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Cooperative Output Regulation for a Class of Switched Linear Multi–Agent Systems with Intermittent Measurements

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Abstract— **This paper addresses the cooperative output regulation via error feedback for a class of switched linear multi–agent systems with asynchronous intermittent transmissions. A leader–follower multi–agent scheme is proposed, where a virtual leader is modeled as an exosystem, and the followers are represented by a class of switched linear systems. A novel hybrid distributed control law is proposed, effectively ensuring leader synchronization. The approach allows heterogeneous agent dynamics, and the switching signal between subsystems is modeled in the hybrid framework. The stability and regulation of the multi–agent system with the hybrid distributed control law are analyzed as a complete system. The effectiveness of the contribution is demonstrated through an illustrative example.**

I. Introduction

In recent years, multi–agent systems (MASs) have attracted growing attention for their effectiveness in performing complex tasks, offering cost–efficient and simplified alternatives compared to single–agent systems. Preliminary studies on the consensus and formation of leader–follower MASs are proposed in several works to overcome the problems of indirect availability of the leader's trajectory, obstacles, and different dynamics between agents [1]–[4].

An approach with high relevance within the field of the consensus problem in heterogeneous leader–follower MASs is the application of output regulation techniques with distributed control laws. This approach, identified as the cooperative output regulation (COR) problem, focuses on the leader modeled as an exosystem within the framework of output regulation theory. Subsequent studies have applied regulation theory to continuous–time MASs in diverse contexts. Distributed observers for the COR problem in linear MASs were developed in [5]–[7], and the challenges of nonlinear systems and unknown parameters were tackled in [8]. Approaches for discrete MASs are also addressed in [9], [10]. However, few results have focused on the case where inter–agent communication occurs intermittently. Although there are COR results with intermittent inter–agent communication using event–triggering approaches [11]–[14], this focuses on the design of efficient consensus mechanisms to reduce communication exchanges, which is not usually possible in systems with intermittent and low–rate transmission.

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These problems are even worse when the structural change in the system dynamics cannot be compensated by robust output control theory, this change can be modeled as a switched linear system (SLS). The works of [15], [16] have presented approaches for output regulation of SLSs, while [17] addressed the COR problem with SLSs via full–state feedback. However, most of these works are focused on the continuous–time COR and on approaches for switched linear multi–agent systems (SLMASs). To the best of the authors' knowledge, *no results are available in the literature that addresses the COR problem for SLMASs with asynchronous intermittent transmissions*. This work fills this gap by designing a novel hybrid distributed control law for the COR problem of SLMASs that only require error feedback on intermittent asynchronous measurements, bounded by a maximum time window. This novel proposal allows the agents to be heterogeneous, admitting varying dynamics, provided that the output dimension remains consistent across all agents. The switching signal between subsystems has also been modeled under the hybrid framework.

This paper is organized as follows. Section II provides preliminary results on the observer design for SLS with intermittent measurements. In Section III, the COR problem for a class of SLMASs with asynchronous intermittent transmissions is formulated, and solved in Section IV. The effectiveness of the result is demonstrated through an example in Section V. Finally, Section VI presents conclusions and future work suggestions.

A. Notations

$\mathbb{N}_0$, $\mathbb{R}$, and $\mathbb{R}^+$ represent the sets of positive integers including zero, real numbers, and non–negative real numbers, respectively. Given a set $\mathcal{X}$, $\mathrm{co}\{\mathcal{X}\}$ denotes the convex hull of $\mathcal{X}$. In partitioned symmetric matrices, the symbol $\star$ represents symmetric blocks.

II. Preliminary Results

A continuous SLS can be denoted as follows

$$\begin{aligned} \dot{x}(t) &= A_r x(t) + B_r u(t), \\ y(t) &= C x(t), \end{aligned} \tag{1}$$

where $r \in \{1, \cdots, h\}$, $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the system input and $y \in \mathbb{R}^p$ is the system output. Furthermore, $A_r \in \mathbb{R}^{n\times n}$ and $B_r \in \mathbb{R}^{n\times m}$ are known matrices, switching according to a known switching signal $\sigma_t\colon [t_0,\infty) \to \{1,\cdots h\}$, where $C \in \mathbb{R}^{p\times n}$ is a known and constant matrix.

In the case when the system's output is measured asynchronously, system (1) can be reformulated in a hybrid framework [18], allowing for modeling the subsystem switching and asynchronous output measurements as a hybrid SLS of the form

$$\begin{aligned} \dot{x}(t) &= A_r x(t) + B_r u(t), && t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ x(t_k^+) &= x(t_k), && t_k \in \mathcal{T}, \\ y(t_k) &= Cx(t_k), && t_k \in \mathcal{T}, \end{aligned} \tag{2}$$

with appropriate states and dimensions, where the output information values $y(t_k) = Cx(t_k)$ are only available for feedback. Note that the state $x(t)$ evolves according to the differential equation with initial condition $x(t_k)$ when $t \in [t_k, t_{k+1})$, called the *flow dynamics*, which undergoes a jump when $t_k \in \mathcal{T}$, called the *jump dynamics* [18].

In this setting, for complete solutions, one considers an unbounded ordered set $\mathcal{T} := \{t_0, t_1, \cdots\}$ of strictly increasing time instants $t_k \in \mathbb{R}^+$, $k \in \mathbb{N}_0$ as an *admissible time basis*, satisfying the property

$$0 < \tau_{\min} \le t_{k+1} - t_k \le \tau_{\max} < \infty, \quad \forall k \in \mathbb{N}_0, \tag{3}$$

where $\tau_{\min}$, $\tau_{\max}$ are the least rational lower and upper bounds of the sequence $\{t_{k+1} - t_k\}$. Thus, the following hybrid observer is designed

$$\begin{aligned} \dot{\hat{x}}(t) &= A_r \hat{x}(t) + B_r u(t), && t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \hat{x}(t_k^+) &= \hat{x}(t_k) + G\left(y(t_k) - C\hat{x}(t_k)\right), && t_k \in \mathcal{T}, \end{aligned} \tag{4}$$

