IEEE Control Systems Letters paper presented at
2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Robust Admittance Control with Complementary Passivity

Jiapeng Xu[1], Xiang Chen[1], Ying Tan[2], and Wulin Zou[3]

***Abstract*— This paper studies a robust admittance control problem with a passivity requirement for stable and unstable linear time-invariant systems, motivated by control issues originated from physical human-robot interaction. A complementary admittance control structure is proposed and analyzed, revealing that the nominal performance (admittance tracking and passivity) is decoupled from robustness. Simulations on the admittance control for human arm strength augmentation with a passivity requirement validate the proposed controller design.**

I. Introduction

Admittance/impedance control is a widely used control approach for interactive systems such as physical human-robot interaction and a robot system involving interaction with the environment [1]–[4]. It has been applied to address admittance/impedance tracking problems in various applications, e.g., force augmentation exoskeleton robots [4]–[6], robot-assisted rehabilitation [7], industrial manipulators [8], [9], and electric power steering system [10]. Reference [3] provides a comprehensive overview of the development and applications of admittance control.

Unlike classical servo control problems, where the goals are command following and disturbance rejection or attenuation, admittance control as an interaction control approach usually has an additional requirement of coupled stability [11], noticing that coupled stability allows poles on the imaginary axis [11]. It is shown in [11] that the coupled stability between a robot and any passive environment can be guaranteed if the robot is designed to behave like a passive system. Hence, in the design of an admittance controller, the controlled system in terms of a specific input-output channel is usually required to be passive to guarantee coupled stability or safe human-robot interaction. Both time-domain and frequency-domain methods have been studied for the passivity-based controller design, e.g., [12]–[14] in the time domain and [2], [11], [15], [16] in the frequency domain by using positive real condition.

Besides admittance tracking performance and passivity requirements, it is recognized that robustness against model uncertainties and external disturbances is also a key performance index in the admittance controller design [3], [11], [15]. However, most existing work considers either passivity [2], [11]–[16] or robust performance only [5], [8], [17]. A transfer function approach is proposed in [15] for interaction control. While passivity and disturbance attenuation are treated separately, a stable plant must be assumed and the robustness issue is actually not well explored. In [18], a multiobjective control synthesis approach is presented, where all objectives are formulated in a common Lyapunov function. However, it is noted that this design strategy inherently introduces conservatism. Hence, it is desired to develop an effective admittance controller design method considering both passivity requirements and robust performance.

In this paper, we delve into the field of human-robot interactive systems and focus on the investigation of a robust admittance control problem for linear time-invariant (LTI) systems, irrespective of whether they are stable or unstable. Our primary objective is to design an admittance controller in the state-space formulation that achieves the H_∞ performance without compromising the nominal performance (admittance tracking and passivity)[1]. In particular, by utilizing the control framework in [19], a robust admittance controller is proposed and shown to achieve complementary nominal performance, i.e., recovering nominal admittance tracking performance and passivity with respect to a specific input-output channel when uncertainties disappear. It is noted that our result (Theorem 2) proves that by adopting the control structure in [19], the nominal performance, i.e., admittance tracking and passivity, is decoupled from robustness. This is an extension of [19].

Notations: The set $\mathbb{R}^n$ consists of all n-dimensional real vectors, with $\mathbb{R} := \mathbb{R}^1$. A positive definite (positive semidefinite) is denoted by X by $X > 0$ ($X \geq 0$) and for a real matrix or vector Y, its transpose is denoted by Y'. The spectral radius of a square matrix is denoted by $\rho(\cdot)$. Let $\|T(s)\|_2$ and $\|T(s)\|_\infty$ represent the H_2 and H_∞ norms of transfer function or matrix $T(s)$, respectively. A transfer matrix in terms of state-space data is simply denoted by $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] := C(sI-A)^{-1}B + D$.

Definition 1 (Passivity): [2] [20, Definition 2.1] A system with input-output pair (h, g) where $h(t)$ and $g(t)$ are vectors of the same dimension is passive if there exists a nonpositive constant α such that

$$\int_0^t h'(\tau)g(\tau) \geq \alpha \tag{1}$$

for all functions h and all $t > 0$.

