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Flatness-Based Control Strategy for n Parallel Connected Boost Choppers and n Sources with Differing Characteristics

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Abstract— **This paper presents a new control strategy based on a differential flatness approach for n boost choppers and n sources connected in parallel with different characteristics, the objective being to make them work on the same voltage bus. We first give an in-depth study of flatness of the n-boost system and propose a flat output for this type of boost choppers configuration, which ensures homogeneous power sharing among the choppers and guarantees continuous current in at least $n-1$ choppers. Another contribution is the establishment of a relation between the proposed flat output and the regulation of the DC bus voltage via an additional power load control loop. To demonstrate the effectiveness of the proposed control strategy, the paper includes simulation results for three boost choppers and sources connected in parallel with different characteristics.**

Keywords: **boost choppers average model, differential flatness, power control.**

I. INTRODUCTION

Boost choppers are widely used in power electronics, and this article aims to address the question of how to make different energy sources and boost choppers work together on the same voltage bus. This issue arises when it becomes necessary, due to power demand, availability, or security reasons, to connect multiple sources to the bus. This seemingly simple problem is, in reality, more complex than it appears. Indeed, it has been established in the literature (see, e.g., [26], [33]), that the dynamical model of a boost chopper has a non-minimum phase zero dynamics if the bus voltage is considered as the output variable. To compensate for this non-minimum phase behavior, many control algorithms have been developed that, in addition to the control loop for the voltage regulation, target the regulation of the current using a cascade structure such as passivity-based control (see, e.g., [5]), PI controllers (see, e.g., [6]), or sliding mode control (see, e.g., [33]). Comparisons of some of these methods have been conducted, for instance, in [11], [25], [32].

A class of control systems that has the property of having no zero dynamics is that of differential flat systems (simply, flat systems). The notion of flatness was introduced in control theory in the 1990s, by Fliess, Lévine, Martin and Rouchon [9] (see also [1], [17], [18], [28]) and has attracted a considerable interest [10], [22], [27], [29], [38] because of its important applications in the problems of motion planning and constructive controllability. In the context of electric power systems, flatness has already been successfully applied to electrical systems in simulations as well as in experimental works (see, e.g., [7], [20], [30]–[32], [34], [35], [39], [40] for some recent works). Although the average dynamic model of a single boost chopper is known to be flat (see, for instance, [11], [34]) and flatness-based control approaches have already been applied for a boost chopper (see, e.g., [12]), the flatness property of an interconnected system formed by several parallel connected boost choppers becomes uncertain and is not a direct consequence of that of each boost chopper. It has been shown (see [24]) that, even in the simplest case of linear systems, if each subsystem is flat, the global interconnected system is not necessarily flat and if it is, then, for the class of interconnected systems considered in [24], the flat output is never the collection of the flat outputs of each subsystem considered independently. In the case of an interconnected system of several boost choppers, this makes it difficult to ensure continuous mode for the boosts and an homogeneous power distribution on each boost to be transmitted to the load, which are important problems when dealing with such systems.

Therefore, the contribution of this work is two-fold. Firstly, we investigate the flatness property of a system consisting of n distinct boost choppers connected in parallel and compute all its flat outputs. We discuss their uniqueness and show that among all possible flat outputs, there exists a flat output whose components are the currents of $n-1$ boost choppers completed by the total stored energy of the n-boost system. This flat output is very interesting from a control point of view because it guarantees that at least $n-1$ boosts are always in continuous mode. Secondly, we introduce a load power control loop that establishes a relationship between the bus voltage and the flat output, our primary goal being to regulate the bus voltage while accomplishing the following objectives: 1) maintaining continuous operation for the maximum number of boost choppers; 2) achieving uniform power distribution; 3) ensuring robust tracking of the constant DC bus voltage in the face of load variations.

Parallel connected boost choppers have already been considered in the literature: [36], [37] deal with the case of parallel-connected boost choppers powered by a single DC source full cell. In [36], the DC bus voltage is handled using the stored energy of the total DC bus capacitor, which is actually considered as a flat output for a subsystem of the control system modeling the whole circuit, and flatness-based control is applied for that subsystem. Also [37] uses flatness

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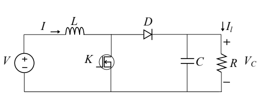
(again of a subsystem and not of the control system modeling the whole network) for the inner fuel cell power regulation, the proposed flat output corresponding to the input power of each boost. In [23], the considered network possesses two distributed generators interfaced to the DC bus by non-isolated boost converters, so it has, like our model, several sources, but it presents a different topology than ours. A nonlinear controller based on flatness of a subsystem, the flat output being the energy of the output voltage capacitor, is proposed. Although all these papers present interesting and effective control strategies for the considered problems, none of them provided an in-depth study of the overall system's flatness.

