IEEE Control Systems Letters paper presented at
2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Bearing-only formation control with bounded disturbances in agents' local coordinate frames

Chinmay Garanayak and Dwaipayan Mukherjee

Abstract— **This paper studies formation control using bearing-only measurements for elevation angle rigid configurations in the presence of time-varying bounded disturbances. Elevation angle rigidity-based control laws ensure bearing-only formation control in agents' local frame of reference sans any orientation synchronization or orientation estimation algorithms. However, existing control laws do not account for bounded disturbances in the agents' dynamics. Motivated by this, we design bearing-only control laws for single integrators in agents' frame of reference and prove local finite-time convergence to the desired formation. Then control laws for double integrators are proposed, and local asymptotic stability is proved when agents' accelerations are affected by bounded disturbances. Simulations are provided to validate the claims.**

Index terms: Bearing-only, double integrators, unknown disturbances

I. INTRODUCTION

Formation control is an important research area due to its applicability to several domains such as source seeking, surveillance, etc. [1], [2]. Depending on the sensing abilities of the agents and constraints specifying the desired formation, we have distance-based, displacement-based, bearing-based, or bearing-only [3], [4] formation control. Rigidity theory helps in characterizing unique formation shapes from a set of constraints [2]. For instance, distance rigidity theory [2], [5], [6] resulted in unique distance-based formations. However, the control laws for distance and displacement-based formation require distance and displacement measurements, which are noisier than relative bearing measurements. Bearing measurements are obtained using vision only sensors like cameras, and hence require simpler sensory arrangements, thereby reducing payload. This is desirable for small UAV applications [4]. Due to these appealing properties, bearing-only formation control has garnered much attention.

To uniquely characterize a formation shape from bearing constraints, bearing rigidity theory was proposed in [7], and then formation stabilization was achieved for single-integrators. Bearing-only formation tracking for single integrators, double integrators, and unicycles for constant velocity leaders was studied in [8]. A finite time bearing-only control law for single integrators was proposed in [9], which used a leader-first-follower interaction topology. Persistence of excitation was exploited in [10] for studying time varying formation control using only inter-agent bearing and velocity measurements. In all of these bearing-only control strategies, the desired formation was specified in terms of bearing constraints. But bearing is co-ordinate dependent. Hence, in all of these bearing-only control strategies, agents either need knowledge of a global reference frame, or some orientation synchronization or orientation estimation algorithms need to be employed in cascade with the formation control law [11]–[13]. Motivated by the above limitations, in [14] an angle-based formation control was studied. In [15] the desired formation was specified in terms of inter-agent angle constraints, which are co-ordinate free, and formation control laws were derived using bearing-only measurements. However, only planar formations were considered. In [16] each agent was attached with a circular disc and only bearing measurements were used for planar formations. A recent paper, [17], proposed an elevation angle-based formation control law, in which desired formation was specified by elevation angle constraints which are co-ordinate free. Each agent was attached with a rod in 2-D and a ball in 3-D. A gradient based formation control law was then proposed for single integrators using elevation angle rigidity. In [18] sign-elevation angle rigidity based bearing-only control law was proposed for single integrators. External bounded disturbances might affect the agents' dynamics adversely, and deteriorate the performance of formation control algorithms. In [19], although bearing-only control was considered in presence of time-varying disturbances, it required knowledge of a global reference frame.

Motivated by this, we consider bounded time-varying disturbances affecting the dynamics of the agents and derive bearing-only control laws in agents' local frames considering elevation angle rigid configurations. The important contributions of this letter are highlighted below:
Firstly, for single integrators, bearing-only formation control laws are designed, and local finite-time stability is proved in the presence of bounded time-varying disturbances. *Secondly*, for double integrators, bearing-only formation control laws are designed to obtain local asymptotic stability in the presence of bounded time-varying disturbances. *Thirdly*, only local bearing measurements are used in the control design, and no other orientation synchronization or estimation algorithms are used. *Finally*, the exact upper bound for the time-varying disturbances need not be known for control design.

Remark 1: In [5], [19], [20], formation control with bounded disturbance was reported. However, [5] considered distance-based formation control which required inter-agent displacements and distances. In [20], a bearing-based ap-

C. Garanayak (chinmayg22@gmail.com) and D. Mukherjee (dm@ee.iitb.ac.in) are with the Department of Electrical Engineering, Indian Institute of Technology Bombay, Mumbai- 400076, India.
The work of DM was supported in part by an ISRO funded project bearing code RD/0120-ISROC00-007.

proach was adopted where the formation control law required inter-agent distances along with bearings. In [19] also a bearing-only control law was reported, albeit with knowledge of a global frame by the agents. In contrast, our laws require only bearing measurements in agents' local frames. While [5] too, like our proposed laws, can only guarantee local stability, [19], [20] do guarantee global stability. However, the laws in [19], [20] require knowledge of a global frame by the agents. In contrast, our approach does not require global orientation information, but this comes at the cost of lacking guaranteed global stability.

Remark 2: Unlike [17], where single integrators were considered, we consider both single and double integrators. Further, [17] did not consider disturbances and local exponential stability was ensured. We guarantee local finite-time stability in the presence of disturbances for similar agents.

Notations: $||.||_p$ denotes the standard p-norm for a vector, or induced p-norm for a matrix. For 2-norm $||.||$ is used. $\otimes$ denotes the Kronecker product. ℓ_∞ is the space of all bounded functions, while ℓ_p is the space of functions with bounded p-norm. For $x \in \mathbb{R}$, $\text{sig}^\beta(x) := \text{sign}(x)|x|^\beta$, where $|\cdot|$ is the absolute value, and $\text{sign}(x) = 0, -1, 1$ for $x = 0, x < 0$ and $x > 0$, respectively. For $x = [x_1 \dots x_n]^T \in \mathbb{R}^n$, $\text{sign}(x) := [\text{sign}(x_1) \dots \text{sign}(x_n)]^T$, $\text{sig}^\beta(x) := [\text{sig}^\beta(x_1) \dots \text{sig}^\beta(x_n)]^T$, and $|x|^{[p]} := \sum_{i=1}^n |x_i|^p$, where $p > 0$. For $x_1, \dots, x_n \in \mathbb{R}^d$, $\text{vec}(x_1, \dots, x_n) := [x_1^T \dots x_n^T]^T \in \mathbb{R}^{dn}$. For $A_1, \dots, A_n \in \mathbb{R}^{d\times d}$, $\text{diag}(A_1 \dots A_n) \in \mathbb{R}^{nd\times nd}$ is block diagonal with $A_1, \dots, A_n$ as diagonal blocks. $I_k \in \mathbb{R}^{k\times k}$ denotes identity.

II. PROBLEM FORMULATION

A. Preliminaries

Consider n agents in $\mathbb{R}^d$, with $d = 2$ or 3. An undirected graph, $\mathcal{G}(\mathcal{V}, \mathcal{E})$, is used to model the interaction among the agents, where $\mathcal{V}$ is the vertex set (representing agents) and $\mathcal{E}$ is the edge set. The set of neighbours of vertex/agent i is defined by $\mathcal{N}_i = \{j|(i,j) \in \mathcal{E}\}$, and $n_i = |\mathcal{N}_i|$. Further details about algebraic graph theory are available in [21].