[1]Jiapeng Xu and Xiang Chen are with the Department of Electrical and Computer Engineering, University of Windsor, Windsor, ON N9B 3P4, Canada. jxu@uwindsor.ca, xchen@uwindsor.ca

[2]Ying Tan is with the Department of Mechanical Engineering, The University of Melbourne, Melbourne, VIC 3010, Australia. yingt@unimelb.edu.au

[3]Wulin Zou is with the Department of Research and Development, Xeno Dynamics Company, Ltd., Shenzhen 518055, China. zouwulin@xeno.com

[1]Nominal performance in this paper represents a target performance in terms of admittance and passivity achieved by the nominal system.

[2]The physical meaning of passivity is that the passive system cannot output more energy than its input energy.

II. A Motivation Example and State-Space Formulation of Problem

A. A motivation example

This example comes from the admittance control for human arm strength augmentation [4]. In this example, the human could lift up a heavy load with the help of the robot (see more details of this setting in [4]). The linearized model of the robot is:

$$m\ddot{\theta} + b\dot{\theta} + k\theta = u + d + \tau, \tag{2}$$

where θ is the joint angle, τ is the human–robot interaction force, u is the control input, and d is the disturbances representing modeling uncertainties, including the gravitational term of the added unknown load and other uncertainties. Here, m, b, and k are the mechanical inertia, damping and stiffness of the robot, respectively. There is sensor measuring the joint angle with measurement noise n, i.e.,

$$y_\theta = \theta + n. \tag{3}$$

Without measurement noise, the admittance model is used to characterize the dynamic relationship between velocity $\dot{\theta}$ and interaction force τ [2], i.e.,

$$Y_2(s) = \frac{s\theta(s)}{\tau(s)} = sY_1(s),\ Y_1(s) = \frac{1}{ms^2 + bs + k}. \tag{4}$$

The admittance controller aims to make the robot's inherent admittance Y_2 match a desired admittance, characterized by the following passive model:

$$Y_{2d}(s) = \frac{s\theta_d(s)}{\tau(s)} = sY_{1d}(s),\ Y_{1d}(s) = \frac{1}{m_d s^2 + b_d s + k_d}, \tag{5}$$

where θ_d is the desired position, and m_d, b_d and k_d are desired inertia, damping and stiffness, respectively.

To guarantee a stable/safe human-robot interaction, the closed-loop admittance from τ to $\dot{\theta}$ is required to be passive. In literature, two control schemes have been proposed to achieve the desired admittance behaviours. One is a position controller K_θ shown in Fig. 1(a). The form of K_θ can be PD controllers. The other scheme is to add a force feed-forward term to achieve a better admittance tracking performance with the passivity constraint [3], as shown in Fig. 1(b). Force feed-forward laws have been frequently used in the admittance controller design, see, for example, [2]–[4], [15], [16], and references therein. When the disturbances or noises are added, the tracking performance, the robustness against disturbances, and the needed passivity should be considered simultaneously. In this work, the design framework proposed in [19] will be adapted in the robust design of admittance control to address these concerns.

B. State-space formulation

Motivated by the robot model (2) and (3), a generalized state-space model is used to describe the controlled system:

$$\begin{aligned} \dot{x} &= Ax + B_1 w_1 + B_2 w_2 + B_3 u, \\ y &= C_3 x + D_{31} w_1, \end{aligned} \tag{6}$$

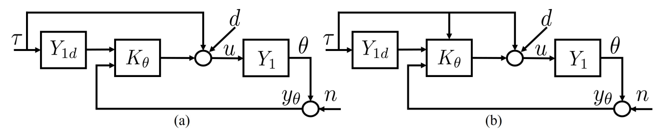

Fig. 1. Two classical admittance control structures: (a) admittance control with position controller; (b) admittance control extended with force feed-forward.

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ are the state, control input and measurement output, respectively, $w_1 \in \mathbb{R}^{q_1}$ is the unknown external disturbance signal, and $w_2 \in \mathbb{R}^{q_2}$ is a known (measured/estimated) exogenous signal such as the interaction force τ in human-robot systems. All matrices have compatible dimensions and $R_3 := D_{31}D_{31}' > 0$. The following is a standing assumption in the controller design for LTI system in the form of (6).

Assumption 1: (A, B_3) is stabilizable and (C_3, A) is detectable.

