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Distributed Adaptive Formation Control for Uncertain Point Mass Agents with Mixed Dimensional Space

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Abstract— **We propose distance-based distributed adaptive formation control of point mass agents in port-Hamiltonian (pH) framework that can deal with parameter uncertainties and with mixed dimensional space (2D, 3D or mixed 2D/3D). Adaptive control mechanism is subsequently proposed to maintain formation of uncertain pH systems with unknown damping parameters. Numerical simulations are presented for both known and uncertain point mass agents in mixed 2D/3D space.**

I. Introduction

Multi-robot systems have been studied and deployed for the past decade in a wide range of robotic applications, such as, in construction works [1], in object transportation [2], and in surveillance and exploration [3]. For completing the group tasks in these applications, the coordination of these multi-robot systems can be done in a centralized or distributed fashion [4]. In the former approach, a centralized robot (or a global coordinator/orchestrator) is typically required to process information from all other robots. In the latter approach, each agent relies only on local measurement and relative information from its neighbors to accomplish the group tasks. Such distributed method provides advantages over the former approach, including resilience against single-node failure, scalability, and robustness [5].

Existing distributed formation control methods can be differentiated based on the type of relative information used to maintain the formation. Some of the well-known methods are the distance-based, position-based, and displacement-based distributed formation control methods. The distance-based formation control has been widely used due to its simplicity and its ease-of-implementation using only the local frame of reference of every agent [6]. The trade-off in the distance-based formation control is the requirement of rigidity and persistency on the underlying graph [7]. Recent works that explore the use of different relative information are bearing-based [8], [9] and internal-angle-based formation control [10].

In most literature, every agent is commonly described as a single-integrator [11] or a double-integrator [12]. The physics-based model has also been considered in the design of distributed formation control to represent the physical systems accurately. One such approach is using the port-Hamiltonian (pH) framework to describe the agent's dynamics [13]. Recent research on the distributed control for moving formation control of pH systems is discussed in [14], and a study that covers both distance-based and displacement-based approaches is discussed in [15]. The incorporation of energy in the pH framework suits well to the formation control problem as it can be formulated as a design problem of virtual mechanical spring coupling where the minimum energy (associated with the equilibrium point) corresponds to the desired formation shape. In the presence of algebraic constraints, which may arise from physical/interconnection constraints, the pH framework leads to pH differential algebraic equations (pHDAE) [16], [17].

As one of our main results, we present a distance-based formation control method for pHDAE systems that is applied to point mass agents moving in a mixed 2D and 3D space, which we will refer to as the mixed 2D/3D space. While the approach works well for known parameters, the presence of uncertainties can negatively affect the performance of closed-loop systems. For instance, temperature-dependent friction constant can greatly influence the dynamics of electro-mechanical systems involving motors that generate heat [18]. In the existing literature, adaptive control can be combined with a distributed distance-based formation control to handle system uncertainties such as unknown and bounded disturbance [19]. As our next contribution, we design an adaptive control for the proposed distance-based formation control of phDAE systems. In summary, the main novelties of our proposed approach are as follows:

- A distributed distance-based formation control design for non-linear point mass agents is defined as pHDAE systems that can deal with heterogeneous pH systems moving in a mixed 2D/3D space.
- Adaptive distance-based formation control design for pHDAE systems with uncertain non-linear damping term.

The paper is organized as follows. In Section II, we present the notation, dynamics of the multi-agent system (MAS) in pH, and a short overview on rigidity graph framework. The proposed distributed distance-based formation control strategy for pH systems is presented in Section III. Subsequently, the development of an adaptive control strategy is given in Section IV. Simulation results are presented in Section V. Finally, we conclude the paper with conclusions in Section VI.

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II. Preliminaries

As usual, we denote the n-dimensional identity matrix by I_n. For a given square matrix R, we denote $\bar{R} = R \otimes I_n$, where $n = 2$ or $n = 3$ for agents that move in the 2D space or 3D space, respectively. For a set of vectors $x_i \in \mathbb{R}^n, i \in \{1, ..., k\}$, we write the corresponding stacked vector $x \in \mathbb{R}^{kn}$ as $x \triangleq \begin{bmatrix} x_1^\top & x_2^\top & \dots & x_k^\top \end{bmatrix}^\top$. For a set of sub-matrices $x_i \in \mathbb{R}^{m \times n}, i \in \{1, ..., k\}$, we define the corresponding block diagonal matrix by $D_x = \text{diag}(x_i)_{i \in \{1,...,k\}} \in \mathbb{R}^{km \times kn}$. The space $\mathscr{L}_2(\mathbb{R}_+)$ is the space of all continuous-time signals $x : \mathbb{R}_+ \to \mathbb{R}^n$ that are square-integrable, i.e. $\int_0^\infty \|x(t)\|^2 \mathrm{d}t < \infty$. The space $\mathscr{L}_\infty(\mathbb{R}_+)$ is the space of all continuous-time signals $x : \mathbb{R}_+ \to \mathbb{R}^n$ that are essentially bounded.

