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Collision Avoidance in Longitudinal Platooning: Graceful Degradation and Adaptive Designs

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***Abstract*— An externally positive system has the property of giving a nonnegative output for any nonnegative input. By making the inter-vehicle spacing a nonnegative output, this system property is significant for collision avoidance in platooning. Yet, existing platooning results based on external positivity just apply to adaptive cruise control (ACC): as ACC uses on-board sensing only, these results do not apply when on-board sensing is integrated with inter-vehicle communication, as in cooperative adaptive cruise control (CACC). This work provides an integrated external positivity design for CACC. When unreliable communication requires transitions between CACC and ACC, the design still guarantees graceful degradation in terms of collision avoidance and disturbance rejection. Such graceful transitions can be attained also in the presence of vehicle parameter uncertainty, via a suitable adaptive control design.**

I. Introduction

Longitudinal platooning refers to automated vehicles driving at desired inter-vehicle spacing. Although a wide range of platooning strategies have been proposed, they can all be categorized along two main technologies: *Adaptive Cruise Control* (ACC), only using on-board sensors like radar, tachometer, accelerometer [1], [2]; and *Cooperative Adaptive Cruise Control* (CACC), where on-board sensing is augmented by wireless inter-vehicle communication [3], [4], [5].

The additional wireless signals offer improved behaviour of CACC as compared to ACC [3], [4], e.g., in terms of string stability [6] and disturbance decoupling [7], [8]: the former refers to attenuating a disturbance propagating throughout the platoon from the preceding vehicles; the latter refers to decoupling such disturbance from the inter-vehicle spacing error. Collision avoidance is another significant behaviour in platooning: in this regard, *external positivity* has been shown as a promising property to avoid collisions [1], [2]: external positivity of a dynamic system is defined as the nonnegativity of the output (i.e., inter-vehicle distance) for any given nonnegative input (i.e., predecessor velocity). This property has been studied only for ACC; its application in CACC is potentially significant but still open.

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In practical platooning scenarios, unreliability of wireless communication may induce transitions between CACC and ACC [9], [10]: thus, notions of *graceful degradation* of platooning performance are needed to guarantee desirable properties even when CACC degenerates to ACC. Unfortunately, available graceful degradation notions only considered string stability [4], giving a first motivation for this work: studying an external positivity design for CACC with desirable properties even when CACC degenerates to ACC.

The uncertainty around vehicle dynamics gives a second motivation to the work: most platooning designs rely on the knowledge of the time needed by the engine of each vehicle to reach a desired acceleration (engine time constant). As the engine time constant is uncertain and affected by velocity, gear, vehicle load, and road slope, *adaptive designs* have been proposed that adjust the control gains online to cope with vehicle uncertainty [11], [12]. Yet, external positivity and graceful degradation remain open problems in such adaptive designs. The main contributions of this work are:

- An augmented external positivity design for CACC, where on-board sensing is augmented by inter-vehicle wireless communication;
- An integrated CACC-ACC design with graceful degradation, i.e. where collision avoidance and string stability are retained in both CACC and ACC modes;
- A novel adaptive design inspired by adaptive switched control [13], [14], [15], [16] with stability proven for arbitrary transitions between CACC and ACC modes.

The problem is presented in Section II. Section III discusses external positivity in CACC and ACC; Section IV discusses the graceful degradation. Section V presents an adaptive design to cope with vehicle uncertainty. Validations are in Section VI, with conclusions in Section VII.

Standard norms are adopted, such as the $\mathcal{L}_2$ norm ($\|v\|_2 = \left[\int_0^\infty v^\top(t)v(t)dt\right]^{1/2}$) and the $\mathcal{L}_\infty$ norm ($\|v\|_\infty = \sup_{t\geq 0}\left[v^\top(t)v(t)\right]^{1/2}$). We say that $v \in \mathcal{L}_2$ or $v \in \mathcal{L}_\infty$ whenever the signal is bounded in the corresponding norm.

II. Problem formulation

Consider a predecessor-follower system with vehicles indexed as $i-1$ (predecessor) and i (follower). Each vehicle has longitudinal dynamics

$$\begin{aligned} \dot{s}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= a_i(t), \\ \tau_i \dot{a}_i(t) &= -a_i(t) + u_i(t), \end{aligned} \tag{1}$$

where s_i, v_i, $a_i \in \mathbb{R}$ are the longitudinal position, velocity, and acceleration of vehicle i (similar for vehicle $i-1$).

Dynamics (1) are standard in the literature [3], [17], [18], [19], with the last equation capturing the engine dynamics, i.e. the time constant $\tau_i > 0$ representing the time required by the engine to reach the desired acceleration $u_i \in \mathbb{R}$.