The control objective of this work is to track a desired state x_d and achieve the desired performance coming from the generalized desired admittance model, which has the following form:

$$\begin{aligned} \dot{x}_d &= A_d x_d + B_d w_2, \quad x_d(0) = x_{d0} \\ y_d &= C_d x_d \end{aligned} \tag{7}$$

where $x_d \in \mathbb{R}^n$ is the desired state corresponding to this given output reference $y_d \in \mathbb{R}^{q_2}$. In the human-robot interaction example, the desired output satisfies $y_d = \dot{\theta}_d$.

The following lemma provides a necessary and sufficient condition to ensure passivity in the frequency domain for a stable LTI system.

Lemma 1: [20, Theorem 2.25] A stable LTI system with a rational square transfer matrix $G(s)$ where all the poles of $G(s)$ have negative real parts, is passive if and only if

$$G(j\omega) + G'(-j\omega) \geq 0,\ \forall \omega \in \mathbb{R}. \tag{8}$$

To apply the above lemma, the following assumption is made.

Assumption 2: A_d is Hurwitz and the admittance from w_2 to y_d is passive.

Remark 1: Assumption 2 holds in the human-robot interaction example if all the parameters m_d, b_d and k_d are positive. The assumption of A_d to be Hurwitz makes the closed-loop system from input w_2 to output z_2 stable, such that the passivity requirement on (w_2, z_2) is equivalent to the condition in the frequency domain provided by Lemma 1. In the case that A_d has eigenvalues on the imaginary axis, the positive real condition [21, Section 6.3] is sufficient to guarantee passivity.

For the given task (tracking the desired state x_d and satisfying the passivity requirement), we can define two controlled outputs:

$$z_1 := C_1(x - x_d) + D_{13}u, \tag{9}$$

$$z_2 := C_d x, \tag{10}$$

where $z_1 \in \mathbb{R}^{q_3}$ is used to optimize the admittance tracking error $x - x_d$ and penalize the control effort, while $z_2 \in \mathbb{R}^{q_2}$ is defined for passivity, requiring the closed-loop system from w_2 to z_2 to be passive. In the human-robot interaction example, $z_2 = \dot{\theta}$. Define $R_1 := D_{13}' D_{13} > 0$.

Let $T_{z_1 w_1}(s)$, $T_{z_1 w_2}(s)$, and $T_{z_2 w_2}(s)$ denote the closed-loop transfer matrices from w_1 to z_1, w_2 to z_1 and w_2 to z_2, respectively. The objectives of this work are to design an admittance controller such that

(a) (*admittance tracking*) the state x can track the desired state x_d, which can be realized by optimizing certain norm of $T_{z_1 w_2}(s)$;
(b) (*passivity*) the closed-loop system from input w_2 to output z_2 is passive, i.e., $T_{z_2 w_2}(j\omega) + T_{z_2 w_2}'(-j\omega) \geq 0$, $\forall \omega \in \mathbb{R}$ by Lemma 1 and Assumption 2;
(c) (*robustness*) the H_∞ performance $\|T_{z_1 w_1}(s)\|_\infty < \gamma$ is satisfied, where $\gamma > 0$ is a prescribed value.

III. Main Results

In this section, we first propose a robust admittance control structure, which includes two parts: one part for nominal performance satisfying the admittance tracking performance and passivity requirement (objectives (a) and (b) in the previous section); the other for robustness to achieve the H_∞ performance (objective (c)). We will show that the nominal performance and robust performance can be two decoupled and are complementary to each other.

A. Robust admittance control structure

Motivated by the new control framework developed in [19], a robust admittance control structure is proposed, depicted in Fig. 2, where the control law is expressed as

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + B_2 w_2 + B_3 u + L(C_3 \hat{x} - y), \\ \dot{x}_d &= A_d x_d + B_d w_2, \ \ x_d(0) = x_{d0}, \\ u_n &= F\hat{x} + F_d x_d + F_{w_2} w_2, \\ u_f &= \tilde{Q}(f), \ f = C_3 \hat{x} - y, \\ u &= u_n + u_f. \end{aligned} \tag{11}$$

The control input u consists of two parts: 1) the nominal control u_n satisfying nominal design objectives including the admittance tracking performance and passivity requirement, characterized by the state feedback and observer gains (F, L) and feed-forward gains (F_d, F_{w_2}); 2) the robust control u_f, generated from the robustification controller $\tilde{Q}$ driven by residual signal f. Given the nominal control u_n, the H_∞ performance index can be achieved by finding a robustification controller $\tilde{Q}$.

