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Robust Fault-tolerant Control based on L_∞ Design for Discrete-time Systems with Parameter Uncertainty

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Abstract— **In this paper, a robust fault-tolerant controller is proposed for discrete-time systems with parameter uncertainty. First, the state and fault are estimated by an observer based on the L_∞ design. The stability of the L_∞ observer is proved and the error system is robust against unknown disturbances. Then, on the basis of fault and state estimation, L_∞ technique is applied to design the robust fault-tolerant controller to recover the performance of the system affected by the actuator fault. Based on the proposed method, the compensated state can be bounded by the designed L_∞ index, which guarantees the safety of the system. Finally, the proposed robust fault-tolerant controller is applied to the simulation of a dual-rotor aero-engine system model, and its effectiveness is verified.**

I. INTRODUCTION

Reliability, safety and stability are important indicators to measure the control performance of aircraft, satellites, chemical systems, etc [1]. The increasing complexity and changes in the working environment may cause system components to encounter some failures during the execution of tasks [2]. Failures may cause control performance degradation or even loss of control. In order to minimize performance degradation and guarantee system safety, timely and effective fault-tolerant control (FTC) is necessary. In recent decades, FTC has been widely concerned and successfully applied [3], [4], [5].

In general, the commonly used FTC strategies can be divided into two categories: passive and active FTC methods [8]. A robust passive FTC based on adaptive fuzzy control has been proposed in [6] such that the robot manipulator has good control performance. In [7], an adaptive fuzzy sliding-mode controller is designed for robust passive FTC of a quadrotor to reduce the estimation error of the nonlinear functions. However, these passive FTC methods need to counteract the effects of uncertain dynamics and maximum failure, which requires high system robustness. In essence, the methods take advantage of the robustness inherent in the designed controller algorithm to mitigate the impact of faults and sacrifice the control performance of the system in the absence of faults. Different from the passive FTC methods, the active FTC methods can reconfigure the controller in real time to compensate for faults based on the fault information provided by the fault estimator. In this case, accurate fault information is helpful to improve the robustness of fault-tolerant control system. Therefore, fault estimation (FE) has received considerable attention [9], [10], [11], [12]. In the existing FE methods, the augmented FE observer is commonly studied, which can achieve state and fault estimation simultaneously. For practical systems, it is particularly important to consider the estimation robustness against unknown disturbances and parameter uncertainties. H_∞ technique is widely used in robust observer design [13], [14], [15]. Note that both H_∞ technique and L_∞ technique can be applied to the FE observer design, the H_∞ technique assumes that disturbances and noise are energy-bounded in the entire time domain, whereas in practical systems disturbances and noise are generally peak-bounded. Fortunately, the L_∞ technology only assumes that disturbances and noise are peak-bounded, which is a more reasonable assumption for the observer design. In addition, different from the FTC based on H_∞ design, the FTC based on L_∞ design can limit the upper bound of the state after fault compensation, which is an effective way to guarantee the safety of the system with uncertainties.

Based on the above analysis, in order to achieve the system security and reliability, in this paper, a novel robust FTC method based on L_∞ design is proposed for discrete-time systems with parameter uncertainty. The main contributions are concluded as follows. First, we propose a robust observer design method for state and fault simultaneous estimation based on L_∞ technique to attenuate the influences of disturbance, measurement noise and parameter uncertainty on estimation error. Second, considering the coupling between the estimation error and the compensated state in FTC introduced by parameter uncertainty, we propose a robust FTC design method with L_∞ performance such that the upper bound of the state after fault compensation satisfies the L_∞ index limitation. Third, the L_∞ index is used to predict the upper bound of the compensated state and to verify the security and reliability of the system with uncertainties.

The rest is organized as follows. Section II is dedicated to the problem formulation. In section III, state and fault estimation based on L_∞ design is presented. Section IV gives the fault-tolerant controller based on L_∞ design. Simulation results and discussions are provided in section V. Finally, conclusions and future works are drawn in section VI.

Notation: The identify matrix with appropriate dimensions is represented as I. Meanwhile, a matrix with appropriate dimensions is represented as 0. In a symmetric matrix, $*$ is used to represent a term that can be include by symmetry.

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Moreover, for a matrix A, $A \prec 0$ ($A \succ 0$) denotes negative (positive) definiteness. For a vector $x \in \mathbb{R}^n$, $\|x\| = \sqrt{x^T x}$ represents its European norm. Given a sequence of signals $t(k)(k = 0, \cdots, \infty)$, $\|t(k)\|_\infty = \sqrt{max_{k=0}^{\infty} t^T(k)t(k)}$.