To establish a platoon, a spacing error is needed. The velocity-dependent spacing error with time-headway $h > 0$

$$e_i(t) = s_{i-1}(t) - s_i(t) - hv_i(t), \tag{2}$$

is commonly adopted to seek desirable properties like string stability [6]. To regulate e_i, consider an input in the form

$$u_i(t) = \underbrace{\overbrace{k_1 e_i(t) + k_2 \nu_i(t) + k_3 a_i(t)}^{\text{ACC mode}} + k_4 a_{i-1}(t)}_{\text{CACC mode}}, \tag{3}$$

with $\nu_i = v_{i-1} - v_i$ being the relative velocity, and k_1, k_2, k_3, k_4 control gains to be designed. The platooning protocol (3) includes feedback from e_i, ν_i, a_i acquired via ACC on-board sensors (radar, tachometer, accelerometer), and a_{i-1} available to vehicle i via CACC wireless communication.

Remark 1 (Transitions between modes): In line with [4], we study transitions between CACC and ACC dictated by communication failures that make a_{i-1} unavailable, cf. Fig. 1. Unavailability of a_{i-1} during failures can be equivalently represented by imposing $k_4 = 0$ in ACC mode.

The following notions of external positivity and string stability are known from the literature:

***Definition** 1 (External positivity [1]):* A system described by the transfer function $G(s)$ from input u to output y is said *externally positive* if and only if its corresponding impulse response satisfies ($\mathfrak{L}^{-1}$ is the inverse Laplace transform)

$$g(t) = \mathfrak{L}^{-1}\{G(s)\} \geq 0, \quad \forall t \geq 0.$$

Thus, assuming zero initial conditions, we have

$$u(t) \geq 0,\ \forall t \geq 0 \Rightarrow y(t) \geq 0,\ \forall t \geq 0.$$

Lemma 1 (External positivity of inter-vehicle distance [1]): For the predecessor-follower system (1), if system $G(s)$ with input a_{i-1} and output a_i (equivalently, input v_{i-1} and output v_i) is externally positive, then the system with input v_{i-1} and output $\epsilon_i = s_{i-1} - s_i$ is also externally positive.

***Definition** 2 (String stability [6]):* For the predecessor-follower system (1), let $G(s)$ be the transfer function from a_{i-1} to a_i. If $G(s)$ satisfies

$$\|G\|_\infty = \sup_{\omega \in \mathbb{R}} |G(j\omega)| \leq 1,$$

then the predecessor-follower system is string stable.

Remark 2 (Collision avoidance): As ϵ_i is the inter-vehicle distance, Definition 1 guarantees collision avoidance since, in view of Lemma 1, any $v_{i-1}(\cdot) \geq 0$ implies $\epsilon_i(\cdot) \geq 0$. With $v_{i-1}(\cdot) \geq 0$, external positivity also allows $v_i(\cdot) \geq 0$ [1].

The problem to be solved can be broken down as follows:

Problem 1: For the predecessor-follower system (1) with spacing error (2), consider:

a) *CACC mode:* design the control gains (k_1, k_2, k_3, k_4) in (3) to achieve external positivity, string stability, and

$$a_{i-1} \in \mathcal{L}_2 \cap \mathcal{L}_\infty \Rightarrow \lim_{t\to\infty} e_i(t) = 0; \tag{4}$$

b) *ACC mode* (equivalently, $k_4 = 0$)*:* design the remaining gains (k_1, k_2, k_3) in (3) to achieve external positivity, string stability, and (4);

c) *Graceful degradation:* design the gain k_4 in CACC and a set of gains (k_1, k_2, k_3) common to CACC and ACC, to guarantee properties in a)-b) in both modes and stability for arbitrary CACC/ACC transitions;

d) *Adaptive design:* with unknown τ_i in (1), design adaptive gains $(\hat{k}_1, \hat{k}_2, \hat{k}_3, \hat{k}_4)$ guaranteeing convergence to externally positive and string stable dynamics, and stability for arbitrary CACC/ACC transitions.

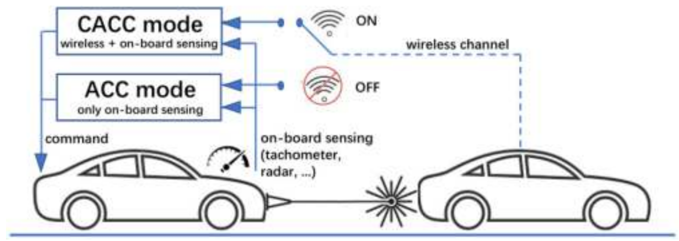


Fig. 1: Integrated CACC/ACC architecture.

