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Adaptive model predictive controller for building thermal dynamics

Seyed Shahabaldin Tohidi[1], Davide Calì[1], and Henrik Madsen[1]

Abstract— **Model predictive controllers are becoming widespread in building thermal dynamic control and energy management systems. Decreasing building energy consumption, load shifting, cost reduction, and indoor air quality improvement are some of the topics that these controllers have been shown to be efficient. However, they rely on accurate models that are hard to develop and can be expensive. Additionally, the model should be time-varying to represent the thermal dynamics in a building. To address this issue, this paper proposes an adaptive model predictive controller for thermal dynamic control in buildings. It includes an adaptive parameter identification algorithm that updates the model parameters and guarantees that the estimated parameters converge to the actual values. Moreover, a model predictive controller with an additional constraint to ensure the boundedness of the system trajectories is introduced. The proposed framework uses a simple linear grey-box model of the thermal dynamics, as a nominal model, and the adaptive parameter identification updates the model. This eliminates the need for an accurate model and an enormous bank of data, while the benefits of the model predictive controller and adaptive controller are retained. Simulation results are also provided to demonstrate the capability to identify the deviations and the efficiency of using the updated model in the model predictive controller design.**

I. INTRODUCTION

heating, i.e. space and water heating, is responsible for around 40% of total final energy consumption in Europe [1]. Efficient management of building energy demand plays a vital role in balancing supply and demand, reducing the costs of extending the power plants, reducing CO_2 emissions, and enhancing indoor thermal comfort [2].

Various strategies are employed for building thermal system control [2]–[5]. Among them are 1) rule-based controllers [6], [7], 2) model-based and optimal controllers [8]–[12], and 3) data-driven machine-learning methods [13]–[15]. Rule-based controllers are suitable for single-input-single-output (SISO) systems, while the other two can be employed in multi-input-multi-output (MIMO) systems relatively easily. Furthermore, guaranteeing optimality cannot be achieved in rule-based methods.

Model predictive controllers (MPCs), which are among model-based and optimal control methods, have been shown to be effective for building energy management in many studies [9], [16]. While non-adaptive MPCs, in their classical form, can overcome the drawbacks of rule-based controllers, they cannot consider uncertainty and dynamic changes. They also rely on accurate models that are hard to develop and can be expensive [16]–[18]. Grey-box models are well-known in building energy management systems since they use prior physical knowledge about the building as well as data. Statistical methods can then be used to select an accurate, but at the same time, low dimensional model of the system appropriate for control purposes [19]–[22].

Machine learning control approaches are capable of dealing with modeling problems using data-driven approaches. Also, using deep reinforcement learning methodologies, they can adapt the control mechanisms once a dynamic change occurs [14], [23]. However, they need an enormous bank of data, high computational power, and a model to learn control policies.

The need for an optimal and adaptive control strategy that does not rely on large data sets and expensive modeling procedures leads to adaptive MPC approaches. The capability of these methods is integrating adaptive parameter estimation with MPC. A discrete adaptive generalized predictive control for a heating, ventilating, and air conditioning (HVAC) system is introduced in [24], [25]. In this paper, parameter adaptation is achieved using the recursive least squares estimator. Stability analysis of the adaptive generalized predictive control is provided in [26], [27]. In [28], a learning-based MPC with a discrete-time model is used to learn and compensate for the heating demand of an HVAC system. An adaptive MPC with a simplified moving-horizon estimation for parameter estimation is proposed in [29].

In this paper, we propose an adaptive MPC to generate optimal control signals while considering the system parameter deviation and constraints using a linear grey-box model of the thermal dynamics. Unlike the method in [29], where the nonlinear structure of the true system is assumed to be known, we considered the true system to have a linear form with an additional nonlinear term. Thus, manual derivation of a first-principles model beforehand and the need for extensive data collection are no longer necessary. In particular, we focus on building thermal dynamics, where a validated grey-box model can be obtained using an automated identification mechanism [22]. Furthermore, different from [29], a continuous-time parameter identification method is employed and convergence analysis is provided. Inspired by [29], a similar constraint is added to the MPC formulation to drive the system trajectories toward the origin.