where $r \in \{1,\cdots,h\}$ and $G \in \mathbb{R}^{n\times p}$ is a common gain matrix to be designed. Note that this observation structure allows us to estimate the state $x(t)$ even in the moments between samples by using only the measurements of the output $y(t_k)$. The observation error between (2) and (4) is defined as $\varepsilon := x - \hat{x}$ such that the hybrid dynamics

$$\begin{aligned} \dot{\varepsilon}(t) &= A_r \varepsilon(t), && t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \varepsilon(t_k^+) &= (I_n - GC)\varepsilon(t_k), && t_k \in \mathcal{T}, \end{aligned} \tag{5}$$

is asymptotically stable by designing an appropriate common matrix G. Note that the time–driven mechanism that triggers the asynchronous output measurements can be modeled according to [19] as a hybrid system of the form

$$\begin{aligned} \dot{\tau}(t) &= -1, && t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \tau(t_k^+) &\in [\tau_{\min}, \tau_{\max}], && t_k \in \mathcal{T}, \end{aligned} \tag{6}$$

with $\tau \in \mathbb{R}_{\ge 0}$ having the form of a sawtooth wave with negative ramp and variable amplitude between $\tau_{\min}$ and $\tau_{\max}$, where $\tau_{\min}$ also serves as a minimum dwell time for the switched subsystems. The switching signal $r \in \{1,\cdots,h\}$ can also be modeled as a hybrid system mechanism of the form

$$\begin{aligned} \dot{r}(t) &= 0, && t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ r(t_k^+) &\in \{1,\cdots,h\}, && t_k \in \mathcal{T}. \end{aligned} \tag{7}$$

The equation (7) exhibits the possibility that if a jump occurs in the system and it subsequently returns to the same value, as denoted by $r(t_k^+) = r(t)$, then a mode transition without switching to another subsystem occurs. Therefore, the stability analysis of the composite system (5)–(6) is demonstrated through the following Theorem.

***Theorem** 1:* Let $\mathcal{T}$ be an admissible time basis with $\tau_{\min} \le \tau_{\max}$ two given positive rational scalars. If there exist a common symmetric positive definite matrix $\varUpsilon \in \mathbb{R}^{n\times n}$ and a common gain matrix $G \in \mathbb{R}^{n\times p}$ such that the inequalities

$$(I_n - GC)^\top e^{A_r^\top \delta} \varUpsilon e^{A_r \delta}(I_n - GC) - \varUpsilon < 0 \tag{8}$$

hold for all $r \in \{1,\cdots,h\}$ and all $\delta \in [\tau_{\min}, \tau_{\max}]$, then the origin of the composite system (5)–(6) is globally uniformly asymptotically stable and the switching between subsystems occurs only during the asynchronous output measurements. $\diamond$

Proof: Case 1: Hybrid error system with no switching subsystems.

Consider the following Lyapunov function candidate for the hybrid error system (ε, τ) defined by

$$V_o(\varepsilon, \tau) = \varepsilon^\top e^{A_o^\top \tau} \varUpsilon e^{A_o \tau} \varepsilon, \tag{9}$$

with A_o being a constant matrix of the set of matrices $\{A_1,\cdots,A_h\}$, the results of [19] show that the variation of $V_o(\varepsilon,\tau)$ during flows is $\dot{V}_o(\varepsilon,\tau) = 0$, while the variation of $V_o(\varepsilon,\tau)$ during jumps is

$$\varDelta V_o(\varepsilon,\tau) = \varepsilon^\top \Big((I_n - GC)^\top e^{A_o^\top \delta} \varUpsilon e^{A_o \delta}(I_n - GC) - \varUpsilon \Big) \varepsilon,$$

for all $\delta \in [\tau_{\min}, \tau_{\max}]$. Therefore, condition (8) when $A_r = A_o$ is necessary to ensure that $\varDelta V_o(\varepsilon,\tau) < 0$ and, since $\dot{V}_o(\varepsilon,\tau) = 0$, the composite system (5)–(6) is globally uniformly asymptotically stable (GUAS) at the origin if the sufficient Lyapunov conditions for persistent jumping systems ([18], Proposition 3.24) are met. These conditions are completed if the condition $k \ge \gamma_r(t) - N_r$ is fulfilled with $\gamma_r(t) = \frac{t}{\tau_{\max}}$ and $N_r = 1$, where $k \in \mathbb{N}_0$ is the subscript of $t_k \in \mathcal{T}$, γ_r is a class–κ_∞ function, and $N_r \ge 0$ is a constant. This implies that hybrid arcs originating from $\varepsilon(0)$ during a jump with form

$$\varepsilon(t_k) = (I_n - GC)e^{A_o(t_k - t_{k-1})} \cdots (I_n - GC)e^{A_o(t_1 - t_0)}\varepsilon(0), \tag{10}$$

as well as hybrid arcs during a flow with form $\varepsilon(t) = e^{A_o(t-t_k)}\varepsilon(t_k)$ are valid solutions for (5).

Case 2: Hybrid error system with switching subsystems.

For the case of switching, a Common Lyapunov function [20, Theorem 2.1] with form (9) can be constructed for each $r \in \{1,\cdots,h\}$ subsystem, such that the set of inequalities (8) must be fulfilled for all $r \in \{1,\cdots,h\}$ and all $\delta \in [\tau_{\min}, \tau_{\max}]$. Now, for simplicity, consider the case of switching only between two subsystems a, b with switching time sequence $0 = t_0 < t_a < t_b = \infty$. Therefore, a hybrid arc that switches and jumps during the critical time point t_a has the form

$$\varepsilon(t_a) = (I_n - GC)e^{A_a(t_a - t_{a-1})} \cdots (I_n - GC)e^{A_a(t_1 - t_0)}\varepsilon(t_0), \tag{11}$$

which is a valid solution of (5) with form (10). Furthermore, the hybrid arcs evolving after this switching have the form

$$\varepsilon(t_k) = (I_n - GC)e^{A_b(t_k - t_{k-1})} \cdots (I_n - GC)e^{A_b(t_{a+1} - t_a)}\varepsilon(t_a) \tag{12}$$