The paper is organized as follows. The averaged modeling of the n-boost system is presented in Section II. Section III gives a flatness analysis of the connected boost choppers and proposes a flat output for the overall system. Then the control laws are presented in Section IV. Finally, simulation results and some future work directions are given in Section V.

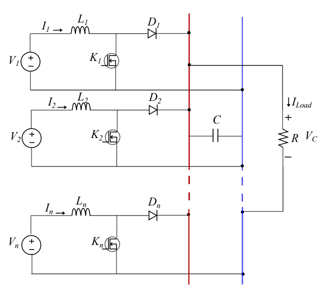
Notations for the n-boost system:
$i = 1, \dots, n$: integer labeling the boost choppers,
I_i: current in the inductance of the i^{th} boost chopper,
$x_i = I_{i_{av}}$: averaged current in the i^{th} inductance,
V_C: bus voltage, and C: bus capacitance,
$x_{n+1} = V_{C_{av}}$: averaged bus voltage,
α_i: duty cycle of the i^{th} boost chopper,
T: switching period, and $F = \frac{1}{T}$: switching frequency,
V_i: source voltage of the i^{th} boost chopper,
L_i: inductance of the i^{th} boost chopper,
P_{ri}: rated power of the i^{th} boost chopper,
R: load resistance, and I_l: load current,
I_{di}: desired current $I_{i_{av}}$ and V_{dC}: desired bus voltage $V_{C_{av}}$,
E_t: total energy, and E_{dt}: desired total energy.

II. n-BOOST SYSTEM MODELING

A boost chopper is a switching power supply that converts a DC voltage into a different DC voltage of higher value [8], [34]. DC/DC power converters, such as boost choppers, are generally used to supply a regulated DC output voltage. The schematic of the boost chopper is shown in Fig. 1a. It contains a diode, a Mosfet switch, an inductance, and a capacitor.



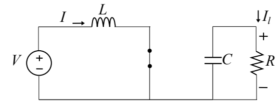
(a) A boost chopper with resistive load.



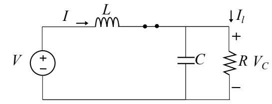
(b) Parallel connected n boost choppers.

Fig. 1: Schematics.

To simplify the writing and the transition to the average model, we assume that the system is driven by pulse width modulation (PWM). The boost chopper operates in two modes. In the on-mode, the switch K is closed (i.e., $K = 1$) from time 0 to αT, with $\alpha \in [0, 1[$, where α and T denote, resp., the duty cycle of the boost chopper and the switching period; then the diode D does not conduct. During this mode, the energy provided by the DC voltage source $V > 0$ is being stored in the inductance L. Then the capacitor C is being discharged through the load resistance R with respect to equation (1) of the on-mode boost chopper configuration (Fig. 2a). In the off-mode (Fig. 2b), K is opened, i.e. $K = 0$, from αT to T and the diode D conducts. During this mode, the energy stored in the inductance L is transferred in the capacitor and the load with respect to equation (2).



(a) On-mode ($0 < t \leq \alpha T$).



(b) Off-mode ($\alpha T < t \leq T$).

Fig. 2: Operation configurations of the boost chopper.

Fig. 2a gives the following equations for $K = 1$:

$$\begin{pmatrix} \dot{I} \\ \dot{V}_C \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \frac{-1}{RC} \end{pmatrix} \begin{pmatrix} I \\ V_C \end{pmatrix} + \begin{pmatrix} \frac{V}{L} \\ 0 \end{pmatrix}, \quad (1)$$

while from Fig. 2b, we deduce the following for $K = 0$:

$$\begin{pmatrix} \dot{I} \\ \dot{V}_C \end{pmatrix} = \begin{pmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{pmatrix} \begin{pmatrix} I \\ V_C \end{pmatrix} + \begin{pmatrix} \frac{V}{L} \\ 0 \end{pmatrix}. \quad (2)$$

