IEEE Control Systems Letters paper presented at
2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Privacy-informed Consensus-based Secondary Control in Cyber-physical DC Microgrids

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***Abstract*— This letter proposes a privacy-informed consensus-based distributed secondary control strategy for cyber-physical dc microgrids. The proposed secondary control approach relies on output masks that transform the physical states of distributed generation (DG) units to some auxiliary states by adding local perturbation signals, whose functional form and parameters are local and chosen independently by each DG unit. The proposed privacy-preserving secondary control scheme ensures voltage balancing and proportional current sharing in dc microgrids without disclosing the physical states of distributed generation units to their neighbors. Simulation case studies confirm the theoretical results of this letter.**

***Index Terms*— DC microgrids, consensus algorithms, distributed secondary control, privacy-preserving control.**

I. Introduction

CONSENSUS-based distributed and cooperative control algorithms have been frequently used in the secondary control of dc microgrids with multiple converter-interfaced distributed generation (DG) units in a distribution grid. Communication and information exchanges amongst DG units are the cornerstone of such consensus-based control schemes. The communication and information exchange however might disclose the local states of DG units, e.g., operating voltage, power generation, and incremental cost. Such an explicit exchange of DG units' state information might lead to two potential problems. First, it might impose privacy concerns, particularly if DG units' states contain private information or if DG operators aim to keep their states confidential. Second, communication and exchanging information increase the vulnerability to eavesdropping attackers who might steal information for the design of a strategic cyber-attack on dc microgrids' control systems. Hence, enhancing the privacy of DG units' state information is critical to their operational confidentiality as potential privacy threats might impede local DG units' participation in coordinated frameworks for voltage regulation and current sharing in microgrids [1].

While the secondary control methods in dc microgrids have received much attention (e.g., [2]–[6] and references therein), there has been little attempt to investigate the privacy-preserving feature of the underlying information exchange on the participating DG units. To ensure the privacy and security of exchanged information, cryptographic solutions based on one or several layers of encryption-based techniques have been widely utilized [7], [8]. The encryption techniques however are computationally costly. Also, most of the existing privacy approaches such as differential privacy rely on the idea of obfuscation to mask the true state information by adding noise with specific statistical properties to the state variables [9], [10]. Nevertheless, the existing privacy methods based the differential privacy characterizes a trade-off between privacy and accuracy [7], as corrupting the transmitted information by a large amount of injected noise might lead to the degradation of control performance and the accuracy of ensuring current sharing and voltage balancing. The problem of privacy-preserving in the consensus problem for multi-agent dynamics has been widely studied (e.g., [11]–[13] and references therein). However, these works mostly focus on the privacy protection of the initial states of the agents and consider multi-agent systems with first-order/second-order integral dynamics.

Motivated by the importance of enhancing privacy in consensus-based control of dc microgrids and the existing gaps in the literature, this letter proposes a novel secondary control mechanism for cyber-physical power-electronics-dominated dc microgrids with the potential to overcome fundamental privacy-preserving limitations of current secondary control systems and enhance them while maintaining acceptable control performance. To this end, this letter develops a consensus-based distributed secondary control approach with built-in privacy-preserving solutions. The proposed distributed secondary controller relies on output masks that are local, designed, and implemented independently by each DG unit. The output maps transform the DGs' physical states to some auxiliary states by adding local perturbation signals, whose functional form and/or numerical parameters are kept hidden from the other DG units. It will be shown that by an appropriate design of the parameters of such a privacy-informed control strategy, an acceptable performance level in voltage regulation and current sharing is achieved while enhancing the privacy of DG units' physical states against other DG units and external eavesdropping attacks. Simulation results on a dc microgrid case study verify the effectiveness of the proposed system-theoretical privacy-informed consensus-based distributed secondary control approach.

Notation. In this letter, $\mathbf{1}_n$, $\mathbf{0}_n$, $\mathbf{I}_n$, and $\mathbf{0}_{n\times m}$ are an $n\times 1$ vector of ones, an $n\times 1$ zero vector, an $n\times n$ Identity matrix, and a zero matrix of dimension $n\times m$, respectively. The sets $\mathbb{R}_+$ and $\mathbb{R}_{\geq 0}$ are respectively defined as the set of positive and

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non-negative real numbers.

Preliminary. Consider an undirected connected graph $\mathscr{G}=(\mathscr{V},\mathscr{E})$, where $\mathscr{V}=\{1,\ldots,n\}$ and $\mathscr{E}\subseteq\mathscr{V}\times\mathscr{V}$ are the set of nodes and edges, respectively. The Laplacian matrix of $\mathscr{G}$ is defined as $\mathbb{L}=\mathbb{D}-\mathbb{A}$, where $\mathbb{A}$ is the adjacency matrix and $\mathbb{D}$ is the degree matrix of $\mathscr{G}$ [14]. For such a graph, $\mathbb{L}\in\mathbb{R}^{n\times n}$ is a symmetric and positive semidefinite with a simple zero eigenvalue; moreover, $\mathbb{L}\mathbf{1}_n=\mathbf{0}_n$ [14]. The Moore-Penrose inverse, referred to as the generalized inverse of $\mathbb{L}$, denoted by $\mathbb{L}^+$, satisfies $\mathbb{L}\mathbb{L}^+=\mathbb{L}^+\mathbb{L}=\mathbf{I}_n-\frac{1}{n}\mathbf{1}_n\mathbf{1}_n^T$ and $\mathbb{L}\mathbb{L}^+\mathbb{L}=\mathbb{L}$ [15].

