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Model Predictive Control Strategies for Electric Endurance Race Cars Accounting for Competitors' Interactions

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Abstract— **This paper presents model predictive control strategies for battery electric endurance race cars accounting for interactions with the competitors. In particular, we devise an optimization framework capturing the impact of the actions of the ego vehicle when interacting with competitors in a probabilistic fashion, jointly accounting for the optimal pit stop decision making, the charge times and the driving style in the course of the race. We showcase our method for a simulated 1 h endurance race at the Zandvoort circuit, using real-life data from a previous event. Our results show that optimizing both the race strategy and the decision making during the race is very important, resulting in a significant 21 s advantage over an always overtake approach, whilst revealing the competitiveness of e-race cars w.r.t. conventional ones.**

I. Introduction

The interest in the electrification of race cars has grown over the last years, with the introduction of both hybrid and fully electric racing classes. In the case of fully electric racing, it is of paramount importance to make optimal use of the available battery energy—as it is the most limiting resource—strategically selecting charging stops and driving style through the course of a race. This challenge gets an additional level of complexity, once interactions with competitors are considered: For instance, the overtaking of competitors are inevitable events that can require a significant amount of additional battery energy. Moreover, competitors can also bring energy savings, as consumption can be reduced by driving in their slipstream, though at the cost of being restricted to their lap times. These disturbances must be dealt with by carefully selecting the best action accounting for its impact on the overall race strategies. To address these challenges, this paper proposes model predictive control (MPC) algorithms that select the most appropriate actions when interacting with competitors, jointly optimizing the race strategies.

Related Literature: This work pertains to two main research streams: race strategy optimization and simulations.

Several authors have optimized the race from a lap perspective, for fully electric race cars [1]–[3] or hybrid-electric race cars [4], either using an offline or adaptive online approach. These works were extended by accounting for competitor interactions in [5], whereby a set of overtaking strategies is defined offline and then used in an online framework employing Stochastic Dynamic Programming. Lastly, competitor decisions are predicted using a non-cooperative

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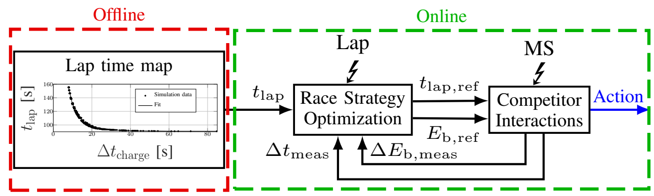

Fig. 1. Overview of the controller architecture. The race strategy optimization uses lap time maps to compute reference trajectories every lap, while the decision making algorithm provides an action and feeds back the actual states.

game approach [6] or Gaussian Processes [7]. Yet these methods only capture the energy management strategy and not the impact on the pit stop strategy over the entire race. Alternatively, game-theoretic approaches are leveraged for decision making [8], but energy management is not accounted for.

Race simulations including probabilistic effects are used to infer the optimal race strategy. These methods simulate entire races with tyre degradation, pit stops and overtake maneuvers [9], and can potentially capture failures [10] or safety cars phases [11]. However, these methods choose the optimal strategy in terms of pit stop allocation and do not consider energy consumption. Furthermore, interactions are modeled as time losses, but no optimal decisions are taken regarding the interactions.

In conclusion, to the best of the authors' knowledge, there are no control algorithms for electric race strategies including interactions with competitors.

Statement of Contributions: This paper presents MPC algorithms that optimize electric race strategies together with competitors' interactions in an online fashion. The online adaptive race strategy optimization framework computes the race strategy in terms of target lap times, energy consumption and pit stop strategy, at the beginning of every lap. When an interaction with a competitor is predicted to occur, the decision-making process is started to find the action that results in the lowest time penalty w.r.t. the original strategy. In contrast to previous works [9], [11], where deterministic overtake time penalties were assigned, we derive time penalties for all possible interaction outcomes and include probabilistic effects within these penalties. A schematic overview of the controller is shown in Fig. 1. Finally, we showcase our framework for a 1 h race on the Zandvoort circuit with a fully electric vehicle competing against internal combustion engine (ICE) race cars.

Organization: The remainder of this paper is structured as follows: Section II presents the online race strategy optimization framework, after which Section III describes the action selection process. We showcase our framework in Section IV and draw conclusions together with an outlook in Section V.