The behavior of a boost chopper corresponds to (2), when the current I in the inductance does not pass through zero. If I passes through zero then the diode is blocked and (2) is no longer verified. If the boost chopper always has a strictly positive current in its inductance, it is said to be in continuous mode. Under the assumption that the boost chopper is in continuous mode, and that relations (1) and (2) hold, the average model of the boost chopper of Fig. 1a is:

$$\dot{I}_{av} = \frac{\alpha - 1}{L} V_{C_{av}} + \frac{V}{L} \quad (3)$$

$$\dot{V}_{C_{av}} = \frac{1 - \alpha}{C} I_{av} - \frac{V_{C_{av}}}{RC}, \quad (4)$$

with I_{av} the averaged current and $V_{C_{av}}$ the averaged voltage.

For several reasons, like multiple energy sources, not enough energy transmitted by a single boost chopper, hardware redundancy for safety, etc., it may be necessary to connect different boost choppers in parallel. In Fig. 1b, n different boost choppers with different sources are connected in parallel. Each boost and its characteristics are labeled using the integer i, with $1 \leq i \leq n$. Note that the parallel connected capacitances C_i of each boost chopper have been regrouped into one total capacitance $C = \sum_{i=1}^{n} C_i$ leading to a state reduction. In the case of a low power demand, all boost choppers may provide a small current that approaches zero. This can potentially cause the boosts choppers to go to a discontinuous mode. To prevent this, we may disconnect one or more boost choppers, allowing the remaining boost choppers to provide a higher current. This ensures continuous mode operation for the boost choppers that are still active.

The new system has the same topology as the original one, only the number n of boost choppers decreases. It is described by system (5) below, with f and g_i given, resp., by (6) and (7), but which now contains, instead of n, at most $n-1$ boost choppers operating in continuous mode. Therefore, from now on, we can suppose the following:

Assumption 1: All boost choppers are in continuous mode.

The average model of the n parallel connected boost choppers of Fig. 1b is of the form:

$$\dot{x} = f(x) + \sum_{i=1}^{n} g_i(x)\alpha_i, \quad (5)$$

where $x := (x_1, x_2, .., x_n, x_{n+1})^\top$ is the state and is equal to $x = (I_{1_{av}}, I_{2_{av}}, .., I_{n_{av}}, V_{C_{av}})^\top$ corresponding resp., to the averaged currents in the inductances L_i, $1 \le i \le n$, and to the averaged bus voltage. The system is controlled by the n duty cycles $\alpha_i \in [0, 1[$ of the boost choppers. The vector fields f and g_i are given, resp., by:

$$\begin{aligned} f(x) &= \sum_{i=1}^{n} \frac{V_i - V_{C_{av}}}{L_i} \frac{\partial}{\partial I_{i_{av}}} + \left(\sum_{i=1}^{n} \frac{I_{i_{av}}}{C} - \frac{V_{C_{av}}}{RC} \right) \frac{\partial}{\partial V_{C_{av}}} \\ &= \sum_{i=1}^{n} \frac{V_i - x_{n+1}}{L_i} \frac{\partial}{\partial x_i} + \left(\sum_{i=1}^{n} \frac{x_i}{C} - \frac{x_{n+1}}{RC} \right) \frac{\partial}{\partial x_{n+1}}, \quad (6) \\ g_i(x) &= \frac{V_{C_{av}}}{L_i} \frac{\partial}{\partial I_{i_{av}}} - \frac{I_{i_{av}}}{C} \frac{\partial}{\partial V_{C_{av}}} \\ &= \frac{x_{n+1}}{L_i} \frac{\partial}{\partial x_i} - \frac{x_i}{C} \frac{\partial}{\partial x_{n+1}}, \quad 1 \le i \le n. \quad (7) \end{aligned}$$

From a mathematical point of view, the state space of the above system is $\mathbb{R}^{n+1}$, but since all source voltages V_i are positive and Assumption 1 is supposed to be always verified, the only possible states, in continuous mode, are in $\mathbb{R}_+^{n+1}$.

The goal of this paper is to present a new control strategy based on a flatness approach for n boost choppers and sources connected in parallel with different characteristics. It is well-known that the control system (3)–(4), describing the dynamics of a single boost chopper, is flat (see, e.g., [11], [34]), so each subsystem forming the interconnected one (5) is flat, and one of the most natural questions is whether the global system (5) is also flat. We will answer that question in the next section.