Next, we provide a detailed design of the nominal performance controller and robustification controller $\tilde{Q}$.

B. Design of nominal performance controller

The nominal performance controller achieving objectives (a) and (b) in Subsection II-B is first designed for the nominal system, i.e., system (6) without w_1. Here, we present a design scheme from an optimization point of view.

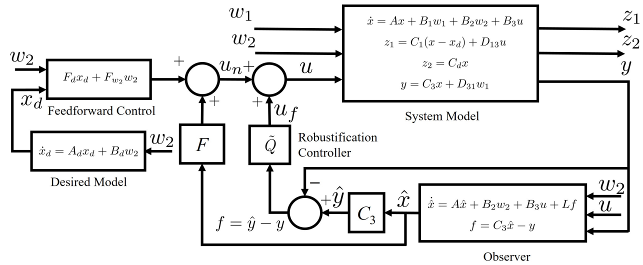

Fig. 2. A robust admittance control structure.

The first step is to design an observer-based output feedback stabilizing controller characterized by (F, L), shown in Fig. 3. There are many classical methods to synthesize controller gains, such as pole placement and LQG/H_2 control. We will adopt a frequently used performance controller: H_2 optimal controller, which requires solving the following optimization problem

$$\min_{F,L} \|T_{z_1 w_1}(s)\|_2. \tag{12}$$

To apply the Riccati equation method to solve the above optimization problem, the following assumption is needed [22, Section 14.5].

Assumption 3: (i) $\begin{bmatrix} A - j\omega I & B_3 \\ C_1 & D_{13} \end{bmatrix}$ has full column rank for all $\omega \in \mathbb{R}$, and (ii) $\begin{bmatrix} A - j\omega I & B_1 \\ C_3 & D_{31} \end{bmatrix}$ has full row rank for all $\omega \in \mathbb{R}$.

Condition (i) in Assumption 3 is equivalent to the statement that $\left((I - D_{13} R_1^{-1} D_{13}')C_1, A - B_3 R_1^{-1} D_{13}' C_1\right)$ has no unobservable modes on the imaginary axis [22, Lemma 13.9]; condition (ii) has a similar explanation. Then the solution is given by $F = -R_1^{-1}(\Pi_1 B_3 + C_1' D_{13})'$ and $L = -(\Pi_2 C_3' + B_1 D_{31}') R_3^{-1}$ [22, Chapter 14], where $\Pi_1 \geq 0$ and $\Pi_2 \geq 0$ are the stabilizing solution to the following two algebraic Riccati equations in the hypothesis of Assumption 3:

$$\begin{aligned} \Pi_1 A + A'\Pi_1 - (\Pi_1 B_3 + C_1' D_{13}) R_1^{-1} (\Pi_1 B_3 + C_1' D_{13})' \\ + C_1' C_1 = 0, \\ \Pi_2 A' + A\Pi_2 - (\Pi_2 C_3' + B_1 D_{31}') R_3^{-1} (\Pi_2 C_3' + B_1 D_{31}')' \\ + B_1 B_1' = 0. \end{aligned} \tag{13}$$

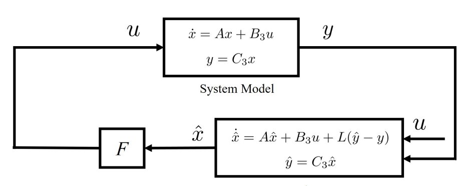

Fig. 3. Diagram for the design of an observer-based stabilizing controller.

Next we design the feed-forward controller for the admittance tracking and passivity requirement (objectives (a) and (b)), which do not involve the disturbance input w_1. Then by setting $w_1 = 0$ in Fig. 2, we obtain a simplified diagram shown in Fig. 4. Then feed-forward control gains F_d and F_{w_2} can be obtained by solving the following optimization problem:

$$\min_{F_d, F_{w_2}} \|T_{z_1 w_2}(s)\|_\infty$$
$$\text{s.t. } T_{z_2 w_2}(j\omega) + T'_{z_2 w_2}(-j\omega) \geq 0,\ \forall \omega \in \mathbb{R}, \tag{14}$$

where $T_{z_1 w_2}(s)$ and $T_{z_2 w_2}(s)$ are given by [3]

$$\begin{bmatrix} T_{z_1 w_2}(s) \\ T_{z_2 w_2}(s) \end{bmatrix} = \left[\begin{array}{cc|c} A + B_3 F & B_3 F_d & B_2 + B_3 F_{w_2} \\ 0 & A_d & B_d \\ \hline C_1 + D_{13} F & -C_1 + D_{13} F_d & D_{13} F_{w_2} \\ C_d & 0 & 0 \end{array}\right]. \tag{15}$$