during jumps or $\varepsilon(t) = e^{A_b(t-t_k)}\varepsilon(t_k)$ during flows, which are also valid solutions of (5). Now, suppose that there exists a hybrid arc with the switch of subsystems during a flow. In the absence of jumps during the evolution, the hybrid arc has the form

$$\varepsilon(t) = e^{A_b(t-t_a)}e^{A_a(t_a-t_0)}\varepsilon(0), \tag{13}$$

while for cases with jumps in the preceding subsystem, the hybrid arc takes the form

$$\varepsilon(t) = e^{A_b(t-t_a)}e^{A_a(t_a-t_{a-1})} \cdots (I_n - GC)e^{A_a(t_1-t_0)}\varepsilon(0), \tag{14}$$

which are not valid solutions for (5) due to $r(t)$ switching the subsystems only at asynchronous measurement times, as detailed in (7). Therefore, the switching between subsystems only during an asynchronous output measurement is a necessary condition for fulfilling the persistent jumping conditions. This completes the proof. ■

Remark 1: Since inequality (8) has infinite solutions due to $\delta \in [\tau_{\min}, \tau_{\max}]$ its solution can be derived by means of a polytopic overapproximation under a finite number of $f = 2^n$ linear matrix inequalities (LMIs) [19]. $\diamond$

III. THE COOPERATIVE OUTPUT REGULATION FOR SLMASs

In the following, a heterogeneous MAS is considered, consisting of a leader labeled as ‘agent 0’ and N agents referred to as ‘followers.’ Since the output of each agent can only be measured in asynchronous intermittent intervals, the dynamics of each i^{th} follower, for $i = 1, \cdots, N$, are described by the following hybrid linear system

$$\begin{aligned} \dot{x}_i(t) &= A_{r,i}x_i(t) + B_{r,i}u_i(t) + P_i w(t), && t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ x_i(t_k^+) &= x_i(t_k), && t_k \in \mathcal{T}, \\ y_i(t_k) &= C_i x_i(t_k), && t_k \in \mathcal{T}, \end{aligned} \tag{15}$$

where $x_i \in \mathbb{R}^{n_i}$ is the follower state, $u_i \in \mathbb{R}^{m_i}$ is the follower control input, and $y_i \in \mathbb{R}^p$ is the follower output. Furthermore, $A_{r,i} \in \mathbb{R}^{n_i \times n_i}$ and $B_{r,i} \in \mathbb{R}^{n_i \times m_i}$ are known matrices, switching according to a known switching signal $\sigma_t : [t_0, \infty) \to \{1, \cdots h\}$, where $r \in \{1, \cdots, h\}$ while $C_i \in \mathbb{R}^{p \times n_i}$ and $P_i \in \mathbb{R}^{n_i \times q}$ are constant and known matrices. The leader dynamics are described within the hybrid framework by the following dynamical system

$$\begin{aligned} \dot{w}(t) &= Sw(t), && t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ w(t_k^+) &= w(t_k), && t_k \in \mathcal{T}, \\ y_r(t_k) &= Qw(t_k), && t_k \in \mathcal{T}, \end{aligned} \tag{16}$$

where $w \in \mathbb{R}^q$ is the state of the virtual leader, $y_r \in \mathbb{R}^p$ is the leader reference, $S \in \mathbb{R}^{q \times q}$ is a known constant neutral matrix and $Q \in \mathbb{R}^{p \times q}$ is a known and constant matrix. Given the heterogeneous nature of the MAS, the following Assumption ensures uniformity in the dimension of the output mapping for each agent.

Assumption 1: $\operatorname{rank}(C_i) = p$, $i = 1, \cdots, N$. $\diamond$

The communication among the $N+1$ agents, when available, is represented by a directed graph $\mathcal{G}_{N+1}$, which is assumed to be free of self–loops, and with a spanning tree rooted on the virtual leader. For further details on the graph theory used, please refer to [1], [2]. Each follower is driven by the following hybrid distributed controller to ensure leader synchronization and achieve COR

$$\begin{aligned} \dot{z}_i(t) &= F_{r,i}z_i(t), && t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ z_i(t_k^+) &= E_i z_i(t_k) + G_i e_i(t_k), && t_k \in \mathcal{T}, \\ u_i(t) &= H_{r,i}z_i(t), && t \in \mathbb{R}^+, \end{aligned} \tag{17}$$

where $r \in \{1, \cdots, h\}$, $z_i \in \mathbb{R}^{\nu_i}$ is the state, $F_{r,i}$, $H_{r,i}$ are switching real matrices to be designed, E_i, G_i are constant real matrices to be designed, and $e_i \in \mathbb{R}^p$ for $i, j = 1, \cdots, N$ is the tracking error of each agent defined as

$$e_i(t_k) := \tfrac{1}{\bar{\rho}_i} \textstyle\sum_{j=1}^{N} d_{ij}\left(\tilde{y}_i(t_k) - \tilde{y}_j(t_k)\right) + d_{i0}\left(\tilde{y}_i(t_k) - y_r(t_k)\right), \tag{18}$$

which is measured only at asynchronous time instants t_k such that only the values $e_i(t_k)$ are available for feedback. Here, d_{ij} represents the elements in the unit weighted adjacency matrix $\mathcal{D}$ of the directed graph $\mathcal{G}_N$ of the followers, while d_{i0} denotes the weight from the leader to the i^{th} follower. Additionally, $\bar{\rho}_i$ denotes the cardinality of the i^{th} agent, which refers to the number of neighborhoods (including the leader), and $\tilde{y}_i(t_k)$ is defined as $\tilde{y}_i(t_k) := y_i(t_k) - \bar{C}_i z_i(t_k)$, where $\bar{C}_i = (C_i \quad 0)$.