The main contributions of this paper are as follows: 1) The overall framework does not need a complicated model. It utilizes a linear grey-box model that can be generated using short-period data collection and an automated identification method, e.g., the method presented in [22]. Thus, the proposed method saves the cost and time that are generally

[1]Shahab Tohidi, Davide Cali, and Henrik Madsen are with Department of Applied Mathematics and Computer Science, Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark {sshto, dcal, hmad}@dtu.dk

required for modeling. 2) After a model is obtained, the proposed approach can be implemented in various zones with similar, but not necessarily identical, thermal dynamics. The adaptive identification method can identify the true parameters and drive the parameters toward them. In parallel MPC takes care of generating optimal control commands based on the updated model. 3) If true dynamics are deviated due to some fault, the adaptive method identifies it and updates the model accordingly. 4) Convergence of the adaptive identification method and the boundedness of the states in MPC are investigated using Lyapunov stability analysis.

This paper is organized as follows. Section II presents the true system dynamics as well as the model and introduces the problem. In Section III, we design an adaptive parameter identification algorithm with convergence analysis. In Section IV, we present the adaptive MPC with a boundedness analysis. Simulation results are added to demonstrate the effectiveness of the identification method and the adaptive MPC approach. A summary is given in Section V.

II. Problem statement

Consider the true system dynamics

$$\dot{x} = h(x, v, A, B) = Ax + B_1 v + g(x, v), \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $v \in \mathbb{R}^{n_v}$ is the control input, $g(x, v) \in \mathcal{G}$ is the unmodeled dynamics, and h represents the nonlinear dynamics. For simplicity of presentation, we assume that there is no measurement noise and forecast error; however, the results can be extended to the case with measurement noise and forecast error by integrating them in g. As introduced in [21], many thermal systems can be modeled as resistors and capacitors (linear RC models), and statistical methods can be employed to validate and find the best model [20]. It is shown in [21] that more complex models may not necessarily lead to a better model. In this paper, we assume that there exists a linear model that behaves similarly as (1) such that

$$\dot{\bar{x}} = A\bar{x} + B_1\bar{v} + B_2 d = A\bar{x} + B\bar{u}, \tag{2}$$

where $\bar{x}$ is the state of the model, $B = [B_1\ B_2]$, $\bar{u} = [\bar{v}\ d]^\top \in \mathbb{R}^{n_v+n_d}$ is the input vector, consists of control input $\bar{v}$ and bounded disturbance d, $A \in \mathbb{R}^{n\times n}$ is the state matrix and $B \in \mathbb{R}^{n\times(n_v+n_d)}$ is the input matrix. In this setup, A and B are considered unknown. To delineate the variables, we add a bar above x and v for the model. It is noted that $B_2 d$ is not necessarily equal to $g(x, v)$.

Consider the thermal dynamics of a building. One simple model consists of an interior, heater, envelope, and ambient, with two storing elements: T_i as interior temperature and T_e as envelope temperature. RC model of the thermal dynamics is shown in Figure 1. The thermal dynamics can be written in the following state-space representation as

$$\frac{dT_x}{dt} = \begin{bmatrix} \frac{-1}{R_\text{ie}C_\text{i}} & \frac{1}{R_\text{ie}C_\text{i}} \\ \frac{1}{R_\text{ie}C_\text{e}} & -(\frac{1}{R_\text{ea}C_\text{e}} + \frac{1}{R_{ie}C_e}) \end{bmatrix} T_x + \begin{bmatrix} \frac{1}{C_\text{i}} & 0 \\ 0 & \frac{1}{R_\text{ea}C_\text{e}} \end{bmatrix} \begin{bmatrix} \Phi_h \\ T_a \end{bmatrix},$$

where $T_x = [T_\text{i}\ T_\text{e}]^\top$, Φ_h is the heat input, and T_a is the ambient temperature as a disturbance. C_e and C_i are the thermal capacitance of the envelope and interior, respectively. Moreover, R_ea and R_ie are the thermal resistance between the building envelope and the ambient, and between the interior and the building envelope, respectively.

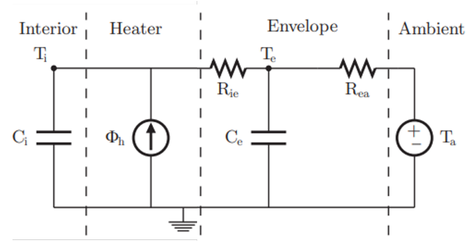

Fig. 1. RC circuit representing a simple model for the heat flows in buildings [21].

Model predictive controllers are able to control thermal and energy systems efficiently, due to their capability to find the optimal solution, constraint consideration, and forecast data usage. Model predictive controllers are based on solving an optimization problem utilizing a mathematical model that represents the dynamics of the real system.