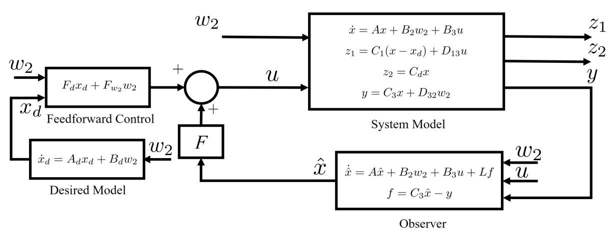

Fig. 4. Diagram for the design of the feed-forward controller.

Remark 2: One may would like to use H_2 norm instead of H_∞ norm in the optimization problem (14). However, it is known that the real rational subspace of H_2 only contains strictly proper transfer matrices [22, Page 98], resulting in zero-value F_{w_2}, following from the expression of $T_{z_1 w_2}$. Therefore, the H_∞ norm optimization is considered here in order to generate a non-zero F_{w_2}.

C. Robustification controller $\tilde{Q}$

The objective here is to find a robustification controller $u_f = \tilde{Q}(f)$ such that the H_∞ performance $\|T_{z_1 w_1}(s)\|_\infty < \gamma$ is achieved. To this end, set $w_2 = 0$ in Fig. 2. Thus a simplified diagram shown in Fig. 5 can be obtained. Define $\bar{x} := \begin{bmatrix} x' & x' - \hat{x}' \end{bmatrix}'$. Then from (6), (9) and (11) we have the following augmented system:

$$\begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}_1 w + \bar{B}_3 u_f, \\ f &= \bar{C}_3 \bar{x} + \bar{D}_{31} w_1, \\ z_1 &= \bar{C}_1 \bar{x} + \bar{D}_{13} u_f. \end{aligned} \tag{16}$$

See [19] for expressions of the system matrices. Clearly, the design of $u_f = \tilde{Q}(f)$ becomes a standard H_∞ controller design [22], [23] for the augmented system (16). Consider the following algebraic Riccati equations:

$$\begin{aligned} & P_1 A + A' P_1 + \gamma^{-2} P_1 B_1 B_1' P_1 + C_1' C_1 - \\ & \quad (P_1 B_3 + C_1' D_{13}) R_1^{-1} (P_1 B_3 + C_1' D_{13})' = 0, \\ & P_2 A' + A P_2 + \gamma^{-2} P_2 C_1' C_1 P_2 + B_1 B_1' - \\ & \quad (P_2 C_3' + B_1 D_{31}') R_3^{-1} (P_2 C_3' + B_1 D_{31}')' = 0. \end{aligned} \tag{17}$$

[3]The formula (15) can be obtained by converting the system in Fig. 4 from state space into transfer function.

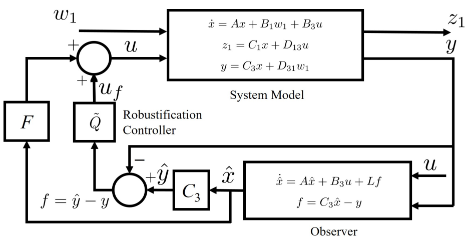

Fig. 5. Diagram for the design of the robustification controller $\tilde{Q}$.

Then the following theorem provides a simple state-space realization for the desired stable controller $\tilde{Q}$.

Theorem 1: Under Assumptions 1 and 3, and given (F, L) and $\gamma > 0$, there exists a stable $\tilde{Q}$ such that $\|T_{z_1 w_1}(s)\|_\infty < \gamma$ if and only if: (i) there exists the stabilizing solution $P_1 \geq 0$ and $P_2 \geq 0$ to Riccati equations (17), and (ii) $\rho(P_1 P_2) < \gamma$. Furthermore, one such $\tilde{Q}$, if exists, can be simply constructed as

$$\tilde{Q} = \left[\begin{array}{c|c} A_q & B_q \\ \hline F_q & 0 \end{array}\right] = \left[\begin{array}{cc|c} A & B_3 F_h & L \\ B_h C_3 & A_h & -B_h \\ \hline -F & F_h & 0 \end{array}\right], \tag{18}$$