II. Modeling and Problem Formulation

A. Dynamic Model of Cyber-physical DC Microgrids

DC microgrids are examples of cyber-physical systems that integrate a physical layer (the layer of DG units with loads and power electronics converters and distribution lines) with a cyber layer (the layer of sensing, control, and communication). The physical layer of a dc microgrid is modeled as an undirected connected graph $\mathscr{G}_e=(\mathscr{V},\mathscr{E})$, where the node set $\mathscr{V}=\{1,\ldots,n\}$ is the set of DG units and the edge set $\mathscr{E}\subseteq\mathscr{V}\times\mathscr{V}$ is the set of distribution lines connecting different DG units [16]. The dynamics of each node $i\in\mathscr{V}$ are covered by the following equations [16]:

$$\begin{aligned} L_{t_i}\dot{I}_{t_i}(t) &= -V_i(t)+V_{dc,i}d_i(t),\\ C_{t_i}\dot{V}_i(t) &= I_{t_i}(t)-Y_iV_i(t)-\sum_{k=1}^{m}\mathscr{B}_{ik}I_k(t), \end{aligned} \tag{1}$$

where $I_{t_i}(t)$, $V_i(t)$, and $d_i(t)$ are the current of the power converter i, the voltage at Point of Common Coupling (PCC) i, and the duty cycle of the dc/dc converter i, respectively. In (1), L_{t_i} is the filter inductance of the dc-dc converter i, Y_i is the load conductance at PCC i, C_{t_i} is the filter capacitance, and $V_{dc,i}$ is the dc input voltage of the converter i. The dynamics of the line (the edge $k\in\mathscr{E}$) can be presented as follows:

$$L_k\dot{I}_k(t)=-R_kI_k(t)+\sum_{j=1}^{n}\mathscr{B}_{jk}V_j(t), \tag{2}$$

where $I_k(t)$ is the line current, (R_k,L_k) are the resistance and inductance of the line k, and $\mathscr{B}_{jk}=1$ if line k leaves DG j ($j\in\mathscr{V}$), $\mathscr{B}_{jk}=-1$ if line k enters DG j; otherwise, $\mathscr{B}_{jk}=0$.

B. Problem Formulation

The two main secondary control objectives in dc microgrids (proportional current sharing and voltage balancing) are mathematically formulated as follows [3]:

$$\left|\lim_{t\to\infty}\left(\frac{I_{t_i}(t)}{I^s_{t_i}}-\frac{I_{t_j}(t)}{I^s_{t_j}}\right)\right|\leq\varepsilon_I,\quad\forall i,j\in\mathscr{V} \tag{3a}$$

$$\left|\lim_{t\to\infty}\frac{1}{n}\mathbf{1}_n^TW(\mathbf{V}(t)-\mathbf{V}^*)\right|\leq\varepsilon_V, \tag{3b}$$

where $\mathbf{V}(t)=[V_1(t),\ldots,V_n(t)]^T\in\mathbb{R}^n$ is a vector of voltage signals at PCCs, $\mathbf{V}^*=[V_1^*,\ldots,V_n^*]^T\in\mathbb{R}^n$ is the given reference voltage vector, $I^s_{t_i}\in\mathbb{R}_+$ is the rated current of DG i, $W=\mathrm{diag}(I^s_{t_1},\ldots,I^s_{t_n})$, and ε_I and ε_V are two small non-negative scalars. Note that the acceptable range of ε_I and ε_V is determined based on available IEEE standards, e.g., [17].

Ensuring the proportional current sharing and voltage balancing in (3a) and (3b) requires a distributed consensus-based control framework where the information of each DG ($I_{t_i}(t)$, $V_i(t)$, and $I^s_{t_i}$) is exchanged with the neighboring DG units based on a neighbor-to-neighbor communication scheme. However, the communication and information exchanges amongst DG units might lead to the divulgence of DG units' states (such as the generated power and operating voltage) to other DG units, as well as external curious attackers (eavesdropping adversaries). The conventional secondary control design of dc microgrids only considers the voltage and current sharing objectives in (3) without taking into account any privacy requirements in the design phase. The main question is, how to achieve the control objectives in (3) while enhancing the privacy of the physical trajectories of DG units (e.g., $I_{t_i}(t)$ and $V_i(t)$; $i\in\mathscr{V}$)?

The main objective of this letter is to provide an answer to the aforementioned question by developing a system-theoretic privacy-informed secondary control mechanism for cyber-physical dc microgrids.

III. System-Theoretic Privacy-informed Secondary Control Design

This section presents a system-theoretical framework for privacy-informed voltage balancing and current sharing in dc microgrids. The proposed privacy-informed secondary control strategy relies on introducing time-varying output maps that function as masks, rendering the physical states of a DG unit indiscernible by the other DGs and eavesdropping adversaries. In this letter, we define the masked outputs as follows:

$$\mathbf{y}_\mathbf{x}(t)=f(\mathbf{x}(t)+p(t,\mathbf{x}(t),\pi_1,\pi_2)), \tag{4}$$

where $f(\cdot)$ is an appropriate output map acting on DGs' states $\mathbf{x}(t)$, $p(\cdot)\in\mathbb{R}^n$ is a continuous-time dynamic additive perturbation signal that has specific dynamics, π_1 is a set of parameters that are known to all DG units, and π_2 is a set of the local parameters/signals for DG units.

A. Proposed Privacy-informed Secondary Control Strategy

To achieve the control objectives in (3) while adding a privacy level to the microgrid control system, we use a privacy-informed consensus-based secondary control layer. To this end, two output maps are defined for each DG $i\in\mathscr{V}$ as follows:

$$\begin{aligned} y_{1,i}(t) &= \beta_1\frac{I_{t_i}(t)}{I^s_{t_i}}-\beta_2z_i(t)+h_{1,i}(t),\\ y_{2,i}(t) &= \beta_3z_i(t)+\beta_4\frac{I_{t_i}(t)}{I^s_{t_i}}+h_{2,i}(t), \end{aligned} \tag{5}$$

where $\beta_l\in\pi_1$, $l=1,\ldots,4$ are known to all DG units, $h_{j,i}(t):\mathbb{R}_{\geq0}\to\mathbb{R}$, $j=1,2$ is a continuously differentiable time-varying bounded additive perturbation for each DG, and $z_i(t)$ is an auxiliary control state whose dynamics are obtained as follows:

$$\tau_{z_i}\dot{z}_i(t)=-\sum_{j=1}^{n}\eta_{i,j}\left(y_{2,i}(t)-y_{2,j}(t)\right), \tag{6}$$

where $\tau_{z_i}\in\mathbb{R}_+$ and $\eta_{i,j}\in\mathbb{R}_{\geq0}$ ($\eta_{i,j}\in\mathbb{R}_+$ if DG i and DG j are connected by a communication link; otherwise, $\eta_{i,j}=0$). Note that the terms $-\beta_2 z_i(t)$ and $\beta_4 \frac{I_{t_i}(t)}{I_{t_i}^s}$ in (5) are considered with opposite signs due to the stability of closed-loop microgrid.