Suppose that the thermal dynamics of a building can be modeled as (2), where A and B are known. Then, the optimization problem can be formulated as

$$\min_{\bar{v}} \int_{t_1}^{t_2} \mathcal{J}(\bar{x}, \bar{v}, d, p)dt \tag{3}$$

$$s.t. \quad \dot{\bar{x}} = A\bar{x} + B\bar{u}, \quad \bar{x}(t_1) = x(t_1), \tag{4}$$

$$\bar{v} \in \mathcal{V}, \tag{5}$$

where $\mathcal{J}$ is a convex function, p is an additional signal, and the control inputs are restricted to the convex set $\mathcal{V} \subset \mathbb{R}^{n_v}$. For example, $\bar{v}$ is the required power for heating ($\bar{v} > 0$) and cooling ($\bar{v} < 0$) of an HVAC system. It is assumed that weather forecasts are available, e.g., for ambient temperature. During each time interval, the optimization problem is formulated as (3)-(5) and can compute the optimal control trajectory that satisfies the constraints. However, only the first element of this trajectory is adopted.

It is seen that the optimization problem (3)-(5) relies on a model of the system. Although, this model is time-invariant in (4), the dynamics of the real system may deviate from it due to aging, faults, human-in-the-loop changes, etc. These changes require to be taken into account so that it does not diminish the controller performance.

An adaptive identification algorithm should be employed to identify the parameters. However, introducing time-varying dynamics to the MPC formulation may cause instability problems. In the sequel, we introduce an adaptive algorithm to collaborate with MPC, leading to an adaptive MPC. A new constraint is also added to the MPC optimization formulation. Moreover, to guarantee that the model variations do not cause problems, a Lyapunov analysis is added.

III. Adaptive parameter identification

In this section, we introduce an adaptive identification algorithm to keep track of the parameters of the system.

Consider that the system can be modeled as (2), where the true values of the elements of A and B matrices are unknown. Also assume that A is stable and state accessible, using some measurement or estimation methods.

Remark 1: The matrix A being stable is a common assumption for the thermal dynamics of buildings. See for example [20], [21]. Also, with more accessibility of the sensors and their price reduction, measuring states seems to be straightforward. Furthermore, the states of the system can be selected based on the measurements of the sensors in hand so that state accessibility is provided.

In the following, we first provide two necessary definitions and then introduce the adaptive parameter identification mechanism for identifying the elements of the unknown matrices A and B. To this end, we use Lyapunov stability analysis to guarantee that the states remain bounded.

Definition 1: The element-wise projection $\mathrm{Proj}(\theta_{i,j}, Y_{i,j})$ is defined as [30], [31]

$$\begin{aligned}&\mathrm{Proj}(\theta_{i,j}, Y_{i,j}) \\ &\equiv \begin{cases} Y_{i,j} - Y_{i,j} f(\theta_{i,j}) & \text{if } f(\theta_{i,j}) > 0 \ \& \ Y_{i,j}(\frac{df}{d\theta_{i,j}}) > 0 \\ Y_{i,j} & \text{otherwise,} \end{cases}\end{aligned} \tag{6}$$

where $f(.)$ is a convex and continuously differentiable function, defined as

$$f(\theta_{i,j}) = \frac{(\theta_{i,j} - \theta_{min_{i,j}} - \zeta_{i,j})(\theta_{i,j} - \theta_{max_{i,j}} + \zeta_{i,j})}{(\theta_{max_{i,j}} - \theta_{min_{i,j}} - \zeta_{i,j})\zeta_{i,j}}, \tag{7}$$

and where $\zeta_{i,j}$ is the projection tolerance. $\theta_{max_{i,j}}$ and $\theta_{min_{i,j}}$ are the upper and lower bound of the $(i,j)^{\text{th}}$ element of θ. This bound form the projection set, $\Omega_{\mathrm{proj}_\theta} = \{\theta : \theta_{i,j} \in [\theta_{min_{i,j}}, \ \theta_{max_{i,j}}]\}$. Also, a measure of the projection set can be provided as $\Delta_\theta = \max_{i,j} |\theta_{max_{i,j}} - \theta_{min_{i,j}}|$.