A. Graph and infinitesimally rigid formation framework

Throughout this paper, we consider a graph $\mathbb{G} = (\mathscr{V}, \mathscr{E})$, where $\mathscr{V} = \{1, ..., |\mathscr{V}|\}$ is a set of nodes and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is the corresponding edge set. Each node of $\mathbb{G}$ is associated to an agent, and together with the position of all agents q, the tuple $(\mathbb{G}, q)$ defines a *framework* for the formation. In this case, each edge represents a relative measurement between two connected nodes in $\mathbb{G}$, which can be distance, bearing, internal angle or other mode of relative measurement. As described in the Introduction, this paper focuses on the distance-based formation control so that each edge $\mathscr{E}_k \in \mathscr{E}$ represents the distance between the two nodes in $\mathscr{E}_k$. For describing the distance-based formation framework, the graph $\mathbb{G}$ is represented by an undirected graph.

In the following, let us present the formulation of a rigid formation framework using the tuple (or framework) $(\mathbb{G}, q)$. Define the relative position at the edge $(i, j) = \mathscr{E}_k$ by $z_k = q_i - q_j$. Using this notation, the associated incidence matrix $B \in \mathbb{R}^{|\mathscr{V}| \times |\mathscr{E}|}$ is used to describe the relative position in all edges and is defined by $b_{ik} = -1$ whenever $i = \mathscr{E}_k^{\text{head}}$, $b_{ik} = 1$ whenever $i = \mathscr{E}_k^{\text{tail}}$, and $b_{ik} = 0$ otherwise, where $\mathscr{E}_k^{\text{head}}$ and $\mathscr{E}_k^{\text{tail}}$ are the head and the tail nodes, respectively, of the edge $\mathscr{E}_k$. Using the incidence matrix B, we can define the relative position z in a compact form by $z = \bar{B}q$, whereas defined before $\bar{B} = B \otimes I_p$ with $p = 2$ or 3 for 2D or 3D space, respectively.

Let us recall the notion of infinitesimally rigid framework, which has been discussed in detail in [20], [21]. For defining a desired formation shape using distance variables, we firstly define an *edge* function by $f_{\mathbb{G}}(q) = \text{col}_{(i,j)\in\mathscr{E}}\{\|q_i - q_j\|\}$. The rigidity matrix $R(z)$ of the framework $(\mathbb{G}, q)$ is defined by the Jacobian of the edge function $f_{\mathbb{G}}(q)$, which satisfies $R(z) = D_z^\top \bar{B}^\top$. For a given desired formation shape where the desired distance on every edges is given by $d^* := \text{col}_{(i,j)\in\mathscr{E}}\{d_{ij}^*\}$ with d_{ij}^* be the desired distance for the edge (i, j), the set of all equilibrium points that satisfy such distance constraint is $E := \{q \,|\, f_{\mathbb{G}}(q) = d^*\}$. The corresponding desired framework $(\mathbb{G}, q^*)$ with the desired distance d^* is said to be *infinitesimally rigid* if the rank of $R(z)$ is $2|\mathscr{V}| - 3$ for 2D formation, and $3|\mathscr{V}| - 6$ for 3D formation. For distance-based formation framework, the admissible infinitesimal displacement is translational and rotational motion.

B. Point-mass Agents as Port-Hamiltonian Systems

In this section, we focus on the design of distributed formation control law for heterogenous MAS described by point mass systems moving in 2D/3D space. In particular, for every agent i, we consider a pH system of the form

$$\begin{bmatrix} \dot{q}_i \\ \dot{p}_i \end{bmatrix} = \begin{bmatrix} 0 & I_{n_i} \\ -I_{n_i} & -R_i(p_i) \end{bmatrix} \begin{bmatrix} \nabla H_{q_i}(q_i, p_i) \\ \nabla H_{p_i}(q_i, p_i) \end{bmatrix} + \begin{bmatrix} 0 \\ I_{n_i} \end{bmatrix} u_i, \quad (1)$$

where $q_i \in \mathbb{R}^{n_i}$ is the generalized position in a n_i-D space with $n_i \in \{2, 3\}$, $p_i \in \mathbb{R}^{n_i}$ is the generalized momentum, H is the Hamiltonian function, and $R_i(p_i) \geq 0$ is the damping matrix. In this pH formulation, the interconnection and input matrices are given by identity matrices. For the point-mass systems, the Hamiltonian function H is given by the kinetic energy and potential energy

$$H(q, p) = \sum_{i=1}^{|\mathscr{V}|} \frac{1}{2m_i} ||p_i||^2 + \sum_{i=1}^{|\mathscr{V}|} P(q_i), \quad (2)$$

where $m_i > 0$ is the mass of agent i, $P(q_i)$ is the potential energy of agent i.