Let Q_i be the orientation of i-th agent with respect to a global frame. The position of the i-th agent, the relative displacement between agents i and j, and the bearing of agent j measured by agent i in its local frame are given by p_i^i, $z_{ij}^i := p_j^i - p_i^i$, and $b_{ij}^i = \frac{z_{ij}^i}{||z_{ij}^i||} \in \mathbb{R}^d$, respectively. We have $p_i = Q_i p_i^i, z_{ij} = Q_i z_{ij}^i$, and $b_{ij} = Q_i b_{ij}^i$, where p_i, z_{ij}, b_{ij} are in global reference frame. The distance between agents i and j is $d_{ij} = ||z_{ij}^i|| = ||z_{ij}||$. We have $p := [p_1^T \ \dots \ p_n^T]^T \in \mathbb{R}^{nd}$, the positions of n agents, and $p^* := [(p_1^*)^T \ \dots \ (p_n^*)^T]^T$ as the desired positions.

1) Elevation angles: These are defined in [17].

a) 2-D case: In 2-D, for obtaining the elevation angles, each agent is attached with a rod of height h_c. Hence, the coordinates of the end point of a rod are $p_i^{'} = p_i + [0 \ 0 \ h_c]^T$. The elevation angle measured by agent i toward agent j is given by $\alpha_{ij} := \angle jij^{'} = \arccos(b_{ij}^T b_{ij^{'}}) = \arccos((b_{ij}^i)^T(b_{ij^{'}}^i)) = \text{arccot}(d_{ij}/h_c) \in (0, \frac{\pi}{2})$. The elevation angles from agent 1 to 2, i.e., α_{12}, and from agent 1 to 3,

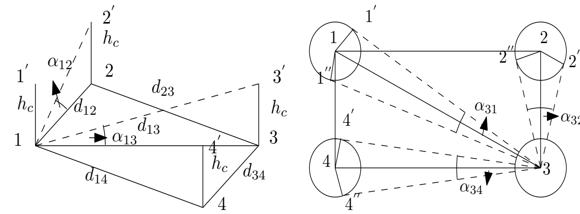


Fig. 1. (a) Elevation angle measurements in 2-D, (b) and in 3-D

i.e., α_{13} are shown in Fig. 1 (a). The elevation constraints in 2-D are $f_{ij} = \cot(\alpha_{ij}) = \frac{d_{ij}}{h_c} = \frac{\cos(\alpha_{ij})}{\sin(\alpha_{ij})} = \frac{b_{ij}^T b_{ij^{'}}}{\sqrt{1-(b_{ij}^T b_{ij^{'}})^2}} = \frac{(b_{ij}^i)^T(b_{ij^{'}}^i)}{\sqrt{1-((b_{ij}^i)^T(b_{ij^{'}}^i))^2}}$, and the desired formation is specified by $f_{ij} = f_{ij}^*$, implying $\cot(\alpha_{ij}) = \cot(\alpha_{ij}^*)$, $\forall (i,j) \in \mathcal{E}$.

b) 3-D case: In 3-D each agent is attached with a ball of radius r_c. The elevation angle is then defined as $\alpha_{ij} := \angle j^{'}ij^{''} = \arccos(b_{ij^{'}}^T b_{ij^{''}}) = 2\angle j^{'}ij = 2\text{arcsin}(\frac{r_c}{d_{ij}}) \in (0, \pi/3)$, where $j, j^{'}, j^{''}, i$ are coplanar points with j and $j^{''}$ being on the surface of agent j's ball, and $b_{ij^{'}}$ and $b_{ij^{''}}$ are perpendicular to $b_{jj^{'}}$ and $b_{jj^{''}}$, respectively. The elevation angles measured by agents 3 to 1, 3 to 2, and 3 to 4 are shown in Fig. 1(b). The elevation constraints are defined by $f_{ij} := \text{cosec}(\alpha_{ij}) = \frac{d_{ij}}{r_c} = \frac{1}{\sin(\frac{\alpha_{ij}}{2})} = \frac{1}{\sqrt{(1-\cos(\alpha_{ij}))/2}} = \frac{1}{\sqrt{(1-b_{ij^{'}}^T b_{ij^{''}})/2}} = \frac{1}{\sqrt{(1-(b_{ij^{'}}^i)^T(b_{ij^{''}}^i))/2}}$.

Remark 3: Note that these constraints, f_{ij}, can be evaluated using only bearing measurements in local frames.

2) Elevation Angle Rigidity: Elevation angle function is constructed by clubbing all the elevation angle constraints, i.e., $f_E := [f_1 \ \dots \ f_m]^T$, where $f_k := (i,j)$-th edge and $k := (i,j) \in \mathcal{E}$. The derivative of the elevation angle function f_E with respect to time is $\frac{df_E}{dt} = \frac{\partial f_E}{\partial p}\dot{p} = R_e(p)\dot{p}$, where $R_e(p)$ is the elevation angle rigidity matrix [17] defined as $R_e(p) := \frac{\partial f_E}{\partial p}$. When we take derivative of (i,j)-th element of f_E (say f_{ij}) we obtain:

$$\frac{df_{ij}}{dt} = \frac{\partial f_k}{\partial p_i}\dot{p}_i + \frac{\partial f_k}{\partial p_j}\dot{p}_j = \rho^{-1}\frac{d||z_k||}{dt} = \rho^{-1} b_{ij}^T(\dot{p}_j - \dot{p}_i),$$

where $\rho := h_c$ or r_c for 2-D and 3-D, respectively. We refer the reader to [17] for details on the notions of rigidity.

Lemma 1: [17, Theorem 1] A framework $\mathcal{F}(\mathcal{G}, p)$ is infinitesimal elevation angle rigid if and only if the rank of its elevation angle rigidity matrix, $R_e(p)$, is $nd-(d+1)d/2$, where d is the dimension of ambient space.

Definition 1: ([17]) A framework, $\mathcal{F}(\mathcal{G}, p)$, is called *minimally rigid* if Lemma 1 is satisfied and $|\mathcal{E}| = nd - d(d+1)/2$, where d is the dimension of the ambient space.

3) Agent dynamics: Consider single integrators and double integrators with bounded disturbances. Single integrators with disturbances in local frames are given by:

$$\dot{p}_i^i = u_i^i + \delta_i^i(t) \quad i \in \mathcal{V}, \tag{1}$$

where $u_i^i \in \mathbb{R}^d$ is control input in agents' local co-ordinate system, and $\delta_i^i(t) \in \mathbb{R}^d$ is the bounded disturbance such that

$||\delta_i^i(t)|| \leq \alpha, \forall t$, where $\alpha > 0$ is some unknown constant. The double integrator dynamics is given by:

$$\dot{p}_i^i = v_i^i;\ \dot{v}_i^i = u_i^i + \delta_i^i(t) \quad i \in \mathcal{V}, \tag{2}$$

where v_i^i is agent i's velocity in its local frame, $u_i \in \mathbb{R}^d$ the control input, $\delta_i^i(t) \in \mathbb{R}^d$ is the bounded disturbance such that $||\delta_i^i(t)|| \leq \alpha, \forall t$, where $\alpha > 0$ is an unknown constant.