Definition 2: The signal u is said to be persistently exciting (PE) if for all $t > t_0$ there exists positive constants T and η, such that [32]

$$\int_t^{t+T} \phi(\tau)^\top \phi(\tau) d\tau > \eta I. \tag{8}$$

Theorem 1: Consider the model (2) with a stable state matrix and assume that the pair (A, B) is controllable. Suppose that the parameters of $\hat{A}$ and $\hat{B}$ are updated using the following adaptive laws,

$$\dot{\hat{A}} = \mathrm{Proj}(\hat{A}, \gamma_A e x^\top), \tag{9}$$

$$\dot{\hat{B}} = \mathrm{Proj}(\hat{B}, \gamma_B e u^\top), \tag{10}$$

where x and $u = [v \ d]^\top$ are measured values from the true system (1), γ_A and γ_B are positive scalars, and Proj is defined in Definition 1. If u is bounded, satisfies the conditions of Definition 2 about PE, and each of its elements contains different frequencies, then the estimation error remains bounded. If in addition $\bar{g}(x, v) = g(x, v) - B_2 d = 0$, then $\hat{A}$ and $\hat{B}$ converges to A and B, respectively.

Proof: Consider an arbitrary matrix Γ_m with negative eigenvalues. Add and subtract $\Gamma_m x$ in (2), then it leads to

$$\dot{x} = \Gamma_m x + (A - \Gamma_m)x + Bu + \bar{g}(x, v). \tag{11}$$

The estimation model can be derived as [32]

$$\dot{\hat{x}} = \Gamma_m \hat{x} + (\hat{A}(t) - \Gamma_m)x + \hat{B}(t)u, \tag{12}$$

where $\hat{A}(t)$ and $\hat{B}(t)$ are the estimates of A and B at time t, respectively. Defining the error as $e(t) = x(t) - \hat{x}(t)$ the error dynamics can be obtained as

$$\dot{e} = \Gamma_m e - \tilde{A}(t)x - \tilde{B}(t)u + \bar{g}(x, v), \tag{13}$$

where $\tilde{A}(t) = \hat{A}(t) - A$ and $\tilde{B}(t) = \hat{B}(t) - B$ are deviations of identified parameters from their actual values. Consider the Lyapunov function

$$V = e^T P e + \mathrm{tr}({\gamma_A}^{-1} \tilde{A}^\top P \tilde{A}) + \mathrm{tr}({\gamma_B}^{-1} \tilde{B}^\top P \tilde{B}) \tag{14}$$

where tr(.) is the trace operator and P is a positive definite symmetric matrix solution of the Lyapunov equation $\Gamma_m^\top P + P\Gamma_m = -Q$, where Q is a symmetric positive definite matrix. The time derivative of the Lyapunov function along the trajectories can be obtained as

$$\begin{aligned}\dot{V} = &-e^\top Q e + 2\mathrm{tr}({\gamma_A}^{-1}(\tilde{A}^\top P(\dot{\hat{A}} - \gamma_A e x^\top))) \\ &+ 2\mathrm{tr}({\gamma_B}^{-1}(\tilde{B}^\top P(\dot{\hat{B}} - \gamma_B e u^\top))) + 2e^\top P \bar{g}.\end{aligned} \tag{15}$$

Using the adaptive laws (9) and (10) and using the properties of projection algorithm [30], it implies that $\dot{V} \le -e^\top Q e + 2e^\top P \bar{g} \le -\lambda_{\min}(Q)||e||^2 + 2||e||G\lambda_{\max}(P)$, where $G \ge ||\bar{g}||$, and $\lambda_{\min}(.)$ and $\lambda_{\max}(.)$ provide the minimum and the maximum eigenvalues. Therefore, $\dot{V} < 0$ if $||e|| > (2G\lambda_{\max(P)})/(\lambda_{\min}(Q))$. This implies that the identification error, e, remains bounded.

If $\bar{g} = 0$, it can be shown that $\dot{V} \le -e^\top Q e < 0$. A negative semi-definite Lyapunov function derivative ensures that the error signal e and the adaptive parameters, $\hat{A}$ and $\hat{B}$, are bounded. This also leads to the boundedness of $\tilde{A}$, $\tilde{B}$, and $\dot{e}$. In addition, it can be shown that $\int_0^t e(t)^\top Q e(t) dt \le -\int_0^t \dot{V}(t) dt \le V(0)$ for all $t \ge 0$, which reveals that $||e||_{L_2}^2 \le V(0)/\lambda_{\min}(Q)$, where $\lambda_{\min}(.)$ stands for minimum eigenvalue of a matrix. Therefore, given that $e \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\dot{e} \in \mathcal{L}_\infty$, and using Barbalat's Lemma, $e(t)$ converges to zero as $t \longrightarrow \infty$, that is, $\hat{A}$ and $\hat{B}$ elements converge to that of A and B, respectively. ■