II. PROBLEM FORMULATION

Consider the following linear discrete-time system with parameter uncertainty:

$$\begin{cases} x(k+1) = (A + \Delta A)x(k) + Bu(k) + D_\omega \omega(k) + Ff(k) \\ y(k) = Cx(k) + D_v v(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$, $u(k) \in \mathbb{R}^{n_u}$ and $y(k) \in \mathbb{R}^{n_y}$ denote the state, control input, measurement output vectors, respectively. $\omega(k) \in \mathbb{R}^{n_\omega}$ denotes the process disturbances. $v(k) \in \mathbb{R}^{n_v}$ denotes the vectors of measurement noise and $f(k) \in \mathbb{R}^{n_f}$ denotes the actuator fault. $A \in \mathbb{R}^{n_x \times n_x}$, $B \in \mathbb{R}^{n_x \times n_u}$, $C \in \mathbb{R}^{n_y \times n_x}$, $F \in \mathbb{R}^{n_x \times n_f}$, $D_\omega \in \mathbb{R}^{n_x \times n_\omega}$ and $D_v \in \mathbb{R}^{n_y \times n_v}$ are known matrices with appropriate dimensions. Moreover, we consider the structure of parameter uncertainty, which will be used later in the observer gain and controller gain design. The modeling errors are represented as parameter uncertainty ΔA satisfies

$$\Delta A = M \Delta N, \tag{2}$$

where the matrices M, N are known, $\Delta \in \mathbb{R}^{n_1 \times n_1}$ is an unknown matrix and $\Delta\Delta^T \preceq I_{n_x}$.

In order to realize the estimation of the fault $f(k)$, the system obtained by considering $f(k)$ as an auxiliary state is formed as follows:

$$\begin{cases} \xi(k+1) = (\bar{A} + \Delta\bar{A})\xi(k) + \bar{B}u(k) + \bar{D}_\omega \omega(k) + \bar{H}h(k) \\ y(k) = \bar{C}\xi(k) + D_v v(k) \end{cases} \tag{3}$$

where $\xi(k) = \begin{bmatrix} x^T(k) & f^T(k) \end{bmatrix}^T$, $h(k) = f(k+1) - f(k)$, $\bar{A} = \begin{bmatrix} A & F \\ 0 & I_{n_f} \end{bmatrix}$, $\Delta\bar{A} = \begin{bmatrix} \Delta A & 0 \\ 0 & 0 \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$, $\bar{D}_\omega = \begin{bmatrix} D_\omega \\ 0 \end{bmatrix}$, $\bar{H} = \begin{bmatrix} 0 \\ I_{n_f} \end{bmatrix}$ and $\bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}$.

In addition, when dealing with parameter uncertainty, we need to use the following lemma.

Lemma 1 ([16]): Let M, N and Δ be matrices of appropriate dimension, and $\Delta^T\Delta \preceq I$, then for any scalar ϵ:

$$M\Delta N + N^T\Delta^T M^T \preceq \frac{1}{\epsilon} MM^T + \epsilon N^T N. \tag{4}$$

This paper aims to design a robust state and fault observer with the estimation error dynamics robust against the unknown disturbance, measurement noise and parameter uncertainty. Based on the robust estimation, we aim to design a robust fault-tolerant controller for system (1) such that the state after fault compensation satisfies the given robust performance, which guarantees the safety of the considered system.

III. STATE AND FAULT ESTIMATION BASED L_∞ DESIGN

Construct the following observer for the augmented system (3):

$$\begin{cases} \hat{\xi}(k+1) = \bar{A}\hat{\xi}(k) + \bar{B}u(k) + L(y(k) - \hat{y}(k)) \\ \hat{y}(k) = \bar{C}\hat{\xi}(k) \end{cases} \tag{5}$$

where $\hat{\xi}(k)$, $\hat{f}(k)$ and $\hat{y}(k)$ are the estimation of $\xi(k)$, $f(k)$ and $y(k)$, respectively. $L = \begin{bmatrix} L_x^T & L_f^T \end{bmatrix}^T$ is the observer gain matrix.

Define the estimation error as $e(k) = \xi(k) - \hat{\xi}(k)$, from (3) and (5), the following error system can be obtained:

$$\begin{aligned} e(k+1) =& (\bar{A} - L\bar{C})e(k) + \Delta\bar{A}\xi(k) + \bar{D}_\omega \omega(k) \\ &+ \bar{H}h(k) - LD_v v(k) \end{aligned} \tag{6}$$

It should be emphasized that $\xi(k) = e(k) + \hat{\xi}(k)$. Therefore, (6) can be rewritten as follows:

$$\begin{aligned} e(k+1) =& (A_c + \Delta\bar{A})e(k) + \Delta\bar{A}\hat{\xi}(k) + \bar{D}_\omega \omega(k) \\ &+ \bar{H}h(k) - LD_v v(k) \end{aligned} \tag{7}$$

where $A_c = \bar{A} - L\bar{C}$.