III. FLATNESS ANALYSIS

The definition of flatness, see [9], for a general control system with s states and m controls, can be stated as follows:

Definition 1: The system $\Xi : \dot{x} = F(x, u)$, where $x \in \mathbb{R}^s$ and $u \in \mathbb{R}^m$, is flat if there locally exist m smooth functions $h_i = h_i(x, u, \dot{u}, \ldots, u^{(\ell)})$, where $1 \le i \le m$ and[1] $\ell \ge -1$, having the following property: there exist an integer q and smooth functions γ_i, $1 \le i \le n$, and δ_j, $1 \le j \le m$, such that locally

$$x_i = \gamma_i(h, \dot{h}, \ldots, h^{(q-1)}) \text{ and } u_j = \delta_j(h, \dot{h}, \ldots, h^{(q)}),$$

where h denotes $h = (h_1, \ldots, h_m)$ and is called a flat output. In the particular case when $h_i = h_i(x)$, for all $1 \le i \le m$, the system is called x-flat.

From the above definition it follows that for a flat system, the evolution in time of all state and control variables can be recovered from that of the flat output components h_i without integration and all trajectories of the system can be completely parameterized. Flatness is closely related to the notion of feedback linearization. The system $\Xi : \dot{x} = F(x, u)$, with $x \in \mathbb{R}^s$ and $u \in \mathbb{R}^m$, is locally linearizable by static feedback if it is equivalent, via a local diffeomorphism $z = \phi(x)$ and an invertible static feedback transformation $u = \psi(x, v)$, to a linear controllable system $\Lambda : \dot{z} = Az + Bv$, with $z \in \mathbb{R}^s, v \in \mathbb{R}^m$. Systems linearizable via invertible static feedback are flat, and for single-input control systems, flatness is actually equivalent to static feedback linearization, see [3], [28], and is thus completely characterized by [15], [19]. Consider again dynamics (3)–(4), describing a single boost chopper, which has only one control. It follows that studying its flatness reduces to checking static feedback linearization and actually to finding a linearizing output, see [19]. For multi-input control systems the equivalence between flatness and static feedback linearization no longer holds and in general, flat systems are not static feedback linearizable but can be seen as a generalization of linear systems (namely they are linearizable via dynamic, invertible and endogenous feedback, see [9], [28]). Nevertheless, we show next that the n-input control system (5), describing the dynamics of n boost choppers connected in parallel, is actually static feedback linearizable and therefore flat (see Proposition 1 below). To system (5), we associate the distribution $\mathcal{D}^0 = \operatorname{span}\{g_1(x), \ldots, g_n(x)\}$ spanned by the control vector fields of (5), given by (7), and define $\mathcal{D}^1 = \mathcal{D}^0 + [f, \mathcal{D}^0] = \operatorname{span}\{g_i(x), [f, g_i](x), 1 \le i \le n\}$, where f is the drift of (5), given by (6), and the bracket represents the Lie bracket.

Proposition 1: The n-input system (5) *is locally static feedback linearizable around any $x^* \in \mathbb{R}^{n+1}$ such that $x^*_{n+1} \neq 0$ and $x^*_i \neq -\frac{RCV_i}{2L_i}$ for at least one integer $1 \le i \le n$, and thus system* (5) *is flat at x^*. Moreover, any n-tuple of smooth functions $(h_1(x), \ldots, h_n(x))$ satisfying*

(i) $(\mathcal{D}^0)^\perp = \operatorname{span}\{\mathrm{d}h_n\}$,

(ii) $(\mathrm{d}h_1 \wedge \ldots \wedge \mathrm{d}h_{n-1} \wedge \mathrm{d}h_n \wedge \mathrm{d}L_f h_n)(x^*) \neq 0$,

forms a flat output of (5) *at x^*.*

Proof: Consider system (5) around any x^* satisfying the hypotheses of Proposition 1, and apply the following (invertible around x^*) static feedback transformation $u_i = \frac{V_i - x_{n+1}}{L_i} + \frac{x_{n+1}}{L_i}\alpha_i$, $1 \le i \le n$, that locally brings (5) into:

$$\begin{aligned} \dot{x}_i &= u_i, \ 1 \le i \le n, \\ \dot{x}_{n+1} &= f_{n+1}(x) - \textstyle\sum_{i=1}^{n} \frac{L_i x_i}{C x_{n+1}} u_i, \end{aligned} \quad (8)$$

where $f_{n+1}(x) = \sum_{i=1}^{n} \frac{V_i x_i}{C x_{n+1}} - \frac{x_{n+1}}{RC}$. By a straightforward computation, it is immediate that the distribution $\mathcal{D}^0 = \operatorname{span}\{\frac{\partial}{\partial x_i} - \frac{L_i x_i}{C x_{n+1}} \frac{\partial}{\partial x_{n+1}}, 1 \le i \le n\}$ is of constant rank n and involutive (indeed we have $[g_i, g_j] = 0$, for any $1 \le i, j \le n$). Moreover, we have $\mathcal{D}^1 = \mathcal{D}^0 + \operatorname{span}\{(\frac{2L_i x_i}{RC^2 x_{n+1}} + \frac{V_i}{C x_{n+1}}) \frac{\partial}{\partial x_{n+1}}, 1 \le i \le n\}$, which is of constant rank $n+1$ around x^*. It follows from [19] that system (8) is static feedback linearizable and hence, so is system (5). Let us now compute its linearizing outputs (which are flat outputs of (8) and thus of (5) as well). The distribution $\mathcal{D}^0$ being involutive of constant corank 1, there exists a non trivial

[1] When $\ell = -1$, we simply have $h_i = h_i(x)$, for all $1 \le i \le m$.

smooth function, denoted by h_n, such that $\mathrm{d}h_n \perp \mathcal{D}^0$ and $L_{[f,g_i]}h_n(x^*) \neq 0$ for at least one integer $1 \leq i \leq n$. This yields the system of equations $\frac{\partial h_n}{\partial x_i} - \frac{L_i x_i}{C x_{n+1}} \frac{\partial h_n}{\partial x_{n+1}} = 0$, for all $1 \leq i \leq n$, whose solution $h_n(x)$ is any function of $\frac{1}{2}\left(\sum_{i=1}^{n} L_i x_i^2 + C x_{n+1}^2\right)$. We put $\tilde{x}_n = h_n$, $\tilde{x}_{n+1} = L_f h_n$ and choose $\tilde{x}_1 = h_1(x), \ldots, \tilde{x}_{n-1} = h_{n-1}(x)$, where $h_i(x)$ are any smooth functions completing h_n and $L_f h_n$ to a coordinate system. In this coordinates, after applying a suitable invertible static feedback, system (5) takes the form

$$\dot{\tilde{x}}_i = \tilde{u}_i,\ 1 \leq i \leq n-1, \qquad \begin{aligned} \dot{\tilde{x}}_n &= \tilde{x}_{n+1}, \\ \dot{\tilde{x}}_{n+1} &= u_n, \end{aligned} \tag{9}$$

and is clearly flat with $h = (\tilde{x}_1, \ldots, \tilde{x}_n)$ a flat output. It follows that $h(x) = (h_1(x), \ldots, h_n(x))$ is a flat output of (5) as well. ■

Remark 1: System (5) is a control-affine system with one input less than the number of states, therefore, in order to decide whether it is flat or not, we could have also applied the results of [2] according to which the system is x-flat as soon as it is strongly accessible for almost every x. Notice also that system (5) is actually a bilinear control system; a sufficient condition for flatness of bilinear control systems can be found in [13], [14].

Remark 2: For the n-boost control system, the singular state $x_{n+1} = 0$ will be forbidden by the control because the system is stabilized around a positive desired bus voltage $x_{d,n+1} = V_{dC}$, so throughout the paper $x_{n+1} = V_{C_{av}}$ is in the vicinity of $V_{C_{av}} > 0$. Moreover, the diode of each boost chopper prevents the current in the inductance to be negative, so $x_i = -\frac{RCV_i}{2L_i}$ can never happen. Therefore, from a physical point of view, the n-boost system is locally static feedback linearizable around any (physically possible) point and from now on, we suppose that $x_i > 0$, for $1 \leq i \leq n+1$.