Remark 2: The adaptive parameter identification method is inspired by the method described in [32]. Different from [32], this study analyzes the case when $\bar{g} \neq 0$ and utilizes the projection operator. Using the projection operator in the adaptation laws (9)-(10) guarantees that the adaptive parameters, i.e., the elements of $\hat{A}$ and $\hat{B}$, are bounded, regardless of any stability condition. In [30], a step-by-step approach is suggested for establishing projection boundaries. Also, [31] presents a modified element-wise projection method that ensures stability while bounding the magnitude and rate of change of adaptive parameters.

Remark 3: This paper focuses on building thermal dynamics. These dynamics represent a dissipative system with stable eigenvalues [33]. Therefore, the assumption on the stability of the state matrix in Theorem 1 holds.

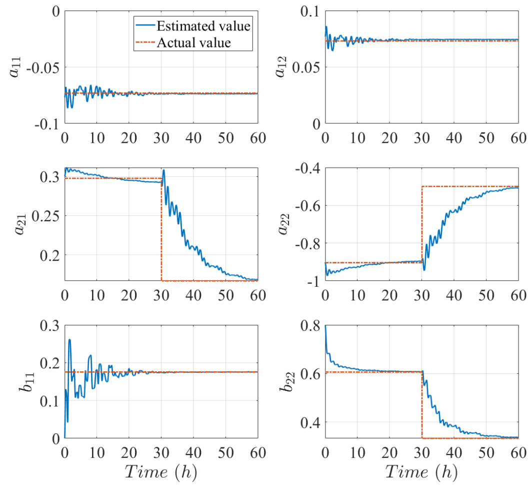

Fig. 2. Identification of the elements of A and B.

A. Simulation results

This section provides a numerical example to demonstrate the adaptive identification algorithm results. Consider the thermal dynamics of a building that can be modeled with two states, like the model previously shown with $C_{\rm e} = 1.4\ kWh/°C$, $C_{\rm i} = 5.7\ kWh/°C$, $R_{\rm ea} = 1.18\ °C/kW$ and $R_{\rm ie} = 2.4\ °C/kW$, that are taken from [34]. With the given parameters, the elements of the state and input matrices are $a_{11} = -0.073$, $a_{12} = 0.073$, $a_{21} = 0.3$, $a_{22} = -0.9$, $b_{11} = 0.17$, $b_{12} = 0$, $b_{21} = 0$, and $b_{22} = 0.6$. Suppose that a landlord wants to add or upgrade the insulation of the outer walls (envelope). This leads to an increase in the envelope resistance and the capacitance between the envelope and the ambient. Specifically, the new values are $R_{\rm ea} = 2\ °C/kW$ and $C_{\rm e} = 2\ kWh/°C$. The process of adding or upgrading the insulation can be completed in a relatively short amount of time compared to the building's thermal dynamics. Therefore, we assume that these changes occur at time $t = 30$ hours and that the parameters of the state and input matrices will change accordingly to $a_{11} = -0.073$, $a_{12} = 0.073$, $a_{21} = 0.166$, $a_{22} = -0.5$, $b_{11} = 0.17$, $b_{12} = 0$, $b_{21} = 0$, and $b_{22} = 0.33$. The goal is to identify the values of the elements of A and B using adaptive identification. Using the estimation model (12) with $\Gamma_m = \text{diag}(-7, -7)$, and employing the adaptation laws (9)-(10), and ensuring the PE of the signals by adding auxiliary inputs $v_1 = 0.1\sin(2t)+\cos(6t)$ and $v_2 = \sin(5t)+0.1\cos(3t)$, Figure 2 demonstrates the identification of the elements of matrices A and B, respectively. It is seen that the method can track the parameters of the state and input matrices, A and B. Figure 2 excludes b_{12} and b_{21} as they are zero.

IV. Adaptive model predictive controller

The previous section introduced the adaptive parameter identification algorithm. In the following, we propose the adaptive model predictive controller which utilizes the parameter identification in the model predictive controller. Stability analysis is provided to guarantee the stability of the proposed approach.