Defining the state $x_i = [e_i\ \nu_i\ a_i]^\top$ and closing the loop with (3), the following error dynamics are obtained

$$\dot{x}_i(t) = \begin{bmatrix} 0 & 1 & -h \\ 0 & 0 & -1 \\ \frac{k_1}{\tau_i} & \frac{k_2}{\tau_i} & \frac{k_3-1}{\tau_i} \end{bmatrix} x_i(t) + \begin{bmatrix} 0 \\ 1 \\ \frac{k_4}{\tau_i} \end{bmatrix} a_{i-1}(t), \tag{5}$$

with $k_4 = 0$ in ACC mode. The rationale of Problem 1 is as follows: dynamics (5) show that a_{i-1} acts as a disturbance. A persistent $a_{i-1}(\cdot)$ prevents in general convergence. Thus, (4) considers asymptotic convergence when $a_{i-1}(\cdot) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$: (4) also accounts for string stability in Definition 2, since $\|G\|_\infty$ is the $\mathcal{L}_2$-induced norm from $a_{i-1} \in \mathcal{L}_2$ to $a_i \in \mathcal{L}_2$. Analogous to [11], [12], convergence to externally positive and string stable dynamics in d) guarantees that the properties in a)-b) are attained asymptotically.

III. External positivity: CACC and ACC modes

When a_{i-1} is available for control, [7], [8] proposed a CACC design inspired by disturbance decoupling[1] in the form

$$u_i(t) = k_1 e_i(t) + k_2 \nu_i(t) + \left(1 - \frac{\tau_i}{h} - hk_2\right) a_i(t) + \frac{\tau_i}{h} a_{i-1}(t), \tag{6}$$

with $k_1, k_2 > 0$ arbitrary, and the other gains designed as $k_3 := 1 - \frac{\tau_i}{h} - hk_2$, $k_4 := \frac{\tau_i}{h}$. Remarkably, the design of (6) attains other properties beyond disturbance decoupling.

Proposition 1 (Externally positive CACC): Consider the predecessor-follower system (1) with spacing error (2). The disturbance-decoupling CACC law (6) solves item a) of Problem 1 for any $h > 0$.

[1]In longitudinal platooning, disturbance decoupling aims to make the controlled variable $e_i(\cdot)$ decoupled from the disturbance $a_{i-1}(\cdot)$.

Proof: By direct calculation from (5), it can be verified that the closed loop with the law (6) gives a Hurwitz state matrix, and the transfer function from a_{i-1} to a_i is

$$G(s)=\frac{\frac{k_4}{\tau_i}s^2+\frac{k_2}{\tau_i}s+\frac{k_1}{\tau_i}}{s^3+\frac{1-k_3}{\tau_i}s^2+\frac{hk_1+k_2}{\tau_i}s+\frac{k_1}{\tau_i}}=\frac{h^{-1}}{s+h^{-1}}, \quad (7)$$

which is externally positive for any $h>0$. In fact, being $G(s)$ a first-order system, its impulse response is $g(t)=h^{-1}e^{-h^{-1}t}$, $t\geq 0$ which satisfies Definition 1.

String stability is verified by Definition 2 as (7) satisfies $\|G\|_\infty=1$. Implication (4) holds from standard input-output properties of stable systems [20, Cor. 3.3.1]. ■

Although external positivity in ACC has been shown in [1], [2], let us propose a different approach here, which later (Sect. IV) allows to integrate CACC and ACC.

Theorem 1 (Externally positive ACC): Consider the predecessor-follower system (1) with spacing error (2). The ACC control law

$$u_i(t)=k_1e_i(t)+\frac{4\tau_i}{h^2}\nu_i(t)+\left(1-\frac{k_1h^2}{4}-\frac{4\tau_i}{h}\right)a_i(t), \quad (8)$$

with $k_1>0$ arbitrary, solves b) of Problem 1 for any $h>0$.

Proof: By direct calculation from (5), it can be verified that the closed loop with the law (8) gives a Hurwitz state matrix, with transfer function from a_{i-1} to a_i being

$$G(s)=\frac{\frac{k_2}{\tau_i}s+\frac{k_1}{\tau_i}}{s^3+\frac{1-k_3}{\tau_i}s^2+\frac{hk_1+k_2}{\tau_i}s+\frac{k_1}{\tau_i}}=\frac{4h^{-2}}{(s+2h^{-1})^2}, \quad (9)$$

which is externally positive for any $h>0$. The control gains in (8) have been obtained as follows: to attain external positivity, let us seek a transfer function in the form

$$G(s)=\frac{p}{s+w}+\frac{q}{(s+w)^2}+\frac{r}{s+v}, \quad (10)$$

with (p,q,r,w,v) and (k_1,k_2,k_3) to be found by equating (10) with (9). We obtain (intermediate steps are omitted)