The Cooperative impulsive output Regulation Problem via Error Feedback (CIORPEF) is formulated as follows. Given the $i = 1, \cdots, N$ followers with form (15) and with virtual leader (16), both evolving in time respecting the condition (3), find, if possible, a hybrid distributed controller (17) such that, for all $i = 1, \cdots, N$ followers with $r \in \{1, \cdots, h\}$ subsystems, the following conditions are guaranteed

(S$_i$) The equilibrium point $(x_i, z_i) = (0, 0)$ of the hybrid feedback system $\zeta_i(t) := \left(x_i(t)^\top \;\; z_i(t)^\top\right)^\top$ without perturbations, i.e., with $w = 0$,

$$\begin{aligned} \dot{\zeta}_i(t) &= A_{L_{r,i}}\zeta_i(t), && t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \zeta_i(t_k^+) &= L_{L_i}\zeta_i(t_k) + \bar{G}_i e_i(t_k), && t_k \in \mathcal{T}, \end{aligned} \tag{19}$$

with

$$A_{L_{r,i}} = \begin{pmatrix} A_{r,i} & B_{r,i}H_{r,i} \\ 0 & F_{r,i} \end{pmatrix}, \quad \bar{G}_i = \begin{pmatrix} 0 \\ G_i \end{pmatrix}, \quad L_{L_i} = \begin{pmatrix} I_{n_i} & 0 \\ 0 & E_i \end{pmatrix}, \tag{20}$$

is GUAS under a switching law $\sigma_t : [t_0, \infty) \to \{1, \cdots h\}$. $\diamond$

(R$_i$) The solution $x_{cl_i}(t) := \left(x_i(t)^\top \;\; z_i(t)^\top \;\; w(t)^\top\right)^\top$ of the closed–loop system

$$\begin{aligned} \dot{x}_{cl_i}(t) &= A_{cl_{r,i}}x_{cl_i}(t), && t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ x_{cl_i}(t_k^+) &= L_{cl_i}x_{cl_i}(t_k) + \tilde{G}_i e_i(t_k), && t_k \in \mathcal{T}, \end{aligned} \tag{21}$$

with

$$A_{cl_{r,i}} = \begin{pmatrix} A_{r,i} & B_{r,i}H_{r,i} & P_i \\ 0 & F_{r,i} & 0 \\ 0 & 0 & S \end{pmatrix}, \quad \tilde{G}_i = \begin{pmatrix} 0 \\ G_i \\ 0 \end{pmatrix},$$
$$L_{cl_i} = \mathrm{diag}\left(I_{n_i}, E_i, I_q\right), \tag{22}$$

is such that $\lim_{t\to\infty} e_i = 0$ uniformly under a switching law $\sigma_t : [t_0, \infty) \to \{1, \cdots h\}$, and for each initial condition $(x_{0_i}, z_{0_i}, w_0) \in \mathbb{R}^{n_i+\nu_i+q}$. ⋄

The following Assumptions are necessary to solve the CIPORPEF.

***Assumption** 2:* The system

$$\begin{aligned} \dot{x}_i(t) &= (A_{r,i} + B_{r,i}K_{r,i})x_i(t), && t \in \mathbb{R}^+ \setminus \mathfrak{T}, \\ x_i(t_k^+) &= x_i(t_k), && t_k \in \mathfrak{T}, \end{aligned}$$

of each i^{th} follower, where $i = 1, \cdots, N$, has a common Lyapunov function [20] for $r \in \{1, \cdots, h\}$, where $K_{r,i} \in \mathbb{R}^{m_i \times n_i}$ ensures that the pair $(A_{r,i}, B_{r,i})$ is stabilizable. ⋄

***Assumption** 3:* There exist mappings $x_{ss,i} = \Pi_{r,i}w$, $z_{ss,i} = \Sigma_{r,i}w$ for each i^{th} follower, where $i = 1, \cdots, N$ and $r \in \{1, \cdots, h\}$, satisfying the following equations

$$\begin{aligned} \Pi_{r,i}S &= A_{r,i}\Pi_{r,i} + B_{r,i}\Gamma_{r,i} + P_i, \\ 0 &= C_i\Pi_{r,i} - Q, \\ \Sigma_{r,i}S &= F_{r,i}\Sigma_{r,i}, \\ \Gamma_{r,i} &= H_{r,i}E_i\Sigma_{r,i}. \end{aligned} \tag{23}$$

⋄

In addition to these assumptions, classical output regulation problems for discrete systems with a fixed sampling time δ typically include a detectability assumption of pairs $\left(e^{\bar{A}_{r,i}\delta}, \bar{C}_i\right)$ where

$$\bar{A}_{r,i} = \begin{pmatrix} A_{r,i} & -B_{r,i}\Gamma_{r,i} \\ 0 & S \end{pmatrix}, \quad \bar{C}_i = \begin{pmatrix} C_i & 0 \end{pmatrix}. \tag{24}$$

However, in the asynchronous and impulsive approach, this assumption is impractical due to $\delta \in [\tau_{\min}, \tau_{\max}]$. Since conditions for fulfilling inequalities with form (8) also derive similar detectability requirements, this feature is embedded in the main result, where a finite set of LMIs are satisfied, thus fulfilling the conditions of Theorem 1. The following presents the main result for solving the CIORFEP.

IV. Solution of the COR for SLMASs

***Theorem** 2:* Suppose that Assumptions 1–3 hold for each follower, and let $\mathfrak{T}$ be an admissible time basis with $\tau_{\min} \leq \tau_{\max}$ two given positive rational scalars. Furthermore, let $\{X_{r,i,1}, X_{r,i,2}, \cdots, X_{r,i,f_i}\}$ be a finite set of $f_i = 2^{n_i+q}$ matrices for $r \in \{1, \cdots, h\}$ and $i = 1, \cdots, N$, such that $e^{\bar{A}_{r,i}[\tau_{\min}, \tau_{\max}]} \in \mathrm{co}\{X_{r,i,1}, X_{r,i,2}, \cdots, X_{r,i,f_i}\}$. If there exist a symmetric positive definite matrix $\Upsilon_i \in \mathbb{R}^{(n_i+q)\times(n_i+q)}$, a matrix $J_i \in \mathbb{R}^{(n_i+q)\times p}$ and a matrix $D_i \in \mathbb{R}^{(n_i+q)\times(n_i+q)}$ for all $r \in \{1, \cdots, h\}$, for each $i = 1, \cdots, N$ and for every $l_i \in \{1, \cdots, f_i\}$ solving the following LMI