Suppose that the system is state accessible, the assumptions of Theorem 1 hold, and states are measured at time t_q, as $x(t_q)$. In addition, the control input and disturbance prediction at time t_q is in hand. Then, using the adaptive parameter identification (see Section III), $\hat{A}$ and $\hat{B}$ are calculated. Having $\hat{A}$ and $\hat{B}$, and $x(t_q)$ as an initial condition, one can formulate the optimization problem for the time interval $t \in [t_q, t_{q+1})$ as

$$\min_{\bar{v}} \int_{t_q}^{t_{q+N}} \mathcal{J}(\bar{x}, \bar{v}, d, p)dt \tag{16}$$

$$s.t.\quad \dot{\bar{x}}(t) = \hat{A}\bar{x}(t) + \hat{B}\bar{u}(t), \quad \bar{x}(t_q) = x(t_q) \tag{17}$$

$$\dot{\hat{A}} = \text{Proj}(\hat{A}, \gamma_A e\bar{x}^\top), \quad \dot{\hat{B}} = \text{Proj}(\hat{B}, \gamma_B e\bar{u}^\top), \tag{18}$$

$$\bar{v} \in \mathcal{V}, \tag{19}$$

$$V_x(t_q)(\hat{A}x(t_q) + \hat{B}u(t_q)) \leq V_x(t_q)(\hat{A}x(t_q) + \hat{B}u^*(t_q)), \tag{20}$$

where (16) includes integration of the cost function $\mathcal{J}$ over the control horizon N, (17) is the dynamic estimate with initial condition $x(t_q)$, measured from the system (1) at sampling time t_q, $u^*(t_q) = [v^*(t_q)\ d(t_q)]^\top$ is the vector consisting of a stabilizing control ($v^*(t_q)$) at sampling time t_q, e.g. linear quadratic regulator (LQR) $v^*(t_q) = K_{LQR}x_p(t_q)$, and a disturbance measurement ($d(t_q)$) at sampling time t_q, $t_{q+i} = t_q + i\Delta t$, and where i is the sample number and Δt is the sample interval. Moreover, using (18), $\hat{A}$ and $\hat{B}$ are calculated as estimates of A and B. The control input u and u^* is also assumed to be constant during each sampling interval, that is, $u(t_q) = u(t)$ and $u^*(t_q) = u^*(t)$, for all $t \in [t_q,\ t_{q+1})$. In addition, the inequality (20) is added to keep the closed-loop system trajectories toward the equilibrium point. The continuously differentiable function V, employed in inequality (20), will be introduced in more detail next. It is noted that V_x stands for $\frac{\partial V}{\partial x}$.

Using the projection algorithm, defined in Definition 1, it is ensured that $\hat{A}(t)$ and $\hat{B}(t)$ belong to Ω_{proj_A} and Ω_{proj_B} for all $t \geq 0$, respectively, with measures Δ_A and Δ_B. For all $\hat{A} \in \Omega_{\text{proj}_A}$ and $\hat{B} \in \Omega_{\text{proj}_B}$, there exists a stabilizing control law, $v^* \in \mathcal{V}$ for the dynamics (2), such that (2) is exponentially stable. Then, using Converse Theorems [35], for all $\hat{A} \in \Omega_{\text{proj}_A}$ and $\hat{B} \in \Omega_{\text{proj}_B}$ there exist a continuously differentiable function $V(t)$ that satisfies the inequalities

$$k_1||x||^2 \leq V(t) \leq k_2||x||^2, \tag{21}$$

$$V_x(t)(\hat{A}x(t) + \hat{B}u^*(t)) \leq -k_3||x||^2, \tag{22}$$

$$||V_x(t)|| \leq k_4||x||, \tag{23}$$

where k_1, k_2, k_3, and k_4 are positive constants. It is noted that $V(t)$ stands for $V(x(t))$. Therefore, with the existence of the stabilizing controller, v^*, the set $\Omega = \{x : \dot{V}(x) < 0\}$ exists. In the following, the stability analysis of the proposed adaptive model predictive controller is investigated.