B. Problem statements

The following assumption is made in our set-up.

Assumption 1: The framework $\mathcal{F}(\mathcal{G}, p^*)$, where $\mathcal{G}$ is the sensing graph and p^* is the desired realization, is minimally infinitesimally elevation-angle rigid.

Problem 1: Consider n single integrators (1) with Assumption 1 satisfied. Design control laws, u_i^i, for $i \in \mathcal{V}$, so that the agents converge to the desired formation shape, i.e., $f_{ij} \to f_{ij}^* \ \ \forall (i,j) \in \mathcal{E}$ in finite time.

Problem 2: Consider n double integrators in (2), with Assumption 1 satisfied. Design control laws, u_i^i, for $i \in \mathcal{V}$, so that the agents converge to the desired formation shape, i.e., $f_{ij} \to f_{ij}^* \ \ \forall (i,j) \in \mathcal{E}$ and $||v_i|| \to 0$ as $t \to \infty$.

The notion of stability refers to $e_{ij}(t) := (f_{ij} - f_{ij}^*) \to 0, \forall (i,j) \in \mathcal{E}$, i.e., stability implies the errors go to zero.

III. MAIN RESULTS

In this section, we present the control laws to solve the two formation control problems and analyze their stability.

A. Single integrators:

The control law proposed for single integrators is:

$$u_i^i = \bar{k} \sum_{j \in \mathcal{N}_i} \text{sig}^\beta(e_{ij}) b_{ij}^i + \hat{\gamma}_i \text{sign}(\hat{\gamma}_i \sum_{j \in \mathcal{N}_i} \text{sig}^\beta(e_{ij}) b_{ij}^i) \tag{3}$$

$$\dot{\hat{\gamma}}_i = \text{Proj}(\mathcal{X}_i, \hat{\gamma}_i), \text{ with } \mathcal{X}_i := || \sum_{j \in \mathcal{N}_i} \text{sig}^\beta(e_{ij}) b_{ij}^i ||_1, \tag{4}$$

where $e_{ij} := f_{ij} - f_{ij}^* \ \ , \forall j \in \mathcal{N}_i$, are the elevation angle error, $\beta \in [\frac{1}{2}, 1)$, $\bar{k} > 0$ is the control gain, and $\hat{\gamma}_i$ is the adaptive gain whose update rule is given by (4). The projection operator ([22]) is defined as $\text{Proj}(\mathcal{X}_i, \hat{\gamma}_i) := (1 - \psi(\hat{\gamma}_i))\mathcal{X}_i$ when $\psi(\hat{\gamma}_i) > 0$ and $\psi^{'}(\hat{\gamma}_i)\mathcal{X}_i > 0$, and $\text{Proj}(\mathcal{X}_i, \hat{\gamma}_i) := \mathcal{X}_i$ otherwise. Here, $\psi(\hat{\gamma}_i) := \frac{\hat{\gamma}_i^2 - \eta^2}{\varepsilon^2 + 2\varepsilon\eta}$, $\psi^{'}(\hat{\gamma}_i) := \frac{\partial \psi(\hat{\gamma}_i)}{\partial \hat{\gamma}_i}$, and η, ε are positive reals. The formation control law (3) has two components, the first of which aids in stabilizing the formation in finite time, whereas the second addresses unknown bounded disturbances. Adaptive gain, $\hat{\gamma}_i$, is used to suppress disturbances. In global frame (3) is:

$$u_i = \bar{k} \sum_{j \in \mathcal{N}_i} \text{sig}^\beta(e_{ij}) b_{ij} + \hat{\gamma}_i Q_i \text{sign}(\hat{\gamma}_i Q_i^T \sum_{j \in \mathcal{N}_i} \text{sig}^\beta(e_{ij}) b_{ij})$$

Defining $Q := \text{diag}(Q_1 \ \ldots \ Q_n)$, we have $Q^T = \text{diag}(Q_1^T \ \ldots \ Q_n^T)$, and $QQ^T = I_{dn}$. Define $\Gamma := \text{diag}(\hat{\Gamma}_1 \ \ldots \ \hat{\Gamma}_n) \in \mathbb{R}^{nd \times nd}$, where $\hat{\Gamma}_i := I_d \otimes \hat{\gamma}_i \in \mathbb{R}^{d \times d}$. As Γ is a diagonal matrix, we have $\Gamma = \Gamma^T$. The following control law, u, results for n agents,

$$u = -kR_e^T \text{sig}^\beta(e) - \Gamma Q \text{sign}(Q^T \Gamma^T R_e^T \text{sig}^\beta(e)), \tag{5}$$

where $k = \bar{k}\rho$. As $\rho > 0$ here, we have $\text{sign}(\rho v) = \text{sign}(v)$, for any $v \in \mathbb{R}^{nd}$. The disturbance vector and velocity vector for n agents in the global frame is given by:

$$\delta = Q\text{vec}(\delta_1^1, \ \ldots, \ \delta_n^n), \ \dot{p} \ = Q\text{vec}(\dot{p}_1^1, \ \ldots, \ \dot{p}_n^n). \tag{6}$$

We have the following result on the single integrators.

Theorem 1: For the single integrators in (1) driven by the control law (3), under Assumption 1, the formation tracking error $e_{ij}, \ \forall (i,j) \in \mathcal{E}$ is bounded globally, and converges to zero locally in finite-time.

Proof: We club the elevation constraints to get $e := [e_1 \ \ldots \ e_m]^T \in \mathbb{R}^{md}$. Consider the Lyapunov candidate:

$$\mathcal{L}(e) = \underbrace{\frac{1}{1+\beta}|e|^{[\beta+1]}}_{\mathcal{L}_1} + \underbrace{\frac{\rho\sqrt{d}}{2}\sum_{i=1}^{n}\tilde{\gamma}_i^2}_{\mathcal{L}_2}, \tag{7}$$

where $\tilde{\gamma}_i := \frac{\hat{\gamma}_i \rho^{-1}}{\sqrt{d}} - \alpha$ (α being the unknown bound on the disturbance). Here, $\mathcal{L}_1$ is motivated by the Lyapunov candidate in [6], and $\mathcal{L}_2$ helps in the Lyapunov analysis to take care for unknown upper bound for the time-varying disturbances. As we have a non-smooth Lyapunov function and a control law containing discontinuous signum function, we use non-smooth analysis in [23, Th. 2.2] to calculate the derivative of $\mathcal{L}_1(e)$, which is given by

$$\dot{\mathcal{L}}_1 \in^{a.e.} \dot{\tilde{\mathcal{L}}}_1 = \bigcap_{\zeta \in \partial \mathcal{L}} \zeta^T K[\dot{p}] = \nabla \mathcal{L}_1^T K[\dot{p}], \tag{8}$$

where $K[\cdot]$ is the set valued map defined in [23], and $a.e$ stands for almost everywhere. We have

$$\frac{\partial \mathcal{L}_1}{\partial p} = \frac{\partial \mathcal{L}_1}{\partial e}\frac{\partial e}{\partial p} = \text{sig}^\beta(e)^T R_e. \tag{9}$$