The flat output components depend on the state variables only, thus the system is x-flat, and we actually have a lot of freedom in choosing the functions h_i. Indeed, among all functions h_i, $1 \leq i \leq n$, only h_n has to verify a structural condition (given by Proposition 1(i)), and $h_1, \ldots, h_{n-1}$ can be any functions of x whose differentials and those of h_n and its derivative are independent at x^* (see condition (ii)). Consequently, a natural question is: *which is the most interesting choice of the flat output from a physical perspective and control objective point of view?* The structural condition (i) requires that h_n is a function of the total energy stored in the n boost choppers. Now recall that one of our goals is to impose currents values high enough in a maximum number of boost choppers inductances such that the associated boost choppers to be in continuous behavior. Hence, we would like the other components of the flat output to involve as much currents variables as possible and we have the following immediate corollary:

Corollary 1: The n-tuple $(h_1, \ldots, h_n)$ of smooths functions given by

$$\begin{cases} h_i(x) = x_i = I_{i_{av}},\ \text{for } 1 \leq i \leq n-1, \\ h_n(x) = \frac{1}{2}\left(\sum_{i=1}^{n} L_i x_i^2 + C x_{n+1}^2\right) \\ \qquad\ \ = \frac{1}{2}\left(\sum_{i=1}^{n} L_i I_{i_{av}}^2 + C V_{C_{av}}^2\right), \end{cases} \tag{10}$$

defines a flat output of system (5) *at any $x^* \in \mathbb{R}^{n+1}$, where $x_i^* > 0$, for $1 \leq i \leq n+1$, with the flat parametrization given by expressions* (11)–(15) *below.*

Proof: Consider the functions $h_i(x)$, $1 \leq i \leq n$, defined by (10). From $h_n(x) = \frac{1}{2}\left(\sum_{i=1}^{n-1} L_i h_i^2 + L_n x_n^2 + C x_{n+1}^2\right)$ and $\dot{h}_n = \sum_{i=1}^{n-1} V_i h_i + V_n x_n - \frac{x_{n+1}^2}{R}$, we deduce that x_n is solution of the second order polynomial equation

$$\frac{L_n}{C} x_n^2 + R V_n x_n + \mu(h, \dot{h}) = 0,$$

where $\mu(h, \dot{h}) = \sum_{i=1}^{n-1}(R V_i h_i + \frac{L_i}{C} h_i^2) - \frac{2}{C} h_n - R\dot{h}_n$, and

$$x_n = \gamma_n(h, \dot{h}) = \frac{-RCV_n + \sqrt{(RCV_n)^2 - 4 L_n C \mu(h, \dot{h})}}{2L_n}. \tag{11}$$

Similarly, from h_n, we conclude that

$$x_{n+1} = \gamma_{n+1}(h, \dot{h}) = \sqrt{\frac{2}{C} h_n - \sum_{i=1}^{n-1} \frac{L_i}{C} h_i^2 - \frac{L_n}{C} \gamma_n^2(h, \dot{h})}. \tag{12}$$

Further more, we have

$$x_i = \gamma_i(h) = h_i,\ 1 \leq i \leq n-1, \tag{13}$$

$$\alpha_i = \delta_i(h, \dot{h}) = 1 + \frac{L_i \dot{h}_i - V_i}{\gamma_{n+1}(h, \dot{h})},\ 1 \leq i \leq n-1. \tag{14}$$

Finally, from $\ddot{h}_n$, we compute:

$$\alpha_n = \delta_n(h, \dot{h}, \ddot{h}) = 1 + \frac{L_n \frac{\mathrm{d}\gamma_n(h, \dot{h},)}{\mathrm{d}t} - V_n}{\gamma_{n+1}(h, \dot{h})}. \tag{15}$$

■

We consider the flat output of Corollary 1 for the flatness-based control of the n-boost system. A first consequence of the fact that h_n is the total energy stored in the n boost choppers is that the control strategy cannot be a decentralized control anymore. For such a system, the currents and the bus voltage are usually measured. The problem is then only one of stabilizing the bus voltage under the constraint of balancing the power in the boost choppers. We show next how this can be achieved by a flatness-based control approach.