A. Stability analysis

In the following, we provide the stability analysis of the adaptive MPC and prove that the states remain bounded for each time interval $[t_q, t_{q+1})$. Notice that the continuous differentiable property of V and the Lipschitz property of $h(x, v, d, A, B)$ provides that there exist positive constants k_x, k_A, k_B, $\bar{k}_x$, $\bar{k}_A$, $\bar{k}_B$, and ρ, such that for each $x_1, x_2 \in \Omega_\rho \equiv \{x : \dot{V}(x) < 0, V(x) \leq \rho\} \subset \Omega$, and $A \in \Omega_{\text{proj}_A}$, and $B \in \Omega_{\text{proj}_B}$, the inequalities

$$||h(x, v, d, A, B)|| \leq R, \tag{24}$$

$$\begin{aligned} &||h(x_1, v, d, A, B) - h(x_2, v, d, \hat{A}, \hat{B})|| \\ &\quad \leq k_x||x_1 - x_2|| + k_A\Delta_A + k_B\Delta_B, \end{aligned} \tag{25}$$

$$\begin{aligned} &||V_x(x_1)h(x_1, v, d, A, B) - V_x(x_2)h(x_2, v, d, \hat{A}, \hat{B})|| \\ &\quad \leq \bar{k}_x||x_1 - x_2|| + \bar{k}_A\Delta_A + \bar{k}_B\Delta_B, \end{aligned} \tag{26}$$

hold, where Δ_A and Δ_B are the projection measures for matrices A and B, respectively. The following lemma is provided before the main stability theorem.

Lemma 1: [36] The matrix exponential norm, $||e^{A(t-t_0)}||$ with Hurwitz A is bounded as $||e^{A(t-t_0)}|| \leq ke^{-\xi(t-t_0)}$ for all $t \geq t_0$, where $k = \sqrt{||X^{-1}||||X||}$, $\xi = 1/(2||X||)$, and X is the solution of the Lyapunov equation $A^TX + XA = -I$, where I is the identity matrix.

From (2) and using Lemma 1, there exists a state transition matrix $\Phi(\tau, t_q)$ for all $\tau \in [t_q, t_{q+1})$ with $||\Phi(\tau, t_q)|| \leq ke^{-\xi(\tau - t_q)}$, such that

$$x(\tau) = \Phi(\tau, t_q)x(t_q) + \int_{t_q}^{\tau} \Phi(\tau, \chi)Bu(t_q)d\chi. \tag{27}$$

Considering $||u|| \leq \bar{U}$ leads to

$$\begin{aligned} ||x(\tau)|| &\leq ||\Phi(\tau, t_q)||||x(t_q)|| + \int_{t_q}^{\tau} ||\Phi(\tau, \chi)||||B||||u(t_q)||d\chi \\ &\leq k||x(t_q)|| + k\Delta B\bar{U}/\xi. \end{aligned} \tag{28}$$

Remark 4: It is noted that k and ξ in inequality (28) are dependant on A. While A is unknown, the range of change of its elements is known and forms the projection boundary. To find an upper bound for $||x(\tau)||$ independent of A, the maximum value of k and the minimum value of ξ can be calculated by using Lemma 1 and by solving the following optimization problems:

$$\begin{aligned} &k_{\max} = \max \sqrt{||X^{-1}||||X||} \\ &\quad s.t.\ A \in \Omega_{\text{proj}_A}, \quad A^\top X + XA = -I, \end{aligned} \tag{29}$$

and

$$\begin{aligned} &\xi_{\min} = \min 1/(2||X||) \\ &\quad s.t.\ A \in \Omega_{\text{proj}_A}, \quad A^\top X + XA = -I. \end{aligned} \tag{30}$$

Thus, $||x(\tau)|| \leq k_M \equiv k_{\max}||x(t_q)|| + k_{\max}\Delta B\bar{U}/\xi_{\min}$.

Theorem 2: Consider the adaptive MPC with optimization problem (16)-(20) and adaptive update laws (18) with projection algorithm given in Definition 1. If there exist positive constants Δt, ρ and $\rho_1 < \rho$ such that

$$-k_3\rho_1/k_2 + \bar{k}_xR\Delta t + \bar{k}_A\Delta_A + \bar{k}_B\Delta_B < 0, \tag{31}$$

$$\Delta t \leq (\rho_1 - \rho)/(-k_3\rho_1/k_2 + \bar{k}_x\bar{k}_M + \bar{k}_A\Delta_A + \bar{k}_B\Delta_B), \tag{32}$$

then for each initial condition $x(t_q) \in \Omega_\rho/\Omega_{\rho_1} = \{x : \dot{V}(x) < 0, \rho_1 \leq V(x) \leq \rho\}$, the solution of (2) remains bounded in $\Omega_\rho/\Omega_{\rho_1}$.

