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Ancillary services provision via aggregation: Joint power flexibility assessment and disaggregation policy design*

Daniel Zamudio[1], Alessandro Falsone[1] *Member, IEEE*,
Federico Bianchi[2] *Member, IEEE*, Maria Prandini[1] *Fellow, IEEE*

***Abstract*— We address the problem of assessing the power flexibility that a pool of prosumers equipped with a generalized storage device can offer to the electrical grid as an ancillary service for balancing power demand and generation. A key feature of the proposed approach is that the disaggregation policy is computed jointly with the aggregate flexibility set, and it is hence readily available for the pool to supply any (feasible) power profile request from the grid. Each prosumer is assumed to provide a contribution which is an affine function of the aggregated power profile. The coefficients of the affine policies are designed by solving a distributed optimization program where the volume of the aggregate flexibility set is maximized while satisfying the power and energy constraints of each storage device and additional constraints involving multiple (possibly all) devices. Simulation results show the superiority of the proposed approach with respect to a state-of-the-art method that inspired our work.**

I. Introduction

The energy sector is facing a transition due to the high penetration of non-programmable renewable energy resources, such as wind and solar power, and the increasing consumption due to the constant electrification of facilities, houses, and vehicles. This transition puts at risk the grid stability by making it more challenging to balance demand and generation. Traditional large-scale, inertia-based facilities, such as gas and pumped-hydro storage power plants, will not be able to offer enough flexibility to the grid to guarantee a safe and reliable operation. Fortunately, the energy sector transition is accompanied by a modernization of the electrical grid, which enables the provision of flexibility (i.e., the capability of adjusting the electric energy exchanged with the grid according to some external signal) through direct involvement of the prosumers, which can be grouped together in a pool by an aggregator to support the grid via explicit demand-response. This ultimately calls for computationally efficient methods to assess the flexibility that the resulting aggregate can offer to the grid, which motivated a significant effort in the literature, including the present work.

Assessing flexibility is equivalent to computing the set of all the feasible power trajectories that the resources in the pool, typically modeled as storage systems, [1], can jointly provide along a given time horizon. It is a challenging task because it implicitly involves mapping any (admissible) power request by the grid back into the power exchange profile of each single storage system (disaggregation policy). Moreover, to fully exploit the available flexibility while reducing the complexity of planning, trading, and control by the aggregator and grid operator, the aggregate flexibility of the pool must be computed and represented as a set with a concise and compact description. Considering the possible heterogeneity of the resources in the pool, the sought form has to be as general as possible to represent different units.

From the observation that computing exactly the aggregated flexibility set is generally intractable [2], approximation methods have been proposed in the literature. In particular, methods seeking for an inner approximation of the flexibility set are briefly reviewed next. Interested readers are referred to [3] for a more comprehensive review.

Some approaches approximate the flexibility set of every single unit independently first and then simply aggregate these sets. The key point here is to choose a suitable geometry for the individual sets to ease their aggregation. According to this rationale, [1] adopts zonotopes, a subclass of polytopes with suitably defined and fixed shapes, to approximate a unit's original polytopic flexibility set along some reference time horizon. However, since zonotopic sets are symmetric and, generally, the sets to be approximated are not, the method tends to leave many feasible trajectories outside the approximation. Also, due to the high computational cost to compute the volume of the zonotopic set - ideally, one wants to find the approximation with the largest volume among all the ones inscribed into the original set -, auxiliary cost functions are used, leading to conservative results when the time horizon increases. [4] presents a different union-based approach that uses homothets of hyper-rectangles. The polytopic set to be approximated is decomposed by recursively inscribing maximum volume hyper-rectangles. The idea is to adopt a set for which it is straightforward to compute the volume and repeat the process recursively with the residual sub-polytopes to cover the whole flexibility set progressively. The accuracy of the result improves at each step. However, as pointed out in [3], the method becomes computationally intractable when more than two steps are used, which limits its performance in practice. [5] tries to reduce the approximation error due to geometrical mismatches by using homothets of a prototype polytope suitably defined to be more flexible than zonotopes and hyper-rectangles. The formulation maximizes the dilation coefficient of the ho-

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[1]Department of Electronics, Information and Bioengineering. Politecnico di Milano, Italy. Email: {name.surname}@polimi.it

[2]Ricerca sul Sistema Energetico - RSE S.p.A. Email: federico.bianchi@rse-web.it

mothets, which indirectly maximizes the volume. However, in all these approaches, conservatism increases substantially when heterogeneity increases due to the propagation of the individual approximation errors towards the aggregate set. Additionally, aggregation of power flexibility is typically limited by constraints over the distribution network, which these methods cannot handle because they approximate the units independently. Disregarding these constraints causes an over-estimation of the aggregated flexibility and, hence, infeasibility.