The additive perturbation signals $h_{1,i}(t) \in \pi_2$ and $h_{2,i}(t) \in \pi_2$ in (5) are local, i.e., they are chosen independently by each DG and kept hidden from the other DGs. Note that $h_{1,i}(t)$ and $h_{2,i}(t)$, $i \in \mathscr{V}$, do not need to be vanishing asymptotically. We then propose the following privacy-informed consensus-based secondary control scheme based on the output maps:

$$\begin{aligned} d_i(t) &= \frac{1}{V_{dc,i}} \left(V_i^* - K_{\theta_i}\left(I_{t_i}(t) - \theta_i(t)\right) - \Omega_i(t)\right), \\ \tau_{z_i}\dot{z}_i(t) &= -\sum_{j=1}^{n} \eta_{i,j}\left(y_{2,i}(t) - y_{2,j}(t)\right), \\ \tau_{\theta_i}\dot{\theta}_i(t) &= -\theta_i(t) + I_{t_i}(t), \end{aligned} \tag{7}$$

for $i \in \mathscr{V}$, where $\tau_{\theta_i} \in \mathbb{R}_+$, $K_{\theta_i} \in \mathbb{R}_+$, and $\Omega_i(t) = \frac{1}{I_{t_i}^s}\sum_{j=1}^{n} \eta_{i,j}\left(y_{1,i}(t) - y_{1,j}(t)\right)$. In (7), $z_i(t)$ and $\theta_i(t)$ are the auxiliary (non-physical) states of the controller of DG i. Note that the dynamics of $\theta_i(t)$ have been added to (7) to prevent the occurrence of oscillation in the current and voltage trajectories of dc microgrids.

Assumption 1. *The communication graph $\mathscr{G}_c = (\mathscr{V}, \mathscr{E})$ in* (7) *is assumed to be undirected and connected.*

Note that Assumption 1 is a common assumption in microgrids, see [3] and [6]. The proposed secondary control scheme in (7) has a distributed structure, as the controller of DG i requires $y_{1,j}(t)$ and $y_{2,j}(t)$, $j \neq i$, from its communication neighbors to compute the control states $z_i(t)$ and $\theta_i(t)$ used to adapt the control input $d_i(t)$ for ensuring (3a) and (3b).

B. Cyber-physical DC Microgrids

Consider a dc microgrid with n DG units with the dynamics given in (1) connecting through m power lines, modeled in (2), and augmented with the proposed secondary controller in (7). Denoting the state vectors $\mathbf{V}(t) = [V_1(t), \ldots, V_n(t)]^T$, $\mathbf{I}_t(t) = [I_{t_1}(t), \ldots, I_{t_n}(t)]^T$, $\mathbf{I}(t) = [I_1(t), \ldots, I_m(t)]^T$, $\mathbf{z}(t) = [z_1(t), \ldots, z_n(t)]^T$, $\theta(t) = [\theta_1(t), \ldots, \theta_n(t)]^T$, and mapped outputs $\mathbf{y}_1(t) = [y_{1,1}(t), \ldots, y_{1,n}(t)]^T$, and $\mathbf{y}_2(t) = [y_{2,1}(t), \ldots, y_{2,n}(t)]^T$, the cyber-physical microgrids are modeled as follows:

$$\begin{aligned} [L_t]\,\dot{\mathbf{I}}_t(t) &= -\left(\mathbf{V}(t) - \mathbf{V}^*\right) - [K_\theta]\left(\mathbf{I}_t(t) - \theta(t)\right) - W^{-1}\mathbb{L}\mathbf{y}_1(t), \\ [\tau_z]\,\dot{\mathbf{z}}(t) &= -\mathbb{L}\mathbf{y}_2(t), \\ [\tau_\theta]\,\dot{\theta}(t) &= -\theta(t) + \mathbf{I}_t(t), \\ [C_t]\,\dot{\mathbf{V}}(t) &= \mathbf{I}_t(t) - [Y]\,\mathbf{V}(t) - \mathbb{B}\mathbf{I}(t), \\ [L]\,\dot{\mathbf{I}}(t) &= -[R]\,\mathbf{I}(t) + \mathbb{B}^T\mathbf{V}(t), \end{aligned} \tag{8}$$

where $\mathbb{L} \in \mathbb{R}^{n\times n}$ is the Laplacian matrix of the communication graph associated with an adjacency matrix with elements η_{ij} and $\mathbb{B} \in \mathbb{R}^{n\times m}$ is the oriented incidence matrix of the physical layer of the dc microgrid in (1) with elements $\mathscr{B}_{ik}$. The dynamical equations of dc microgrids in (8) can be rewritten in a state-space framework as follows:

$$\left\{ \begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{V}^* + \mathbf{B}_h\mathbf{h}(t), \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) \end{aligned} \right. \tag{9}$$

where $\mathbf{x}(t) = \left[\mathbf{I}_t^T(t), \mathbf{z}^T(t), \theta^T(t), \mathbf{V}^T(t), \mathbf{I}^T(t)\right]^T \in \mathbb{R}^{4n+m}$ is the closed-loop state vector, $\mathbf{h}(t) = \left[\mathbf{h}_1^T(t), \mathbf{h}_2^T(t)\right]^T \in \mathbb{R}^{2n}$ is the perturbation signal, $\mathbf{h}_1(t) = [h_{1,1}(t), \ldots, h_{1,n}(t)]^T \in \mathbb{R}^n$, and $\mathbf{h}_2(t) = [h_{2,1}(t), \ldots, h_{2,n}(t)]^T \in \mathbb{R}^n$. The state space matrices $(\mathbf{A}, \mathbf{B}, \mathbf{B}_h, \mathbf{C})$ defined in (10) are of appropriate dimensions.

C. Analysis of Equilibria and Stability

In the following lemma, the equilibria of the cyber-physical dc microgrid in (9) in the absence of the perturbation $\mathbf{h}(t)$ will be obtained. We will later show the impact of $\mathbf{h}(t)$ on the current sharing and voltage regulation objectives in (3).