Proof: Suppose that there exist ρ and ρ_1 such that $x(t_q) \in \Omega_\rho/\Omega_{\rho_1} = \{x : \dot{V}(x) < 0, \rho_1 \leq V(x) \leq \rho\}$. Also, consider the continuously differentiable function $V(\tau)$ for $\tau \in [t_q, t_{q+1})$. From (20), (22) and (26), we have

$$\begin{aligned} \dot{V}(\tau) &= V_x(x(\tau))(Ax(\tau) + Bu(t_q)) \\ &\leq V_x(x(\tau))(Ax(\tau) + Bu(t_q)) - k_3||x(t_q)||^2 \\ &\quad - V_x(x(t_q))(Ax(t_q) + Bu(t_q)) \\ &\leq -k_3\rho_1/k_2 + \bar{k}_x||x(\tau) - x(t_q)|| + \bar{k}_A\Delta_A + \bar{k}_B\Delta_B. \end{aligned} \tag{33}$$

Also, using the Lipschitz property (24), it obtains that $||x(\tau) - x(t_q)|| \leq R\Delta t$. Thus, an upper bound for (33) is provided as

$$\dot{V}(\tau) \leq \beta \equiv -k_3\rho_1/k_2 + \bar{k}_xR\Delta t + \bar{k}_A\Delta_A + \bar{k}_B\Delta_B. \tag{34}$$

Using (31) and (34), we get $\dot{V}(\tau) < 0, \forall\tau \in [t_q,\ t_{q+1})$. Thus,

$$V(x(\tau)) \leq V(x(t_q)) + \beta(\tau - t_q) \leq \rho + \beta(\tau - t_q), \tag{35}$$

which implies that V reduces to ρ_1 in the time interval $[t_q,\ t_q + \frac{\rho_1 - \rho}{\beta})$. Therefore, (32) should also be satisfied. This guarantees that the trajectories cannot leave $\Omega_\rho/\Omega_{\rho_1}$. ■

B. Simulation results

The simulation results of the adaptive MPC in the presence of parameter deviations are demonstrated using the same numerical values and simulation scenario as those in Section III-A. These results are provided in Figure 3. The two top panels show the indoor temperature and heat input with adaptive and conventional MPCs. The conventional MPC performs incredibly well for the period $t < 30$ since it is aware of the building's thermal dynamics during this period. On the other hand, for the period $t < 30$, the indoor temperature by the adaptive MPC oscillates at the beginning and converges to the reference value ultimately. By increasing the insulation of the outer walls at $t = 30$, the ambient temperature has less effect on the indoor temperature. As a result, the building needs less energy to maintain the indoor temperature after $t = 30$. While the conventional MPC is not aware of this dynamic change, the adaptive MPC identifies this and adjusts the heat input. Furthermore, except for a relatively short transient period, the temperature deviation from the reference is within $\pm 1\ ^\circ C$ bound, demonstrating that the adaptive MPC can provide thermal comfort. The third panel shows the ambient temperature while the bottom panel displays the trajectories of Figure 2.

V. Summary

An adaptive MPC in the presence of uncertain system dynamics is proposed for the thermal dynamics of buildings. The method needs neither an expensive model nor historical data and can be implemented in buildings with similar thermal dynamics. Lyapunov analysis for parameter convergence and state boundedness is also provided. Simulation results show the effectiveness of the proposed method.

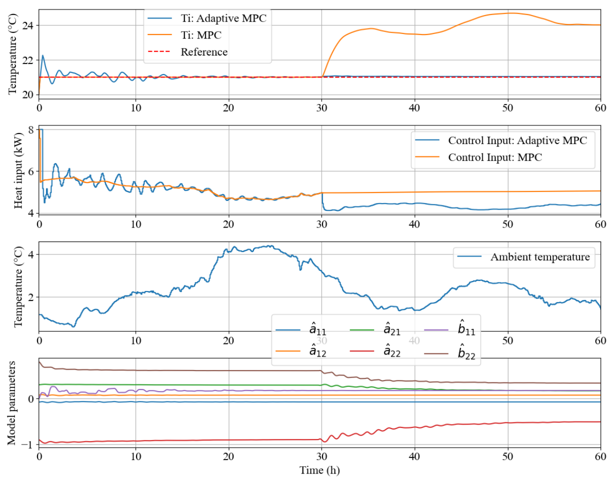

Fig. 3. Control of indoor temperature using Adaptive MPC.

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