II. Online Race Strategy Optimization

This section presents a model predictive controller to optimize the race strategies online. Specifically, we devise the framework inspired by [1], extending it to optimize the current stint on a lap basis instead of once every stint to allow for adaptive pit stop decision making, and implement it online in a shrinking horizon fashion. Hence, we trigger the model predictive controller (MPC) once every lap while driving to adapt the pit stop strategy online. The goal is to choose the optimal stint lengths N_{laps} (being the number of laps in between two pit stops), charge times t_{charge} (the duration of charging during each individual pit stop, directly related to the charged energy) and number of charging stops b_{pit} over the course of the entire race and adapt them online at every lap. We set the elapsed time t_{tot} as the main state variable and convert charged energy to an equivalent charge time t_{fc}.

In endurance racing, we typically aim to maximize the driven distance S_{race} within a certain total race time t_{race}. Optimizing the race strategy online during every lap, we need to decide the amount of energy to spend per lap and whether to make a pit stop to charge the car. To optimize from a race perspective and to maintain a computationally tractable solution, we devise a hybrid formulation whereby we optimize the current stint on a lap basis and the rest of the race on a stint basis. In this context, the objective of maximizing the driven distance can then be formulated as

$$\max S_{\mathrm{race}} = \max \sum_{i=1}^{n_{\mathrm{stops}}} S_{\mathrm{lap}} \cdot N_{\mathrm{laps}}(i) + \sum_{j=1}^{n_{\mathrm{laps}}} S_{\mathrm{lap}} \cdot b_{\mathrm{lap}}(j), \quad (1)$$

where n_{stops} is the pre-defined maximum number of pit stops, $N_{\mathrm{laps}}(i) \in \mathbb{N},\ \forall\ i \in [1,\ldots,\ n_{\mathrm{stops}}\ -\ 1]$ is the stint length with $\mathbb{N}$ the set of natural numbers, and S_{lap} is the length of one lap. Similarly, we have n_{laps} as the pre-defined maximum number of laps in the current stint, and $b_{\mathrm{lap}}(j)$ is a binary variable representing whether lap j is driven. Since the vehicle stops at the pit box, the stint length should be an integer number of laps. Only at the last stint we allow the length to be a non-integer number, i.e., $N_{\mathrm{laps}}(n_{\mathrm{stops}}) \in \mathbb{R}_{+}$, since the vehicle position is free when the final race time is reached.

A. Current Stint

As the current stint is modeled on a lap basis, we make use of pre-computed lap time maps, generated using the framework in [1], that correspond to the minimum achievable lap time for a given energy budget per lap. However, since the main state is defined as a charge time rather than a charged energy, we reformulate the energy budget per lap to an equivalent charge time using a pre-defined charging profile. The lap time maps are then approximated using the piecewise affine and convex functions as

$$t_{\mathrm{lap},k} = \max_{n\in[1,\ldots,n_{\mathrm{fits}}]} \{c_{1,k,n} \cdot \Delta t_{\mathrm{charge}} + c_{0,k,n}\},\ \forall n \in [1,\ldots,n_{\mathrm{fits}}], \quad (2)$$

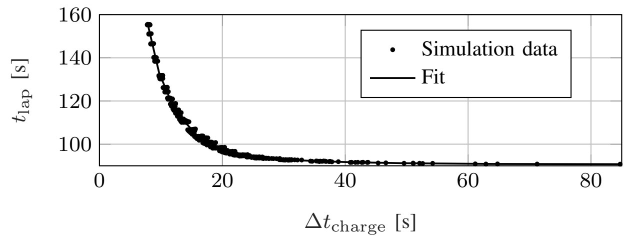

Fig. 2. Lap time map for the base-lap using $n_{\mathrm{fits}} = 50$ piecewise affine functions. The normalized root-mean-square error (RMSE) of the model is 0.012% w.r.t. the maximum lap time.

where n_{fits} is the number of piecewise affine functions used, $t_{\mathrm{lap},k}$ is the obtained lap time, with $k \in \{\text{base,in}\}$, $\Delta t_{\mathrm{charge}}$ is the equivalent charge time corresponding to the energy used per lap and $c_{1,k,n}, c_{0,k,n}$ are fitting parameters. Note that we distinguish between normal base-laps and in-laps, whereby the latter includes driving into the pit lane to charge the car. Out-lap maps are not included, since they are always part of a future stint, removing the need to model them on a lap basis. Fig. 2 shows the lap time map for a base lap. We observe that the slight misalignment of the data with the affine fit is due to the dependency of both the charge power and battery efficiency on the battery energy.