Other approaches try to overcome these limitations by looking for an approximation of the aggregated set directly. [6] and [7] reformulate the involved sum operation as a set projection. The aggregate flexibility is hence considered as the projection of a higher dimensional polytope onto the subspace representing the aggregate power of the units. Therefore, instead of approximating the sum directly by its definition, authors approximate the associated projection operation with respect to the homothets of a given polytope. This problem is solved by means of a robust optimization problem, which, however, introduces conservatism to the solution. The idea has been recently extended in [8] and [9] to reduce conservativeness. However, all these methods use several reformulations of the original optimization problem, often introducing approximations to recover tractability. An interesting approach to approximate the aggregated flexibility is the one proposed in [10], where the aggregated flexibility set is described through an equivalent battery model, whose parameters are determined by assuming a disaggregation policy which is linear as a function of the power profile requested by the grid. Later in [11], an expansion method of the set resulting from [10] is proposed while also accounting for network constraints.

Inspired by [10], in this paper the disaggregation policy structure is fixed a priori and its parameters are chosen so as to maximize the volume of the aggregate flexibility set directly. Differently from [10], where some restrictive assumptions are imposed on the storage-like devices that are aggregated, our framework is more general since it allows to account for time-varying power and energy constraints, as well as for an arbitrary initial energy content. Indeed, a specific initialization cannot be guaranteed, in general, and it would fail to be met as soon as the offered flexibility is actually exploited. Also, we assume an affine disaggregation policy (as opposed to a linear one) and take a box to model the aggregate flexibility set, so as to comply with the requirements of the energy service market, where the offered flexibility has to be given in terms of (constant) downward and upward power made available along some reference time horizon. We show that the problem of maximizing the volume of the aggregate flexibility box while accounting for global constraints related to, e.g., minimum levels of upward and downward services and network constraints, is convex and has a multi-agent constraint-coupled structure. It can then be solved via a distributed scheme, which allows to cope with the growth of the computational effort as the population size increases, while guaranteeing information privacy. The latter is a key feature since, typically, prosumers are willing to offer flexibility but not to disclose their private information encoded in their local constraints.

II. Problem Setting and Background

In this section, we first formalize the addressed problem and then briefly recall the approach in [10], which inspired our methodology.

A. Problem Formulation

Consider a pool of N prosumers, indexed by i, $i = 1, \dots, N$, and a time-horizon composed of M time-slots, each one of duration τ. Each prosumer is equipped with an energy storage device, which can be a battery, a thermostatically controlled load, or another type of load, see [7]. Nominal operating conditions will correspond to a certain usage of the storage device, thus possibly making the residual capacity and power that are available for the flexibility service time-varying quantities.

In each time-slot k, $k = 0, \dots, M-1$, prosumer i can vary its baseline power profile by an amount $p_i(k)$ to absorb ($p_i(k) > 0$) or supply ($p_i(k) < 0$) some constant power. The power $p_i(k)$ exchanged within the k-th time-slot must satisfy

$$l_i(k) \le p_i(k) \le u_i(k), \quad k = 0, \dots, M-1, \tag{1}$$

to comply with the prosumer's (possibly time-varying) power limitations. Denote as $e_i(k)$ the energy content of the storage device with respect to the baseline content at the beginning of time-slot k. Then, the evolution of $e_i(k)$ is described by the recursive equation

$$e_i(k+1) = \zeta_i \, e_i(k) + \tau \, p_i(k), \quad k = 0, \dots, M-1, \tag{2}$$

where the self-discharge coefficient $\zeta_i \in (0, 1]$ models storage energy losses in a time-slot and $e_i(M)$ denotes the storage energy content at the end of time-slot $M-1$. The energy quantity that can be stored into or retrieved from the storage is constrained to be within a (possibly time-varying) minimum $e_i^{\min}(k)$ and a maximum $e_i^{\max}(k)$ values, i.e.,

$$e_i^{\min}(k) \le e_i(k) \le e_i^{\max}(k), \quad k = 1, \dots, M. \tag{3}$$

By collecting the time-evolution of the introduced quantities of the i-th prosumer into the following vectors