$$v=k_1k_2^{-1},\ w^2=k_2\tau_i^{-1},$$
$$\frac{2k_1}{\sqrt{\tau_ik_2}}+\frac{k_2}{\tau_i}=\frac{hk_1+k_2}{\tau_i}\Rightarrow k_2=\frac{4\tau_i}{h^2},$$
$$\tau_i\frac{k_1}{k_2}+2\sqrt{\tau_ik_2}=1-k_3\Rightarrow k_3=1-\frac{k_1h^2}{4}-\frac{4\tau_i}{h},$$

where k_1 remains an arbitrary parameter. Being $G(s)$ in (9) a second-order critically-damped system, external positivity is verified as the impulse response is $g(t)=h^{-2}te^{-h^{-2}t}$, $t\geq 0$. As (9) satisfies $\|G\|_\infty=1$, string stability is also attained for any $h>0$. Implication (4) holds from standard input-output properties of stable systems [20, Cor. 3.3.1]. ■

Remark 3 (String stable ACC): String stability of ACC in Theorem 1 for any $h>0$ seems to contradict known results about string instability of ACC [21], [22], [23], [24]: however, such results use $h=0$. Meanwhile, as some literature reports string instability of ACC for small h [4], [25], [26], let us remark that there is no contradiction as well: there, a different ACC control law with less degrees of freedom is adopted

$$h\dot{u}_i(t)=-u_i+k_1e_i(t)+k_2\nu_i(t)-hk_2a_i(t). \quad (11)$$

As gain $-hk_2$ in (11) is merely chosen to obtain a derivative action $k_2(\nu_i-ha_i)=k_2\dot{e}_i$, the degree of freedom of k_3 is lost in the design, making ACC string unstable for small h.

IV. External positivity: graceful degradation

Communication failures trigger switches between CACC and ACC. A question arises if CACC and ACC can be seamlessly integrated in the presence of such failures: an answer is found by exploiting the freedom to choose arbitrary $k_1,k_2>0$ in (6) and arbitrary $k_1>0$ in (8).

Theorem 2 (Seamless CACC/ACC integration): Consider the predecessor-follower system (1) with spacing error (2). The integrated CACC-ACC law

$$u_i(t)=\frac{4\tau_i}{h^3}e_i(t)+\frac{4\tau_i}{h^2}\nu_i(t)+\left(1-\frac{5\tau_i}{h}\right)a_i(t)+\begin{cases}\frac{\tau_i}{h}a_{i-1}(t) & \text{in CACC mode}\\ 0 & \text{in ACC mode}\end{cases} \quad (12)$$

solves item c) of Problem 1 for any $h>0$.

Proof: The result is obtained noting that (12) is constructed to satisfy both CACC design (6) (Proposition 1) and ACC design (8) (Theorem 1). We need $k_4=\frac{\tau_i}{h}$ in CACC and $k_4=0$ in ACC. We need $k_2=\frac{4\tau_i}{h^2}$ in both CACC and ACC, possible as k_2 is arbitrary in (6). Then, k_1 and k_3 remain to be determined. For CACC, we need

$$k_3=1-\frac{\tau_i}{h}-hk_2=1-\frac{\tau_i}{h}-\frac{4\tau_i}{h}, \quad (13)$$

where we have substituted $k_2=\frac{4\tau_i}{h^2}$. For ACC, we need

$$k_3=1-\frac{k_1h^2}{4}-\frac{4\tau_i}{h}. \quad (14)$$

As k_1 is arbitrary in (6) and (8), there is freedom to impose $\frac{k_1h^2}{4}=\frac{\tau_i}{h}$, i.e. $k_1=\frac{4\tau_i}{h^3}$. To prove stability for arbitrary transitions, closing the loop with (12) gives

$$\dot{x}_i=\underbrace{\begin{bmatrix}0 & 1 & -h\\ 0 & 0 & -1\\ \frac{4}{h^3} & \frac{4}{h^2} & -\frac{5}{h}\end{bmatrix}}_{A}x_i+\begin{cases}\begin{bmatrix}0\\1\\\frac{1}{h}\end{bmatrix}a_{i-1} & \text{in CACC}\\ \begin{bmatrix}0\\1\\0\end{bmatrix}a_{i-1} & \text{in ACC}\end{cases} \quad (15)$$

Being the state matrix A Hurwitz and common to CACC and ACC modes, a common Lyapunov function can be adopted

$$V(x_i)=\frac{1}{2}x_i^\top Px_i, \quad (16)$$

where $P>0$ solves the Lyapunov equation

$$A^\top P+PA=-Q, \qquad Q>0. \quad (17)$$

Stability of the origin of (15), not shown due to space limits, follows from standard results on common Lyapunov functions [27, Sect. 2.1], noting that $V(x_i)$ is continuous at any transition between CACC and ACC modes. ■

V. External positivity: adaptive design

In case τ_i is unknown, none of the laws (6), (8), (12) can be implemented. The works [11], [12] recently studied adaptive tools to handle uncertainty in τ_i. These tools are extended here in a switched-systems sense.