The time to complete the remainder of the current stint $t_{\mathrm{stint,current}}$ is defined as the sum of all remaining lap times

$$t_{\mathrm{stint,current}} = \sum_{j=1}^{n_{\mathrm{laps}}-1} (t_{\mathrm{lap,base}}(j)) + t_{\mathrm{lap,in}}. \quad (3)$$

where we constrain the base lap time as

$$\begin{aligned} t_{\mathrm{lap,base}}(j) &\geq c_{1,\mathrm{base},n} \cdot \Delta t_{\mathrm{charge}}(j) + c_{0,\mathrm{base},n} \\ &\quad - M \cdot (1 - b_{\mathrm{lap}}(j)), \quad \forall j \in [1,\ldots,n_{\mathrm{laps}} - 1], \end{aligned} \quad (4)$$

with $M \gg t_{\mathrm{lap,base,max}}$. This constraint will then hold with equality at the optimum, since it is optimal to minimize lap time. We constrain the in-lap in a similar fashion, but using $j = n_{\mathrm{laps}}$, since the in-lap is always the final lap of the current stint. By introducing the binary variable b_{lap} and constraining the lap time to be non-negative

$$t_{\mathrm{lap},k} \geq 0, \quad (5)$$

we can model *driven* laps and *non-driven* laps, thereby jointly optimizing the laps and the number of laps remaining until charging. In this way we have a fixed length optimization horizon of n_{stops} on future stint level and n_{laps} on the current stint level, with possible zeros as entries to optimize the number of stops and laps, respectively.

To capture the battery energy during the current stint, we keep track of a time to full charge t_{fc}, defined as

$$t_{\mathrm{fc}}(j+1) = t_{\mathrm{fc}}(j) + \Delta t_{\mathrm{charge}}(j) \quad \forall j \in [1,\ldots,n_{\mathrm{laps}}], \quad (6)$$

and bound it through

$$t_{\mathrm{fc}}(j) \geq 0 \quad (7)$$
$$t_{\mathrm{fc}}(j) \leq t_{\mathrm{charge,max}}, \quad (8)$$

where $t_{\mathrm{charge,max}}$ represents the charge time corresponding to charging the battery from the lower energy bound to the upper energy bound. Since we directly linked this equivalent charge time to the battery energy, we are guaranteed to remain within the battery energy operating limits. To account for the current battery energy, we initialize the time to full charge as

$$t_{\text{fc}}(1) = t_{\text{fc,meas}}, \tag{9}$$

where $t_{\text{fc,meas}}$ is the time to full charge calculated from the current measured battery energy.

Finally, we ensure that the in-lap is part of the optimal solution and order the driven lap vector with non-driven and driven laps first and last, respectively, with

$$b_{\text{lap}}(j+1) \geq b_{\text{lap}}(j), \quad \forall j \in [1, n_{\text{laps}}]. \tag{10}$$

B. Future Stints

The remainder of the endurance race is captured on a stint basis, whereby we use stint time maps as a function of the stint length and the charge time, taken from [1]. Similar to the current stint, we keep track of the elapsed time t_{tot} through

$$t_{\text{tot}}(i+1) = t_{\text{tot}}(i) + t_{\text{stint}}(i) + t_{\text{charge}}(i) \; \forall i \in [2, \dots, n_{\text{stops}} - 1], \tag{11}$$

whereby t_{stint} is the time to complete the stint and t_{charge} is the charge time after the stint. Since we do not have a pit stop after the final stint, the elapsed time after the final stint is defined as

$$t_{\text{tot}}(n_{\text{stops}}+1) = t_{\text{tot}}(n_{\text{stops}}) + t_{\text{stint}}(n_{\text{stops}}). \tag{12}$$

As endurance races are time-limited, we bound the total time:

$$t_{\text{tot}}(j) \geq 0 \tag{13}$$

$$t_{\text{tot}}(j) \leq t_{\text{race}} - t_{\text{meas}}, \tag{14}$$

whereby t_{race} represents the race time limit and t_{meas} is the current time, thereby obtaining a shrinking horizon from the current situation to the end of the race. We link the current stint and future stints by setting the initial total time to

$$t_{\text{tot}}(1) = t_{\text{stint,current}} + t_{\text{fc}}(n_{\text{laps}}+1). \tag{15}$$

We model the individual stint times as a positive semi-definite constraint [12] as

$$t_{\text{stint}}(i) \geq x_{\text{s}}(i)^\top Q_{\text{s}} x_{\text{s}}(i) - M \cdot (1 - b_{\text{pit}}(i)), \tag{16}$$

where $x_{\text{s}}(i) = \left[\frac{1}{\sqrt{t_{\text{charge}}(i)}} \; \sqrt{t_{\text{charge}}(i)} \; \frac{N_{\text{laps}}(i)}{\sqrt{t_{\text{charge}}(i)}}\right]^\top$ and $Q_{\text{s}} \in \mathbb{S}^3_+$ is a symmetric positive semi-definite matrix of coefficients. Since it is optimal to minimize stint time, this constraint will hold with equality at the optimum. For further information on the derivation and how to rewrite it as a second-order cone, we refer the reader to [1].