$$\begin{aligned} p_i &= [p_i(0) \ \cdots \ p_i(M-1)]^\top, \\ l_i &= [l_i(0) \ \cdots \ l_i(M-1)]^\top, \\ u_i &= [u_i(0) \ \cdots \ u_i(M-1)]^\top, \\ e_i &= [e_i(1) \ \cdots \ e_i(M)]^\top, \\ e_i^{\min} &= [e_i^{\min}(1) \ \cdots \ e_i^{\min}(M)]^\top, \\ e_i^{\max} &= [e_i^{\max}(1) \ \cdots \ e_i^{\max}(M)]^\top, \end{aligned}$$

and unrolling the recursive equation (2), constraints (1) and (3) can be compactly rewritten as

$$l_i \le p_i \le u_i \ \wedge \ e_i^{\min} \le A_i e_i(0) + B_i p_i \le e_i^{\max},$$

where

$$A_i = \begin{bmatrix} \zeta_i \\ \vdots \\ \zeta_i^M \end{bmatrix}, \qquad B_i = \tau \begin{bmatrix} 1 & 0 & \dots & 0 \\ \zeta_i & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \zeta_i^{M-1} & \zeta_i^{M-2} & \dots & 1 \end{bmatrix}.$$

By further defining $\underline{e}_i = e_i^{\min} - A_i e_i(0)$ and $\overline{e}_i = e_i^{\max} - A_i e_i(0)$, the set of all power trajectories p_i that the i-th prosumer can exchange on top of its nominal operation is given by

$$\mathcal{P}_i = \{p_i \in \mathbb{R}^M : \; l_i \le p_i \le u_i \wedge \underline{e}_i \le B_i p_i \le \overline{e}_i\}. \quad (4)$$

Set $\mathcal{P}_i$ describes the flexibility that prosumer i can provide to the grid and belongs to a class of polytopes known in the literature as PE-polytopes [1] (or resource polytopes [2]), as they are defined by Power constraints (i.e., $l_i \le p_i \le u_i$) and Energy constraints (i.e., $\underline{e}_i \le B_i p_i \le \overline{e}_i$).

Since the power profile $p = [p(0) \; \cdots \; p(M-1)]^\top$ that the whole pool can absorb/supply is given by

$$p = \sum_{i=1}^{N} p_i, \quad (5)$$

then, the flexibility set of the prosumer pool can be expressed as

$$\mathcal{P} = \left\{ p = \sum_{i=1}^{N} p_i : \; p_i \in \mathcal{P}_i, \; i = 1, \dots, N \right\}. \quad (6)$$

Note that the exact description of $\mathcal{P}$ in (6) as a polytope given by the intersection of multiple half-planes is hard to use for assessing the flexibility offered to the grid. In fact, relevant properties like, e.g., the minimum upward and downward power that the pool of storage devices can provide to the grid, are not readily available. Much effort has been spent in the literature to find a simpler and more explicit inner approximation of $\mathcal{P}$, which is easy to compute also when the number of prosumers is large. This is also the objective of the present work, where a distributed scheme for computing a hyper-box inner approximation – together with the local prosumer charge/discharge policy – is proposed.

Starting from the observation that $\mathcal{P}$ can be computed as

$$\mathcal{P} = \mathcal{P}_1 \oplus \cdots \oplus \mathcal{P}_N,$$

where $\oplus$ denotes the Minkowski sum between sets, some approaches in the literature exploit an inner set representation of the local flexibility sets in the form of axis-aligned boxes or zonotopes with specific generators aiming at reproducing the PE-polytopes shape to ease the Minkowski sum inner-approximation, see, e.g., [4], [1]. Besides the conservativeness of the resulting flexibility set, a further step for disaggregating the grid power request is needed. Motivated by these observations in [10], a different approach is proposed where the aggregated flexibility set is directly inner-approximated while designing the disaggregation strategy.

B. The Generalized Battery Model (GBM) Approach

The authors of [10] impose the following simplifying assumptions on each prosumer i, $i = 1, \dots, N$,

$$l_i(k) = L_i \le 0 \qquad u_i(k) = U_i \ge 0 \quad (7a)$$

$$e_i^{\min}(k) = -C_i \le 0 \qquad e_i^{\max}(k) = C_i \ge 0 \quad (7b)$$

$$e_i(0) = 0 \quad (7c)$$

and try to inner-approximate the overall flexibility set $\mathcal{P}$ with a PE-polytope $\tilde{\mathcal{P}}$ defined as

$$\tilde{\mathcal{P}} = \{p \in \mathbb{R}^M : \; L\mathbf{1} \le p \le U\mathbf{1} \wedge -C\mathbf{1} \le \tilde{B}p \le C\mathbf{1}\}, \quad (8)$$