Step 1) Reference dynamics: Define a reference engine time constant $\bar{\tau}_i$, used to form vehicle reference dynamics

$$\begin{aligned} \dot{\bar{e}}_i(t) &= v_{i-1}(t) - \bar{v}_i(t) - h\bar{a}_i(t), \\ \dot{\bar{v}}_i(t) &= \bar{a}_i(t), \\ \tau_i \dot{\bar{a}}_i(t) &= -\bar{a}_i(t) + \bar{u}_i(t), \end{aligned} \tag{18}$$

with control law in the same structure as (12)

$$\bar{u}_i(t) = \frac{4\bar{\tau}_i}{h^3}\bar{e}_i(t) + \frac{4\bar{\tau}_i}{h^2}\bar{\nu}_i(t) + \left(1 - \frac{5\bar{\tau}_i}{h}\right)\bar{a}_i(t) + \begin{cases} \frac{\bar{\tau}_i}{h} a_{i-1}(t) & \text{in CACC mode} \\ 0 & \text{in ACC mode} \end{cases} \tag{19}$$

but with $\bar{\tau}_i$ in place of τ_i. Note that $v_{i-1} = \nu_{i-1} + v_i$ is available with on-board sensing. Define the reference model state $\bar{x}_i = [\bar{e}_i \ \bar{\nu}_i \ \bar{a}_i]^\top$ with $\bar{\nu}_i = v_{i-1} - \bar{v}_i$.

Step 2) Ideal model matching: By design, $\bar{x}_i$ has dynamics as in (15). We find now an ideal control u_i^* making the predecessor-follower (1) match the reference predecessor-follower dynamics (18)-(19): direct calculations give

$$u_i^*(t) = k_1^* e_i(t) + k_2^* \nu_i(t) + k_3^* a_i(t) + \begin{cases} k_4^* a_{i-1}(t) & \text{in CACC mode} \\ 0 & \text{in ACC mode} \end{cases} \tag{20}$$

with $k_1^* = \frac{4\tau_i}{h^3}$, $k_2^* = \frac{4\tau_i}{h^2}$, $k_3^* = 1 - \frac{5\tau_i}{h}$, $k_4^* = \frac{\tau_i}{h}$. With (1) and (20), x_i has also dynamics as in (15).

Step 3) Adaptive model matching: As u_i^* in (20) cannot be implemented, an adaptive version is designed hereafter.

Theorem 3 (Adaptive design): Consider the predecessor-follower system (1) with spacing error (2) and adaptive law

$$u_i(t) = \hat{k}_1(t) e_i(t) + \hat{k}_2(t) \nu_i(t) + \hat{k}_3(t) a_i(t) + \begin{cases} \hat{k}_4(t) a_{i-1}(t) & \text{in CACC mode} \\ 0 & \text{in ACC mode} \end{cases} \tag{21}$$

where $\hat{k}_1$, $\hat{k}_2$, $\hat{k}_3$, $\hat{k}_4$ are adaptive gains updated as

$$\begin{aligned} \dot{\hat{k}}_1(t) &= -\gamma_1 B^\top P \tilde{x}_i(t) e_i(t), \\ \dot{\hat{k}}_2(t) &= -\gamma_2 B^\top P \tilde{x}_i(t) \nu_i(t), \\ \dot{\hat{k}}_3(t) &= -\gamma_3 B^\top P \tilde{x}_i(t) a_i(t), \\ \dot{\hat{k}}_4(t) &= \begin{cases} -\gamma_4 B^\top P \tilde{x}_i(t) a_{i-1}(t) & \text{in CACC mode} \\ 0 & \text{in ACC mode} \end{cases} \end{aligned} \tag{22}$$

where $\gamma_1, \gamma_2, \gamma_3, \gamma_4 > 0$ are update gains, $\tilde{x}_i = x_i - \bar{x}_i$ is the error between the system and the reference model state, $B^\top = \begin{bmatrix} 0 & 1 & h^{-1} \end{bmatrix}$ and $P > 0$ solves the Lyapunov equation (17). Then, the adaptive law (21)-(22) solves item d) of Problem 1 for any $h > 0$. In particular,

$$\lim_{t \to \infty} \tilde{x}_i(t) = 0, \tag{23}$$

i.e. the adaptive closed loop converges to the externally positive and string stable reference model dynamics.