Distance-based distributed formation control design problem of pH systems with mixed 2D/3D space: For a given infinitesimally rigid framework with the agents be as in (1) and with the desired distance vector d^*, design a distributed control law u_i for all $i \in \mathscr{V}$ such that $f_{\mathbb{G}}(q(t)) \to d^*$ as $t \to \infty$.

III. Distance-Based Distributed Formation Control

Corresponding to the distance-based distributed formation control problem above, we define the distance error at every edge k by $e_k = \|z_k\|^\ell - d_k^\ell$ where d_k is the desired distance for the k-th edge, $\ell \geq 1$ can be any positive integer number [22]. By utilizing distance error e_k and relative position z, one can obtain that distance error time-derivative satisfies

$$\dot{e} = \ell \bar{D}_{\tilde{z}} D_z^\top \bar{B}^\top \dot{q}, \quad (3)$$

where $D_z = \text{diag}(z_k)$, $D_{\tilde{z}} = \text{diag}(\|z_k\|^{\ell-2})$. Following [13], we will solve the formation control problem by assigning virtual springs between paired agents. In this case, let us consider the following potential energy of the virtual springs at every edge

$$H_s = \frac{1}{2\ell} \sum_{k=1}^{|\mathscr{E}|} K_k (||z_k||^\ell - d_k^{*\ell})^2, \quad (4)$$

where K_k is a positive constant for every $k \in \{1, ..., |\mathscr{E}|\}$. This setup provides flexibility in designing the parameter ℓ. In the case where ℓ is equal to one, we have a linear virtual spring, and for values of ℓ greater than one, we have non-linear virtual springs.

Proposition III.1. *For a given infinitesimally rigid framework with the agents be as in* (1) *and with the desired* *distance vector d^*, the following distributed control law*

$$u = \nabla P(q) + \big(R(p) - R_d\big)\bar{D}_{\tilde{m}}p - \bar{B}D_z\bar{D}_{\tilde{z}}^\top D_K^\top e, \quad (5)$$

where $D_{\tilde{m}} = D_{\tilde{m}}^\top = diag(\frac{1}{m_i})$, $D_K = D_K^\top = diag(K_k)$, $R_d = diag(R_{di}) > 0$ with R_{di} be the desired damping matrix for each agent i solves the problem of distance-based distributed formation control of pH systems locally and exponentially. Particularly, for all initial conditions $(q(0), p(0))$ in the neighborhood of the desired shape with zero momentum $E \times (0, 0)$, the distance error e_k converge exponentially to zero for all $k \in \{1, ..., |\mathscr{E}|\}$, all agents' position $q_i(t)$ is bounded and converges exponentially to the desired formation shape, i.e., $f_{\mathbb{G}}(q(t)) \to d^*$ as $t \to \infty$, and all agents' momentum $p_i(t)$ converge exponentially to zero for all i in $\{1, ..., |\mathscr{V}|\}$.

The design of distributed control law in (5) is inspired by the Interconnection and Damping Assignment Passivity-Based Control (IDA-PBC) method as presented in [23]. The topic of non-adaptive distributed implementation of IDA-PBC for heterogeneous, underactuated, and non-holonomic systems has been explored in a recent study [24]. By employing the IDA-PBC approach, the closed-loop systems' Hamiltonian function, interconnection, and damping matrices can be shaped by the assignment of suitable control laws. This method is particularly noteworthy as it preserves both the passivity and the pH structure of the systems. In our proposed control law, we assign the damping of the closed-loop systems to be equal to R_d and add the virtual spring potential energy H_s to the Hamiltonian H.

Proof. Firstly, we will show the asymptotic convergence of error e and momenta p to zero by using the following Lyapunov function, which combines the Hamiltonian function (2) and the potential energy of the virtual spring (4),

$$V(p, e) = \underbrace{\frac{1}{2}p^\top \bar{D}_{\tilde{m}}p}_{H(p)} + \underbrace{\frac{1}{2\ell}e^\top D_K e}_{H_s}, \quad (6)$$

where $D_K = \text{diag}_{k\in\{1,...,|\mathscr{E}|\}}(K_k)$. A routine computation to the time-derivative of (6) along the trajectory gives

$$\begin{aligned}\dot{V} &= p^\top \bar{D}_{\tilde{m}}\dot{p} + e^\top D_K \bar{D}_{\tilde{z}} D_z^\top \bar{B}^\top \dot{q},\\ &= p^\top \bar{D}_{\tilde{m}}\big(\nabla P(q) - R\bar{D}_{\tilde{m}}p + u\big) + e^\top D_K \bar{D}_{\tilde{z}} D_z^\top \bar{B}^\top \bar{D}_{\tilde{m}}p.\end{aligned}$$