We take the time derivative of $\mathcal{L}_1$ next, and use (9) with (5), (6), (8) to get:

$$\begin{aligned}\dot{\mathcal{L}}_1 \in^{a.e.} \dot{\tilde{\mathcal{L}}}_1 = &-k||R_e^T \text{sig}^\beta(e)||^2 + \text{sig}^\beta(e)^T R_e \delta \\ &- \text{sig}^\beta(e)^T R_e \Gamma Q K[\text{sign}(Q^T \Gamma^T R_e^T \text{sig}^\beta(e))].\end{aligned}$$

As $\forall x \in \mathbb{R}$ we have $xK[\text{sign}(x)] = \{|x|\}$, which is a singleton set, we get

$$\begin{aligned}\dot{\mathcal{L}}_1 \leq &-k||R_e^T \text{sig}^\beta(e)||^2 + ||\text{sig}^\beta(e)^T R_e \delta|| \\ &- ||Q^T \Gamma^T R_e^T \text{sig}^\beta(e)||_1. \end{aligned} \tag{10}$$

Also, we have $||\delta_i^i|| \leq \alpha$, and $||Q_i|| = 1 \ \forall i \in \mathcal{V}$ leading to

$$||\delta||_\infty = \max_i ||Q_i \delta_i^i||_\infty \leq \max_i ||Q_i \delta_i^i|| \leq \alpha. \tag{11}$$

Using (11) with Hölder's inequality [24], we get:

$$|\text{sig}^\beta(e)^T R_e \delta| \leq ||R_e^T \text{sig}^\beta(e)||_1 ||\delta||_\infty \leq \alpha ||R_e^T \text{sig}^\beta(e)||_1. \tag{12}$$

Note that $||Q^T \Gamma^T R_e^T \text{sig}^\beta(e)||_1 \geq ||Q^T \Gamma^T R_e^T \text{sig}^\beta(e)|| = ||\Gamma^T R_e^T \text{sig}^\beta(e)||$. Let $R_e^T \text{sig}^\beta(e) = [\psi_1^T \ \ldots \ \psi_n^T]^T$, which implies $\Gamma^T R_e^T \text{sgn}^\beta(e) = [\psi_1^T \hat{\Gamma}_1 \ \dots \ \psi_n^T \hat{\Gamma}_n]^T$. Hence,

$$||Q^T\Gamma^T R_e^T \text{sig}^\beta(e)||_1 \geq ||\Gamma^T R_e^T \text{sig}^\beta(e)|| = \sum_{i=1}^n \hat{\gamma}_i ||\psi_i||$$
$$\geq \frac{1}{\sqrt{d}} \sum_{i=1}^n \hat{\gamma}_i ||\psi_i||_1 = \frac{1}{\sqrt{d}} ||\Gamma^T R_e^T \text{sig}^\beta(e)||_1 \quad (13)$$

Now, using inequalities (12), (13), and (10), we get

$$\begin{aligned}
\dot{\mathcal{L}}_1 &\leq -k||R_e^T \text{sig}^\beta(e)||^2 - \frac{1}{\sqrt{d}} ||\Gamma^T R_e^T \text{sig}^\beta(e)||_1 \\
&+ \alpha ||R_e^T \text{sig}^\beta(e)||_1 \\
&\leq -k||R_e^T \text{sig}^\beta(e)||^2 - \sum_{i=1}^n \frac{\hat{\gamma}_i \rho^{-1}}{\sqrt{d}} || \sum_{j\in\mathcal{N}_i} \text{sig}^\beta(e_{ij}) b_{ij}^i ||_1 \\
&+ \alpha \sum_{i=1}^n || \sum_{j\in\mathcal{N}_i} \text{sig}^\beta(e_{ij}) b_{ij}^i ||_1 \\
&\leq -k||R_e^T \text{sig}^\beta(e)||^2 - \sum_{i=1}^n \tilde{\gamma}_i || \sum_{j\in\mathcal{N}_i} \text{sig}^\beta(e_{ij}) b_{ij}^i ||_1. \quad (14)
\end{aligned}$$

We have $\dot{\mathcal{L}} = \dot{\mathcal{L}}_1 + \dot{\mathcal{L}}_2$, and $\dot{\mathcal{L}}_2 := \sum_{i=1}^n \tilde{\gamma}_i \dot{\hat{\gamma}}_i$. Using the expression of $\dot{\hat{\gamma}}_i$ from (4) and $\dot{\mathcal{L}}_1$ from (14) we get:

$$\dot{\mathcal{L}} = \dot{\mathcal{L}}_1 + \dot{\mathcal{L}}_2 \leq -k||R_e^T \text{sig}^\beta(e)||^2 - \sum_{i=1}^n \tilde{\gamma}_i (\mathcal{X}_i - \text{Proj}(\mathcal{X}_i, \hat{\gamma}_i)).$$

Now, from [22], we have $\tilde{\gamma}_i(\mathcal{X}_i - \text{Proj}(\mathcal{X}_i, \hat{\gamma}_i)) \geq 0$, and hence $\dot{\mathcal{L}}$ is negative semi-definite. Hence, the system is Lyapunov stable, and all the signals are bounded, i.e., $e_{ij}, \hat{\gamma}_i, \mathcal{L}, u \in \ell_\infty$. Moreover, from Assumption 1 and Lemma 1, we have a compact neighborhood, $\mathcal{S}$ around $e = 0$, so that $R_e R_e^T$ is positive definite in $\mathcal{S}$. So, for $e \in \mathcal{S}$ initially,

$$\dot{\mathcal{L}} \leq -k||R_e^T \text{sig}^\beta(e)||^2 \leq -\lambda ||\text{sig}^\beta(e)||^2 = -\lambda |e|^{[2\beta]}, \quad (15)$$

where $\lambda = k\bar{\lambda}$ and $\bar{\lambda} > 0$ is the smallest eigenvalue of $R_e R_e^T$ inside $\mathcal{S}$. Integrating both sides of (15) we get

$$\mathcal{L}\big|_{t\to\infty} - \mathcal{L}\big|_{t=0} \leq -\lambda \int_0^\infty |e|^{[2\beta]} dt. \quad (16)$$