where $L \le 0$, $U \ge 0$, and $C \ge 0$ are three scalar parameters to be determined so that $\tilde{\mathcal{P}} \subseteq \mathcal{P}$, while $\mathbf{1}$ is the all-one vector in $\mathbb{R}^M$ and

$$\tilde{B} = \tau \begin{bmatrix} 1 & 0 & \dots & 0 \\ \zeta & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \zeta^{M-1} & \zeta^{M-2} & \dots & 1 \end{bmatrix},$$

with $\zeta \in (0, 1]$ a-priori chosen based on the values of $\zeta_1, \dots, \zeta_N$, e.g., set equal to $\zeta = \frac{1}{N}\sum_{i=1}^{N} \zeta_i$. The assumption underlying the choice of $\tilde{\mathcal{P}}$ in (8) is that the prosumer pool can be regarded as an equivalent battery model (hence the name of the approach) $e(k+1) = \zeta\, e(k) + \tau\, p(k)$ with $e(0) = 0$ and constraints

$$L \le p(k) \le U \;\wedge\; |e(k)| \le C, \quad k = 0, \dots, M-1.$$

In order to ensure that $\tilde{\mathcal{P}} \subseteq \mathcal{P}$, for any $p \in \tilde{\mathcal{P}}$, there must exist $p_1, \dots, p_N$, with $p_i \in \mathcal{P}_i$, $i = 1, \dots, N$, such that $\sum_{i=1}^{N} p_i = p$. To enforce this condition, the authors of [10] propose to parameterize each prosumer's power profile p_i as follows

$$p_i(k) = \beta_i p(k), \quad k = 0, \dots, M-1, \quad (9)$$

with $\beta_i \ge 0$ and $\sum_{i=1}^{N} \beta_i = 1$, and then impose $\beta_i p = p_i \in \mathcal{P}_i$ by requiring, for all $i = 1, \dots, N$,

$$L_i\mathbf{1} \le \beta_i p \le U_i\mathbf{1} \;\wedge\; -C_i\mathbf{1} \le \beta_i B_i p \le C_i\mathbf{1}, \; \forall p \in \tilde{\mathcal{P}}, \quad (10)$$

which, according to [10], is satisfied if, for all $i = 1, \dots, N$,

$$\beta_i C \le \frac{C_i}{\Phi_i} \;\wedge\; \beta_i L \ge L_i \;\wedge\; \beta_i U \le U_i \quad (11)$$

with $\Phi_i = 1 + |(\zeta - \zeta_i)/\zeta_i|$, $\beta_i \ge 0$, and $\sum_{i=1}^{N} \beta_i = 1$. While there are several combinations of the free parameters satisfying (11), the authors of [10] provide, among other alternatives, the following explicit formulas for maximizing the capacity (which is related to flexibility) of the equivalent battery model:

$$C = \sum_{i=1}^{N} \frac{C_i}{\Phi_i}, \; \beta_i = \frac{C_i}{\Phi_i C}, \; L = \max_i \frac{L_i}{\beta_i}, \; U = \min_i \frac{U_i}{\beta_i}. \quad (12)$$

Although inspiring, this approach has three major shortcomings: i) it assumes time-independent power and energy bounds (cf. (7a) and (7b)), which excludes those scenarios in which the baseline power and energy profiles are not constant, ii) it assumes symmetric energy bounds for each prosumer (cf. (7b)), and iii) it assumes a zero initial condition (cf. (7c)).

Regarding the third shortcoming, given that the amount of energy that the storage device can absorb or deliver (depending on the sign of $e_i(0)$) is reduced by an amount equal to $|e_i(0)|$, zero initial conditions can be recovered while preserving symmetric energy bounds by reducing the capacity boundary parameter C_i in (10) of an amount $|e_i(0)|$ and, hence, using $C_i - |e_i(0)|$ in place of C_i in (12).

III. Proposed Methodology

In this section, we build on [10] to propose a new approach to approximate the overall flexibility set $\mathcal{P}$. Specifically, instead of using the linear disaggregation policy (9), we parameterize each prosumer's power profile with the affine map

$$p_i(k) = \beta_i p(k) + \alpha_i, \quad k = 0, \ldots, M-1, \tag{13}$$

and, instead of inner-approximating $\mathcal{P}$ with a PE-polytope, we inner-approximate it with a box

$$\mathcal{B} = \{p \in \mathbb{R}^M : \ c\mathbf{1} - d\mathbf{1} \le p \le c\mathbf{1} + d\mathbf{1}\}, \tag{14}$$

where $c \in \mathbb{R}$ affects the center of the box and $d \in \mathbb{R}$ is half-length of the cube edge.