By substituting (5) to the above equation, we obtain

$$\begin{aligned}\dot{V} &= -\nabla P(q) - p^\top \bar{D}_{\tilde{m}} R \bar{D}_{\tilde{m}} p + e^\top D_K \bar{D}_{\tilde{z}} D_z^\top \bar{B}^\top \bar{D}_{\tilde{m}} p\\ &\quad + p^\top \bar{D}_{\tilde{m}}\big(\nabla P(q) + (R - R_d)\bar{D}_{\tilde{m}}p - \bar{B}D_z\bar{D}_{\tilde{z}}^\top D_K^\top e\big),\\ &= -p^\top \bar{D}_{\tilde{m}} R_d \bar{D}_{\tilde{m}} p \le -\lambda_{\min}\|p\|^2. \quad (7)\end{aligned}$$

where $\lambda_{\min}$ is the smallest eigenvalue of $\bar{D}_{\tilde{m}} R_d \bar{D}_{\tilde{m}}$. From this inequality, it follows that $p \in \mathscr{L}_2$. Furthermore, it follows from this inequality also that V is non-increasing and bounded for all time $t \ge 0$. In particular, $|e(t)| \le \frac{1}{K_{k,\min}}\sqrt{V(0)}$ for all t. In the following, we consider the initial condition $p(0)$ in the neighborhood of 0, $q(0)$ in the neighborhood of E such that $\frac{1}{K_{k,\min}}\sqrt{V(0)} < d^*_{k,\min}$. Hence, we have $|e_k(t)| < d_k^*$ for all t so that $\|z_k(t)\| > 0$ holds for all t.

By the definition of V, the inequality (7) implies that $p, e \in \mathscr{L}_\infty$. Correspondingly, from the control law (5), we have that u_i is bounded (by the boundedness of p and e and the boundedness of z follows from the relation $\|z_k\| = e_k + d_k^*$ for all k). As a result of having all closed-loop signals bounded and inequality (7), we have $p \in \mathscr{L}_2$. It follows from the state equation (1) and the boundedness of ∇H_{p_i} that $\dot{p}$ is also bounded. Consequently, we have $\ddot{V} \in \mathscr{L}_\infty$, $p \in \mathscr{L}_2$, and $\dot{p} \in \mathscr{L}_\infty$, which, by applying the generalized Barbalat's lemma [25], implies that $p(t) \to 0$ as $t \to \infty$. Let us now analyse the closed-loop system given by

$$\dot{q} = \bar{D}_{\tilde{m}}p \quad (8)$$

$$\dot{p} = -R_d\bar{D}_{\tilde{m}}p - \bar{B}D_z\bar{D}_{\tilde{z}}^\top \bar{D}_K^\top e \quad (9)$$

$$\dot{e} = \bar{D}_{\tilde{z}} D_z^\top \bar{B}^\top \bar{D}_{\tilde{m}} p \quad (10)$$

where D_z can be expressed as a function of the state q. Note that the convergence of p to zero is exponential as V is quadratic w.r.t. p, and its derivative in (7) is also bounded by a quadratic term of p. Accordingly, we can conclude from (9) that e also converges to zero exponentially and from (8) that q is bounded. □

The closed-loop systems of pH agent with control law (5) can be described as a Hamiltonian systems of the form

$$\begin{bmatrix}\dot{q}\\ \dot{p}\\ \dot{e}\end{bmatrix} = \begin{bmatrix}0 & I_n & 0\\ -I_n & -R_d & -\bar{B}D_z\bar{D}_{\tilde{z}}^\top\\ 0 & \bar{D}_{\tilde{z}}D_z^\top\bar{B}^\top & 0\end{bmatrix}\begin{bmatrix}\nabla V_q(p,e)\\ \nabla V_p(p,e)\\ \nabla V_e(p,e)\end{bmatrix}.$$

In the mixed 2D/3D pH agents, the 2D agents can only move on the (x, y)-axis, while the 3D agents can move on (x, y, z)-axis. Thus, the distributed formation control among these sub-systems is subjected to the kinematic constraint $\dot{q}_{iz} = 0$ for all 2D agent i. In this case, the 2D agent is described as a pHDAE system of the form

$$\begin{bmatrix}I & 0 & 0\\ 0 & I & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\dot{q}_i\\ \dot{p}_i\\ \dot{\lambda}_i\end{bmatrix} = \begin{bmatrix}0 & I_{ni} & 0\\ -I_{ni} & -R_i(p_i) & C_i\\ 0 & -C_i^\top & 0\end{bmatrix}\begin{bmatrix}\nabla H_{q_i}(q_i,p_i)\\ \nabla H_{p_i}(q_i,p_i)\\ \lambda_i\end{bmatrix} + \begin{bmatrix}0\\ I_{ni}\\ 0\end{bmatrix}u_i, \quad (11)$$

where $C_i = \begin{bmatrix}0 & 0 & 1\end{bmatrix}^\top$, and λ represents the Lagrange multipliers associated with the kinematic constraints. The kinematic constraint on the z-axis is given by $\dot{q}_{iz} = 0$, i.e. $C_i^\top \nabla H_{p_i} = 0$ and $C_i\lambda_i$ is the vector of constraint force in the z direction.