IV. Control Design for Homogenous Power Distribution

The bus voltage needs to be stabilized and set at a desired constant level V_{dC}, and the power transmitted by each boost chopper must be evenly distributed in proportion to the power rating of each source P_{ri}. Then, with respect to the nominal power consumed by the load P_l, the desired power P_{di} assigned to the i^{th} boost chopper is:

$$P_{di} = \frac{P_l P_{ri}}{\sum_{j=1}^{n} P_{rj}},\ 1 \leq i \leq n. \tag{16}$$

Recall that the first $n-1$ components of the flat output are currents, so the desired powers P_{di} must be translated into desired currents I_{di}:

$$I_{di} = \frac{P_{di}}{V_i},\ 1 \leq i \leq n, \tag{17}$$

leading to

$$h_{di} = I_{di} = \frac{P_{di}}{V_i},\ 1 \leq i \leq n-1. \tag{18}$$

The last component h_n of the flat output being the energy stored in the n-boost system, it is necessary to calculate it from I_{di}, $1 \leq i \leq n$, and the desired bus voltage V_{dC}. Therefore, the desired energy h_{dn} is given by:

$$h_{dn} = \sum_{i=1}^{n} \frac{L_i}{2} I_{di}^2 + \frac{C}{2} V_{dC}^2. \tag{19}$$

For the flat n-boost system, the tracking of the above desired flat output components may be designed thanks to Proposition 1, establishing the equivalence of the considered system to the linear one (9), by setting the controls α_i as follows (see also [16]):

$$\begin{aligned}
\alpha_i =& \frac{L_i}{\gamma_{n+1}} \left(\frac{\gamma_{n+1} - V_i}{L_i} - \lambda_i (h_i - h_{di}) + \dot{h}_{di} \right), \ 1 \leq i \leq n-1, \\
\alpha_n =& \left(\frac{V_n \gamma_{n+1}}{L_n} + \frac{2\gamma_{n+1}\gamma_n}{RC} \right)^{-1} \left(-\sum_{i=1}^{n} \frac{V_i(V_i - \gamma_{n+1})}{L_i} \right. \\
&+ \frac{2\gamma_{n+1}}{R} \left(\frac{\sum_{i=1}^{n}\gamma_i}{C} - \frac{\gamma_{n+1}}{RC} \right) - \sum_{i=1}^{n-1} \left(\frac{V_i \gamma_{n+1}}{L_i} + \frac{2\gamma_{n+1}\gamma_i}{RC} \right) \alpha_i \\
&\left. - \lambda_{n,0}(h_n - h_{dn}) - \lambda_{n,1}(\dot{h}_n - \dot{h}_{dn}) + \ddot{h}_{dn} \right),
\end{aligned} \tag{20}$$

where the constant gains λ_i, $1 \leq i \leq n-1$, and $\lambda_{n,j}$, $j = 0, 1$, are calculated via a classical pole placement, and guarantee that the tracking error $h_i - h_{di}$, $1 \leq i \leq n$, exponentially converges to 0.

In a second step, a control loop between the bus voltage and the power assignment P_a, which will replace P_l in (16) (that is, (16) becomes $P_{di} = \frac{P_a P_{ri}}{\sum_{j=1}^{n} P_{rj}}$, $1 \leq i \leq n$, with P_a solution of (21) below), is carried out to increase the robustness of the control laws with respect to load variations. The dynamics of the controlled P_a is described by:

$$\dot{P}_a = -\lambda_{n+1}(V_{C_{av}} - V_{dC}), \tag{21}$$

where λ_{n+1} is a constant gain to be selected based on singular disturbance arguments [21], because the current and energy loops must be faster than the power load control loop (PLCL) to ensure current continuity in each boost chopper. Since the control of P_a is slower than the other control loops, allowing for temporal decoupling, P_{di} and h_{di} can be thus considered constant when P_l is replaced by P_a in (16).

To sum up, it is the desired powers P_{di} (see (16)), controlled through P_a given by (21), which allow to compute the desired trajectories for the flat output components, see (18) and (19). With the PLCL (21), when considering variations of the load resistance, the desired powers P_{di} change and therefore the desired flat output trajectories has also to be updated accordingly (that is, at each variation of the load resistance). Without the PLCL (21), once the desired trajectories of h_{di} have been computed (for the power P_l associated to the nominal load resistance), they no longer change (even in the presence of load variations). This explains the different desired trajectories h_{di} for the cases with and without the PLCL (21), see the simulation results presented in Section V.