The introduction of the affine term in (13) is to add a further degree of freedom to the policy proposed in [10], enabling a net-zero (see (15)) energy exchange among prosumers, while the choice of a box in place of a PE-polytope is motivated by market requirements. As explained next, the use of a box for the inner approximation enables us to find the values of the design parameters c, d, β_i, and α_i, $i = 1, \ldots, N$, that maximize the volume of $\mathcal{B}$ while ensuring $\mathcal{B} \subseteq \mathcal{P}$ via a simple convex optimization problem, without imposing any of the assumptions in (7).

Clearly, (5) must hold, therefore, given (13), one has

$$p = \sum_{i=1}^N p_i = p\sum_{i=1}^N \beta_i + \mathbf{1}\sum_{i=1}^N \alpha_i,$$

for any $p \in \mathcal{B}$, which can be satisfied if and only if

$$\sum_{i=1}^N \beta_i = 1 \ \wedge \ \sum_{i=1}^N \alpha_i = 0. \tag{15}$$

On the other hand, for all $i = 1, \ldots, N$, we must ensure $p_i \in \mathcal{P}_i$, which, using (13) in (4), translates into

$$l_i \le \beta_i p + \alpha_i \mathbf{1} \le u_i, \tag{16a}$$

$$\underline{e}_i \le B_i\beta_i p + \alpha_i B_i \mathbf{1} \le \overline{e}_i. \tag{16b}$$

Since (16) must hold for all $p \in \mathcal{B}$ we have

$$\begin{aligned} l_i &\le \min_{p\in\mathcal{B}} \beta_i p + \alpha_i\mathbf{1}, \\ &\max_{p\in\mathcal{B}} \beta_i p + \alpha_i \mathbf{1} \le u_i, \end{aligned} \tag{17a}$$

$$\begin{aligned} \underline{e}_i &\le \min_{p\in\mathcal{B}} B_i\beta_i p + \alpha_i B_i\mathbf{1}, \\ &\max_{p\in\mathcal{B}} B_i\beta_i p + \alpha_i B_i \mathbf{1} \le \overline{e}_i, \end{aligned} \tag{17b}$$

which can be equivalently posed as

$$\begin{aligned} l_i &\le \beta_i c\mathbf{1} - |\beta_i d|\mathbf{1} + \alpha_i\mathbf{1}, \\ &\beta_i c\mathbf{1} + |\beta_i d|\mathbf{1} + \alpha_i\mathbf{1} \le u_i, \end{aligned} \tag{18a}$$

$$\begin{aligned} \underline{e}_i &\le B_i\beta_i c\mathbf{1} - |B_i\beta_i d|\mathbf{1} + \alpha_i B_i\mathbf{1}, \\ &B_i\beta_i c\mathbf{1} + |B_i\beta_i d|\mathbf{1} + \alpha_i B_i\mathbf{1} \le \overline{e}_i, \end{aligned} \tag{18b}$$

where the minimum and maximum operators with vector arguments and the absolute value of vectors and matrices must be intended component-wise. Unfortunately, constraints (18) contain the products between β_i and c and between β_i and d, which are all decision variables, thus rendering (18) non-convex. However, considering the following change of variables

$$\delta_i = \beta_i d, \tag{19a}$$

$$\mu_i = \beta_i c + \alpha_i, \tag{19b}$$

enables to reformulate (18) as

$$\begin{aligned} l_i &\le \mu_i\mathbf{1} - |\delta_i|\mathbf{1}, \\ &\mu_i\mathbf{1} + |\delta_i|\mathbf{1} \le u_i, \end{aligned} \tag{20a}$$

$$\begin{aligned} \underline{e}_i &\le \mu_i B_i\mathbf{1} - |\delta_i B_i|\mathbf{1}, \\ &\mu_i B_i\mathbf{1} + |\delta_i B_i|\mathbf{1} \le \overline{e}_i, \end{aligned} \tag{20b}$$

which is now convex in μ_i and δ_i. The box parameters can be easily recovered as

$$d = \underbrace{\sum_{i=1}^N \beta_i}_{1} d = \sum_{i=1}^N \delta_i, \tag{21a}$$

$$c = \underbrace{\sum_{i=1}^N \beta_i}_{1} c + \underbrace{\sum_{i=1}^N \alpha_i}_{0} = \sum_{i=1}^N \mu_i, \tag{21b}$$

and (if the feasibility set has non-zero volume) the policy parameters can be computed as

$$\beta_i = \frac{\delta_i}{d} \ \text{ and } \ \alpha_i = \mu_i - \beta_i c. \tag{22}$$