$$\begin{pmatrix} -D_i - D_i^\top & D_i - J_i\bar{C}_i & X_{r,i,l_i}^\top \Upsilon_i \\ \star & -\Upsilon_i & 0 \\ \star & \star & -\Upsilon_i \end{pmatrix} < 0, \tag{25}$$

then the CIORPEF is solvable by the hybrid distributed controller (17) only if the switching between subsystems occurs during jumps, with $G_i = D_i^{-1}J_i \in \mathbb{R}^{(n_i+q)\times p}$, $G_{i1} \in \mathbb{R}^{n_i \times p}$, $G_{i2} \in \mathbb{R}^{q\times p}$ and

$$\begin{aligned} F_{r,i} &= \mathrm{diag}(A_{r,i} + B_{r,i}K_{r,i}, S), & E_i &= \mathrm{diag}(I_{n_i}, I_q), \\ G_i &= \begin{pmatrix} G_{i1}^\top & G_{i2}^\top \end{pmatrix}^\top, & H_{r,i} &= \begin{pmatrix} K_{r,i} & \Gamma_{r_i} \end{pmatrix}, \end{aligned} \tag{26}$$

if and only if $\Pi_{a,i} = \Pi_{b,i}$ for $a, b \in \{1, \cdots, h\}$. ⋄

Proof: For the stability property (S$_i$), each controller state is partitioned as $z_i = \begin{pmatrix} z_{i1}^\top & z_{i2}^\top \end{pmatrix}^\top \in \mathbb{R}^{\nu_i}$, where $z_{i1} \in \mathbb{R}^{n_i}$ and $z_{i2} \in \mathbb{R}^q$, $\nu_i = n + q$. Now, note that the error signal (18), for each i^{th} agent $i = 1, \cdots, N$, involves

$$e_i(t_k) = \tilde{y}_i(t_k) = C_i\left(x_i(t_k) - z_{i1}(t_k)\right),$$

such that each hybrid feedback system (19) becomes

$$\begin{aligned} \dot{\zeta}_i(t) &= A_{L_{r,i}}\zeta_i(t), && t \in \mathbb{R}^+ \setminus \mathfrak{T}, \\ \zeta_i(t_k^+) &= E_{L_i}\zeta_i(t_k), && t_k \in \mathfrak{T}, \end{aligned} \tag{27}$$

where $\zeta_i = \begin{pmatrix} x_i^\top & z_{i1}^\top & z_{i2}^\top \end{pmatrix}^\top$ and

$$\begin{aligned} A_{L_{r,i}} &= \begin{pmatrix} A_{r,i} & B_{r,i}K_{r,i} & B_{r,i}\Gamma_{r,i} \\ 0 & A_{r,i} + B_{r,i}K_{r,i} & 0 \\ 0 & 0 & S \end{pmatrix}, \\ E_{L_i} &= \begin{pmatrix} I_{n_i} & 0 & 0 \\ G_{i1}C_i & I_{n_i} - G_{i1}C_i & 0 \\ G_{i2}C_i & -G_{i2}C_i & I_q \end{pmatrix}. \end{aligned} \tag{28}$$

To analyze the stability of (19), the coordinate transformation $\tilde{\zeta}_i$ is used, where $\tilde{\zeta}_i = \begin{pmatrix} \tilde{\zeta}_{i1}^\top & \tilde{\zeta}_{i2}^\top & \tilde{\zeta}_{i3}^\top \end{pmatrix}^\top = \begin{pmatrix} x_i^\top & (z_{i1} - x_i)^\top & z_{i2}^\top \end{pmatrix}^\top$, so obtaining the dynamics

$$\begin{aligned} \dot{\tilde{\zeta}}_i(t) &= \tilde{A}_{L_{r,i}}\tilde{\zeta}_i(t), && t \in \mathbb{R}^+ \setminus \mathfrak{T}, \\ \tilde{\zeta}_i(t_k^+) &= \tilde{E}_{L_i}\tilde{\zeta}_i(t_k), && t_k \in \mathfrak{T}, \end{aligned} \tag{29}$$

where

$$\begin{aligned} \tilde{A}_{L_{r,i}} &= \begin{pmatrix} A_{r,i} + B_{r,i}K_{r,i} & B_{r,i}K_{r,i} & B_{r,i}\Gamma_{r,i} \\ 0 & A_{r,i} & -B_{r,i}\Gamma_{r,i} \\ 0 & 0 & S \end{pmatrix}, \\ \tilde{E}_{L_i} &= \begin{pmatrix} I_{n_i} & 0 & 0 \\ 0 & I_{n_i} - G_{i1}C_i & 0 \\ 0 & -G_{i2}C_i & I_q \end{pmatrix}. \end{aligned}$$

Assumption 2 ensures that each dynamics $\tilde{\zeta}_{i1}$ is asymptotically stable with a common Lyapunov function, maintaining GUAS behavior through switchings. Furthermore, the dynamics of $\bar{\zeta}_i = \begin{pmatrix} \tilde{\zeta}_{i2}^\top & \tilde{\zeta}_{i3}^\top \end{pmatrix}^\top$ are given by

$$\begin{aligned} \dot{\bar{\zeta}}_i(t) &= \bar{A}_{r,i}\bar{\zeta}_i(t), && t \in \mathbb{R}^+ \setminus \mathfrak{T}, \\ \bar{\zeta}_i(t_k^+) &= (I_{n_i+q} - G_i\bar{C}_i)\bar{\zeta}_i(t_k), && t_k \in \mathfrak{T}. \end{aligned} \tag{30}$$

Note that (30) has the dynamic structure of (5), such that similar conditions (8) for (30) are embedded via a polytopic overapproximation in the set of LMIs (25) that ensure the existence of $G_i = D_i^{-1}J_i \in \mathbb{R}^{(n_i+q)\times p}$ and thus, Theorem 1 holds. Therefore, each switched system (30) is GUAS with switching between subsystems only at the asynchronous output measurements. Consequently, (29) is GUAS. This implies that also (19) is GUAS, thus verifying the stability condition.