It can be clearly seen that due to the existence of parameter uncertainty, the state estimation $\hat{\xi}(k)$ have influence on the error system (7). Moreover, since ΔA is unknown matrix, the effects of $\hat{\xi}(k)$ can be regarded as unknown disturbances, and the error system can be expressed as follows:

$$e(k+1) = A_d e(k) + B_d d(k) \tag{8}$$

where $A_d = A_c + \Delta\bar{A}$, $B_d = D_x + \Delta D$, $D_x = \begin{bmatrix} \bar{D}_\omega & -LD_v & \bar{H} & 0 \end{bmatrix}$, $\Delta D = \begin{bmatrix} 0 & 0 & 0 & \Delta\bar{A} \end{bmatrix}$ and $d(k) = \begin{bmatrix} \omega^T(k) & v^T(k) & h^T(k) & \hat{\xi}^T(k) \end{bmatrix}^T$. Next, we will design the gain matrix L of state and fault observer to make the error system (8) robust to unknown disturbances $d(k)$ (including the effects of state estimates on the system through parameter uncertainty). Unfortunately, the H_∞ method often needs to assume that the disturbances and noise are energy-bounded in the full time domain. Instead, the peak-bounded case is considered in this paper. Fortunately, the proposed L_∞ technique can effectively deal with the peak-bounded disturbances and noise. We aim to design the observer parameter such that the error system satisfies the L_∞ index as follows:

$$\|e(k)\| < \sqrt{\kappa_1^2 \alpha^k \mathcal{J}(0) + \kappa_1^2 \|d(k)\|_\infty^2} \tag{9}$$

where $\kappa_1 > 0$, $0 < \alpha < 1$, $\mathcal{J}(0) = e^T(0)\Theta e(0)$, $\Theta \in \mathbb{R}^{(n_x+n_f)\times(n_x+n_f)}$ is a positive definite matrix, and $\|d(k)\|_\infty = max_{k\geq 0}\|d(k)\|$ is the L_∞ norm of $d(k)$. To this end, we propose the following theorem:

Theorem 1: Given scalar $\kappa_1 > 0$, if there exists a positive-definite matrix $\Theta \in \mathbb{R}^{(n_x+n_f)\times(n_x+n_f)}$, an invertible matrix $G \in \mathbb{R}^{(n_x+n_f)\times(n_x+n_f)}$, a matrix $W \in \mathbb{R}^{(n_x+n_f)\times n_y}$ and a scalar $\epsilon > 0$ such that the following inequalities are satisfied:

$$\begin{bmatrix} \Psi & * \\ M^T & -\epsilon I_{n_1+n_2} \end{bmatrix} \prec 0, \tag{10}$$

$$\varrho I_{n_x+n_f} - \Theta \prec 0, \tag{11}$$

where $\Psi = \Sigma + \epsilon N^T N, \quad \varrho = \frac{1}{\kappa_1^2}$,

$\Sigma = \begin{bmatrix} \Sigma_{11} & * & * \\ \Sigma_{21} & \Sigma_{22} & * \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} \end{bmatrix}$, $\Sigma_{11} = -\alpha\Theta$, $\Sigma_{21} = \begin{bmatrix} 0; 0; 0; 0; 0 \end{bmatrix}$,

$\Sigma_{22} = \begin{bmatrix} Z_{11} & * & * & * & * \\ 0 & Z_{22} & * & * & * \\ 0 & 0 & Z_{33} & * & * \\ 0 & 0 & 0 & Z_{44} & * \\ 0 & 0 & 0 & 0 & Z_{55} \end{bmatrix}$, $Z_{11} = -(1-\alpha)I_{n_\omega}$,

$Z_{22} = -(1-\alpha)I_{n_v}$, $Z_{33} = -(1-\alpha)I_{n_f}$, $Z_{44} = -(1-\alpha)I_{n_x+n_f}$, $Z_{55} = -(1-\alpha)I_{n_u}$, $\Sigma_{31} = G\bar{A} - W\bar{C}$, $\Sigma_{32} = \begin{bmatrix} G\bar{D}_\omega & -WD_v & G\bar{H} & 0 & 0 \end{bmatrix}$, $\Sigma_{33} = \Theta - G - G^T$, $M = \begin{bmatrix} 0; 0; 0; 0; 0; 0; G\bar{M}_1 \end{bmatrix}$, $\bar{M}_1 = \begin{bmatrix} M_1 \\ 0 \end{bmatrix}$, $N = \begin{bmatrix} N_1 & 0 & 0 & 0 & N_1 & 0 & 0 \end{bmatrix}$,
and let $L = G^{-1}W$, then the error system satisfies the L_∞ index (9).

Proof: According to the Schur complement lemma, (10) is equivalent to

$$\Sigma + \frac{1}{\epsilon} MM^T + \epsilon N^T N \prec 0. \tag{12}$$

And, according to Lemma 1, (12) yields

$$\Sigma + M\Delta N + N^T \Delta^T M^T \prec 0. \tag{13}$$

Since $L = G^{-1}W$, we have $W = GL$. Substituting it into Σ yields

$$\Sigma = \begin{bmatrix} -\alpha\Theta & * & * \\ 0 & -(1-\alpha)I_{n_t} & * \\ GA_c & GD_x & \Theta - G - G^T \end{bmatrix}, \tag{14}$$

where $n_t = n_w + n_v + n_f + n_x + n_u$.