We have $|e|^{[2\beta]} \in \ell_1$ since $\int_0^\infty |e|^{[2\beta]} dt$ is upper bounded (from (16)). As $e \in \ell_\infty$, we have $|e|^{[2\beta]} \in \ell_\infty$. Hence, $|e|^{[2\beta]} \in \ell_1 \cap \ell_\infty$. Differentiating $|e|^{[2\beta]}$, we get $\frac{d|e|^{[2\beta]}}{dt} \in^{a.e.} 2\beta \text{sig}^{2\beta-1}(e)^T R_e K[\dot{p}] \in \ell_\infty$. As $2\beta \in [1, 2)$, we have $2\beta - 1 \in [0, 1)$. So $e^{[2\beta]}$ is uniformly continuous. Using Barbalat's lemma [25, Lemma 8.2], $|e|^{[2\beta]} \to 0$ as $t \to \infty$, which implies $e \to 0$ as $t \to \infty$. Hence, local asymptotic convergence results. As boundedness of all signals and local asymptotic convergence are established, consider the Lyapunov candidate $\mathcal{L}_1$. As $\hat{\gamma}_i \in \ell_\infty$, let $\kappa := \max_{t>0} ||\tilde{\gamma}||_\infty$, where $\tilde{\gamma} := [\tilde{\gamma}_1 \ \dots \ \tilde{\gamma}_n]^T$. Hence, from (14), we have:

$$\begin{aligned}
\dot{\mathcal{L}}_1 &\leq -k||R_e^T \text{sig}^\beta(e)||^2 + \kappa \sum_{i=1}^n || \sum_{j\in\mathcal{N}_i} \text{sig}^\beta(e_{ij}) b_{ij}^i ||_1 \\
&\leq -\lambda |e|^{[2\beta]} + \kappa ||R_e^T \text{sig}^\beta(e)||_1 \quad (17) \\
&\leq -\lambda |e|^{[2\beta]} + \kappa\sqrt{d} ||R_e^T \text{sig}^\beta(e)|| \leq -\lambda |e|^{[2\beta]} + \kappa_1 |e|^{[\beta]},
\end{aligned}$$

where $\kappa_1 := \kappa\sqrt{d}\lambda_m$ and $\lambda_m > 0$ is the largest eigenvalue of $R_e R_e^T$ inside $\mathcal{S}$. Now, there exist a set $\mathcal{S}_1$ near $e = 0$, and $0 < \lambda_1 < \lambda$, such that $|e|^{[2\beta]} \geq \frac{\kappa_1}{\lambda_1}|e|^{[\beta]}$ holds in $\mathcal{S}_1$. Such an $\mathcal{S}_1$ must exist because $2\beta > \beta$, and as e is in the neighborhood of the origin, we can choose sufficiently small e, i.e., $|e_k| \leq \varepsilon < 1, \ \forall k \in \mathcal{E}$. Hence, inside $\mathcal{S}_1$,

$$\dot{\mathcal{L}}_1 \leq -(\lambda - \lambda_1)|e|^{[2\beta]} = -\lambda_2 |e|^{[2\beta]}, \quad (18)$$

where $\lambda_2 = (\lambda - \lambda_1) > 0$. Due to [6, Lemma 4], $|e|^{[2\beta]} = |\text{sig}^{\beta+1}(e)|^{[\frac{2\beta}{1+\beta}]} \geq (|\text{sig}^{\beta+1}(e)|^{[1]})^{\frac{2\beta}{\beta+1}} = (|e|^{\beta+1})^{\frac{2\beta}{1+\beta}}$. Hence, $\dot{\mathcal{L}}_1 \leq -\lambda_2(\beta+1)^{\frac{2\beta}{1+\beta}} \mathcal{L}_1^{\frac{2\beta}{1+\beta}} = -\beta_1 \mathcal{L}_1^{\beta_2}$, with $\beta_1 = \lambda_2(\beta+1)^{\frac{2\beta}{1+\beta}}$ and $\beta_2 = \frac{2\beta}{1+\beta} \in (0, 1)$. From [13, Lemma 2], we have finite time convergence within $\mathcal{S}_1$. Starting within $\mathcal{S}$, due to asymptotic convergence, e enters set $\mathcal{S}_1$ after finite time, say τ, and thereafter reaches 0 in time $T = \frac{\mathcal{L}^{1-\beta_2}(\tau)}{\beta_1(1-\beta_2)}$. Hence, time to reach origin is $T_1 = \tau + T < \infty$. ■

Remark 4: The expression for convergence time is $T_1 = \tau + T = \tau + \frac{\mathcal{L}^{\frac{1-\beta}{1+\beta}}(\tau)}{(\bar{k}\rho\bar{\lambda}-\lambda_1)(1+\beta)^{\frac{1-\beta}{1+\beta}}(1-\beta)}$. So, increasing $\bar{k}$ reduces T, but it may saturate actuators due to increased gain.

Remark 5: Instead of the controller in (3) and (4), containing a sig(·) and sign(·) functions, we may use the controller $u_i^i = \bar{k}\text{sign}(\sum_{j\in\mathcal{N}_i} e_{ij} b_{ij}^i) + \hat{\gamma}_i \text{sign}(\hat{\gamma}_i \sum_{j\in\mathcal{N}_i} e_{ij} b_{ij}^i)$, with $\dot{\hat{\gamma}}_i = \text{Proj}(||\sum_{j\in\mathcal{N}_i} e_{ij} b_{ij}^i||_1, \hat{\gamma}_i)$ containing only sign(·) functions, and prove similar result as in Theorem 1. The Lyapunov candidate may be chosen as $\mathcal{L} := \frac{1}{2}e^T e + \mathcal{L}_2$. However, in either case chattering may result due to the presence of discontinuous sign(·) function in the control law.

B. Double integrators:

Next, we propose control laws for double integrators ((2)).

$$\begin{aligned}
\dot{r}_i^i &= k_p \rho^{-1} \sum_{j\in\mathcal{N}_i} e_{ij} b_{ij}^i - k_v v_i^i \\
u_i^i &= 2k_p \rho^{-1} \sum_{j\in\mathcal{N}_i} e_{ij} b_{ij}^i - 2k_v v_i^i + \hat{\gamma}_i \text{sign}(\hat{\gamma}_i (r_i^i - v_i^i))
\end{aligned} \quad (19)$$

$$\dot{\hat{\gamma}}_i = \text{Proj}(\mathcal{X}_i, \hat{\gamma}_i), \text{ with } \mathcal{X}_i := ||r_i^i - v_i^i||_1, \quad (20)$$

Here, $k_p > 0$ and $k_v > 0$ are the control gains, and $\rho := h_c$ or r_c for 2-D and 3-D, respectively. The projection operator is as defined in Section III A. By adjusting the control gains k_p and k_v we can improve transient response, i.e., to increase the damping, we may choose a higher value of k_v than k_p. By increasing the ratio of k_v to k_p we may reduce oscillations, and even eliminate them, though very high values of k_v may lead to higher amplitude of control signal. However, we should not choose k_p to be very small either, as it will increase convergence time for the error. Since the system is nonlinear, analytical quantification of damping with variation of k_v to k_p is intractable. Using the orthogonal matrices, Q_i, we write (19) in global frame of reference as:

$$\begin{aligned}
\dot{r}_i &= \rho^{-1} k_p \sum_{j\in\mathcal{N}_i} e_{ij} b_{ij} - k_v v_i \\
u_i &= 2k_p \rho^{-1} \sum_{j\in\mathcal{N}_i} e_{ij} b_{ij} - 2k_v v_i + \hat{\gamma}_i Q_i \text{sign}(Q_i^T \hat{\gamma}_i (r_i - v_i)).
\end{aligned}$$