V. Simulation results and discussions

Simulation tests are carried out for a system composed of three boost choppers in the Matlab/Simulink environment. The performance of the proposed flatness-based control method (20), with $n = 3$, is evaluated under different voltage sources and variation of the load resistance, with and without the PLCL (21). TABLE I below presents the various parameters of the boost choppers (obtained following [4]) and of the control loops used in the simulations. To obtain only the average values, all measured variables underwent filtering using a second order Butterworth low-pass filter with a cutoff frequency F_{cut}. The abrupt load variation is:

$$R = \begin{cases} 50\ \Omega, & 0s \leq t < 0.5s \text{ and } 2s \leq t < 2.5s, \\ 100\ \Omega, & 0.5s \leq t < 1s \text{ and } 1.5s \leq t < 2s, \\ 200\ \Omega, & 1s \leq t < 1.5s. \end{cases}$$

TABLE I: 3-Boost system and control parameters.

Symbols	Values	Symbols	Values	Symbols	Values
L_1	79.7 mH	V_1	25 V	λ_1	150
L_2	267.7mH	V_2	30 V	λ_2	150
L_3	106.3 mH	V_3	50 V	$\lambda_{3,0}$	22500
C	900 μF	P_{r1}	100 W	$\lambda_{3,1}$	300
T	50 μs	P_{r2}	40 W	λ_4	2
P_l	100 W	P_{r3}	200 W	F_{cut}	1.5 kHz

Fig. 3a depicts the measured bus voltage $V_{C_{av}}$, and demonstrates that flatness-based control driven by a PLCL ensures precise tracking of a given reference constant trajectory V_{dC}. Moreover, its response does not exceed a 7.3% overshoot during the load variations. In contrast, when there is no PLCL, the DC-bus voltage tracks the reference voltage only when the power load is equal to the nominal one (i.e., for $t \in]0.5s, 1s[$ and $t \in]1.5s, 2s[$). In Fig. 3b (top and middle), it can be seen that the measured currents $I_{1_{av}}$ and $I_{2_{av}}$ accurately track their respective reference currents I_{d1} and I_{d2} (which are constant without the flatness-based control driven by the PLCL, and are load-dependent when we add the PLCL). Fig. 3b (bottom) highlights that the measured current $I_{3_{av}}$ is close to zero only at $t = 1.01s$, indicating that the third boost chopper is close to discontinuous mode. Lower is the load power demand, closer to zero is the current $I_{3_{av}}$. Fig. 3c shows that the measured energy E_t follows the desired flat output trajectory E_{dt} (which, similarly to I_{d1} and I_{d2}, is constant without the flatness-based control driven by the PLCL, and load-dependent when the PLCL is added). The above results highlight that the desired flat output trajectories are well followed in both cases (with and without the PLCL) and to have a well bus tracking objective, i.e., $V_{Cav} = 100V$, the flatness-based control driven by the power control loop is necessary.

Future work will consider experimental studies, discontinuous operation mode, boost choppers with different switching frequencies, and the inclusion of energy storage elements such as batteries or super-capacitors.

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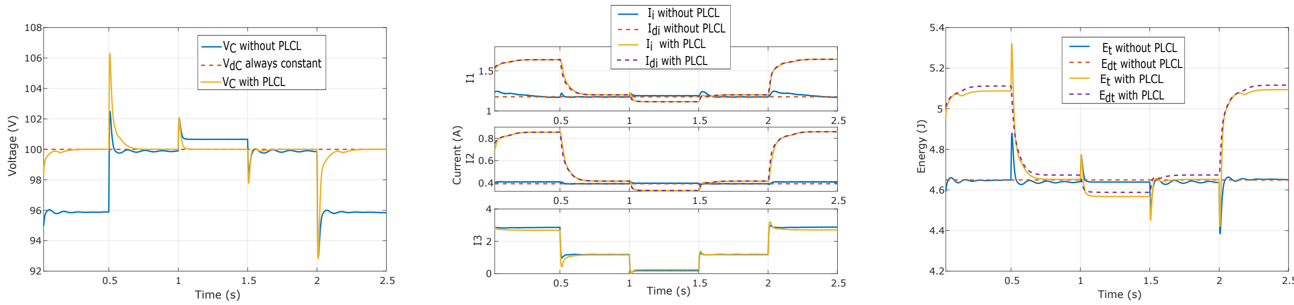

(a) Average bus voltage.

(b) Average currents.

(c) Average global energy.

Fig. 3: Simulation results.

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