Taking M, N and Δ into $M\Delta N$, we get

$$M\Delta N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ G\Delta\bar{A} & G\Delta D & 0 \end{bmatrix} \tag{15}$$

Substituting (14) and (15) into (13) yields

$$\begin{bmatrix} -\alpha\Theta & * & * \\ 0 & -(1-\alpha)I_{n_t} & * \\ GA_d & GB_d & \Theta - G - G^T \end{bmatrix} \prec 0. \tag{16}$$

By pre-multiplying and post-multiplying (16) with $\begin{bmatrix} I_{n_x+n_f} & 0 & A_d^T \\ 0 & I_{n_t} & B_d^T \end{bmatrix}$ and its transpose, respectively, we can get

$$\begin{bmatrix} -\alpha\Theta & * \\ B_d^T\Theta A_d & -(1-\alpha)I_{n_t} + B_d^T\Theta B_d \end{bmatrix} \prec 0. \tag{17}$$

Furthermore, by pre-multiplying and post-multiplying (17) with $\begin{bmatrix} e^T(k) & d^T(k) \end{bmatrix}$ and its transpose, respectively, we have

$$\begin{aligned} &-\alpha e^T(k)\Theta e(k) + e^T(k)A_d^T\Theta B_d d(k) + d^T(k)B_d^T\Theta A_d e(k) \\ &+ d^T(k)B_d^T\Theta B_d d(k) - (1-\alpha)d^T(k)d(k) \le 0. \end{aligned} \tag{18}$$

Consiser the Lyapunov function as follows:

$$\mathcal{J}(k) = e^T(k)\Theta e(k), \tag{19}$$

then according to (19), (18) can be rewritten as follows:

$$\mathcal{J}(k+1) \le \alpha\mathcal{J}(k) + (1-\alpha)d^T(k)d(k), \tag{20}$$

it follows that

$$\begin{aligned} \mathcal{J}(k) &\le \alpha^k\mathcal{J}(0) + (1-\alpha)\sum_{i=0}^{k-1}\alpha^i d_{k-1-i}^T d_{k-1-i} \\ &\le \alpha^k\mathcal{J}(0) + (1-\alpha)\sum_{i=0}^{k-1}\alpha^i \|d(k)\|_\infty^2 \\ &< \alpha^k\mathcal{J}(0) + \|d(k)\|_\infty^2. \end{aligned} \tag{21}$$

By pre-multiplying and post-multiplying (11) with $e^T(k)$ and $e(k)$, respectively, we can get

$$e^T(k)e(k) \le \kappa_1^2 e^T(k)\Theta e(k). \tag{22}$$

Eventually, combining (21) and (22), we have

$$\|e(k)\|^2 < \kappa_1^2\alpha^k\mathcal{J}(0) + \kappa_1^2\|d(k)\|_\infty^2, \tag{23}$$

which implies the L_∞ index (9). ■

To optimize the estimation accuracy, the maximum ϱ can be obtained by solving the following optimization problem:

$$\begin{aligned} &\max \quad \varrho \\ &s.t \quad (10), (11). \end{aligned} \tag{24}$$

and $\kappa_1^2 = \frac{1}{\varrho}$ can be minimized, the feasible solution of the observer gain can be obtained by $L = G^{-1}W$.

In the sequel, the objective is to design a fault-tolerant controller for the considered system (1).

IV. FAULT-TOLERANT CONTROL BASED ON L_∞ DESIGN

According to (5), the observed result of the state can be expressed as follows:

$$\hat{x}(k+1) = A\hat{x}(k) + F\hat{f}(k) + Bu(k) + L_x(y(k) - \hat{y}(k)). \tag{25}$$

In order to reduce the impact of actuator fault on the considered system, the fault-tolerant control law based on fault compensation is designed as follows:

$$u(k) = -K_x\hat{x}(k) - K_f\hat{f}(k), \tag{26}$$

where K_x and K_f are the gain matrices to be designed.

Submitting the fault-tolerant control law (26) into (25), we can get

$$\begin{aligned} \hat{x}(k+1) =& (A - BK_x)\hat{x}(k) + (F - BK_f)\hat{f}(k) \\ &+ L_x(y(k) - \hat{y}(k)). \end{aligned} \tag{27}$$

Furthermore, the effects of the actuator fault can be compensated if

$$F - BK_f = 0. \tag{28}$$

Then, (27) can be rewritten as follows:

$$\hat{x}(k+1) = (A - K_xB)\hat{x}(k) + L_x(y(k) - \hat{y}(k)). \tag{29}$$

According to $x(k) = \hat{x}(k) + \begin{bmatrix} I & 0 \end{bmatrix} e(k)$, combining (8) and (29), the state can be rewritten as follows:

$$x(k+1) = (A_{x1} + \Delta A)x(k) + B_{e1}e(k) + D_1\tilde{d}(k) \quad (30)$$

where $A_{x1} = A - BK_x$, $B_{e1} = BK_xS_1 + BK_fS_2$, $S_1 = \begin{bmatrix} I & 0 \end{bmatrix}$, $S_2 = \begin{bmatrix} 0 & I \end{bmatrix}$, $\tilde{d}(k) = \begin{bmatrix} \omega^T(k) & v^T(k) & h^T(k) & f^T(k) \end{bmatrix}^T$ and $D_1 = \begin{bmatrix} D_\omega & 0 & 0 & F - BK_f \end{bmatrix}$.