Additional relevant constraints can also be included in the problem formulation like, e.g.,

i) the constraint

$$-\sum_{i=1}^N \delta_i \le \sum_{i=1}^N \mu_i \le \sum_{i=1}^N \delta_i \tag{23}$$

to ensure $0 \in \mathcal{B}$, so that a zero request (i.e., no deviation from the baseline profile) can be accommodated,

ii) constraints on the minimum levels of downward ($u^{\min}$) and upward ($l^{\min}$) services requested by the grid

$$-\sum_{i=1}^N \delta_i + u^{\min} \le \sum_{i=1}^N \mu_i \le \sum_{i=1}^N \delta_i + l^{\min}, \tag{24}$$

iii) network constraints of the form

$$\sum_{i\in\mathcal{N}_\ell} \mu_i + \sum_{i\in\mathcal{N}_\ell} \delta_i \le u_\ell^{\max}, \tag{25a}$$

$$l_\ell^{\max} \le \sum_{i\in\mathcal{N}_\ell} \mu_i - \sum_{i\in\mathcal{N}_\ell} \delta_i, \tag{25b}$$

where the power injected by a set $\mathcal{N}_\ell$ of neighboring prosumers in a point ℓ of the grid is subject to congestion constraints.

Finally, since we are inner-approximating the overall flexibility set $\mathcal{P}$ with an M-dimensional cube, maximizing its volume is equivalent to maximizing $d = \sum_{i=1}^N \delta_i$. We can thus find the largest-volume box $\mathcal{B} \subseteq \mathcal{P}$ by solving the following convex optimization program

$$\begin{aligned} \max_{\{\delta_i,\mu_i\}_{i=1}^N} \quad & \sum_{i=1}^N \delta_i \\ \text{subject to:} \quad & l_i \leq \mu_i \mathbf{1} - |\delta_i| \mathbf{1}, \\ & \mu_i \mathbf{1} + |\delta_i| \mathbf{1} \leq u_i, \\ & \underline{e}_i \leq \mu_i B_i \mathbf{1} - |\delta_i B_i| \mathbf{1}, \\ & \mu_i B_i \mathbf{1} + |\delta_i B_i| \mathbf{1} \leq \overline{e}_i, \\ & i = 1, \dots, N, \\ & (23),\ (24),\ (25)\ \forall \ell, \end{aligned} \tag{26}$$

and recovering the box and the policy with (21) and (22).

Note that $d = \sum_{i=1}^N \delta_i$ obtained by solving (26) will necessarily be non-negative because the δ_i's appear in the constraints through their absolute value, and their sum is maximized. Indeed, each single δ_i will be non-negative and, hence, β_i recovered in (22) will also be non-negative, a condition that was enforced a priori in [10] and here is instead an outcome of the optimization.

A slightly different cost function can be used to promote some solutions over others, but still preserving convexity. For example, among all the different boxes with the same volume, one may be interested in finding the one with the center close to zero, which can be easily achieved by using

$$\sum_{i=1}^N \delta_i - \varepsilon \Big\| \sum_{i=1}^N \mu_i \Big\|_2^2$$

where $\varepsilon > 0$ is a sufficiently small coefficient to ensure that the primary objective is still maximizing the volume of $\mathcal{B}$.

Note that the optimization problem (26) is coupled due to the constraints (23)-(25). Yet, it is characterized by a constraint-coupled multi-agent structure (that can also be recovered in the case when the penalization term $\| \sum_{i=1}^N \mu_i \|_2^2$ is introduced by adding an auxiliary decision variable for the aggregator to upper bound it and treating the aggregator as a further agent), and it can then be solved by applying distributed optimization schemes like [12], for achieving scalability and preserving privacy.

If the constraints (23), (24), (25) and the penalization term over μ_i are not of interest, then (26) has a separable structure and each prosumer can solve the local maximization problem

$$\begin{aligned} \max_{\delta_i,\mu_i} \quad & \delta_i \\ \text{subject to:} \quad & l_i \leq \mu_i \mathbf{1} - |\delta_i| \mathbf{1}, \\ & \mu_i \mathbf{1} + |\delta_i| \mathbf{1} \leq u_i, \\ & \underline{e}_i \leq \mu_i B_i \mathbf{1} - |\delta_i B_i| \mathbf{1}, \\ & \mu_i B_i \mathbf{1} + |\delta_i B_i| \mathbf{1} \leq \overline{e}_i, \end{aligned} \tag{27}$$

and then coordinate with the others to recover the box parameters and the policy using (21) and (22). For instance, one of them or an external entity could act as an aggregator, and collect all the δ_i, μ_i solutions, compute the center parameter c and half-size d of the box, and then send them to all the prosumers so that each one can derive its own policy parameters without sharing its local information (loss coefficient, energy and power bounds, initial energy content).