For the regulation (R_i), the first two equations of (23) for each $i = 1, \cdots, N$ hold by Assumption 3, note that the second equation of (23) can only be satisfied if and only if $\Pi_{a,i} = \Pi_{b,i}$, for $a, b = 1, \cdots, h$. Furthermore, the last two equations of (23) that constitute the internal model principle are fulfilled with $\Sigma_{r,i} := \begin{pmatrix} 0 & I_q \end{pmatrix}^\top$.

Moreover, the matrices (22) of the closed–loop system (21) become

$$A_{cl_{r,i}} = \begin{pmatrix} A_{r,i} & B_{r,i}K_{r,i} & B_{r,i}\Gamma_{r,i} & P_i \\ 0 & A_{r,i} + B_{r,i}K_{r,i} & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & S \end{pmatrix}, \tag{31}$$
$$L_{cl_i} = I_{(2n_i+2q)}, \quad \tilde{G}_i = \begin{pmatrix} 0 & G_i & 0 \end{pmatrix}^\top,$$

with $x_{cl_i} = \begin{pmatrix} x_i^\top & z_{i1}^\top & z_{i2}^\top & w^\top \end{pmatrix}^\top$. Now, defining the error system $\xi_{r,i} := \begin{pmatrix} \xi_{r,i1}^\top & \xi_{r,i2}^\top & \xi_{r,i3}^\top \end{pmatrix}^\top$ for $i = 1, \cdots, N$, where $\xi_{r,i1} = x_i - \Pi_{r,i}w$, $\xi_{r,i2} = z_{i1}$ and $\xi_{r,i3} = z_{i2} - w$, one gets

$$\dot{\xi}_{r,i}(t) = A_{\xi_{r,i}}\xi_{r,i}(t), \qquad t \in \mathbb{R}^+ \setminus \mathcal{T},$$
$$\xi_{r,i}(t_k^+) = E_{\xi_{r,i}}\xi_{r,i}(t_k) + \begin{pmatrix} 0 \\ e_i \end{pmatrix}, \quad t_k \in \mathcal{T},$$

where $A_{\xi_{r,i}} = A_{L_{r,i}}$, $E_{\xi_{r,i}} = I_{(2n_i+q)}$. Now, defining

$$\begin{aligned}
\xi_{r,\mathcal{G}1} &:= (\xi_{r,11}^\top, \cdots, \xi_{r,N1}^\top)^\top, & A_{r,\mathcal{G}} &:= \mathrm{diag}(A_{r,1}, \cdots, A_{r,N}), \\
\xi_{r,\mathcal{G}2} &:= (\xi_{r,12}^\top, \cdots, \xi_{r,N2}^\top)^\top, & B_{r,\mathcal{G}} &:= \mathrm{diag}(B_{r,1}, \cdots, B_{r,N}), \\
\xi_{r,\mathcal{G}3} &:= (\xi_{r,13}^\top, \cdots, \xi_{r,N3}^\top)^\top, & C_{\mathcal{G}} &:= \mathrm{diag}(C_1, \cdots, C_N), \\
\Gamma_{r,\mathcal{G}} &:= \mathrm{diag}(\Gamma_{r,1}, \cdots, \Gamma_{r,N}), & K_{r,\mathcal{G}} &:= \mathrm{diag}(K_{r,1}, \cdots, K_{r,N}), \\
S_{\mathcal{G}} &:= I_N \otimes S, & G_{\mathcal{G}1} &:= \mathrm{diag}(G_{11}, \cdots, G_{N1}), \\
I_{n_{\mathcal{G}}} &:= \mathrm{diag}(I_{n_1}, \cdots, I_{n_N}), & G_{\mathcal{G}2} &:= \mathrm{diag}(G_{12}, \cdots, G_{N2}), \\
\mathcal{M}_{\mathcal{G}} &:= \bar{\rho}_{\mathcal{G}}(\mathcal{M} \otimes I_p), & I_{q_{\mathcal{G}}} &:= I_N \otimes I_q, \\
\mathcal{M} &:= \mathcal{L} + \mathrm{diag}(d_{10}, \cdots, d_{N0}), & \bar{\rho}_{\mathcal{G}} &:= \mathrm{diag}\left(\frac{1}{\bar{\rho}_1}, \cdots, \frac{1}{\bar{\rho}_N}\right) \otimes I_p.
\end{aligned}$$

Then, the overall closed-loop error system, defined by $\xi_{r,\mathcal{G}} := \begin{pmatrix} \xi_{r,\mathcal{G}1}^\top & \xi_{r,\mathcal{G}2}^\top & \xi_{r,\mathcal{G}3}^\top \end{pmatrix}^\top$ can be arranged as follows

$$\begin{aligned} \dot{\xi}_{r,\mathcal{G}}(t) &= A_{\xi_{r,\mathcal{G}}}\xi_{r,\mathcal{G}}(t), \quad t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \xi_{r,\mathcal{G}}(t_k^+) &= E_{\xi_{\mathcal{G}}}\xi_{r,\mathcal{G}}(t_k), \quad t_k \in \mathcal{T}, \end{aligned} \tag{32}$$

where

$$\begin{aligned} A_{\xi_{r,\mathcal{G}}} &= \begin{pmatrix} A_{r,\mathcal{G}} & B_{r,\mathcal{G}}K_{r,\mathcal{G}} & B_{r,\mathcal{G}}\Gamma_{r,\mathcal{G}} \\ 0 & A_{r,\mathcal{G}} + B_{r,\mathcal{G}}K_{r,\mathcal{G}} & 0 \\ 0 & 0 & S_{\mathcal{G}} \end{pmatrix}, \\ E_{\xi_{\mathcal{G}}} &= \begin{pmatrix} I_{n_{\mathcal{G}}} & 0 & 0 \\ G_{\mathcal{G}_1}\mathcal{M}_{\mathcal{G}}C_{\mathcal{G}} & I_{n_{\mathcal{G}}} - G_{\mathcal{G}_1}\mathcal{M}_{\mathcal{G}}C_{\mathcal{G}} & 0 \\ G_{\mathcal{G}_2}\mathcal{M}_{\mathcal{G}}C_{\mathcal{G}} & -G_{\mathcal{G}_2}\mathcal{M}_{\mathcal{G}}C_{\mathcal{G}} & I_{q_{\mathcal{G}}} \end{pmatrix}. \end{aligned} \tag{33}$$