Note that $u(k)$ (26) is designed to reduce the impact of actuator faults, which leads to the coupling between $x(k)$ and $e(k)$. Then, $\Delta\bar{A}\hat{\xi}(k)$ in the error system (7) cannot be regarded as unknown disturbances, the error system can be rewritten as follows:

$$e(k+1) = (A_{x2} + \Delta A_{x2})x(k) + B_{e2}e(k) + D_2\tilde{d}(k) \quad (31)$$

where $A_{x2} = 0$, $\Delta A_{x2} = \begin{bmatrix} \Delta A \\ 0 \end{bmatrix}$, $B_{e2} = \bar{A} - L\bar{C}$ and $D_2 = \begin{bmatrix} \bar{D}_\omega & -LD_v & \bar{H} & 0 \end{bmatrix}$.

In this case, define a new augmented variable $\tilde{\varepsilon}(k) = \begin{bmatrix} x^T(k) & e^T(k) \end{bmatrix}^T$, $\tilde{\varepsilon}(k)$ can be formulated by combining (30) and (31) as follows:

$$\tilde{\varepsilon}(k+1) = \tilde{A}_t\tilde{\varepsilon}(k) + \tilde{D}_t\tilde{d}(k), \quad (32)$$

where $\tilde{A}_t = A_t + \Delta A_t$, $A_t = \begin{bmatrix} A_{x1} & B_{e1} \\ A_{x2} & B_{e2} \end{bmatrix}$, $\Delta A_t = \begin{bmatrix} \Delta A & 0 \\ \Delta A_{x2} & 0 \end{bmatrix}$ and $\tilde{D}_t = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$. Note that $x(k) = \tilde{C}\tilde{\varepsilon}(k)$, where $\tilde{C} = \begin{bmatrix} I & 0 \end{bmatrix}$.

Consequently, we can get the fault-tolerant control system of the considered system as follows:

$$\begin{cases} \tilde{\varepsilon}(k+1) = \tilde{A}_t\tilde{\varepsilon}(k) + \tilde{D}_t\tilde{d}(k) \\ x(k) = \tilde{C}\tilde{\varepsilon}(k) \end{cases} \quad (33)$$

We aim to design the control gain matrix K_x such that $x(k)$ is robust to $\tilde{d}(k)$. Specifically, the following L_∞ index is satisfied:

$$\|x(k)\| < \lambda\sqrt{\tilde{\alpha}^k V(0) + \|\tilde{d}(k)\|_\infty^2} \quad (34)$$

where $\lambda > 0$, $0 < \tilde{\alpha} < 1$, $V(0) = \tilde{\varepsilon}^T(0)P\tilde{\varepsilon}(0)$, $P \in \mathbb{R}^{(n_x+n_x+n_f)\times(n_x+n_x+n_f)}$ is a positive definite matrix and $\|\tilde{d}(k)\|_\infty = max_{k\geq 0}\|\tilde{d}(k)\|$ is the L_∞ norm of $\tilde{d}(k)$.

In order to design the fault-tolerant controller gain matrix K_x such that the system (33) satisfies the L_∞ index (34), the following theorem is proposed.

Theorem 2: Given a scalar $\lambda > 0$, $0 < \tilde{\alpha} < 1$, if there exist positive-definite matrices $P_1 \in \mathbb{R}^{n_x\times n_x}$, $P_2 \in \mathbb{R}^{n_x\times n_x}$ and $P_3 \in \mathbb{R}^{n_f\times n_f}$, a matrix $Y_1 \in \mathbb{R}^{n_x\times n_x}$ and a scalar $\tilde{\epsilon} > 0$ such that

$$\begin{bmatrix} \tilde{\phi} + \tilde{M}\tilde{M}^T & * \\ \tilde{N} & -\tilde{\epsilon}I \end{bmatrix} \prec 0, \quad (35)$$