The final stint of the race is not followed by a pit stop, which means that the battery can be fully depleted. Therefore, we separately model the final stint with a fixed energy budget corresponding to $t_{\text{charge}}(n_{\text{stops}}+1) = t_{\text{charge,max}}$. With the charge time being fixed, we can then model the final stint time by a quadratic function with the stint length as

$$t_{\text{stint}}(n_{\text{stops}}+1) \geq D_{\text{s,f}}^\top x_{\text{s,f}}, \tag{17}$$

where $D_{\text{s,f}}$ is a vector of coefficients and $x_{\text{s,f}} = [N_{\text{laps}}^2(n_{\text{stops}}+1) \; N_{\text{laps}}(n_{\text{stops}}+1) \; 1]^\top$. Whenever a non-driven stint is taken instead of a real stint, i.e., $b_{\text{pit}}(i) = 0$, we define an upper bound on stint length as

$$N_{\text{laps}}(i) \leq N_{\text{laps,max}} \cdot b_{\text{pit}}(i), \tag{18}$$

thereby excluding it from the objective function.

Finally, we ensure that the final stint is part of the optimal solution by writing

$$b_{\text{pit}}(i+1) \geq b_{\text{pit}}(i), \quad \forall i \in [1, n_{\text{stops}}]. \tag{19}$$

Fig. 3. Mini-sector definition for the Zandvoort reference track.

C. Online Race Strategy Optimization Problem

This section presents the online race strategy optimization problem of the electric race car. Given a race time limit, current race time and battery energy, we formulate the problem using the state variables $x = (t_{\text{tot}}, t_{\text{fc}})$ and the control variables $u = (\Delta t_{\text{charge}}, b_{\text{pit}}, t_{\text{charge}}, N_{\text{laps}}, b_{\text{lap}})$ and decision variables $(t_{\text{tot}}, t_{\text{fc}}, \Delta t_{\text{charge}}, t_{\text{charge}}, N_{\text{laps}}(n_{\text{stops}})) \in \mathbb{R}^5_+$, $(b_{\text{lap}}, b_{\text{pit}}) \in \{0,1\}$ and $N_{\text{laps}}(i) \in \mathbb{N}$, $\forall\, i \in [1, \dots, n_{\text{stops}} - 1]$ as:

Problem 1 (Maximum-race-distance Strategies)**.** *The maximum-race-distance strategies are the solution of*

$$\max_u \sum_{i=1}^{n_{\text{stops}}} S_{\text{lap}} \cdot N_{\text{laps}}(i) + \sum_{j=1}^{n_{\text{laps}}} S_{\text{lap}} \cdot b_{\text{lap}}(j),$$

s.t. (3) – (19).

Problem 1 is a mixed-integer second-order conic program providing global optimality guarantees [13], [14].

III. Competitor Interactions

This section analyzes various types of competitor interactions and models them with time penalties representing possible actions. During a race, there are many other vehicles driving along the track, each of them with a different pace. These competitors can then act as disturbances on the optimal race strategy. In this analysis, we consider three main interactions, being the ego vehicle approaching a competitor (*Attack*), the ego vehicle being approached by a competitor from behind (*Defend*) and track position prediction after a pit stop (*Pit-lane Exit Traffic*). For each of these interactions, there is a set of actions that can be taken by the ego vehicle, with the optimal action resulting in the lowest estimated time penalty. The time penalties are estimated with a limit of one single interaction per mini-sector (MS) m. We decompose the Zandvoort reference track into a total of 9 MS based on the three main sectors and the most common locations with interactions, shown in Fig. 3.

A. Attack

The *Attack* interaction happens when the ego vehicle has the chance to overtake the car in front during the upcoming MS. This is triggered by predicting the virtual gap at the end of it, e.g., if the ego vehicle is predicted to be in front at the end of the MS according to the free-flow strategy, we are *virtually overtaking* the car ahead and the decision-making process is triggered. The set of possible actions by the ego vehicle is composed by *Overtake*, *Stay Behind* and *Box*.