Proposition III.2. *Consider a set of point mass agents in a port-Hamiltonian (pH) framework composed of mixed 2D/3D agents interacted under a framework $(\mathbb{G}, q)$ that is infinitesimally rigid with the desired distance constant d^*. Let each 2D agent i described by a pHDAE as in (11) with the Hamiltonian H in (2). Then using the distributed control law as in (5) which also accounts for the kinematic constraints on the relevant axes, the closed-loop system under the presence of kinematic constraints solves the problem of distance-based distributed formation control of pH systems locally and exponentially.*

Proof. We will show the asymptotic convergence of error e and momenta p for all the phDAE agents to zero by using the Lyapunov function (6) whose time-derivative along the trajectory is given by

$$\begin{aligned}\dot V &= p^\top \bar D_{\tilde m}\dot p + e^\top D_K \bar D_{\tilde z} D_z^\top \bar B^\top \dot q,\\ &= p^\top \bar D_{\tilde m}\big(-\nabla P(q) - R\bar D_{\tilde m}p + u + C_i\lambda\big) + e^\top D_K \bar D_{\tilde z} D_z^\top \\ &\quad \bar B^\top \bar D_{\tilde m}p,\\ &= -\nabla P(q) - p^\top \bar D_{\tilde m} R \bar D_{\tilde m} p + p^\top \bar D_{\tilde m} u + p^\top \bar D_{\tilde m} C_i\lambda \\ &\quad + e^\top D_K \bar D_{\tilde z} D_z^\top \bar B^\top \bar D_{\tilde m} p.\end{aligned}$$

Due to the constraint force $C_i^\top \nabla H_{p_i} = 0$ (as noted after (11)), we have $p^\top \bar D_{\tilde m} C_i = 0$. By substituting (5) and this constraint force relation to the above equation, we obtain

$$\begin{aligned}\dot V &= -\nabla P(q) - p^\top \bar D_{\tilde m} R \bar D_{\tilde m} p + e^\top D_K \bar D_{\tilde z} D_z^\top \bar B^\top \bar D_{\tilde m} p \\ &\quad + p^\top \bar D_{\tilde m}\big(\nabla P(q) + (R - R_d)\bar D_{\tilde m} p - \bar B^\top D_z \bar D_{\tilde z} D_K^\top e\big),\\ &= -p^\top \bar D_{\tilde m} R_d \bar D_{\tilde m} p \le -\lambda_{\min}\|p\|^2. \qquad (12)\end{aligned}$$

Following the same proof of Proposition III.1, we can conclude the proof of the boundedness of all signals and local exponential convergence of $p \to 0$ and $e \to 0$ as $t \to \infty$ for formation of mixed 2D/3D pH agents. □

We remark that the closed-loop system of pHDAE agents with the control law (5) can be described as a Hamiltonian system of the form

$$\begin{bmatrix}\dot q\\ \dot p\\ \dot\lambda\\ \dot e\end{bmatrix} = \begin{bmatrix} 0 & I_n & 0 & 0\\ -I_n & -R_d & C_i & -\bar B D_z \bar D_{\tilde z}^\top\\ 0^\top & -C_i^\top & 0 & 0\\ 0 & \bar D_{\tilde z} D_z^\top \bar B^\top & 0 & 0\end{bmatrix}\begin{bmatrix}\nabla V_q(p,e)\\ \nabla V_p(p,e)\\ \lambda\\ \nabla V_e(p,e)\end{bmatrix}.$$