$$\tau\tilde{C}^T\tilde{C} - P \prec 0, \quad (36)$$

where

$P = \begin{bmatrix} P1 & * & * \\ 0 & P2 & * \\ 0 & 0 & P3 \end{bmatrix}$, $\tilde{\phi} = \begin{bmatrix} \Pi_{11} & * & * \\ \Pi_{21} & \Pi_{22} & * \\ \Pi_{31} & \Pi_{32} & \Pi_{33} \end{bmatrix}$, $\Pi_{11} = \begin{bmatrix} \tilde{\phi}_{11} & * & * \\ 0 & \tilde{\phi}_{22} & * \\ 0 & 0 & \tilde{\phi}_{33} \end{bmatrix}$, $\Pi_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\Pi_{22} = \begin{bmatrix} \tilde{\phi}_{44} & * & * & * \\ 0 & \tilde{\phi}_{55} & ** & \\ 0 & 0 & \tilde{\phi}_{66} & * \\ 0 & 0 & 0 & \tilde{\phi}_{77} \end{bmatrix}$, $\Pi_{31} = \begin{bmatrix} \tilde{\phi}_{81} & \tilde{\phi}_{82} & \tilde{\phi}_{83} \\ 0 & \tilde{\phi}_{92} & \tilde{\phi}_{93} \\ 0 & \tilde{\phi}_{102} & \tilde{\phi}_{103} \end{bmatrix}$, $\Pi_{32} = \begin{bmatrix} \tilde{\phi}_{84} & 0 & 0 & \tilde{\phi}_{87} \\ \tilde{\phi}_{94} & \tilde{\phi}_{95} & 0 & 0 \\ 0 & \tilde{\phi}_{105} & \tilde{\phi}_{106} & 0 \end{bmatrix}$, $\Pi_{33} = \begin{bmatrix} -P_1 & * & * \\ 0 & -P_2 & * \\ 0 & 0 & -P_3 \end{bmatrix}$, $\tilde{\phi}_{11} = -\tilde{\alpha}P_1$, $\tilde{\phi}_{22} = -\tilde{\alpha}P_2$, $\tilde{\phi}_{33} = -\tilde{\alpha}P_3$, $\tilde{\phi}_{44} = -(1-\tilde{\alpha})I_{n_\omega}$, $\tilde{\phi}_{55} = -(1-\tilde{\alpha})I_{n_v}$, $\tilde{\phi}_{66} = \tilde{\phi}_{77} = -(1-\tilde{\alpha})I_{n_f}$, $\tilde{\phi}_{81} = P_1A - Y_1$, $\tilde{\phi}_{82} = Y_1$, $\tilde{\phi}_{83} = P_1BK_f$, $\tilde{\phi}_{84} = P_1D_\omega$, $\tilde{\phi}_{87} = P_1(F - BK_f)$, $\tilde{\phi}_{92} = P_2A - P_2L_xC$, $\tilde{\phi}_{93} = P_2F$, $\tilde{\phi}_{94} = P_2D_\omega$, $\tilde{\phi}_{95} = -P_2L_xD_v$, $\tilde{\phi}_{102} = -P_3L_fD_v$, $\tilde{\phi}_{103} = P_3$, $\tilde{\phi}_{105} = -P_3L_fD_v$, $\tilde{\phi}_{106} = P_3$, $\tau = \frac{1}{\lambda^2}$.

$\tilde{M} = \begin{bmatrix} 0 \\ P_1M_1 \\ P_2M_1 \\ 0 \end{bmatrix}$, $\tilde{N} = \begin{bmatrix} N_1 & 0 & 0 & 0 \end{bmatrix}$, and let $K_x = (P_1B)^{-1}Y_1$, then the fault-tolerant control system satisfies the L_∞ index (34).

Proof: According to the Schur complement lemma, (35) is equivalent to

$$\tilde{\phi} + \frac{1}{\tilde{\epsilon}}\tilde{M}\tilde{M}^T + \tilde{\epsilon}\tilde{N}^T\tilde{N} \prec 0. \quad (37)$$

According to Lemma 1, (37) implies

$$\tilde{\phi} + \tilde{M}\Delta\tilde{N} + \tilde{N}^T\Delta^T\tilde{M}^T \prec 0. \quad (38)$$

Submitting A_t, D_t and P into $\tilde{\phi}$, we can get

$$\tilde{\phi} = \begin{bmatrix} -\tilde{\alpha}P & * & * \\ 0 & -(1-\tilde{\alpha})I_{n_d} & * \\ PA_t & P\tilde{D}_t & -P \end{bmatrix} \quad (39)$$

where $I_{n_d} = n_\omega + n_v + n_f + n_h$.

Taking $\tilde{M}$, $\tilde{N}$ and Δ into $\tilde{M}\Delta\tilde{N}$, we get

$$\tilde{M}\Delta\tilde{N} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ P\Delta A_t & 0 & 0 \end{bmatrix} \quad (40)$$

Substituting (39) and (40) into (38) yields

$$\begin{bmatrix} -\tilde{\alpha}P & * & * \\ 0 & -(1-\tilde{\alpha})I_{n_d} & * \\ P\tilde{A}_t & P\tilde{D}_t & -P \end{bmatrix} \prec 0. \quad (41)$$

By pre-multiplying and post-multiplying (41) with $\begin{bmatrix} I_{2n_x+n_f} & 0 & \tilde{A}_t^T \\ 0 & I_{n_d} & \tilde{D}_t^T \end{bmatrix}$ and its transpose, respectively, we can get

$$\begin{bmatrix} -\tilde{\alpha}P + \tilde{A}_t^TP\tilde{A}_t & * \\ \tilde{D}_t^TP\tilde{A}_t & -(1-\tilde{\alpha})I_{n_d} + \tilde{D}_t^TP\tilde{D}_t \end{bmatrix} \prec 0. \quad (42)$$