Proof: To get the dynamics of $\tilde{x}_i$, close the loop of (1) with (21): then, add and subtract the ideal control (20):

$$\dot{\tilde{x}}_i(t) = A\tilde{x}_i(t) + B\frac{1}{k_4^*}\big(\tilde{k}_1(t) e_i(t) + \tilde{k}_2(t)\nu_i(t) + \tilde{k}_3(t) a_i(t) + \begin{cases} \tilde{k}_4(t) a_{i-1}(t)\big) & \text{in CACC mode} \\ 0\big) & \text{in ACC mode} \end{cases} \tag{24}$$

with $\tilde{k}_1 = \hat{k}_1 - k_1^*$, $\tilde{k}_2 = \hat{k}_2 - k_2^*$, $\tilde{k}_3 = \hat{k}_3 - k_3^*$, $\tilde{k}_4 = \hat{k}_4 - k_4^*$. Consider the Lyapunov function (common to CACC and ACC)

$$V(\tilde{x}_i, \tilde{k}) = \frac{1}{2}\tilde{x}_i^\top P \tilde{x}_i + \frac{\tilde{k}_1^2}{2\gamma_1 k_4^*} + \frac{\tilde{k}_2^2}{2\gamma_2 k_4^*} + \frac{\tilde{k}_3^2}{2\gamma_3 k_4^*} + \frac{\tilde{k}_4^2}{2\gamma_4 k_4^*} \tag{25}$$

with $\tilde{k}^\top = \begin{bmatrix} \tilde{k}_1 & \tilde{k}_2 & \tilde{k}_3 & \tilde{k}_4 \end{bmatrix}$. The time derivative of V gives

$$\dot{V}(\tilde{x}_i, \tilde{k}) = \frac{1}{2}\tilde{x}_i^\top \big(A^\top P + PA\big)\tilde{x}_i + \frac{\tilde{k}_1 \dot{\tilde{k}}_1}{\gamma_1 k_4^*} + \frac{\tilde{k}_2 \dot{\tilde{k}}_2}{\gamma_2 k_4^*} + \frac{\tilde{k}_3 \dot{\tilde{k}}_3}{\gamma_3 k_4^*} + \frac{\tilde{k}_4 \dot{\tilde{k}}_4}{\gamma_4 k_4^*} + B^\top P \tilde{x}_i \frac{1}{k_4^*}\big(\tilde{k}_1 e_i + \tilde{k}_2 \nu_i + \tilde{k}_3 a_i + \begin{cases} \tilde{k}_4 a_{i-1}\big) & \text{in CACC mode} \\ 0\big) & \text{in ACC mode} \end{cases} \tag{26}$$

where dynamics (24) is used. Substitution of the Lyapunov equation $A^\top P + PA = -Q$ and rearranging terms gives

$$\begin{aligned} \dot{V}(\tilde{x}_i, \tilde{k}) = &-\frac{1}{2}\tilde{x}_i^\top Q \tilde{x}_i + \frac{\tilde{k}_1}{k_4^*}(B^\top P \tilde{x}_i e_i + \frac{\dot{\hat{k}}_1}{\gamma_1}) \\ &+ \frac{\tilde{k}_2}{k_4^*}(B^\top P \tilde{x}_i \nu_i + \frac{\dot{\hat{k}}_2}{\gamma_2}) + \frac{\tilde{k}_3}{k_4^*}(B^\top P \tilde{x}_i a_i + \frac{\dot{\hat{k}}_3}{\gamma_3}) \\ &+ \begin{cases} \frac{\tilde{k}_4}{k_4^*}(B^\top P \tilde{x}_i a_{i-1} + \frac{\dot{\hat{k}}_4}{\gamma_4}) & \text{in CACC mode} \\ \frac{\tilde{k}_4}{k_4^*}\frac{\dot{\hat{k}}_4}{\gamma_4} & \text{in ACC mode} \end{cases} \end{aligned} \tag{27}$$

where we have also used the fact that the ideal gains in (20) are constant. Substitution of the adaptive laws (22) gives

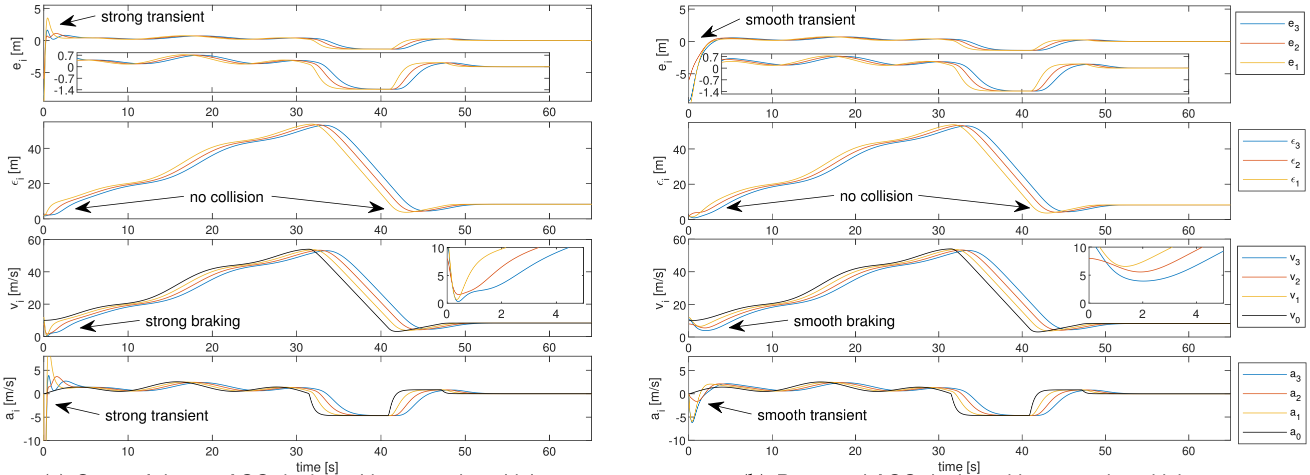
$$\dot{V}(\tilde{x}_i, \tilde{k}) = -\frac{1}{2}\tilde{x}_i^\top Q \tilde{x}_i \leq 0. \tag{28}$$