IV. Distributed Adaptive Distance-Based Formation Control

In practice, we may encounter parameter uncertainties in the modeling of electro-mechanical systems. A nice review on the design of adaptive control for a *single* pH system is presented in [26] that can handle uncertain pH systems. In this letter, we investigate the design of distributed adaptive formation control for pH agents where the uncertainties come from the nonlinear damping term R_i. The uncertainties in the nonlinear damping function R_i have been investigated in literature, and they can arise from various physical phenomena, such as, temperature-dependent friction constants as studied in [18]. In this case, the controller (5) cannot be implemented as it requires precise knowledge on the nonlinear damping terms. In order to address this, we propose the following modified control law

$$u = \nabla P(q) + (\Theta_R \xi_R(p) - R_d)\bar D_{\tilde m} p - \bar B^\top D_z \bar D_{\tilde z}^\top D_K^\top e, \quad (13)$$

where $\hat R = \Theta_R \xi_R(p)$ is the estimated nonlinear damping term that is split in a linear-in-the-parameter form, and it comprises of block diagonal matrices as in $R(p)$ in Proposition III.1; thus, they can be described in a distributed way. Here, the regressand and the regressor are defined by Θ and ξ, respectively, and correspondingly, we can define the linear-in-the-parameter form of the actual damping parameters by $R^* = \Theta_R^* \xi_R$. We note here that Θ_R and ξ_R is a square block matrix that combines the regressand and the regressor for each agent, respectively. The error between the estimated and real damping parameters satisfies $\tilde\Theta_R = \Theta_R - \Theta_R^*$.

Proposition IV.1. *Consider a set of point mass agents in the port-Hamiltonian (pH) framework interacted under a rigid formation framework* $(\mathbb{G}, q)$ *that is infinitesimally rigid with the desired distance constant* d^**. Suppose that each agent* i *is described by a pH system in* (1) *with uncertain nonlinear damping term* R_i *and the Hamiltonian* H *is as in* (2)*. Then the closed-loop uncertain MAS with the distributed control law defined in* (13)*, and the distributed adaptive law*

$$\dot{\tilde\Theta}_R = -\Gamma_R p p^\top \bar D_{\tilde m}^\top \xi_R^\top \bar D_{\tilde m}^\top, \quad (14)$$

where $\Gamma_R = \Gamma_R^\top > 0$ *is the adaptive gain, solves the problem of distance-based distributed formation control of pH systems locally and exponentially.*

Proof. Let us consider the following Lyapunov function

$$V(e, p, \tilde\Theta_R) = \frac{1}{2} p^\top \bar D_{\tilde m} p + \frac{1}{2\ell} e^\top D_K e + \frac{1}{2}\mathrm{tr}(\tilde\Theta_R \Gamma_R^{-1} \tilde\Theta_R^\top). \quad (15)$$

The time-derivative of (15) satisfies

$$\begin{aligned}\dot V &= -\nabla P(q) - p^\top \bar D_{\tilde m} R \bar D_{\tilde m} p + p^\top \bar D_{\tilde m} u + e^\top D_K \bar D_{\tilde z} D_z^\top \\ &\quad \bar B^\top \bar D_{\tilde m} p + \mathrm{tr}(\tilde\Theta_R \Gamma_R^{-1} \dot{\tilde\Theta}_R^\top).\end{aligned}$$

By substituting the control law (13) to the above equation, we obtain

$$\begin{aligned}\dot V &= -\nabla P(q) - p^\top \bar D_{\tilde m} R \bar D_{\tilde m} p + e^\top D_K \bar D_{\tilde z} D_z^\top \bar B^\top \bar D_{\tilde m} p \\ &\quad + p^\top \bar D_{\tilde m}\big(\nabla P(q) + (\Theta_R \xi_R - R_d)\bar D_{\tilde m} p - \bar B^\top D_z \bar D_{\tilde z}, \\ &\quad D_K^\top e\big) + \mathrm{tr}(\tilde\Theta_R \Gamma_R^{-1} \dot{\tilde\Theta}_R^\top),\\ &= p^\top \bar D_{\tilde m} \tilde\Theta_R \xi_R \bar D_{\tilde m} p + \mathrm{tr}(\tilde\Theta_R \Gamma_R^{-1} \dot{\tilde\Theta}_R^\top) - p^\top \bar D_{\tilde m} R_d \bar D_{\tilde m} p,\\ &= -p^\top \bar D_{\tilde m} R_d \bar D_{\tilde m} p + \mathrm{tr}(\tilde\Theta_R(\Gamma_R^{-1}\dot{\tilde\Theta}_R^\top + p p^\top \bar D_{\tilde m}^\top \xi_R^\top \bar D_{\tilde m}^\top)). \quad (16)\end{aligned}$$

Here we use the property $a^\top b = \mathrm{tr}(ab^\top)$ in the above computation. By substituting the adaptive laws (14), we obtain

$$\dot V = -p^\top \bar D_{\tilde m} R_d \bar D_{\tilde m} p \le -\lambda_{\min}\|p\|^2. \quad (17)$$