Lemma 1. *Let us assume that $\mathbf{h}(t) = \mathbf{0}_{2n}$. Under Assumption 1, the cyber-physical dc microgrid in* (8) *admits a unique equilibrium $\bar{\mathbf{x}} = \left[\bar{\mathbf{I}}_t^T, \bar{\mathbf{z}}^T, \bar{\theta}^T, \bar{\mathbf{V}}^T, \bar{\mathbf{I}}^T\right]^T$ that satisfies the following equations:*

$$\begin{aligned} &\bar{\mathbf{V}} = \Delta_V^+ \begin{bmatrix} \beta^{-1}\sigma_W W\mathbb{L}^+ W\mathbf{V}^* \\ \mathbf{1}_n^T W\mathbf{V}^* \end{bmatrix}, \ \bar{\mathbf{I}}_t = \mathbb{L}_e\bar{\mathbf{V}}, \ \bar{\theta} = \mathbb{L}_e\bar{\mathbf{V}}, \\ &\bar{\mathbf{z}} = \mathbf{1}_n z^* - \beta_4\beta_3^{-1}W^{-1}\bar{\mathbf{I}}_t, \ \bar{\mathbf{v}} = \bar{\mathbf{I}}_t, \ \bar{\mathbf{I}} = [R]^{-1}\mathbb{B}^T\bar{\mathbf{V}}, \end{aligned} \tag{11}$$

where

$$\begin{aligned} &\beta = \beta_1 + \frac{\beta_2\beta_4}{\beta_3}, \ \Delta_V = \begin{bmatrix} \mathbb{B}[R]^{-1}\mathbb{B}^T + \sigma_W[Y] + \beta^{-1}\sigma_W W\mathbb{L}^+ W \\ \mathbf{1}_n^T W \end{bmatrix}, \\ &\sigma_W = \mathbf{I}_n - W\mathbf{1}_n\left(\mathbf{1}_n^T W\mathbf{1}_n\right)^{-1}\mathbf{1}_n^T, \ \mathbb{L}_e = [Y] + \mathbb{B}[R]^{-1}\mathbb{B}^T, \\ &z^* = (\mathbf{1}_n^T[\tau_z]^{-1}\mathbf{1}_n)^{-1}\mathbf{1}_n^T[\tau_z]^{-1}(z(0) + \beta_4\beta_3^{-1}W^{-1}\bar{\mathbf{I}}_t). \end{aligned} \tag{12}$$

Furthermore, $\bar{\mathbf{V}}$ and $\bar{\mathbf{I}}_t$ satisfy the following conditions:

$$\begin{aligned} &\frac{1}{n}\mathbf{1}_n^T W\left(\bar{\mathbf{V}} - \mathbf{V}^*\right) = 0, \\ &\mathbb{L}W^{-1}\bar{\mathbf{I}}_t = -\beta^{-1}W\left(\bar{\mathbf{V}} - \mathbf{V}^*\right). \end{aligned} \tag{13}$$

Proof: The equilibria of the closed-loop dc microgrid in (8) are obtained by solving the following algebraic equations:

$$\mathbf{0}_n = \left(\mathbf{V}^* - \bar{\mathbf{V}}\right) - \beta_1 W^{-1}\mathbb{L}W^{-1}\bar{\mathbf{I}}_t + \beta_2 W^{-1}\mathbb{L}\bar{\mathbf{z}}, \tag{14a}$$

$$\mathbf{0}_n = \mathbb{L}\left(\beta_3\bar{\mathbf{z}} + \beta_4 W^{-1}\bar{\mathbf{I}}_t\right), \tag{14b}$$

$$\mathbf{0}_n = \bar{\theta} - \bar{\mathbf{I}}_t, \tag{14c}$$

$$\mathbf{0}_n = \bar{\mathbf{I}}_t - [Y]\,\bar{\mathbf{V}} - \mathbb{B}\bar{\mathbf{I}}, \tag{14d}$$

$$\mathbf{0}_m = -[R]\,\bar{\mathbf{I}} + \mathbb{B}^T\bar{\mathbf{V}}. \tag{14e}$$

where $\left[\bar{\mathbf{I}}_t^T, \bar{\mathbf{z}}^T, \bar{\theta}^T, \bar{\mathbf{V}}^T, \bar{\mathbf{I}}^T\right]^T$ are the equilibria of (8). By left multiplying (14a) by $\mathbf{1}_n^T W$, one can obtain that

$$\mathbf{1}_n^T W\left(\bar{\mathbf{V}} - \mathbf{V}^*\right) = 0. \tag{15}$$

Replacing $\mathbb{L}\bar{\mathbf{z}}$ with $-\frac{\beta_4}{\beta_3}\mathbb{L}W^{-1}\bar{\mathbf{I}}_t$ in (14a) results into:

$$\mathbb{L}W^{-1}\bar{\mathbf{I}}_t = -\beta^{-1}W\left(\bar{\mathbf{V}} - \mathbf{V}^*\right), \tag{16}$$

where β is defined in (12). By left multiplying both sides of the above equation by the generalized inverse of $\mathbb{L}$ (denoted by $\mathbb{L}^+$), it follows that:

$$\mathbb{L}(\underbrace{W^{-1}\bar{\mathbf{I}}_t - \beta^{-1}\mathbb{L}^+\left(W\left(-\bar{\mathbf{V}} + \mathbf{V}^*\right)\right)}_{I_{kernel}}) = \mathbf{0}_n.$$

As a result, I_{kernel} belongs to the kernel of $\mathbb{L}$. Therefore, $\bar{\mathbf{I}}_t$ can be obtained as follows:

$$\bar{\mathbf{I}}_t = W\mathbf{1}_n i^* + \beta^{-1}W\mathbb{L}^+\left(W\left(-\bar{\mathbf{V}} + \mathbf{V}^*\right)\right). \tag{17}$$