As the Lyapunov function (25) is common to CACC and ACC, it is continuous at any arbitrary transition instants. Continuity and (28) imply that the origin $(\tilde{x}_i, \tilde{k}) = 0$ is stable, i.e. the signals $\tilde{x}_i(\cdot)$, $\tilde{k}(\cdot)$ are bounded ($\tilde{x}_i, \tilde{k} \in \mathcal{L}_\infty$).

We obtain convergence of $\tilde{x}_i$ using Barbalat's Lemma. Recall that $a_{i-1}(\cdot) \in \mathcal{L}_\infty$ (cf. Problem 1), implying $\bar{x}_i \in \mathcal{L}_\infty$ as a result of the stable reference model (18). It follows that $x_i = \bar{x}_i + \tilde{x}_i \in \mathcal{L}_\infty$. Then, $\dot{\tilde{x}}_i \in \mathcal{L}_\infty$ from the error dynamics (24). To apply Barbalat's Lemma, we need $\tilde{x}_i \in \mathcal{L}_2$. This can be shown by integrating $V(t) = V(\tilde{x}_i(t), \tilde{k}(t))$ in (28)

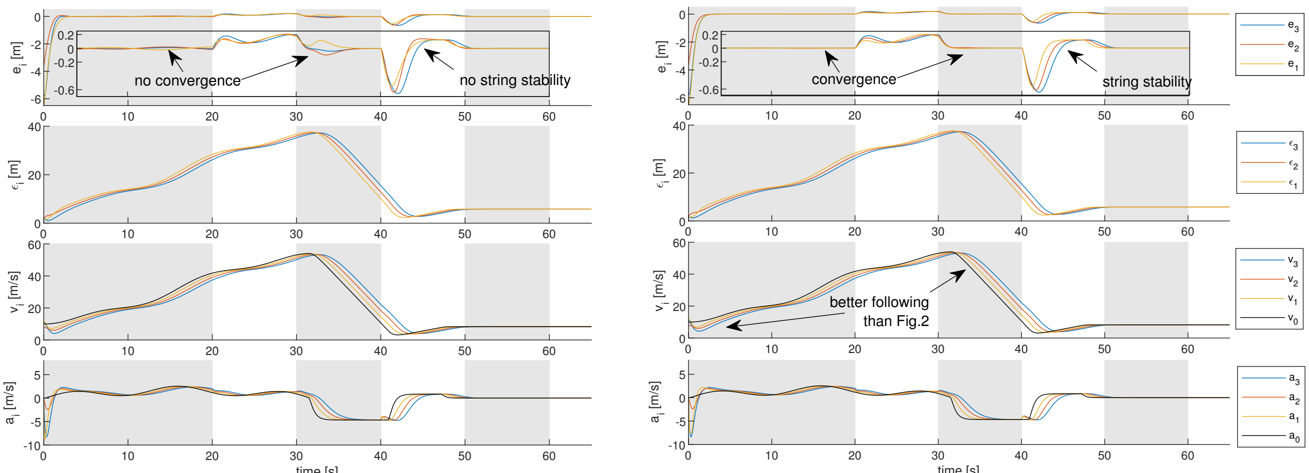
$$\frac{1}{2}\int_0^\infty \tilde{x}_i^\top(t) Q \tilde{x}_i(t)\, dt = V(0) - V_\infty, \tag{29}$$

where $V_\infty = \lim_{t \to \infty} V(t)$ is bounded. Consequently, $\tilde{x}_i \in$



(a) State-of-the-art ACC design with external positivity. (b) Proposed ACC design with external positivity.

Fig. 2: Spacing errors, inter-vehicle distances, velocities and accelerations with different ACC designs. Note the strong initial braking of the state of the art as compared to the proposed design. For both designs, positive inter-vehicle distances validate collision avoidance.