IV. Simulation Results

In this section, we compare the performance of the proposed method and the GBM approach in [10]. To this aim, we consider a pool of $N = 50$ prosumers, each one equipped with a storage device that satisfies the simplifying assumptions (7a) and (7b) adopted in [10], with the lower and upper bounds on the power exchange in (7a) taken with the same absolute value ($L_i = -U_i$, $i = 1, \dots, N$) and the time-slot duration $\tau = 1$. In order to generate the population of prosumers, we extract at random the parameters C_i, U_i, ζ_i from a uniform distribution $\mathcal{U}(I)$ over some interval I, respectively set equal to $[8, 12]$, $[5.5, 7.5]$, and $[0.6, 1]$.

Given the extracted population, we then compute the flexibility sets according to the two approaches. We consider $M = 2, \dots, 7$ as values for the number M of time-slots, while we maintain the time slot duration constant and equal to τ, with an initial condition $e_i(0)$ extracted according to $\mathcal{U}(\gamma[-C_i, C_i])$ with $\gamma = 0, 0.2, 0.4, \dots, 1$, so as to impose an initial state dispersion around zero of $\gamma \cdot 100\%$.

For each pair $(M, \gamma \cdot 100)$, Figure 1 reports the ratio

$$V_{rel} = \frac{V_{GBM}}{V_{box}} \tag{28}$$

where V_{GBM} and V_{box} are the volumes of the M-dimensional flexibility sets obtained with the GBM approach in [10] and our method, respectively. Volume V_{GBM} has been computed using the COntinuous Reachability Analysis (CORA) Toolbox, [13]. Darker colors correspond to values of V_{rel} close to zero (meaning that the flexibility set provided by our approach is much larger than the one returned by [10]), while lighter colors correspond to higher ratios, for which the GBM approach is more effective.

Note that V_{rel} is strictly smaller than 1 in most instances. Also, V_{rel} takes smaller and smaller values as γ grows, which indicates that the approximation adopted in [10] is more conservative as the dispersion in the initial condition increases. For a dispersion larger than 20%, V_{rel} decreases as the number M of time slots increases.

Just to appreciate the level of conservatism introduced in the GBM approach by a growth in the initial condition dispersion $\gamma \cdot 100\%$ within the capacity range $-[C_i, C_i]$, we consider $M = 2$ and report the flexibility sets for $\gamma \in \{0, 0.4\}$ in Figure 2a ($\gamma = 0$) and 2b ($\gamma = 0.4$). Note that our method is more robust against the dispersion of the initial condition. In fact, in both scenarios, the box reaches the boundaries of the exact aggregated flexibility set and, thus, the error of the approximation is mainly given by geometric mismatches, whereas the region obtained using [10], despite

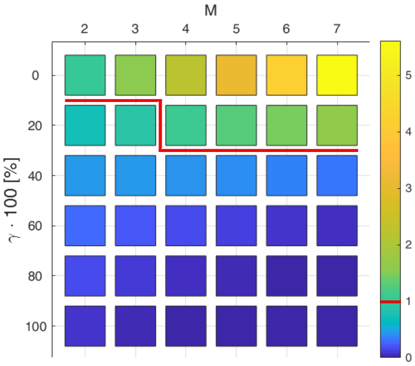
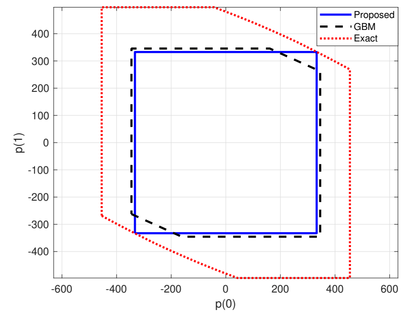
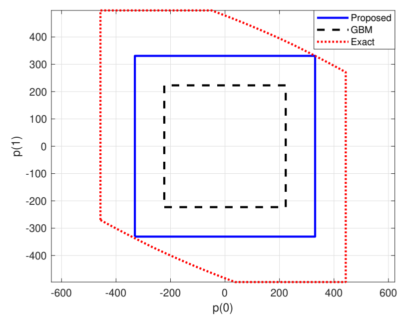


Fig. 1: Volume ratio as a function of the number M of time-slots and the dispersion $\gamma \cdot 100\%$ of the initial state. The red line separates the $(M, \gamma \cdot 100\%)$ pairs for which the volume ratio is larger than 1 from those where it is smaller than 1.