$$
\mathbf{A}=\left[\begin{array}{ccccc}
-\beta_1[L_t]^{-1}W^{-1}\mathbb{L}W^{-1}-[L_t]^{-1}K_\theta & \beta_2[L_t]^{-1}W^{-1}\mathbb{L} & K_\theta[L_t]^{-1} & -[L_t]^{-1} & \mathbf{0}_{n\times m}\\
-\beta_4[\tau_z]^{-1}\mathbb{L}W^{-1} & -\beta_3[\tau_z]^{-1}\mathbb{L} & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times m}\\
[\tau_v]^{-1} & \mathbf{0}_{n\times n} & -[\tau_v]^{-1} & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times m}\\
[C_t]^{-1} & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & -[C_t]^{-1}[Y] & -[C_t]^{-1}\mathbb{B}\\
\mathbf{0}_{m\times n} & \mathbf{0}_{m\times n} & \mathbf{0}_{m\times n} & [L]^{-1}\mathbb{B}^T & -[L]^{-1}[R]
\end{array}\right],\ \mathbf{B}=\left[\begin{array}{c}
[L_t]^{-1}\\ \mathbf{0}_{n\times n}\\ \mathbf{0}_{n\times n}\\ \mathbf{0}_{n\times n}\\ \mathbf{0}_{m\times n}
\end{array}\right],
$$
$$
\mathbf{B}_h=\left[\begin{array}{cc}
-[L_t]^{-1}W^{-1}\mathbb{L} & \mathbf{0}_n\\
\mathbf{0}_{n\times n} & -[\tau_z]^{-1}\mathbb{L}\\
\mathbf{0}_{n\times n} & \mathbf{0}_{n\times n}\\
\mathbf{0}_{n\times n} & \mathbf{0}_{n\times n}\\
\mathbf{0}_{m\times n} & \mathbf{0}_{m\times n}
\end{array}\right],\ \mathbf{C}=\left[\begin{array}{ccccc}
\mathbf{I}_n & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times m}\\
\mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & \mathbf{I}_n & \mathbf{0}_{n\times m}
\end{array}\right]. \tag{10}
$$

where i^* is a scalar. Left multiplying both sides of (14d) and (17) by $\mathbf{1}_n^T$ yields that

$$
\begin{aligned}
&\mathbf{1}_n^T\bar{\mathbf{I}}_t=\mathbf{1}_n^T[Y]\bar{\mathbf{V}},\\
&\mathbf{1}_n^T\bar{\mathbf{I}}_t=\mathbf{1}_n^T W\mathbf{1}_n i^*+\beta^{-1}\mathbf{1}_n^T W\mathbb{L}^+\left(W\left(-\bar{\mathbf{V}}+\mathbf{V}^*\right)\right).
\end{aligned}
$$

From the above equations, i^* is obtained as follows:

$$
i^*=(\mathbf{1}_n^T W\mathbf{1}_n)^{-1}\left(\mathbf{1}_n^T[Y]\bar{\mathbf{V}}-(\beta^{-1}\mathbf{1}_n^T W\mathbb{L}^+(W(-\bar{\mathbf{V}}+\mathbf{V}^*)))\right).
$$

Therefore, $\bar{\mathbf{I}}_t$ is obtained as follows:

$$
\bar{\mathbf{I}}_t=(\mathbf{1}_n^T W\mathbf{1}_n)^{-1}\left(W\mathbf{1}_n\mathbf{1}_n^T[Y]\bar{\mathbf{V}}\right)+\beta^{-1}\sigma_W W\mathbb{L}^+W(-\bar{\mathbf{V}}+\mathbf{V}^*), \tag{18}
$$

where σ_W is defined in (12). Moreover, from (14e) and (14d), one obtains that

$$
\bar{\mathbf{I}}=[R]^{-1}\mathbb{B}^T\bar{\mathbf{V}},\ \bar{\mathbf{I}}_t=\left([Y]+\mathbb{B}[R]^{-1}\mathbb{B}^T\right)\bar{\mathbf{V}}. \tag{19}
$$

According to (15), (18), and (19), it follows that

$$
\bar{\mathbf{V}}=\Delta_V^+\left[\begin{array}{c}\beta^{-1}\sigma_W W\mathbb{L}^+W\mathbf{V}^*\\ \mathbf{1}_n^T W\mathbf{V}^*\end{array}\right],
$$

where Δ_V is defined in (12). From (14b), we have $\bar{\mathbf{z}}=\mathbf{1}_n z^*-\beta_4\beta_3^{-1}W^{-1}\bar{\mathbf{I}}_t$, where z^* is a scalar. Moreover, it can be shown that $\mathbf{1}_n^T[\tau_z]^{-1}(\bar{\mathbf{z}}-\mathbf{z}(0))=0$. Therefore, z^* is obtained as $z^*=\frac{\mathbf{1}_n^T[\tau_z]^{-1}}{\mathbf{1}_n^T[\tau_z]^{-1}\mathbf{1}_n}(z(0)+\frac{\beta_4}{\beta_3}W^{-1}\bar{\mathbf{I}}_t)$. Moreover, in Lemma 2, it will be shown than $\mathbf{A}$ in (10) is Hurwitz; therefore, invertible. As a result, $\bar{\mathbf{x}}=-\mathbf{A}^{-1}\mathbf{B}\mathbf{V}^*$ is a unique equilibrium.

The following lemma illustrates the rigorous stability analysis of the cyber-physical dc microgrid in (9).

Lemma 2. *Under Assumption 1,* $\mathbf{A}$ *in (10) is Hurwitz.*

Proof: Let $\mathbf{h}(t)=\mathbf{0}_{2n}$ in (9). Then, it suffices to show that the equilibrium $\bar{\mathbf{x}}$ of (9) is globally asymptotically stable. To this end, we consider the following Lyapunov function:

$$
\mathscr{V}=\frac{1}{2}(\mathbf{x}(t)-\bar{\mathbf{x}})^T\mathscr{P}(\mathbf{x}(t)-\bar{\mathbf{x}}),
$$

where $\mathscr{P}=\operatorname{diag}\left([L_t],\frac{\beta_2}{\beta_4}[\tau_z],[\tau_\theta],[C_t],[L]\right)$. The time derivative of $\mathscr{V}$ along with the trajectories (9) is expressed as

$$
\dot{\mathscr{V}}=\frac{1}{2}(\mathbf{x}(t)-\bar{\mathbf{x}})^T\underbrace{(\mathbf{A}^T\mathscr{P}+\mathscr{P}\mathbf{A})}_{-\mathscr{Q}}(\mathbf{x}(t)-\bar{\mathbf{x}}), \tag{20}
$$

where $\mathscr{Q}=\operatorname{diag}(\mathscr{Q}_1,[Y],[R])$ and

$$
\mathscr{Q}_1=\left[\begin{array}{cc|c}
\beta_1W^{-1}\mathbb{L}W^{-1}+[K_\theta] & \mathbf{0}_{n\times n} & -[K_\theta]\\
\mathbf{0}_{n\times n} & \beta_2\beta_3\beta_4^{-1}\mathbb{L} & \mathbf{0}_{n\times n}\\ \hline
-[K_\theta] & \mathbf{0}_{n\times n} & [K_\theta]
\end{array}\right]. \tag{21}
$$