1) Overtake: We define the time penalty $t_{\text{pen,ov}}$ as the sum of fixed time losses from the *Overtake* attempt $t_{\text{ov,c}}$, time gain upon a successful overtake $t_{\mathrm{ov,g}}$ and time losses upon a failed overtake $t_{\mathrm{ov,l}}$:

$$t_{\mathrm{pen,ov}}(\Delta E_{\mathrm{b}}, m) = \frac{(t_{\mathrm{ov,c}}(\Delta E_{\mathrm{b}}) - t_{\mathrm{ov,g}}(\Delta E_{\mathrm{b}}, m) + t_{\mathrm{ov,l}}(m))}{P_{\mathrm{ov}}(\Delta t_{\mathrm{d}}, m)}, \tag{20}$$

$$t_{\mathrm{ov,c}}(\Delta E_{\mathrm{b}}) = \Delta t_{\mathrm{charge,ov}}(\Delta E_{\mathrm{b}}) + t_{\mathrm{rl}}, \tag{21}$$

$$t_{\mathrm{ov,g}}(\Delta E_{\mathrm{b}}, m) = (\Delta t_{\mathrm{d}}(\Delta E_{\mathrm{b}}) - \Delta t_{\mathrm{p}}(m)) \cdot P_{\mathrm{ov}}(\Delta t_{\mathrm{d}}, m), \tag{22}$$

$$t_{\mathrm{ov,l}}(m) = (\Delta t_{\mathrm{p}}(m) + t_{\mathrm{gap,min}}) \cdot (1 - P_{\mathrm{ov}}(\Delta t_{\mathrm{d}}, m)), \tag{23}$$

where $\Delta t_{\mathrm{charge,ov}}$ captures the increase in charge time due to the additional battery energy used ΔE_{b}, t_{rl} is the time penalty for deviating from the optimal racing line (estimated based on data from previous races), Δt_{d} and Δt_{p} are the desired and predicted gaps with the competitor, respectively, P_{ov} is the probability of a successful overtake and $t_{\mathrm{gap,min}}$ is the minimum following gap to a competitor in front. For the time gain and loss estimation, we essentially take the gap w.r.t. the original strategy and weigh it by the respective probability. Finally, we divide by P_{ov} to prevent overtakes whenever the chance of succeeding is minor. The probability of a successful overtake both depends on the predicted gap to the competitor and the track layout, and is estimated from previous race results. Since we can choose to drive faster, thereby increasing the odds of a successful overtake, we jointly optimize the time penalty, battery energy and the vehicle operation using a modified minimum-lap-time framework [1] and adding (21)-(23) as constraints to obtain a perturbation around the original solution.

2) Stay Behind: The aforementioned *Overtake* maneuver potentially requires more energy, which means that staying behind the competitor and saving energy can be beneficial. The time penalty $t_{\mathrm{pen,sb}}$ is computed as

$$\begin{aligned} t_{\mathrm{pen,sb}}(\Delta E_{\mathrm{b}}, m) = \Delta t_{\mathrm{charge,sb}}(\Delta E_{\mathrm{b}}) + \Delta t_{\mathrm{p}}(m) + \\ t_{\mathrm{gap,min}} + t_{\mathrm{buff,sb}}(m) + \Delta t_{\mathrm{p}}(m+1), \end{aligned} \tag{24}$$

$$t_{\mathrm{buff,sb}}(m+1) = \begin{cases} t_{\mathrm{buff,sb}}(m) + t_{\mathrm{pen,sb}}(\Delta E_{\mathrm{b}}, m), & \text{if stay behind} \\ 0, & \text{otherwise,} \end{cases} \tag{25}$$

where $\Delta t_{\mathrm{charge,sb}}$ is the decrease in charge time due to energy saved and $t_{\mathrm{buff,sb}}$ is a time buffer that keeps track of the total time lost during the *Stay Behind* action. Since the race strategy is only updated every lap, it could occur that a considerable amount of time is lost if the *Stay Behind* action is chosen for consecutive MS. Yet this would be unnoticed until the race strategy is triggered. We prevent this by adding an accumulating buffer, capturing the total time lost during the action, instead of only the local time loss. Furthermore, we include the predicted gap of the consecutive MS to add more incentive to overtake in the case where this consecutive MS provides little overtaking opportunities. We again optimize the vehicle operation to obtain the minimum time penalty for this action.

3) Box: Whenever there is an interaction in the final MS, we can decide to prematurely stop to charge the vehicle, thereby avoiding possible time loss from the interaction. However, since this often means deviating from the race strategy, we have to evaluate the impact on the race by solving Problem 1 and enforcing the current lap to be an in-lap. If the distance covered exceeds the distance covered in the original solution, this action can be beneficial. In motorsports, this is often referred to as *undercut* and is most likely to be viable when vehicles have a similar pace and when we are close to making a pit stop.