The dynamics of all the agents in compact form is:

$$\begin{aligned}\dot{r} &= -k_p R_e^T e - k_v v\\ u &= -2k_p R_e^T e - 2k_v v + \Gamma Q \text{sign}(Q^T\Gamma^T(r-v)),\end{aligned} \quad (21)$$

where $r = [r_1^T \ \dots \ r_n^T]^T$, $v = [v_1^T \ \dots \ v_n^T]^T$ and $u = [u_1^T \ \dots \ u_n^T]^T \in \mathbb{R}^{nd}$. Let $\hat{\gamma} := [\hat{\gamma}_1 \ \dots \ \hat{\gamma}_n]^T$.

Theorem 2: For double integrators in (2) driven by (19), subject to Assumption 1, the formation tracking errors $e_{ij}, \ \forall(i,j) \in \mathcal{E}$, and agent velocities $v_i, \ i \in \mathcal{V}$ are bounded globally, and converge to zero locally asymptotically.

Proof: Consider the following Lyapunov candidate:

$$\mathcal{L} = \frac{k_p}{2}||e||^2 + \frac{1}{2}||r||^2 + \frac{1}{2}||r-v||^2 + \frac{\sqrt{d}}{2}||\tilde{\gamma}||^2 \quad (22)$$

where $\tilde{\gamma}_i := (\frac{\hat{\gamma}_i}{\sqrt{d}} - \alpha - 1)$ and $\tilde{\gamma} = [\tilde{\gamma}_1 \ \dots \ \tilde{\gamma}_n]^T$. Thereafter,

$$\begin{aligned}&\dot{\mathcal{L}} \in^{a.e.} \dot{\tilde{\mathcal{L}}} = k_p e^T \dot{e} + r^T\dot{r} + (r-v)^T(\dot{r} - K[u] - \delta) + \tilde{\gamma}^T\dot{\hat{\gamma}}\\ &\implies \dot{\mathcal{L}} \in^{a.e.} \dot{\tilde{\mathcal{L}}} = k_p e^T R_e v - k_p r^T R_e^T e - k_v r^T v + \tilde{\gamma}^T\dot{\hat{\gamma}}\\ &+ (r-v)^T(k_p R_e^T e + k_v v + \Gamma Q K[\text{sign}(Q^T\Gamma^T(r-v))] - \delta)\\ &\implies \dot{\mathcal{L}} = -k_v||v||^2 - ||Q^T\Gamma(r-v)||_1 - \delta^T(r-v) + \tilde{\gamma}^T\dot{\hat{\gamma}}.\end{aligned}$$

We have, $||\delta||_\infty = \max_i ||Q_i\delta_i^i||_\infty \le \alpha$. Also,

$$\begin{aligned}&||Q^T\Gamma(r-v)||_1 \ge ||Q^T\Gamma(r-v)|| = ||\Gamma(r-v)||\\ &\ge \frac{1}{\sqrt{d}}||\Gamma(r-v)||_1 = \frac{1}{\sqrt{d}}\sum_{i=1}^n \hat{\gamma}_i||(r_i - v_i)||_1.\end{aligned}$$

Also, $|\delta^T(r-v)| \le ||r-v||_1||\delta||_\infty \le \alpha||(r-v)||_1$. Hence,

$$\begin{aligned}\dot{\mathcal{L}} &\le -k_v||v||^2 - ||Q^T\Gamma(r-v)||_1 + |\delta^T(r-v)| + \tilde{\gamma}^T\dot{\hat{\gamma}}\\ &\le -k_v||v||^2 - \frac{1}{\sqrt{d}}||\Gamma(r-v)||_1 + \alpha||(r-v)||_1 + \tilde{\gamma}^T\dot{\hat{\gamma}}\\ &\le -k_v||v||^2 - \sum_{i=1}^n||r_i - v_i||_1 - \sum_{i=1}^n\tilde{\gamma}_i||r_i - v_i||_1 + \sum_{i=1}^n\tilde{\gamma}_i\dot{\hat{\gamma}}_i\\ &\le -k_v||v||^2 - ||r-v||_1 - \sum_{i=1}^n \tilde{\gamma}_i(\mathcal{X}_i - \text{Proj}(\mathcal{X}_i, \hat{\gamma}_i))\end{aligned} \quad (23)$$

From (23) and using the fact $\tilde{\gamma}_i(\mathcal{X}_i - \text{Proj}(\mathcal{X}_i, \hat{\gamma}_i)) \ge 0$ [22],

$$\dot{\mathcal{L}} \le -k_v||v||^2 - ||r-v||_1, \quad (24)$$

which is negative semi-definite. Hence, all the signals are bounded, i.e., $e_{ij}, \hat{\gamma}_i, v_i, \mathcal{L}, u_i \in \ell_\infty$. It readily follows that

$$\mathcal{L}\big|_{t\to\infty} - \mathcal{L}\big|_{t=0} \le -\int_0^\infty k_v||v||^2 dt - \int_0^\infty ||r-v||_1 dt. \quad (25)$$

As the L.H.S. of (25) is bounded, the R.H.S. is also bounded. Hence, $\int_0^\infty ||v||^2 dt < \infty$ and $\int_0^\infty ||r-v||_1 dt < \infty$, implying $||v|| \in \ell_2$ and $||r-v|| \in \ell_1$. Also, $\dot{v} = u + \delta \le ||u||_\infty + ||\delta||_\infty \in \ell_\infty$, and $\dot{r} - \dot{v} \in \ell_\infty$, implies $||v||$ and $||r-v||$ are uniformly continuous. Together with $||v|| \in \ell_2 \cap \ell_\infty$ and $||v-r|| \in \ell_1 \cap \ell_\infty$, using Barbalat's lemma [25], we have $||v|| \to 0$ and $||r-v|| \to 0$ as $t \to \infty$. Hence, $r \to 0$, as $t \to \infty$, implying $R_e^T e \to 0$, as $t \to \infty$. However, due to minimally infinitesimal rigidity, there exists a neighbourhood, $\mathcal{S}$, of $e = 0$ where $R_e^T e \equiv 0$, implies $e \equiv 0$, because inside $\mathcal{S}$, rank(R_e)= rank(R_e^T)= $nd - d(d+1)/2$. So R_e^T has full row rank, and null space of R_e^T is trivial. Hence, starting inside $\mathcal{S}$, e converges to zero asymptotically. ■

Remark 6: Introduce a parameter ϵ in the control laws and redefine the elevation constraint error as $e_{ij} := f_{ij} - \epsilon f_{ij}^*$. Then the desired equilibrium corresponds to $f_{ij} = \epsilon f_{ij}^*$, implying $\frac{d_{ij}}{\rho} = \frac{\epsilon d_{ij}}{\rho}$, i.e., $d_{ij} = \epsilon d_{ij}^*$. Hence, for $\epsilon > 1$, the formation scales up and for $0 < \epsilon < 1$ it scales down.