In the following, it will be shown that $\mathscr{Q}\succeq 0$. Due to the block diagonal structure of $\mathscr{Q}$, and the fact that $[Y]\succ 0$, and $[R]\succ 0$, it is enough to show that $\mathscr{Q}_1=\left[\begin{array}{c|c}\mathscr{Q}_{11} & \mathscr{Q}_{12}\\ \hline \mathscr{Q}_{21} & \mathscr{Q}_{22}\end{array}\right]$ in (21) is positive semi-definite. The Weyl's inequality [18] implies that $\beta_1W^{-1}\mathbb{L}W^{-1}+[K_\theta]\succ 0$. Moreover, according to the Schur Complement Lemma [19], $\mathscr{Q}_1\succeq 0$ as $\mathscr{Q}_{11}\succeq 0$ and its Schur's complement $\mathscr{Q}_{11}-\mathscr{Q}_{12}\mathscr{Q}_{22}^{-1}\mathscr{Q}_{21}\succeq 0$. As a result, $\dot{\mathscr{V}}\leq 0$. In the next step, LaSalle's Invariance Theorem [20] is used to show that $\bar{\mathbf{x}}$ of (9) is globally asymptotically stable. To this end, we define $\mathscr{S}=\left\{\mathbf{x}(t):\ \dot{\mathscr{V}}(\mathbf{x})=0\right\}$. It can be shown that $\dot{\mathscr{V}}(\mathbf{x})=0$ implies that $\mathbf{V}=\bar{\mathbf{V}}$, $\mathbf{I}=\bar{\mathbf{I}}$, $\mathbf{z}-\bar{\mathbf{z}}=\mathbf{1}_n e_z^*$, $\mathbf{I}_t=\theta$, $\mathbf{I}_t-\bar{\mathbf{I}}_t=W^{-1}\mathbf{1}_n e_I^*$, where $e_z^*\in\mathbb{R}$ and $e_I^*\in\mathbb{R}$. The closed-loop trajectories in (8) imply that $\mathbf{I}_t-\bar{\mathbf{I}}_t=\theta-\bar{\theta}=\mathbf{0}_n$. Furthermore, due to the conservation law on the dynamics of $\mathbf{z}(t)$ in (8), $\mathbf{1}_n^T[\tau_z]^{-1}\mathbf{z}(t)$ is constant $\forall t\geq 0$. Hence, $\mathbf{1}_n^T[\tau_z]^{-1}(\mathbf{z}-\bar{\mathbf{z}})=0$. It follows that $e_z^*=0$ and therefore, $\mathbf{z}=\bar{\mathbf{z}}$. Hence, the only solution that can stay identically in $\mathscr{S}$ is $\bar{\mathbf{x}}$. Therefore, $\bar{\mathbf{x}}$ of (9) is globally asymptotically stable and $\mathbf{A}$ in (10) is Hurwitz.

IV. Theoretical Analysis of Convergence

As discussed in Section III, introducing the output maps $y_{1,i}(t)$ and $y_{2,i}(t)$; $i\in\mathscr{V}$, in (5) enhances the privacy of the exchanged data in dc microgrids' secondary control scheme. However, it is required to show that the proposed privacy-informed secondary control protocol in (7) guarantees the voltage balancing and current sharing objectives in (3). The following theorem discusses the conditions on the control parameters in (7) for ensuring the control objectives (3).

Theorem 1. *Let Assumption 1 hold. The proposed consensus-based distributed control framework in* (7) *ensures the proportional current sharing and voltage balancing objectives in* (3) *provided that the value of* β *is sufficiently large.*

Proof: Denoting $\mathbf{e}_\mathbf{x}(t)=\mathbf{x}(t)-\bar{\mathbf{x}}$, the dynamics of the cyber-physical dc microgrids (9) are presented as follows:

$$
\left\{\begin{array}{l}
\dot{\mathbf{e}}_\mathbf{x}(t)=\mathbf{A}\mathbf{e}_\mathbf{x}(t)+\mathbf{B}_h\mathbf{h}(t),\\
\mathbf{e}_\mathbf{y}(t)=\mathbf{C}\mathbf{e}_\mathbf{x}(t)
\end{array}\right. \tag{22}
$$

From the above error dynamics, $\mathbf{e}_\mathbf{y}(t)$ is obtained as follows:

$$
\mathbf{e}_\mathbf{y}(t)=\mathbf{C}\exp(\mathbf{A}t)\mathbf{e}_\mathbf{x}(0)+\int_0^t\mathbf{C}\exp(\mathbf{A}(t-\tau))\mathbf{B}_h\mathbf{h}(\tau)d\tau. \tag{23}
$$

In the next step, the upper bound of $\lim_{t\to\infty}\|\mathbf{e}_\mathbf{y}(t)\|$ will be obtained. From the above equation and the use of the Cauchy-Schwartz inequality, one can obtain that

$$
\lim_{t\to\infty}\|\mathbf{e}_\mathbf{y}(t)\|\leq\lim_{t\to\infty}\left\|\int_0^t\mathbf{C}\exp(\mathbf{A}(t-\tau))\mathbf{B}_h\mathbf{h}(\tau)d\tau\right\|. \tag{24}
$$

In the above inequality, we have used the fact that $\mathbf{A}$ is a Hurwitz matrix and $\lim_{t\to\infty}\|\mathbf{C}\exp(\mathbf{A}t)\mathbf{e_x}(0)\| = 0$. Given the uniform boundedness of the perturbation signals $\mathbf{h}(t)$, there exists a constant vector $\bar{\mathbf{h}} \in \mathbb{R}^{2n}$ so that:

$$\lim_{t\to\infty}\left\|\int_0^t \mathbf{C}\exp(\mathbf{A}(t-\tau))\mathbf{B}_h\mathbf{h}(\tau)d\tau\right\| \leq \lim_{t\to\infty}\left\|\int_0^t \mathbf{C}\exp(\mathbf{A}(t-\tau))\mathbf{B}_h\bar{\mathbf{h}}d\tau\right\| \quad (25)$$