Furthermore, by pre-multiplying and post-multiplying (42) with $\begin{bmatrix}\tilde{\varepsilon}^T(k) & \tilde{d}^T(k)\end{bmatrix}$ and its transpose, respectively, we can get

$$\begin{aligned}&\tilde{\varepsilon}^T(k)\tilde{A}_t^T P\tilde{A}_t\tilde{\varepsilon}(k)-\tilde{\alpha}\tilde{\varepsilon}^T(k)P\tilde{\varepsilon}(k)+\tilde{\varepsilon}^T(k)\tilde{A}_t^T P\tilde{D}_t\tilde{d}(k)\\&+\tilde{d}^T(k)\tilde{D}_t^T P\tilde{A}_t\tilde{\varepsilon}(k)+\tilde{d}^T(k)\tilde{D}_t^T P\tilde{D}_t\tilde{d}(k)\\&-(1-\tilde{\alpha})\tilde{d}^T(k)\tilde{d}(k)\le 0.\end{aligned} \tag{43}$$

Define the Lyapunov function as follows:

$$V(k)=\tilde{\varepsilon}^T(k)P\tilde{\varepsilon}(k), \tag{44}$$

then according to (44), (43) can be rewritten as follows:

$$V(k+1)\le\tilde{\alpha}V(k)+(1-\tilde{\alpha})\tilde{d}^T(k)\tilde{d}(k), \tag{45}$$

it follows that

$$\begin{aligned}V(k)&\le\tilde{\alpha}^k V(0)+\sum_{i=0}^{k-1}\tilde{\alpha}^i\tilde{d}_{k-1-i}^T\tilde{d}_{k-1-i}\\&\le\tilde{\alpha}^k V(0)+\sum_{i=0}^{k-1}\tilde{\alpha}^i\|\tilde{d}(k)\|_\infty^2\\&<\tilde{\alpha}^k V(0)+\|\tilde{d}(k)\|_\infty^2.\end{aligned} \tag{46}$$

By pre-multiplying and post-multiplying (36) with $\tilde{\varepsilon}^T(k)$ and $\tilde{\varepsilon}(k)$, respectively, we can get

$$x^T(k)x(k)\le\lambda^2\tilde{\varepsilon}^T(k)P\tilde{\varepsilon}(k). \tag{47}$$

Eventually, combining (46) and (47), we have

$$\|x(k)\|^2<\lambda^2(\tilde{\alpha}^k V(0)+\|\tilde{d}(k)\|_\infty^2), \tag{48}$$

which implies the L_∞ index (34). ■

The maximum τ can be obtained by solving the following optimization problem:

$$\begin{aligned}&\max\ \ \tau\\&s.t\ \ (35),(36).\end{aligned} \tag{49}$$

and $\lambda^2=\frac{1}{\tau}$ can be minimized, the feasible solution of the controller gain can be obtained by $K_x=(P_1B)^{-1}Y_1$.

V. SIMULATION RESULTS AND DISCUSSIONS

The effectiveness of the proposed fault tolerant control method is verified by an aero-engine model [17], which can be expressed as follows:

$$\begin{cases}\begin{bmatrix}\dot{N}_2-\dot{N}_2^o\\\dot{\pi}_T-\dot{\pi}_T^o\end{bmatrix}=A_p\begin{bmatrix}\dot{N}_2-\dot{N}_2^o\\\dot{\pi}_T-\dot{\pi}_T^o\end{bmatrix}+B_p\begin{bmatrix}WFM-WFM^o\\A8-A8^o\end{bmatrix}\\\begin{bmatrix}N_2-N_2^o\\\pi_T-\pi_T^o\end{bmatrix}=C_p\begin{bmatrix}N_2-N_2^o\\\pi_T-\pi_T^o\end{bmatrix}\end{cases} \tag{50}$$

where WFM is the main fuel flow rate, $A8$ is the nozzle throat area, N_2 is the speed of compressor and π_T is the turbine exit pressure ratio. Moreover, WFM^o, $A8^o$, N_2^o and π_T^o are values at the nominal state operating point. A_p, B_p and C_p represent the matrices of linearized model between the idle and intermediate state at $H=0km$, $M_a=0$.

The above continuous model is discretized using the Euler one-step method with sampling time $T_s=0.025s$. And considering the effects of model uncertainties and actuator fault, we can obtain a discrete-time system in the form of (1) with the following parameters:

$$\begin{aligned}&A=\begin{bmatrix}0.9450&-0.0441\\-0.0170&0.8815\end{bmatrix},\ B=\begin{bmatrix}0.0315&0.0206\\0.0077&0.0526\end{bmatrix},\\&C=\begin{bmatrix}1.0000&0\\0&1.0000\end{bmatrix},\ D_\omega=\begin{bmatrix}0.0500&0\\0&0.0500\end{bmatrix},\\&F=\begin{bmatrix}0.0315&0.0206\\0.0077&0.0526\end{bmatrix},\ D_v=\begin{bmatrix}0.0500&0\\0&0.0500\end{bmatrix},\\&M=\begin{bmatrix}0.2000&0\\0&0.2000\end{bmatrix},\ N=\begin{bmatrix}0.2000&0\\0&0.2000\end{bmatrix},\\&\Delta=\begin{bmatrix}0.04\delta_1^k&0\\0&0.04\delta_2^k\end{bmatrix},\ \omega(k)=\begin{bmatrix}0.1\delta_3^k&0\\0&0.1\delta_4^k\end{bmatrix},\\&v(k)=\begin{bmatrix}0.02\delta_5^k&0\\0&0.02\delta_6^k\end{bmatrix},\end{aligned}$$

where δ_1^k, δ_2^k, δ_3^k, δ_4^k, δ_5^k and δ_6^k are random signals between 0 and 1.