Remark 7: Projection-based update rule ensures $\hat{\gamma}_i$ is always bounded [22]. Instead we can also use $\dot{\hat{\gamma}}_i = \mathcal{X}_i$ in (4) and (20). However, due to the discrete implementation and chattering around the origin, the adaptive gains might slowly increase. Define the set $\Omega_\zeta := \{e : |e| = \sum_{k=1}^m |e|_k < \zeta\}$ with $\zeta > 0$ being a small positive number. We can stop updating $\hat{\gamma}_i \ \ \forall i \in \mathcal{V}$ when $e \in \Omega_\zeta$, i.e., $\dot{\hat{\gamma}}_i = 0 \ \ \forall i \in \mathcal{V}$, else we use $\dot{\hat{\gamma}}_i = \mathcal{X}_i$. Then e will be bounded below ζ. We may thus observe the expression above (24) , i.e., $\dot{\mathcal{L}} \le -k_v||v||^2 - \sum_{i=1}^n ||r_i - v_i||_1 - \sum_{i=1}^n \tilde{\gamma}_i||r_i - v_i||_1 + \sum_{i=1}^n \tilde{\gamma}_i\dot{\hat{\gamma}}_i$. When $e \in \Omega_\zeta$ we have $\dot{\hat{\gamma}}_i = 0 \ \ \forall i \in \mathcal{V}$, hence we have $\dot{\mathcal{L}} \le -k_v||v||^2 - \sum_{i=1}^n ||r_i - v_i||_1 - \sum_{i=1}^n \tilde{\gamma}_i||r_i - v_i||_1$. Now, if the term $-k_v||v||^2 - \sum_{i=1}^n ||r_i - v_i||_1 - \sum_{i=1}^n \tilde{\gamma}_i||r_i - v_i||_1$ is negative, we have $e \in \Omega_\zeta$, and if it is positive the error may start to increase and as soon as it leaves the set Ω_ζ, the update law $\dot{\hat{\gamma}}_i = \mathcal{X}_i$ becomes active. We then have $\dot{\mathcal{L}} \le -k_v||v||^2 - \sum_{i=1}^n ||r_i - v_i||_1$, and hence e_i decreases.

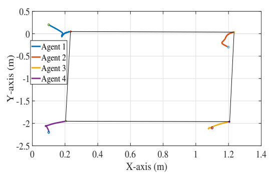
IV. ILLUSTRATIVE EXAMPLE

A. Single integrators

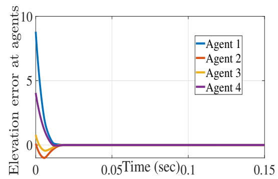
For single integrators we have 4 agents in 2-D, and $\mathcal{E} = \{(1,2),(2,3),(3,4),(1,4),(1,3)\}$. The desired elevation constraints are $f_{12}^* = f_{34}^* = 20, f_{14}^* = f_{23}^* = 10, f_{13}^* = 10\sqrt{5}$. The initial positions of the agents are $p_1(0) = [0.1 \ \ 0.2]^T, p_2(0) = [1.2 \ \ -0.3]^T, p_3(0) = [1.1 \ \ -2.1]^T$, and $p_4(0) = [0.1 \ \ -2.2]^T$. The disturbances are $\delta_1 = [0.2\text{sin}(2\pi t) \ \ e^{-3t}]^T, \delta_2 = [0.3\text{sin}(2\pi t) \ \ 0.4\text{cos}(\pi t)]^T, \delta_3 = [0.5\text{sin}(3\pi t) \ \ \text{cos}(2\pi t)]^T, \delta_4 = [e^{-3t} \ \ 0.3\text{cos}(\pi t)]^T$. The local frame of the i-th agent has $\frac{(i-1)\pi}{4}$ rotation about the global frame of reference.The control parameters are $\bar{k} = 10, \beta = 0.5$. Also, $h_c = 0.1$m. Fig. 2(a) shows the trajectories of the agents converging to the desired shape in finite time. Fig. 2(b) shows the plot of elevation constraint errors for different agents which go to zero around 0.017 sec. In Fig. 2(c) the sum of all edge errors are plotted for different parameter values (i.e. $\bar{k}$ and β). Fig. 2(d) shows persistent oscillations in error when control law in [17] is used instead.

B. Double integrators

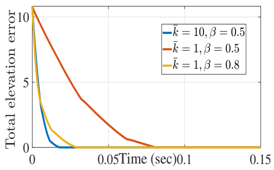
For double integrators we have 5 agents in 3-D, and $\mathcal{E} = \{(1,2),(2,3),(3,1),(1,4),(2,4),(3,4),(1,5),(2,5),(3,5)\}$. The desired elevation constraints are $f_{ij}^* = 10, \ \forall(i,j) \in \mathcal{E}$. The initial positions of the agents are $p_1(0) = [0.1 \ \ -0.2 \ \ 0.1]^T, p_2(0) = [0.8 \ \ -0.1 \ \ 0.1]^T, p_3(0) = [1.2 \ \ -1.2 \ \ -0.1]^T, p_4(0) = [0.2 \ \ -1.15 \ \ -0.2]^T$, and $p_5(0) = [0.5 \ \ -0.5 \ \ 1.1]^T$, and the corresponding disturbances are $\delta_1 = [0.3\text{sin}(2\pi t) \ \ e^{-5t} \ \ 0.2\text{cos}(\pi t)]^T, \delta_2 = [0.4\text{cos}(2\pi t) \ \ \ e^{-4t} \ \ \ 0.3\text{sin}(2\pi t)]^T, \delta_3 =$



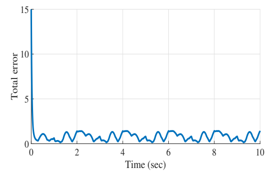
(a) Trajectories for single integrators



(b) Elevation errors at agents

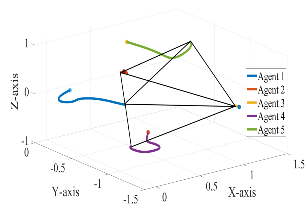


(c) Total error in present case

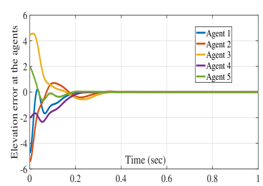


(d) Total error with [17]

Fig. 2. (a) Single integrators achieving formation in finite time, (b) Elevation errors $\sum_{j\in\mathcal{N}_i} e_{ij}$, $i \in \mathcal{V}$, (c) Elevation error ($\sum_{k\in\mathcal{E}} |e_k|$) for different parameters, (d) Elevation error ($\sum_{k\in\mathcal{E}} |e_k|$) with [17].