Considering (24) and (25), one can obtain that

$$\lim_{t\to\infty}\|\mathbf{e_y}(t)\| \leq \left\|-\mathbf{C}\mathbf{A}^{-1}\mathbf{B}_h\bar{\mathbf{h}}\right\|. \quad (26)$$

Applying the matrix inversion lemma [18], it can be shown that $\mathbf{C}\mathbf{A}^{-1}\mathbf{B}_h$ can be obtained as follows:

$$\mathbf{C}\mathbf{A}^{-1}\mathbf{B}_h = \begin{bmatrix} WJ^{-1}\mathbb{L} & \frac{\beta_2}{\beta_3}WJ^{-1}\mathbb{L}\mathbb{L}^{+}\mathbb{L} \\ \mathbb{L}_e^{-1}WJ^{-1}\mathbb{L} & \frac{\beta_2}{\beta_3}\mathbb{L}_e^{-1}WJ^{-1}\mathbb{L} \end{bmatrix}, \quad (27)$$

where $J = \beta\tilde{J}$ and $\tilde{J} = \mathbb{L} + \beta^{-1}W\mathbb{L}_e^{-1}W$. Note that according to Weyl's inequality [18], it can be shown that $\mathbb{L}_e \succ 0$ and $J \succ 0$; hence, they are invertible. Assuming that $\nu_1 \leq \cdots \leq \nu_n$, $0 = \rho_1 \leq \cdots \leq \rho_n$, $\tilde{\rho}_1 \leq \cdots \leq \tilde{\rho}_n$, and $\lambda_n \leq \cdots \leq \lambda_1$, are respectively the eigenvalues of $W\mathbb{L}_e^{-1}W$, $\mathbb{L}$, $\tilde{J}$, and J^{-1}, from the Weyl's inequality [18], the following conditions hold:

$$\beta^{-1}\nu_1 + \rho_i \leq \tilde{\rho}_i \leq \beta^{-1}\nu_n + \rho_i. \quad (28)$$

Consequently,

$$(\nu_n + \beta\rho_i)^{-1} \leq \lambda_i \leq (\nu_1 + \beta\rho_i)^{-1} \quad (29)$$

for $i = 1,\ldots,n$. As one can observe from (29), for a sufficiently large value of β, $\lambda_i \approx 0$, for $i = 2,\ldots,n$, and $\frac{1}{\nu_n} \leq \lambda_1 \leq \frac{1}{\nu_1}$. Let us assume that $x_1 = \frac{1}{\sqrt{n}}\mathbf{1}_n$ is an eigenvector of $\mathbb{L}$ corresponding to the zero eigenvalue $\rho_1 = 0$ and $\tilde{x}_1$ is an eigenvector of $\tilde{J}$ corresponding to the eigenvalue $\tilde{\rho}_1$. The Davis-Kahan theorem [21] bounds the angle θ_1 between x_1 and $\tilde{x}_1$ as $|\sin(\theta_1)| \leq \frac{\|\beta^{-1}W\mathbb{L}_e^{-1}W\|_2}{\min_{j\neq 1}|\tilde{\rho}_j - \rho_1|}$. Thus, for a sufficiently large value of β, θ_1 is very small. Hence, $\tilde{x}_1 = \frac{1}{\sqrt{n}}\mathbf{1}_n$ is an eigenvector of J corresponding to the eigenvalue of $\beta\tilde{\rho}_1$. Moreover, $\tilde{x}_1$ is the eigenvector of J corresponding to λ_1. The eigendecomposition of J^{-1} can be written as follows:

$$J^{-1} = \sum_{i=1}^{n}\lambda_i v_i v_i^T = \lambda_1\tilde{x}_1\tilde{x}_1^T + \sum_{i=2}^{n}\lambda_i v_i v_i^T, \quad (30)$$

where v_i is an eigenvector associated with the eigenvalue λ_i. For a sufficiently large value of β, from the above equation, we have $J^{-1} \approx \lambda_1\tilde{x}_1\tilde{x}_1^T = \frac{\lambda_1}{n}\mathbf{1}_n\mathbf{1}_n^T$. This implies that $J^{-1}\mathbb{L} \approx \mathbf{0}_{n\times n}$; thus, $\mathbf{C}\mathbf{A}^{-1}\mathbf{B}_h \approx \mathbf{0}_{2n\times 2n}$. Thus, $\lim_{t\to\infty}\|\mathbf{e_y}(t)\| \leq \varepsilon$, where ε is a small non-negative scalar. As a result, for a sufficiently large value of β, the current sharing and average voltage regulation objectives in (3a) and (3b) are guaranteed with a very small value of ε_I and ε_V irrespective of the existence of the additive perturbation signals $\mathbf{h}(t)$.

Remark 1. *In the proof of Theorem 1, it is required to show the condition on $\beta > 0$ so that $\lambda_i \approx 0$ for $i = 2,\ldots,n$. As the eigenvalues of J^{-1} are positive (according to Weyl's inequality), it is enough to show that $\lambda_2 \approx 0$ and then $\lambda_i \approx 0$ for $i = 3,\ldots,n$. According to (29), one can obtain that $(\nu_n + \beta\rho_2)^{-1} \leq \lambda_2 \leq (\nu_1 + \beta\rho_2)^{-1}$ where $\rho_2 > 0$ is the second smallest eigenvalue of $\mathbb{L}$, also called the algebraic connectivity of the communication graph [14]. For the proposed privacy-informed secondary controller in (7) with a communication network with an algebraic connectivity ρ_2, if the value of β is selected so that $(\nu_1 + \beta\rho_2)^{-1} \approx 0$, then $\lambda_2 \approx 0$.*

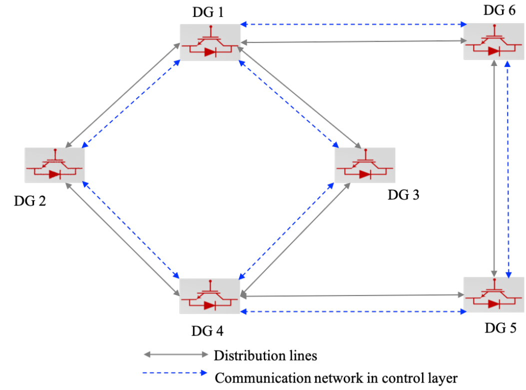

Fig. 1. Diagram of a dc microgrid with its communication network.