(a) Non-adaptive CACC-ACC design with uncertainty in τ_i. (b) Adaptive CACC-ACC design with uncertainty in τ_i.

Fig. 3: CACC-ACC designs with different knowledge of τ_i. Shaded areaa indicate active CACC mode. Spacing errors, inter-vehicle distances, velocities and accelerations for non-adaptive and adaptive designs. The proposed adaptive design improves the spacing error behavior.

$\mathcal{L}_2$, which implies from Barbalat's Lemma that $\tilde{x}_i \to 0$ as $t \to \infty$. This finally implies that the system state x_i converges to the state $\bar{x}_i$ of the reference model, which is externally positive and string stable by design. ∎

Remark 4 (Degradation in disturbance decoupling): Differently from CACC, no ACC design (8) *can achieve disturbance decoupling [7].*

VI. Numerical Examples

Let us first validate ACC, to allow a comparison with the state-of-the-art externally positive design [2]. For the homogeneous scenario in the numerical example of [2], i.e. $h = 1$, $\tau_i = 0.1$, $\forall i$, the state-of-the-art design is

$$u_i(t) = -k^\top \begin{bmatrix} v_i(t) \\ a_i(t) \\ \epsilon_i(t) \\ z_i(t) \end{bmatrix}, \quad \begin{aligned} k &= [18.225 \quad 1.3 \quad -5.625 \quad 50.625] \\ \dot{z}_i(t) &= hv_i(t) - \epsilon_i(t) \end{aligned} \tag{30}$$

We compare (30) with the design (12) in Theorem 2, specialized to ACC ($k_4 \equiv 0$). The other gains are $\theta_1 = 1.167$ and $\theta_1 = 0.816$. We consider a platoon with 1 leading and 3 following vehicles. We let the leader accelerate in a sinusoidal fashion with $u_0 = \sin(0.1t) + 0.5\sin(0.5t)$ up to 5π seconds, followed by a braking phase with $u_0 = -5.5$ up to 6.5π seconds, an acceleration phase with $u_0 = 1$ up to 7.5π seconds, and $u_0 = 0$ thereafter. The braking phase is designed to check collision avoidance. The initial conditions of the vehicles, reported in Table I, have also been selected to induce an initial braking phase. Although the zoomed spacing errors e_i in Fig. 2 show the same response at regime, the velocity response in Fig. 2a shows a strong initial braking phase for the state-of-the-art design (30), in contrast with the smooth response of the proposed design in Fig. 2b. The acceleration responses suggest a more comfortable behaviour for the proposed design. Yet, the positive inter-vehicle distances validate collision avoidance for both designs. As long as u_0 is non-vanishing, no ACC design can regulate e_i to zero, cf. Remark 4: regulation becomes possible at the end of the scenario when $u_0 = 0$.

We now consider an integrated CACC-ACC scenario, us-

TABLE I: Initial conditions and ideal engine constants

i	0	1	2	3
$s_i(0)$	0	-2	-4	-6
$v_i(0)$	10	12	8	11
$a_i(0)$	0	0	0	0
τ_i	0.2	0.1	0.3	0.25

ing another platoon with 1 leading and 3 following vehicles, with the same initial conditions as before and heterogeneous engine constants as in Table I and $h = 0.7$. To simulate uncertain engine time constants, we consider

1) the non-adaptive CACC-ACC design wrongly assuming that $\tau_i = \tau_0 = 0.2$, for $i \in \{1, 2, 3\}$;
2) the adaptive CACC-ACC design in Theorem 3, with $\gamma = 0.1$ and $Q = 10^3 I$.

The leading vehicle has the same u_0 as before. The comparative results are in Fig. 3 for various transitions between CACC and ACC (the shaded area indicates active CACC mode): one of such transitions occurs during braking. By comparing Fig. 3a with Fig. 3b, it can be seen that uncertainty in τ_i creates non-zero spacing errors e_i in the non-adaptive design, even when CACC is active. In the adaptive design, thanks to the disturbance decoupling property, the spacing errors e_i converge to zero when CACC is active, despite $u_0 \neq 0$: ACC makes e_i converge to zero only at the end of the scenario when $u_0 = 0$. Despite performance degradation of ACC, collision avoidance is achieved, even in the non-adaptive design. The comparison between Fig. 2 and Fig. 3 show better vehicle-following behaviour of CACC-ACC, with smoother acceleration/deceleration than the state-of-the-art ACC in [2].

VII. Conclusions

This work provided an external positivity framework for longitudinal platooning where in the transitions between cooperative adaptive cruise control (CACC) and adaptive cruise control (ACC) desirable properties of collision avoidance and string stability can be guaranteed. Such graceful transitions have been studied also in the presence of vehicle parameter uncertainty, via a suitable adaptive control design.

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