B. Defend

Similar to the *Attack* interaction, the *Defend* interaction occurs when we get *virtually overtaken* by a competitor for a position, i.e., the predicted gap between the ego vehicle and the competitor t_{p} is negative at the end of the following MS. The set of possible actions is composed by *Block*, *Let Through* and *Box*, where the latter is the same as explained in Section III-A.3. We assume that the competitor is always taking the most aggressive action, i.e., *Overtake*.

1) Block: Similar to the *Overtake* action, the *Block* action impact $t_{\mathrm{pen,blk}}$ is defined as a sum of fixed time losses $t_{\mathrm{blk,c}}$, time gain upon a successful blocking maneuver $t_{\mathrm{blk,g}}$ and time losses upon a failed attempt $t_{\mathrm{blk,l}}$:

$$t_{\mathrm{pen,blk}}(\Delta E_{\mathrm{b}}, m) = \frac{(t_{\mathrm{blk,c}}(\Delta E_{\mathrm{b}}) - t_{\mathrm{blk,g}}(\Delta E_{\mathrm{b}}, m) + t_{\mathrm{blk,l}}(m))}{P_{\mathrm{def}}(\Delta t_{\mathrm{d}}, m)}, \tag{26}$$

$$t_{\mathrm{blk,c}}(\Delta E_{\mathrm{b}}) = (\Delta t_{\mathrm{charge,blk}}(\Delta E_{\mathrm{b}}) + t_{\mathrm{rl}}, \tag{27}$$

$$t_{\mathrm{blk,g}}(\Delta E_{\mathrm{b}}, m) = (\Delta t_{\mathrm{d}}(\Delta E_{\mathrm{b}})) \cdot P_{\mathrm{def}}(\Delta t_{\mathrm{d}}, m), \tag{28}$$

$$t_{\mathrm{blk,l}}(m) = \max(\Delta t_{\mathrm{p}}(m) + t_{\mathrm{gap,min}}, 0) \cdot (1 - P_{\mathrm{def}}(\Delta t_{\mathrm{d}}, m)), \tag{29}$$

where $\Delta t_{\mathrm{charge,blk}}$ is the additional charge time due to the defending maneuver and P_{def} is the probability of successfully blocking the competitor, thereby preventing the ego vehicle from being overtaken. Note that the definition of this time penalty is similar to the penalty defined in Section III-A.1, with the addition of a saturation for the time losses as $\Delta t_{\mathrm{p}} \leq 0$ during *Defend* interactions. Again, to find the optimal trade-off between driving faster and increasing the probability of a successful block, and the energy consumption, we optimize the vehicle operation.

2) Let Through: Instead of attempting to block the competitor, the ego vehicle can also decide to continue following the original strategy and let the competitor pass. Although this action might seem counter-intuitive in the first place, this can be very beneficial in the long term. The blocking action discussed in the previous section can require a considerable amount of additional energy if the competitor is significantly faster. Instead, it can be beneficial to follow the original strategy and let the competitor pass, which is also often seen in races involving pit stops. For example, if the ego vehicle spends more energy on preventing the *Overtake*, it may happen that it has to charge one lap earlier and thereby lose the interaction. In this case, the energy is essentially wasted, since the position is not preserved. To detect interference with the original strategy of the ego vehicle after a pass from the competitor, we check for possible interactions in the three upcoming MS. If no other interaction occurs with the same competitor, we always decide to let the competitor pass. However, if another interaction is predicted, we have to estimate the time penalty of letting the competitor through and staying behind for the entire horizon.

The time penalty is defined as the difference between the original strategy $t_{\mathrm{MS,p}}$ and the target time t_{target}, where the latter is set such that we follow the competitor:

$$t_{\mathrm{pen,lt}}(\Delta E_{\mathrm{b}}, m) = \max\left(t_{\mathrm{gap,min}} + \Delta t_{\mathrm{p}}(m), 0\right) + \sum_{l=m+1}^{m+3} \left(t_{\mathrm{target}}(l) - t_{\mathrm{MS,p}}(l) + \Delta t_{\mathrm{charge,lt}}(\Delta E_{\mathrm{b}}, l)\right), \tag{30}$$

where $\Delta t_{\mathrm{charge,lt}}(\Delta E_{\mathrm{b}}, m)$ is the change in charging time w.r.t. the original strategy in MS m. The change in battery energy is then computed by optimizing the vehicle operation such that the target MS are met.