The values of the observer parameters and the fault-tolerant controller parameters are set as $\alpha=0.6$, $\epsilon=0.9$, $\tilde{\alpha}=0.5$, $\tilde{\epsilon}=0.66$. By solving the optimization problem (24) and (49), we can obtain $\kappa_1=0.4371$, $\lambda=0.0543$. The corresponding observer and the controller gain matrices are obtained as follows:

$$L=\begin{bmatrix}1.3580&0.0135\\0.0387&1.3568\\14.8408&-5.0922\\-1.2837&10.1091\end{bmatrix},K_x=\begin{bmatrix}15.8956&-6.9707\\-2.7954&8.2884\end{bmatrix},$$
$$K_f=\begin{bmatrix}1.0000&0\\0&1.0000\end{bmatrix}.$$

We give three fault forms, which are fault 1: $f(k)=[0.5\sin(0.1k-3);0.5\sin(0.1k-3)],30\le k\le 125$; fault 2: $f(k)=[0.5\sin(0.08k-0.8);0.5\sin(0.08k-0.8)],10\le k\le 128$; fault 3: $f(k)=[0.0125k-0.25;0.0125k-0.25,20\le k\le 100$. Among them, fault 1 is used to simulate Figs. 1-3 to verify the FTC performance, while fault 1, fault 2 and fault 3 are applied in Fig. 3 to verify the safety of the system under L_∞ index respectively. The estimation results of the two state components and the two fault components via the proposed observer are given in Fig. 1 and Fig. 2. They show that the proposed observer can obtain very accurate estimation of state and relatively accurate estimation of fault. There is a lag between true fault and estimated fault mainly caused by the variation of the fault signal between two adjacent instants, the estimation accuracy of state is higher than that of fault. The state dynamics with using fault-tolerant controller (26) are shown in Fig. 3. Different from the method in [13], the influence of parameter uncertainty on the system is considered in the observer design and fault-tolerant controller design based on L_∞ technology. It can be seen that the effect of actuator fault has been attenuated and we get better system performance than the method in [13]. In order to test possible scenarios of uncertainties, we have made 500 independent simulations with different disturbances and noise under three fault forms in Fig. 4. It can be seen that the Euclidian norm of the system state can be kept under the L_∞ index by the proposed fault-tolerant control method, ensuring that the system operates within secure limits. However, the Euclidian norm of the state by the method in [13] exceeds the preset L_∞ index.

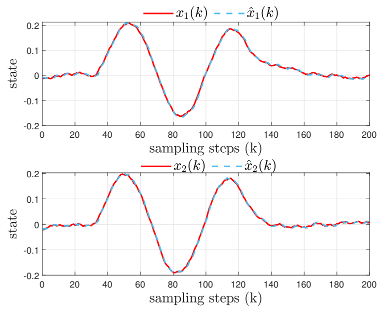

Fig. 1. State $x(k)$ and its estimation.

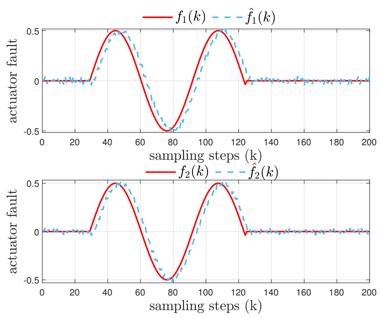

Fig. 2. Fault $f(k)$ and its estimation.

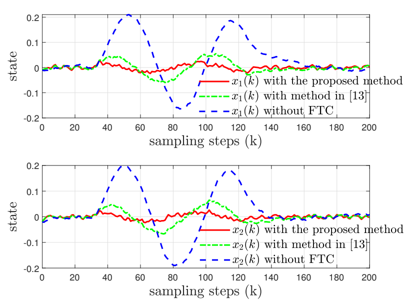

Fig. 3. The state $x(k)$ with and without fault-tolerant control.

VI. CONCLUSIONS AND FUTURE WORKS

A robust FTC approach has been proposed for discrete-time systems with parameter uncertainty, which has been mathematically proved and demonstrated by simulation results of a dual-rotor aero-engine system model. State and fault estimations for robust fault-tolerant controller design are obtained by the robust observer with L_∞ performance. The proposed robust FTC method can not only realize the performance recovery under actuator fault, but also realizes the state limitation to ensure the system works in a safe and reliable range. In the future, more complex systems will be considered.

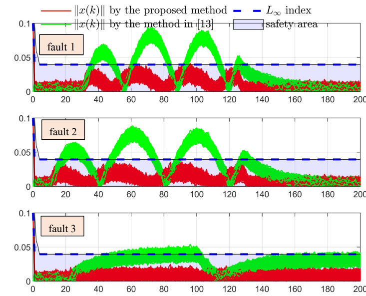

Fig. 4. The norm of state $x(k)$ with fault-tolerant control and its L_∞ index.

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