From (17), it follows that $p \in \mathscr{L}_2$ and $V \in \mathscr{L}_\infty$. The boundedness of V implies that $p, e, \tilde\Theta_R \in \mathscr{L}_\infty$. Because the actual parameters in Θ_R^* are bounded, the boundedness of Θ_R follows suit. It is obvious that the control law (13) is bounded by the boundedness of p, e, Θ_R. Then from state equation (1) and boundedness of ∇H_{p_i}, we can conclude that $\dot p_i$ is also bounded. By generalized Barbalat's lemma [25], $p \in \mathscr{L}_2$ and $\dot p \in \mathscr{L}_\infty$ imply that $p(t) \to 0$ as $t \to \infty$. Subsequently, let us analyse the closed-loop system

$$\dot q = \bar D_{\tilde m} p \quad (18)$$

$$\dot p = -R_d D_m p + \tilde\Theta_R \xi_R \bar D_{\tilde m} p - \bar B^\top D_z \bar D_{\tilde z} D_K^\top e \quad (19)$$

$$\dot e = \bar D_{\tilde z} D_z^\top \bar B^\top \bar D_{\tilde m} p. \quad (20)$$

As before, the convergence of p to zero is exponential since V is quadratic w.r.t. p and its derivative in (17) is also bounded by a quadratic term of p. Accordingly, we can conclude from (19) that e also converges to zero exponentially, and from (18) that q is bounded. □

In mixed 2D/3D case with uncertain pHDAE agent, we can obtain the same conclusion, which is similar to the results presented in Proposition III.2.

V. Numerical Simulations

In this section, we validate our main results in the previous two sections via numerical simulations. The formation control simulations are performed using four agents that move in mixed 2D/3D space. We consider three pH systems that can only move in 2D space (x, y)-axis, which are labeled as pH1, pH2 and pH3, respectively, and another pH system that can move in 3D space (x, y, z)-axis. For simulation setup, we consider the setup of heterogeneous agents where the initial conditions and nonlinear functions of R_i for each agent are presented in Table I.

TABLE I

pH System initial conditions and parameters.

	Init.cond. $q_i[x,y,z]$	Init.cond. $p_i[p_x,p_y,p_z]$	$R_i(p_i)$	R_{di}
pH 1	[0,0,0]	[0,0,0]	$\begin{bmatrix} \dot{q}_{1x} & 0 & 0 \\ 0 & \dot{q}_{1y} & 0 \\ 0 & 0 & \dot{q}_{1z} \end{bmatrix}$	$\begin{bmatrix} 1&0&0\\0&1&0\\0&0&1 \end{bmatrix}$
pH 2	[3,4,0]	[1,0,0]	$\begin{bmatrix} 2\dot{q}_{2x} & 0 & 0 \\ 0 & 2\dot{q}_{2y} & 0 \\ 0 & 0 & 2\dot{q}_{2z} \end{bmatrix}$	$\begin{bmatrix} 2&0&0\\0&2&0\\0&0&2 \end{bmatrix}$
pH 3	[4,3,0]	[0,1,0]	$\begin{bmatrix} 3\dot{q}_{3x} & 0 & 0 \\ 0 & 3\dot{q}_{3y} & 0 \\ 0 & 0 & 3\dot{q}_{3z} \end{bmatrix}$	$\begin{bmatrix} 3&0&0\\0&3&0\\0&0&3 \end{bmatrix}$
pH 4	[3,1,3]	[1,1,1]	$\begin{bmatrix} 4\dot{q}_{4x} & 0 & 0 \\ 0 & 4\dot{q}_{4y} & 0 \\ 0 & 0 & 4\dot{q}_{4z} \end{bmatrix}$	$\begin{bmatrix} 4&0&0\\0&4&0\\0&0&4 \end{bmatrix}$

In this research, only the 3D agent that has potential energy along the z-axis corresponding to the gravity. An exemplary application to this setup is the collaboration of autonomous ground vehicles with an autonomous aerial vehicle that conduct a joint task, such as goods transportation or retrieval. Here, we define $P(q_i) = m_i g_i^\top q_i$, and the vector gravitational $g_i = 0$ for agent $i \in \{1, 2, 3\}$, and $g_4 = \begin{bmatrix} 0 & 0 & 9.8 \end{bmatrix}^\top$. The incidence matrix B, the masses m_i, and the virtual spring constant K_k are set as

$$B = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}, \ell = 2, m_1 = 1, m_2 = 2, m_3 = 3,$$
$$m_4 = 4, K_1 = 1, K_2 = 2, K_3 = 3, K_4 = 4, K_5 = 5, K_6 = 6.$$

A. Non-adaptive pHDAE Distributed Formation Control

In this subsection, we show the first numerical result where we use the non-adaptive version of our distributed formation control for mixed 2D/3D pH systems. We set the desired distance $d_k^* = 5$ for all six edges k that represent a tetrahedron shape.