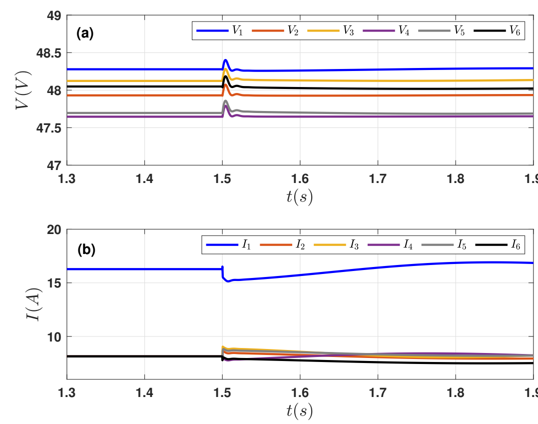

Fig. 2. (a) Voltage and (b) current trajectories of the dc microgrid.

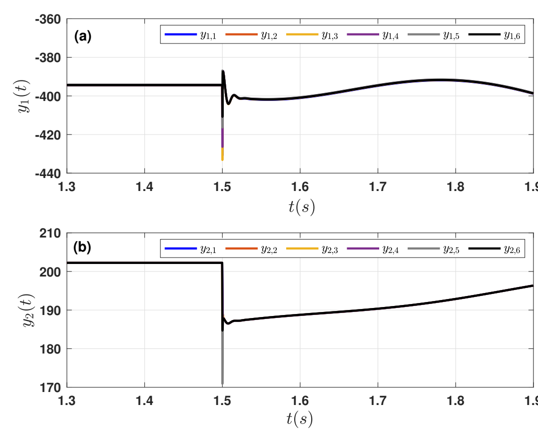

Fig. 3. Masked outputs: (a) $y_{1,i}(t)$ and (b) $y_{2,i}(t)$.

V. Simulation Results

We consider a low-voltage dc microgrid consisting of $n = 6$ DG units that are connected via $m = 7$ distribution lines whose topology is depicted in Fig. 1. The DG units include bidirectional dc/dc buck converters fed by dc voltage sources of $V_{dc,i} = 80\ V$, $i = 1,\ldots,n$. The parameters of distribution lines, loads, and converters' filters are given in [22]. It is assumed that $V_i^* = 48\ V$ and the rating current of DG 1 is twice the other DG units' rating.

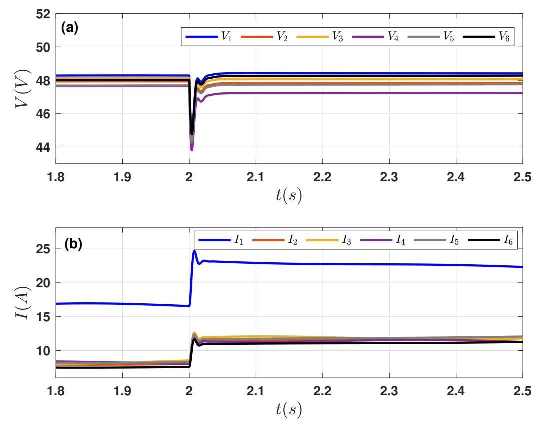

Fig. 4. Dynamic response of the dc microgrid to a load change at $t = 2\ s$: (a) PCC voltage and (b) current trajectories of dc/dc converters.

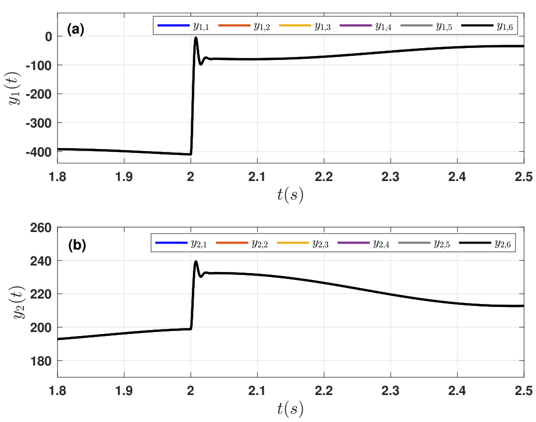

Fig. 5. Masked outputs: (a) $y_{1,i}(t)$ and (b) $y_{2,i}(t)$.

The first case study shows the performance of the proposed privacy-informed distributed control in voltage balancing and current sharing. To this end, it is assumed that the additive perturbation (local) signals $h_{1,i}(t)$ and $h_{2,i}(t)$, $i = 1, \dots, n$ are activated at $t = 1.5\ s$. These signals are chosen by each DG and are unknown to other DG units and selected to be a combination of sinusoidal signals of different amplitude, frequency, and phase. Fig. 2 shows the voltage and current trajectories of the dc microgrid while the masked outputs $y_{1,i}(t)$ and $y_{2,i}(t)$ are depicted in Fig. 3. As one can observe from Fig. 2, the activation of the additive perturbation signals affects the current sharing performance in the dc microgrid; however, the impact is not significant. Furthermore, this characterizes the trade-off between control performance specifications and the privacy-informed feature of the secondary controller. In addition, Fig. 3 highlights that the exchanged data amongst DG units are different from the current and voltage trajectories.

In the second case study, the robustness of the proposed control scheme to load change is verified. To this end, a load change is conducted at PCC 4. The dynamic response of the dc microgrid and the masked outputs are respectively shown in Fig. 4 and Fig. 5. The results demonstrate the robustness of the proposed secondary controller against load changes.

VI. Conclusion

This letter develops a system-theoretic privacy-informed consensus-based distributed secondary control protocol for cyber-physical dc microgrids. The proposed control approach relies upon a data-obfuscation-based mechanism by adding uniformly bounded local (in the sense of "local to each DG", that is, chosen and implemented independently by each DG unit's controller) perturbation signals to exchanged data. It is shown that the proposed privacy-informed secondary control approach ensures an accepted level of voltage balancing and current sharing in dc microgrids.

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