C. Pit-exit Traffic

After a charging stop and exiting the pit-lane, it can occur that the ego vehicle returns to the track behind a slower competitor, resulting in a possible time loss. This scenario is referred to as pit-exit traffic and it is a common challenge in motorsports. Of course, it is possible to attempt to *Overtake* the competitor during the out-lap, but it could be advantageous to shorten the charging stop and attempt to arrive in front of the competitor. Hereby, we would trade charged energy for a better track position. To analyze the interaction, we compare the decision of *Shorten Pit-stop* with *Overtake during Out-lap*, assuming that the competitor is always defending the position.

1) Short Pit-stop: To investigate whether a shorter pit stop is beneficial, we have to check the increase in lap time due to the reduced amount of battery energy available. Therefore, the time penalty $t_{\mathrm{pen,sp}}$ is defined as the difference in lap time between the original strategy and the predicted lap times with the reduced battery energy, whereby we assume to spread the gap in battery energy across the entire stint:

$$t_{\mathrm{pen,sp}} = \sum_{j=1}^{N_{\mathrm{laps}}} \left(t_{\mathrm{lap}}\left(\Delta \bar{t}_{\mathrm{charge}}(j) - \frac{\Delta t_{\mathrm{charge,sp}}}{N_{\mathrm{laps}}}\right) - \bar{t}_{\mathrm{lap}} \right), \tag{31}$$

where $\Delta \bar{t}_{\mathrm{charge}}$ is the equivalent charge time per lap in the original strategy, $\bar{t}_{\mathrm{lap}}$ is the predicted lap time of the original strategy, $\Delta t_{\mathrm{charge,sp}}$ is the deduction of charge time needed in order to exit the pit-lane in front of the competitor and N_{laps} is the predicted stint length. In this case, we do not need an optimization framework, since we can use pre-computed lap time maps.

2) Overtake during the Out-lap: In the case where the ego vehicle is predicted to catch up with the competitor during the out-lap, we calculate the time penalty for an *Overtake* maneuver as described in Section III-A.

IV. Results

This section presents the numerical results for the combined MPC framework and competitor interaction decision making. As a use case, we choose the InMotion electric endurance race car as our ego vehicle and the 2023 Supercar Challenge at the Zandvoort circuit as the race event. This event consists of a 1 h race with a total of 31 participants, where all other cars are equipped with an ICE. To simulate the race, we assume that the position and lap times of the competitors are not influenced by the ego vehicle. For example, in the case of an overtake, we assume that the competitor does not react to the ego vehicle. This behavior is common in high level motorsports that involve pit stops and significant pace differences, since defending the position would only result in time loss for the competitor. Furthermore, we include probabilistic effects to model the interactions, i.e., an attempt to overtake is not guaranteed to be successful. However, since this would result in a stochastic race outcome, we simplify the analysis by imposing the outcome of interactions to be the outcome with the highest probability. Hence, we obtain the most likely race outcome and the analysis becomes deterministic, allowing for a fair comparison between different strategies.

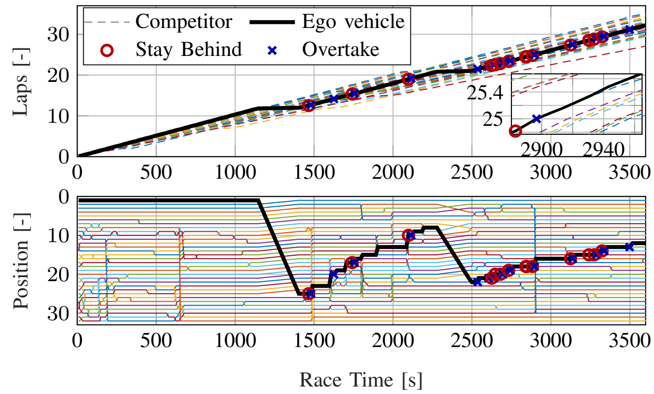

Fig. 4. Evolution of driven distance and position as a function of time for the ego vehicle and competitors, together with a zoom of an overtake. The ego vehicle starts at the first position, but has to charge before the competitors make their pit stop, resulting in a loss of several positions. Thereafter, the ego vehicle starts overtaking the competitors, resulting in the 13th overall finishing position among 32 participants.

We set the maximum number of stops $n_{\mathrm{stops}} = 10$ and the maximum number of laps in a stint $n_{\mathrm{laps}} = 15$ and solve Problem 1 every lap with a shrinking horizon. The online race strategy problem is parsed using YALMIP [15] and solved using Gurobi [16], while the time penalties and vehicle operation in the MS are solved and parsed using CasADi [17] and IPOPT [18], respectively. The average fraction of solver time over the MS time is 15 %, while the worst case is 27 % of the corresponding MS time on an Intel Core i7-4710MQ 2.5 GHz processor with 8GB of RAM, demonstrating the real-time capabilities of the framework.