where $\left[\begin{array}{c|c} A_h & B_h \\ \hline F_h & 0 \end{array}\right]$ is the central H_∞ controller achieving $\|T_{z_1 w_1}(s)\|_\infty < \gamma$ for system (6) and (9) with $w_2 = 0$, and

$$\begin{aligned} & A_h = A + \gamma^{-2} B_1 B_1' P_1 + B_3 F_h - B_h (C_3 + \gamma^{-2} D_{31} B_1' P_1), \\ & B_h = -(I - \gamma^{-2} P_1 P_2)^{-1} L_h,\ L_h = -(P_2 C_3' + B_1 D_{31}') R_3^{-1}, \\ & F_h = -R_1^{-1} (P_1 B_3 + C_1' D_{13})'. \end{aligned}$$

Proof: It follows from the central H_∞ controller [22], [23] and Theorem 1 of [19] that the expressions of (A_q, B_q, F_q) have the same form as (A_h, B_h, F_h), using the system matrices in (16) , i.e.,

$$\begin{aligned} & A_q = \bar{A} + \gamma^{-2} \bar{B}_1 \bar{B}_1' \bar{P}_1 + \bar{B}_3 F_q - B_q (\bar{C}_3 + \gamma^{-2} \bar{D}_{31} \bar{B}_1' \bar{P}_1), \\ & B_q = -(I - \gamma^{-2} \bar{P}_1 \bar{P}_2)^{-1} L_q,\ L_q = -(\bar{P}_2 \bar{C}_3' + \bar{B}_1 \bar{D}_{31}') R_3^{-1}, \\ & F_q = -R_1^{-1} (\bar{P}_1 \bar{B}_3 + \bar{C}_1' \bar{D}_{13})', \end{aligned}$$

with $\bar{P}_1 = \begin{bmatrix} P_1 & 0 \\ 0 & 0 \end{bmatrix}$, $\bar{P}_2 = \begin{bmatrix} P_2 & P_2 \\ P_2 & P_2 \end{bmatrix}$. After some tedious algebraic manipulations, (A_q, B_q, F_q) can be simply expressed as

$$\begin{aligned} A_q &= \begin{bmatrix} A_h + B_{\mathrm{h}} C_3 & -B_h C_3 \\ A_h - A - B_3 F_h + B_h C_3 & A - B_h C_3 \end{bmatrix}, \\ B_q &= \begin{bmatrix} -B_h \\ -B_h - L \end{bmatrix},\ F_q = \begin{bmatrix} F_h - F & F \end{bmatrix}. \end{aligned}$$

By a linear transformation with transformation matrix $T = \begin{bmatrix} I & -I \\ I & 0 \end{bmatrix}$, the expressions of (A_q, B_q, F_q) in the theorem can be obtained, which completes the proof.

D. Decoupled performances

Now we will reveal an interesting property of the proposed robust admittance control design: decoupled nominal and robust performances.

Theorem 2: For the robust control structure shown in Fig. 2, where the nominal performance controller and robustification controller $\tilde{Q}$ are designed by Subsection III-B and Theorem 1, respectively, the following two statements holds:

(i) The nominal performance (admittance tracking and passivity) is determined solely by (F, F_d, F_{w_2}), regardless of $\tilde{Q}$.

(ii) The closed-loop transfer matrix $T_{z_1 w_1}(s)$ is the same as that under the central H_∞ controller $\left[\begin{array}{c|c} A_h & B_h \\ \hline F_h & 0 \end{array}\right]$ for system (6) and (9) with $w_2 = 0$, which is apparently independent of selection of (F, F_d, F_{w_2}).

Proof: We prove statement (i) first. To investigate the nominal performance, set $w_1 = 0$ in Fig. 2. Notice that the admittance tracking performance and passivity requirement come from the transfer matrices $T_{z_1 w_2}$ and $T_{z_2 w_2}$, whose expressions are given by (15). Clearly, $T_{z_1 w_2}$ and $T_{z_2 w_2}$ are determined solely by (F, F_d, F_{w_2}), not related to $\tilde{Q}$.