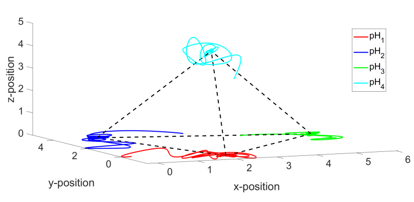

Fig. 1. Simulation result of a distributed formation control of 4 pHDAE systems in mixed 2D/3D space.

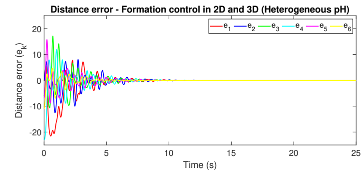

Fig. 2. The plot of distance error of 4 pHDAE systems in the numerical simulation of non-adaptive pHDAE distributed formation control.

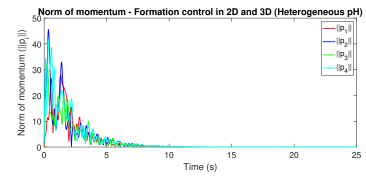

Fig. 3. The plot of momentum norm of 4 pHDAE systems in the numerical simulation of non-adaptive pHDAE distributed formation control.

It can be seen in Figure 1 that the 2D agents pH1, pH2 and pH3 remain on the 2D plane due to the kinematic constraints $\dot{q}_{iz}$, and only the pH4 that moves freely in x, y, z-axis. All agents converge to the desired tetrahedron shape as desired. The plots of the distance and momentum errors are shown in Figure 2 and Figure 3 where all of them converge to zero

B. Adaptive pHDAE Distributed Formation Control

After validating the non-adaptive version of pHDAE distributed formation control, let us evaluate the performance of the adaptive one. We use the same pH systems setup as before and use the following setup for the regressor

$$\Theta_R(0) = \text{diag}(0_{1\times12}),\ \xi_R = \text{diag}(\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4) \qquad (21)$$

with the adaptive gains given by $\Gamma_R = I_{12\times12}$.

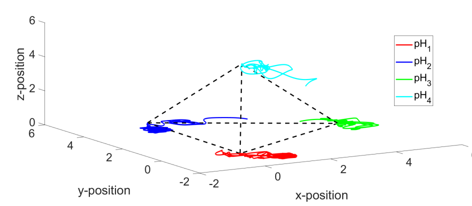

Fig. 4. Simulation result of a distributed adaptive formation control of 4 uncertain pHDAE systems in mixed 2D/3D space.

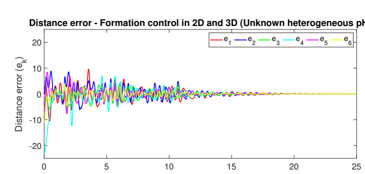

Fig. 5. The plot of distance error of 4 uncertain pHDAE systems in the numerical simulation of distributed adaptive formation control.

Figure 4 shows that all agents always move within the space that they are constrained to and converge to the desired tetrahedron shape as expected. The distance and momentum

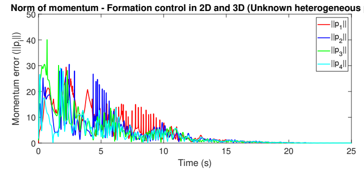

Fig. 6. The plot of momentum norm of 4 uncertain pHDAE systems in the numerical simulation of distributed adaptive formation control.

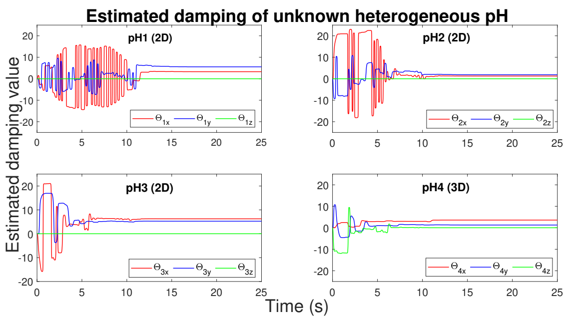

Fig. 7. The evolution of estimated damping parameters for each of 4 uncertain pHDAE systems.

error plots (Figure 5 and Figure 6) both converge to zero. Figure 7 shows the behaviour of the regressand, where the estimated damping parameters of every pH agent converge to a constant. Due to the presence of kinematic constraints, the damping parameters on the z-axis of pH1, pH2, and pH3 systems remain constant for all time.

VI. Conclusions

We present the design of distance-based distributed formation control in a port-Hamiltonian framework with mixed dimensional space. Particularly, in the presence of kinematic constraints, it leads to a pHDAE systems. When uncertain nonlinear damping terms are present in the systems, we propose the adaptive version of the distributed controller. The local exponential stability analyses are provided along with numerical simulation results using nonlinear heterogeneous pHDAE systems. For future works, we will consider the incorporation of obstacles and collision avoidance [27], [28], and the safety analysis of the closed-loop systems [29].

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