The evolution of the race as a function of time is shown in Fig. 4 for both the ego vehicle and the competitors. Since the ego vehicle is capable of performing the fastest lap times, it starts in 1st place. Furthermore, no defensive interactions are observed, since the ego vehicle is the fastest during a lap. At the beginning of the race, the ego vehicle creates a gap to the competitors, until it requires charging. Since the charging time is considerably longer than the pit stop time of the ICE-driven competitors, the ego vehicle drops back in position, resulting in several interactions for the remainder of the race. Ultimately, the ego vehicle finishes the race in 13th place out of the 32 participants, revealing that e-race cars could race alongside ICE race cars in the near future.

To validate the decision making, we compare our proposed strategy with a baseline strategy that always attempts an overtake. Fig. 5 shows the time gap and difference in battery

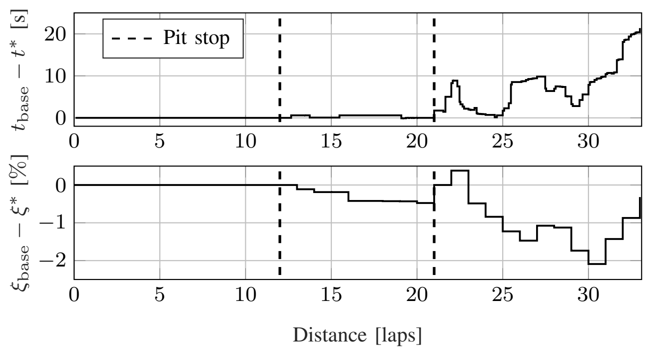

Fig. 5. Comparison between the baseline strategy and the optimal solution for the total time gap and the battery State of Charge (SoC) ξ, where the baseline strategy represents always overtake. The optimal strategy saves energy by staying behind competitors and overtaking at more favorable locations on the track, resulting in a total time saving of about 21.4 s.

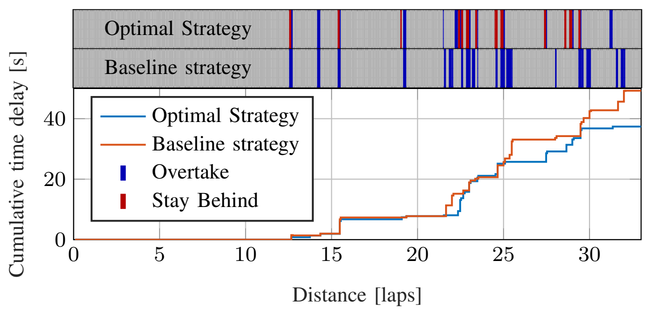

Fig. 6. Analysis of the cumulative time delay due to competitor interactions for the baseline and optimal strategy. The timelines at the top represent the actions taken by the respective strategy. The optimal strategy shows a lower time delay compared to the baseline at the end of the race, since attempting an overtake does not guarantee a successful overtake maneuver.

energy between the baseline and optimal strategy. The time gap at the end of the race is around 21.4 s, demonstrating that optimizing both the race strategy as well as the decision making has significant benefits. By staying behind, the optimal strategy saves energy, resulting in a shorter charging time at the second pit stop. Furthermore, in the last stint after lap 30, the surplus of battery energy can be used to drive faster lap times, compared to the baseline.

Lastly, we investigate the cumulative time delay due to the interactions for both strategies. Fig. 6 shows the interaction events for both strategies together with the cumulative time delay. We observe that the optimal strategy has a lower time delay overall, which further contributes to the time gap with the baseline. For example, at lap 25, the time gap between both strategies is almost negligible, yet the optimal strategy has more battery energy reserve. Therefore, it succeeds in a successful overtake, while the baseline strategy makes a series of unsuccessful overtake attempts, significantly increasing the time delay. This shows that staying behind competitors instead of attempting to overtake them can be faster and extremely important to account for in the course of a race.

V. Conclusion

In this paper, we have presented a control framework to optimize the endurance race strategy for a fully electric vehicle online, accounting for competitors' interactions. To this end, we reformulated an existing race strategy optimization framework to an online approach, capturing the pit stop decision on a lap-basis. In addition, we derived time penalty functions for all major interactions and decisions to obtain the best action w.r.t. the competitors. Our results showed that accounting for uncertainty in overtaking maneuvers and staying behind competitors, although possibly counter-intuitive, was 21 s faster over the course of a 1 h race. Ultimately, these methods bring e-race cars one step closer to ICE race cars. In future work, we plan to implement these strategies in a real-life situation on the race track and include game-theoretic approaches to improve the interaction models [8].

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