(a) Trajectories for double integrators



(b) Elevation errors at agents

Fig. 3. (a) Double integrators achieving the formation, (b) Elevation errors $\sum_{j\in\mathcal{N}_i} e_{ij}$ of different agents $i \in \mathcal{V}$ with $k_p = 20$, $k_v = 30$.

$[e^{-5t} \ 0 \ 0.4\sin(\pi t)]^T, \delta_4 = [0.5\cos(\pi t) \ e^{-5t} \ 0]^T, \delta_5 = [0.3\sin(2\pi t) \ 0.2\cos(\pi t) \ e^{-5t}]^T$. The local frame of the i-th agent has a rotation of $\frac{i\pi}{6}$, and the axis of rotation for 1-st and 2-nd agent is x-axis, i.e., $[1 \ 0 \ 0]^T$, the agent 3 and 4 is y-axis, i.e., $[0 \ 1 \ 0]^T$, and the 5-th agent is z-axis, i.e., $[0 \ 0 \ 1]^T$. Also, $r_c = 0.1m$. Fig. 3(a) shows the trajectories of the agents converging to the desired shape, and Fig. 3(b) shows the elevation errors of the agents converging to zero.

V. CONCLUSIONS

In this paper, formation control for single and double integrators was studied in presence of bounded disturbances. The control laws used only bearing measurements in agents' local frame, and an adaptive gain was used to address time-varying disturbances with unknown upper bounds. A minimally elevation-angle rigid graph modeled the interaction among the agents. Local finite time stability result was obtained for single integrators, and local asymptotic stability was obtained for double integrators. Obtaining global stability results in this setup, as well as finite time convergent laws for double integrators are future research goals.

REFERENCES

[1] L. Brinón-Arranz, A. Renzaglia, and L. Schenato, "Multirobot symmetric formations for gradient and hessian estimation with application to source seeking," *IEEE Transactions on Robotics*, vol. 35, no. 3, pp. 782–789, 2019.

[2] B. D. Anderson, C. Yu, B. Fidan, and J. M. Hendrickx, "Rigid graph control architectures for autonomous formations," *IEEE Control Systems Magazine*, vol. 28, no. 6, pp. 48–63, 2008.

[3] K.-K. Oh, M.-C. Park, and H.-S. Ahn, "A survey of multi-agent formation control," *Automatica*, vol. 53, pp. 424–440, 2015.

[4] S. Zhao and D. Zelazo, "Bearing rigidity theory and its applications for control and estimation of network systems: Life beyond distance rigidity," *IEEE Control Systems Magazine*, vol. 39, no. 2, pp. 66–83, 2019.

[5] D. Van Vu, M. H. Trinh, P. D. Nguyen, and H.-S. Ahn, "Distance-based formation control with bounded disturbances," *IEEE Control Systems Letters*, vol. 5, no. 2, pp. 451–456, 2020.

[6] Z. Sun, S. Mou, M. Deghat, B. D. Anderson, and A. S. Morse, "Finite time distance-based rigid formation stabilization and flocking," *IFAC Proceedings Volumes*, vol. 47, no. 3, pp. 9183–9189, 2014.

[7] S. Zhao and D. Zelazo, "Bearing rigidity and almost global bearing-only formation stabilization," *IEEE Transactions on Automatic Control*, vol. 61, no. 5, pp. 1255–1268, 2015.

[8] S. Zhao, Z. Li, and Z. Ding, "Bearing-only formation tracking control of multiagent systems," *IEEE Transactions on Automatic Control*, vol. 64, no. 11, pp. 4541–4554, 2019.

[9] M. H. Trinh and H.-S. Ahn, "Finite-time bearing-based maneuver of acyclic leader-follower formations," *IEEE Control Systems Letters*, vol. 6, pp. 1004–1009, 2021.

[10] Z. Tang, R. Cunha, T. Hamel, and C. Silvestre, "Bearing-only formation control under persistence of excitation," in *2020 59th IEEE Conference on Decision and Control (CDC)*. IEEE, 2020, pp. 4011–4016.

[11] C. Garanayak and D. Mukherjee, "Distributed fixed-time orientation synchronization with application to formation control," in *2021 60th IEEE Conference on Decision and Control (CDC)*. IEEE, 2021, pp. 7130–7135.

[12] J. Zhao, X. Yu, X. Li, and H. Wang, "Bearing-only formation tracking control of multi-agent systems with local reference frames and constant-velocity leaders," *IEEE Control Systems Letters*, vol. 5, no. 1, pp. 1–6, 2020.

[13] Q. Van Tran, M. H. Trinh, D. Zelazo, D. Mukherjee, and H.-S. Ahn, "Finite-time bearing-only formation control via distributed global orientation estimation," *IEEE Transactions on Control of Network Systems*, vol. 6, no. 2, pp. 702–712, 2018.

[14] L. Chen, M. Cao, and C. Li, "Angle rigidity and its usage to stabilize planar formations," *arXiv preprint arXiv:1908.01542*, 2019.

[15] ——, "Angle rigidity and its usage to stabilize multi-agent formations in 2d," *IEEE Transactions on Automatic Control*, 2020.

[16] N. P. K. Chan, B. Jayawardhana, and H. G. de Marina, "Angle-constrained formation control for circular mobile robots," *IEEE Control Systems Letters*, vol. 5, no. 1, pp. 109–114, 2021.

[17] L. Chen and Z. Sun, "Gradient-based bearing-only formation control: An elevation angle approach," *Automatica*, vol. 141, p. 110310, 2022.

[18] C. Garanayak and D. Mukherjee, "Bearing-only formation control using sign-elevation angle rigidity for avoiding formation ambiguities." in *2023 American Control Conference*. IEEE, 2023, pp. 2684–2689.

[19] M. H. Trinh, Q. Van Tran, D. Van Vu, P. D. Nguyen, and H.-S. Ahn, "Robust tracking control of bearing-constrained leader–follower formation," *Automatica*, vol. 131, p. 109733, 2021.

[20] Y.-B. Bae, S.-H. Kwon, Y.-H. Lim, and H.-S. Ahn, "Distributed bearing-based formation control and network localization with exogenous disturbances," *International Journal of Robust and Nonlinear Control*, vol. 32, no. 11, pp. 6556–6573, 2022.

[21] C. Godsil and G. F. Royle, *Algebraic graph theory*. Springer Science & Business Media, 2001, vol. 207.

[22] J.-B. Pomet, L. Praly *et al.*, "Adaptive nonlinear regulation: Estimation from the lyapunov equation," *IEEE Transactions on automatic control*, vol. 37, no. 6, pp. 729–740, 1992.

[23] D. Shevitz and B. Paden, "Lyapunov stability theory of nonsmooth systems," *IEEE Transactions on Automatic control*, vol. 39, no. 9, pp. 1910–1914, 1994.

[24] G. H. Hardy, J. E. Littlewood, and G. Pólya, *Inequalities*. Cambridge university press, 1952.

[25] H. K. Khalil, "Nonlinear systems(3rd edn)," 2002.