Next we prove statement (ii). By setting $w_2 = 0$, the augmented system (16) can be obtained. Then from the augmented system (16) and robustification controller (18), we have

$$T_{z_1 w_1}(s) = \left[\begin{array}{cccc|c} A+B_3F & -B_3F & -B_3F & B_3F_h & B_1 \\ 0 & A+LC_3 & 0 & 0 & B_1+LD_{31} \\ 0 & -LC_3 & A & B_3F_h & -LD_{31} \\ 0 & B_hC_3 & B_hC_3 & A_h & B_hD_{31} \\ \hline C_1+D_{13}F & -D_{13}F & -D_{13}F & D_{13}F_h & 0 \end{array}\right]$$

The state of the above closed-loop system is $\left[\begin{array}{cc} \bar{x}' & x_q' \end{array}\right]'$ with x_q being the state of $\tilde{Q}$. By a linear transformation

$$x_t = T_1 \left[\begin{array}{c} \bar{x} \\ x_q \end{array}\right], \; T_1 = \left[\begin{array}{cccc} I & -I & -I & 0 \\ 0 & I & 0 & 0 \\ 0 & I & I & 0 \\ 0 & 0 & 0 & I \end{array}\right],$$

the closed-loop system can be transformed to the following simple form:

$$T_{z_1 w_1}(s) = \left[\begin{array}{cccc|c} A+B_3F & 0 & 0 & 0 & 0 \\ 0 & A+LC_3 & 0 & 0 & B_1+LD_{31} \\ 0 & 0 & A & B_3F_h & B_1 \\ 0 & 0 & B_hC_3 & A_h & B_hD_{31} \\ \hline C_1+D_{13}F & 0 & C_1 & D_{13}F_h & 0 \end{array}\right].$$

Hence, the above equality can be simplified as

$$T_{z_1 w_1}(s) = \left[\begin{array}{cc|c} A & B_3F_h & B_1 \\ B_hC_3 & A_h & B_hD_{31} \\ \hline C_1 & D_{13}F_h & 0 \end{array}\right], \qquad (19)$$

which is in fact the closed-loop transfer matrix from w_1 to z_1 for system (6) and (9) under the central H_∞ controller. Clearly, $T_{z_1 w_1}(s)$ in (19) does not depend on (F, F_d, F_{w_2}). The proof is thus complete.

Theorem 2 implies that if there is no disturbance, i.e., $w_1 = 0$, the nominal admittance performance is recovered, such that there is no tradeoff between the nominal and robust performances under the proposed robust admittance control design and they are complementary to each other. Here, the robustification controller $\tilde{Q}$ that provides an extra degree of freedom in the controller design plays a key role in achieving the complementarity.

IV. A Simulation Example

To verify the effectiveness of the proposed controller, we consider the example of admittance control for human arm strength augmentation described in Section II.

Let $x = \left[\begin{array}{cc} \theta & \dot{\theta} \end{array}\right]'$, $x_r = \left[\begin{array}{cc} \theta_r & \dot{\theta}_r \end{array}\right]'$ $w_1 = \left[\begin{array}{cc} d & n \end{array}\right]'$, $w_2 = \tau$ and $y = y_\theta$. The two controlled outputs are selected as $z_1 = \left[\begin{array}{ccc} 1000\theta & 10\dot{\theta} & u \end{array}\right]'$ and $z_2 = \dot{\theta}$, respectively. The system state-space model is thus given as follows:

$$A = \left[\begin{array}{cc} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{array}\right], \; B_1 = \left[\begin{array}{cc} 0 & 0 \\ \frac{1}{m} & 0 \end{array}\right], \; B_2 = B_3 = \left[\begin{array}{c} 0 \\ \frac{1}{m} \end{array}\right],$$

$$C_1 = \left[\begin{array}{cc} 1000 & 0 \\ 0 & 10 \\ 0 & 0 \end{array}\right], \; D_{13} = \left[\begin{array}{c} 0 \\ 0 \\ 1 \end{array}\right], \; C_d = \left[\begin{array}{cc} 0 & 1 \end{array}\right],$$

$$C_3 = \left[\begin{array}{cc} 1 & 0 \end{array}\right], \; D_{31} = \left[\begin{array}{cc} 0 & 0.01 \end{array}\right],$$

$$A_d = \left[\begin{array}{cc} 0 & 1 \\ -\frac{k_d}{m_d} & -\frac{b_d}{m_d} \end{array}\right], \; B_d = \left[\begin{array}{c} 0 \\ \frac{1}{m_d} \end{array}\right].$$

The parameter values are given by $m = 1.2$ Nm·s^2/rad, $b = 7.232$ Nm·s/rad and $k = 10.0$ Nm·/rad.