The coordinate transformation $\tilde{\xi}_{r,\mathcal{G}}$ is applied for the stability analysis of (32) where $\tilde{\xi}_{r,\mathcal{G}} = \begin{pmatrix} \tilde{\xi}_{r,\mathcal{G}1}^\top & \tilde{\xi}_{r,\mathcal{G}2}^\top & \tilde{\xi}_{r,\mathcal{G}3}^\top \end{pmatrix}^\top = \begin{pmatrix} \xi_{r,\mathcal{G}1}^\top & (\xi_{r,\mathcal{G}2} - \xi_{r,\mathcal{G}1})^\top & \xi_{r,\mathcal{G}3}^\top \end{pmatrix}^\top$, so obtaining the rearranged dynamics

$$\begin{aligned} \dot{\tilde{\xi}}_{r,\mathcal{G}}(t) &= \tilde{A}_{\xi_{r,\mathcal{G}}}\tilde{\xi}_{r,\mathcal{G}}(t), \quad t \in \mathbb{R}^+ \setminus \mathcal{T}, \\ \tilde{\xi}_{r,\mathcal{G}}(t_k^+) &= \tilde{E}_{\xi_{\mathcal{G}}}\tilde{\xi}_{r,\mathcal{G}}(t_k), \quad t_k \in \mathcal{T}, \end{aligned} \tag{34}$$

where

$$\begin{aligned} \tilde{A}_{\xi_{r,\mathcal{G}}} &= \begin{pmatrix} A_{r,\mathcal{G}} + B_{r,\mathcal{G}}K_{r,\mathcal{G}} & B_{r,\mathcal{G}}K_{r,\mathcal{G}} & B_{r,\mathcal{G}}\Gamma_{r,\mathcal{G}} \\ 0 & A_{r,\mathcal{G}} & -B_{r,\mathcal{G}}\Gamma_{r,\mathcal{G}} \\ 0 & 0 & S_{\mathcal{G}} \end{pmatrix}, \\ \tilde{E}_{\xi_{\mathcal{G}}} &= \begin{pmatrix} I_{n_{\mathcal{G}}} & 0 & 0 \\ 0 & I_{n_{\mathcal{G}}} - G_{\mathcal{G}_1}\mathcal{M}_{\mathcal{G}}C_{\mathcal{G}} & 0 \\ 0 & -G_{\mathcal{G}_2}\mathcal{M}_{\mathcal{G}}C_{\mathcal{G}} & I_{q_{\mathcal{G}}} \end{pmatrix}. \end{aligned}$$

Assumption 2 ensures that the system dynamics $\tilde{\xi}_{r,\mathcal{G}1}$ is asymptotically stable. Furthermore, the dynamics of $\bar{\xi}_{r,\mathcal{G}} = (\tilde{\xi}_{r,\mathcal{G}2}^\top \quad \tilde{\xi}_{r,\mathcal{G}3}^\top)^\top$ can be written in block form as

$$\begin{aligned} \dot{\bar{\xi}}_{r,\mathcal{G}}(t) &= \begin{pmatrix} A_{r,\mathcal{G}} & -B_{r,\mathcal{G}}\Gamma_{r,\mathcal{G}} \\ 0 & S_{\mathcal{G}} \end{pmatrix}\bar{\xi}_{r,\mathcal{G}}(t), & t &\in \mathbb{R}^+ \setminus \mathcal{T}, \\ \bar{\xi}_{r,\mathcal{G}}(t_k^+) &= \left(I_{n_{\mathcal{G}}+q_{\mathcal{G}}} - \begin{pmatrix} G_{\mathcal{G}1}\mathcal{M}_{\mathcal{G}} \\ G_{\mathcal{G}2}\mathcal{M}_{\mathcal{G}} \end{pmatrix}\begin{pmatrix} C_{\mathcal{G}} & 0 \end{pmatrix}\right)\bar{\xi}_{r,\mathcal{G}}(t_k), & t_k &\in \mathcal{T}. \end{aligned} \tag{35}$$

Since the directed graph $\mathcal{G}_{N+1}$ includes a spanning tree rooted in the virtual leader, and without loss of generality, system (35) can be reorganized such that $\mathcal{M}_{\mathcal{G}}$ takes a lower triangular block form with identity matrices in its diagonal so that Theorem 1 holds. Furthermore, the solution to the LMIs (25) guarantees the existence of the block matrix gain $\bar{G}_{\mathcal{G}}$ such that (35) is GUAS. Consequently, (34) is also GUAS, implying that the overall error vector $e_{\mathcal{G}} := \begin{pmatrix} e_1^\top, \cdots, e_N^\top \end{pmatrix}^\top$ with the form $e_{\mathcal{G}} = -\mathcal{M}_{\mathcal{G}}C_{\mathcal{G}}\tilde{\xi}_{r,\mathcal{G}2}$, and composed of each tracking error (18) asymptotically converges to zero, satisfying the regulation condition. This completes the proof. ■

V. Simulation Results

Consider a heterogeneous MAS composed of three followers, as shown in the communication graph of Figure 1.

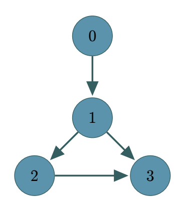

Fig. 1. Communication topology graph.