(a) Flexibility sets for $\gamma = 0$.



(b) Flexibility sets for $\gamma = 0.4$.

Fig. 2: Comparison of the GBM method and our method: corresponding flexibility sets against the exact one ($M = 2$).

having the potential of adopting the "right" shape, has a smaller volume when $\gamma = 0.4$.

Note that, when $\gamma = 0$, the approach in [10] performs better than ours ($V_{rel} > 1$) because of its PE-polytopic shape. In this respect we are favoring [10] in the comparison since the PE-polytope obtained by [10] should be inner-approximated by a box to comply with the energy service market requirements. If we compare the volume of our box and the volume of the box contained in the PE-polytope of [10], then we are always better by construction, since our box is the one which maximizes the volume.

V. Conclusion

We propose a new method to assess the power flexibility of a population of storage systems, which rests on the adoption of an affine disaggregation policy and on the approximation of the flexibility set with a box. Notably, the resulting optimization problem for determining the box and the coefficients of the disaggregation policy is amenable for a distributed implementation. Numerical results reveal that our method has a better performance than an inspiring contribution in the literature as the time horizon length and the dispersion of the initial condition grow. Future work will focus on further improving the results by extending the degrees of freedom of the affine policy. Note that the proposed method was derived referring to resources that are described as PE-polytopes. Resources modeled as general convex polytopes (see [7]) could also be considered, whereas the non-convex case remains open.

References

[1] F. L. Mller, J. Szab, O. Sundstrm, and J. Lygeros, "Aggregation and disaggregation of energetic flexibility from distributed energy resources," *IEEE Trans. on Smart Grid*, vol. 10, pp. 1205–1214, 2019.

[2] K. Trangbaek and J. Bendtsen, "Exact constraint aggregation with applications to smart grids and resource distribution," in *IEEE Conf. on Decision and Control*, 2012, pp. 4181–4186.

[3] E. ztrk, K. Rheinberger, T. Faulwasser, K. Worthmann, and M. Preiinger, "Aggregation of demand-side flexibilities: A comparative study of approximation algorithms," *Energies*, vol. 15, no. 7, 2022.

[4] M. S. Nazir, I. A. Hiskens, A. Bernstein, and E. Dall'Anese, "Inner approximation of minkowski sums: A union-based approach and applications to aggregated energy resources," in *IEEE Conf. on Decision and Control*, 2018, pp. 5708–5715.

[5] L. Zhao, W. Zhang, H. Hao, and K. Kalsi, "A geometric approach to aggregate flexibility modeling of thermostatically controlled loads," *IEEE Trans. on Power Systems*, vol. 32, pp. 4721–4731, 2017.

[6] L. Zhao, H. Hao, and W. Zhang, "Extracting flexibility of heterogeneous deferrable loads via polytopic projection approximation," in *IEEE Conf. on Decision and Control*, 2016, pp. 6651–6656.

[7] S. Barot and J. A. Taylor, "A concise, approximate representation of a collection of loads described by polytopes," *International Journal of Electrical Power and Energy Systems*, vol. 84, pp. 55–63, 2017.

[8] X. Chen and N. Li, "Leveraging two-stage adaptive robust optimization for power flexibility aggregation," *IEEE Trans. on Smart Grid*, vol. 12, pp. 3954–3965, 2021.

[9] B. Cui, A. Zamzam, and A. Bernstein, "Network-cognizant time-coupled aggregate flexibility of distribution systems under uncertainties," *IEEE Control Systems Letters*, vol. 5, pp. 1723–1728, 2021.

[10] H. Hao, B. M. Sanandaji, K. Poolla, and T. L. Vincent, "Aggregate flexibility of thermostatically controlled loads," *IEEE Trans. on Power Systems*, vol. 30, pp. 189–198, 2015.

[11] G. Wang, Z. Li, and F. Wang, "Enhanced sufficient battery model for aggregate flexibility of thermostatically controlled loads considering coupling constraints," *IEEE Trans. on Sustainable Energy*, vol. 12, pp. 2493–2496, 2021.

[12] A. Falsone, I. Notarnicola, G. Notarstefano, and M. Prandini, "Tracking-ADMM for distributed constraint-coupled optimization," *Automatica*, vol. 117, p. 108962, 2020.

[13] M. Althoff, "An introduction to CORA 2015." [Online]. Available: http://systems.cs.colorado.edu/research/cyberphysical/taylormodels/