{i\}}|R_iK_{ij}|$ inspired by (18c). For the case in Fig. 4a, $J = 0.8235$, while for the case in Fig. 3a, $J = 0.9053$. Thus, the addition of DSS constraints not only provides a formal guarantee of string stability, but also

[1]Publicly available at https://github.com/NDzsong2/Longitudinal-Vehicular-Platoon-Simulator.git



(a) A baseline without the DSS constraints (b) A platoon with 7 vehicles (c) Add the 8th vehicles at the end

Fig. 3: Vehicular merging using decentralized co-design: (a) a baseline; (b) begin with 7 vehicles; (c) add one at the end.



(a) Add the last two vehicles (b) Velocity tracking results (c) Position tracking errors

Fig. 4: Decentralized co-design results: (a) topology configuration; (b) velocities of vehicles; (c) position tracking errors.

gives more margin towards 1. Moreover, from Fig. 4b-4c, it is readily seen that the velocity and position tracking is well achieved but with minor deviations from the reference signals. Such deviations are caused by external disturbances which have been sufficiently compensated by the enforced l_2-stability in the co-design.

VI. Conclusion

In this paper, we studied the merging and splitting of vehicular platoons and proposed a dissipativity-based distributed controller and communication topology co-design method with a formal DSS guarantee. A centralized co-design technique was first presented with a local controller design technique. Besides, the analysis of the DSS was provided, resulting in a centralized DSS constraint. Next, we showed the decentralization of these DSS constraints by proposing a sufficient alternative condition. Then, using a decentralization technique inspired by Sylvester's criterion, the centralized co-design technique was made decentralized, enabling vehicular merging/splitting. More importantly, the resulting control and topology co-design process maintained the compositionality and DSS. Simulation results illustrated the effectiveness of the proposed method. Moreover, a comparison to the decentralized co-design without the DSS constraints indicates that back-to-front communication links in the platoon play the most crucial role in enhancing the string stability.

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