Each agent is considered as an SLS in the form (15) given by the following subsystem structures

$$A_{1,1} = \begin{pmatrix} 1 & 2 & 3 & 6 \\ 9 & -2 & 4 & 0 \\ 0 & 0 & 1 & -3 \\ 7 & 4 & -1 & -1 \end{pmatrix}, A_{1,2} = \begin{pmatrix} 1 & 2 & 4 \\ 4 & -6 & 3 \\ -5 & 4 & -2 \end{pmatrix}, B_{1,1} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 2 \\ 7 & 0 \end{pmatrix}, B_{r,2} = \begin{pmatrix} 0 & 0 \\ 1 & -3 \\ 2 & 1 \end{pmatrix}, P_1 = \begin{pmatrix} 0 & -0.1 \\ 0.2 & 0 \\ -0.8 & -0.2 \\ 0.6 & 0.3 \end{pmatrix},$$
$$A_{2,1} = \begin{pmatrix} 1 & 2 & 3 & 6 \\ 9 & -2 & 4 & 0 \\ 4 & -2 & 5 & 1 \\ 6 & 2 & 5 & -3 \end{pmatrix}, A_{2,2} = \begin{pmatrix} 1 & 2 & 4 \\ 4 & -6 & 3 \\ 3 & 2 & -1 \end{pmatrix}, B_{2,1} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 4 & 0 \\ 0 & 1 \end{pmatrix}, B_{1,3} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ -1 & 3 \end{pmatrix}, P_2 = \begin{pmatrix} -0.2 & 0.4 \\ 0.6 & 0 \\ 0.5 & -0.1 \end{pmatrix},$$
$$A_{1,3} = \begin{pmatrix} -3 & -1 & 0 & -2 \\ -3 & 1 & -1 & 3 \\ 1 & 4 & -2 & 1 \\ 3 & -2 & -2 & -1 \end{pmatrix}, A_{3,2} = \begin{pmatrix} 1 & 2 & 4 \\ 4 & -6 & 3 \\ 1 & -3 & -1 \end{pmatrix}, C_1 = \begin{pmatrix} 0 & 1 \\ 0 & 2 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}^\top, B_{2,3} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -2 & 4 \\ 0 & 1 \end{pmatrix}, P_3 = \begin{pmatrix} 0 & -0.1 \\ 0.2 & 0 \\ -0.3 & -0.2 \\ 0.6 & 0.3 \end{pmatrix},$$
$$A_{2,3} = \begin{pmatrix} -3 & -1 & 0 & -2 \\ -3 & 1 & -1 & 3 \\ 1 & -5 & -2 & 1 \\ 1 & -1 & 0 & 1 \end{pmatrix}, C_2 = \begin{pmatrix} 1 & 0 \\ 1 & 2 \\ 0 & 0 \end{pmatrix}^\top, C_3 = \begin{pmatrix} 1 & -3 \\ 0 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}^\top,$$

where the asynchronous output measurements and the signal $r \in \{1, \cdots, h\}$ to switch between subsystems are transmitted according to (6) with $\tau_{\min} = 0.01s$ and $\tau_{\max} = 0.1s$. Both sequences are shown in Figure 2. Note that switching during systems transient is permissible due to Assumption 2. The initial conditions for each agent are $x_1(0) = (1\ \ 0\ {-1}\ \ 3)^\top$, $x_2(0) = (5\ \ 1\ \ -6)^\top$ and $x_3(0) = (3\ \ 7\ \ -9\ \ 2)^\top$. The virtual leader is modeled by (16) with

$$S = \begin{pmatrix} 0 & 1 \\ -\alpha^2 & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} \beta & 0 \\ 0 & -\beta/\alpha \end{pmatrix},$$

with initial condition $w(0) = (1\ \ 0)^\top$, where $\alpha = \pi$ is the frequency and $\beta = 3$ the amplitude. The initial conditions for (17) are $z_1(0) = (0\ \ 1\ -\ 1\ \ 2\ \ 1\ -1)^\top$, $z_2(0) = (1\ -\ 2\ \ 3\ \ 0\ -1)^\top$ and $z_3(0) = (1\ \ 2\ -2\ \ 0\ -1\ \ 1)^\top$. Note that the condition of Theorem 2 involving $\Pi_{a,i} = \Pi_{b,i}$ for $a, b \in \{1, \cdots, h\}$ is fulfilled such that one can find $\Gamma_{r,i}$ for the distributed controllers (17), where the gains $K_{r,i}$ for $i = 1, \cdots N$ and $r = \{1, \cdots, h\}$ were computed using the Linear Quadratic Regulator (LQR) with matrices $Q_{Kr,i} = I_{n_i}$ and $R_{Kr,i} = 0.01$. Furthermore, the gains G_i were calculated by solving a finite set of LMIs (25) (see Remark 1).

The tracking error norms e_i and the MAS output trajectories are shown in Figures 2 and 3, respectively. Agents track the signal of the virtual leader, achieving regulation and asymptotic error stabilization to zero.

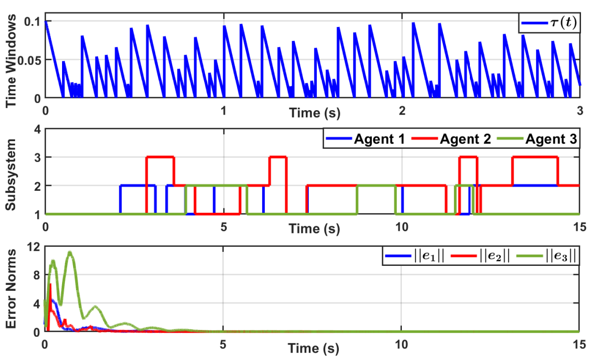

Fig. 2. Top: Time windows of the time-driven transmission mechanism (6); Center: Switching between subsystems commanded by the switching signal mechanism (7); Bottom: Tracking error norm of followers.

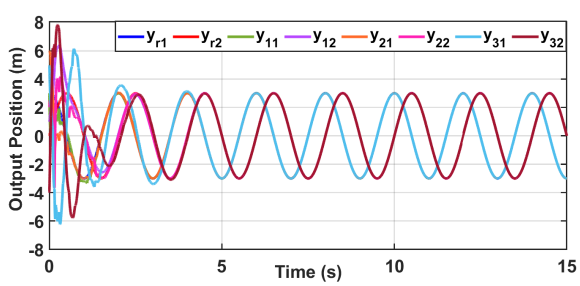

Fig. 3. Cooperative output regulation of the MAS.

VI. Conclusions and Future Work

In this paper, a novel approach to the COR problem in SLMASs, particularly under intermittent measurements, is introduced and analyzed. Its effectiveness in maintaining cooperative regulation despite measurement irregularities is showcased through simulations. Future research directions include exploring robust COR in switched linear systems, adapting the framework to nonlinear systems, and examining the impact of time–varying delays [21].

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