By solving Riccati equations (13) for the H_2 optimal controller, we obtain $F = \left[\begin{array}{cc} -990.0500 & -43.0514 \end{array}\right]$ and $L = \left[\begin{array}{cc} -7.6537 & -29.2894 \end{array}\right]$. Set $m_d = 0.2m$, $b_d = 0.2b$ and $k_d = 0.2k$. Solving the optimization problem (14) by MATLAB function "systune" using the nonsmooth optimization [24], the control gains for the feed-forward controller are obtained as $F_d = \left[\begin{array}{cc} 989.9736 & 43.0125 \end{array}\right]$ and $F_{w_2} = 3.9982$. Setting $\gamma = 11$, then the robustification controller $\tilde{Q}$ can be computed according to Theorem 1. The interactive force τ is set as a sinusoidal signal $4\sin(\pi t)$ Nm. The disturbance d is the sum of a gravitational signal ($20\sin\theta$ Nm) representing a kind of modelling uncertainties, a step signal (amplitude: -60 Nm), and a pulse signal (amplitude: 90 Nm, period: 0.6 s, pulse width: 50%). Two controllers are considered: with and without $\tilde{Q}$. The simulation results are shown in Fig. 6, where the disturbance d is added after $t = 10s$. It is clearly seen that when there is no disturbance, the tracking results of the two controllers are almost the same, which means that the robustification controller $\tilde{Q}$ does not influence the nominal performance generated by the H_2 and the feed-forward controller. However, as shown in Fig. 6, the nominal performance can be easily destroyed by the disturbances, while it can be restored by $\tilde{Q}$. Hence, the robustification controller can significantly reduce effect of disturbances on the admittance error.

The Bode plot of the closed-loop admittance is shown in Fig. 7. The passivity requirement from τ to $\dot{\theta}$ is satisfied since the phases are in the range of $[-90°, 90°]$ over the full frequency range [11]. It is also shown that the closed-loop admittance model $T_{\dot{\theta}\tau}(s)$ under the proposed controller matches closely with the desired admittance model $T_{\dot{\theta}_d\tau}(s)$.

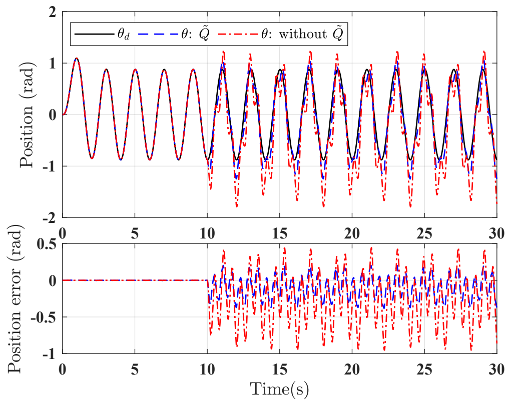

Fig. 6. Simulation results of the proposed method when $m_d = 0.2m$, $b_d = 0.2b$ and $k_d = 0.2k$. The disturbance d is added after $t = 10s$.

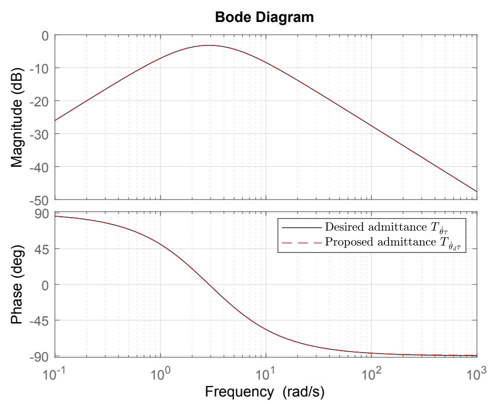

Fig. 7. Bode plots of the closed-loop admittance transfer function $T_{\dot{\theta}\tau}(s)$ and the desired transfer function $T_{\dot{\theta}_d\tau}(s)$.

V. Conclusion

In this study, we have developed a robust admittance control framework for linear time-invariant systems. The proposed complementary control structure decouples nominal performance from robust performance, allowing for effective handling of modeling uncertainties and external disturbances. Simulations have demonstrated the effectiveness and robustness of the proposed controller.

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