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Covariance Steering for Systems Subject to Unknown Parameters

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Abstract— **This work considers the optimal covariance steering problem for stochastic systems subject to both additive noise and uncertain parameters which may enter multiplicatively with the state and the control. The unknown parameters are modeled as constant random variables sampled from distributions with known moments. The optimal covariance steering problem is formulated so as to include dependence between the unknown parameters and future states and is solved using sequential convex programming. The proposed approach is demonstrated numerically using a spacecraft control application.**

I. Introduction

While existing stochastic optimal control methods typically require detailed knowledge of the system being optimized, many systems arising in practice include uncertain parameters. In such cases, these parameters may be modeled as constant random variables sampled from a particular probability distribution.

In this paper, we examine the problem of steering a stochastic linear system, in finite time, from an initial mean and covariance to a terminal mean and covariance, when the system is subject to parametric uncertainties (i.e., the disturbances enter both multiplicatively with the state and control, as well as additively). The covariance steering problem has previously been studied for both the infinite horizon [1]–[3] and the finite horizon [4]–[7] cases, in the presence of chance constraints [8]–[10], and for systems subject to purely additive Gaussian i.i.d. disturbances.

The literature on multiplicative disturbances is much less developed. In particular, the authors of [11], [12] investigated the covariance steering problem with parametric uncertainties. However, whereas previous works assumed the disturbances were independently, and identically distributed in time, this work assumes that the disturbances are unknown but constant, which is a more realistic assumption for modeling uncertainty. Furthermore, the proposed formulation allows for the dependence between the states and the disturbance realization, whereas an assumption of state-disturbance independence is critical to the approach of [11], [12]. In particular, we make no assumptions of the underlying distribution of the disturbances beyond that of all moments being known (e.g., the disturbances may be sampled from a Gaussian or uniform distribution).

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The proposed problem formulation has connections to the literature of robust control and set-based methods, in particular, in the case that the disturbances are assumed to be sampled from a uniform distribution. The robust control literature primarily considers unknown, but deterministic disturbances which may enter the dynamics both multiplicatively or additively, similar to the proposed problem [13]–[17]. The difference being that set-based methods upper bound the reachable set of the state for all possible disturbance realizations, requiring the disturbances to be drawn from bounded sets (i.e., distributions with bounded support such as the uniform distribution) [18]. Stochastic approaches as used in this work, on the other hand, have the advantage of being able to deal with disturbances having bounded or unbounded support by considering the likelihood of disturbance realizations and by imposing probabilistic bounds [18], [19].

Prior works on covariance steering have utilized semidefinite programming (SDP) to solve the optimal covariance steering problem [4], [8], [9], [20]–[22]. However, the problem considered in this work includes a state-disturbance dependence which greatly complicates the moment dynamics and prevents the use of prior techniques for the derivation of an SDP. Instead, the proposed approach utilizes sequential convex programming (SCP), which has been shown to have convergence guarantees to a local optimum under mild conditions [23], [24] (see, for example, [13], [25], [26]). It is shown that if the proposed solution method converges to a stationary point, then a solution of the nonconvex covariance steering problem has been found.

Notation: A random variable drawn from a normal distribution with mean μ and covariance matrix Σ is denoted by $x \sim \mathcal{N}(\mu, \Sigma)$, and a variable drawn from a uniform distribution with bounds a and b is denoted by $\mathcal{U}(a, b)$. $\mathbb{E}[\cdot]$ denotes the expectation operator, and $\Pr(x)$ denotes the probability of event x. I_n denotes the $n \times n$ identity matrix, $\mathtt{diag}(a_1, \ldots, a_n)$ denotes a square diagonal matrix with entries $a_1, \ldots, a_n$ on the diagonal, and $\mathtt{tr}(\cdot)$ denotes the trace operation. A symmetric positive (semi)-definite matrix is denoted by $M \succ 0$ ($M \succeq 0$).

II. Problem Formulation

Consider the system

$$x_{k+1} = Ax_k + Bu_k + Dw_k, \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$, $u_k \in \mathbb{R}^{n_u}$, and $w_k \in \mathbb{R}^{n_w}$, where $\mathbb{E}[w_k] = 0$, $\mathbb{E}[(w_k - \mathbb{E}[w_k])(w_k - \mathbb{E}[w_k])^\top] = I_{n_w}$, and $\mathbb{E}[w_{k_1} w_{k_2}^\top] = \mathbb{E}[w_{k_1}]\mathbb{E}[w_{k_2}^\top] = 0, \ \forall\ k_1 \neq k_2$. Let the initial conditions be given as $\mathbb{E}[x_0] = \mu_0$ and $\mathbb{E}[(x_0 - \mathbb{E}[x_0])(x_0 - \mathbb{E}[x_0])^\top] = \Sigma_0$, where $\Sigma_0 \succeq 0$. Additionally, the system matrices are comprised of a known component and a time-invariant stochastic component, which depends on a set of constant, but unknown parameters $\{p_1,\dots,p_{n_p}\}$, given by $A=\bar{A}+\sum_{j=1}^{n_p}\tilde{A}_j p_j$, $B=\bar{B}+\sum_{j=1}^{n_p}\tilde{B}_j p_j$, $D=\bar{D}+\sum_{j=1}^{n_p}\tilde{D}_j p_j$, where, for all $j=1,\dots,n_p$, $p_j:\Omega\to\mathbb{R}$ is a random variable with corresponding probability triple $(\Omega,\mathcal{F},P)$ and $\mathbb{E}[p_j]=0$. The zero-mean assumption is not restrictive because the mean can always be accounted for by adding an appropriate offset to $\bar{A}$, $\bar{B}$, and/or $\bar{D}$ accordingly. Additionally, we assume that x_0, w_k, and p_j are all mutually independent for all $k=0,\dots,N-1$ and $j=1,\dots,n_p$, yielding $\mathbb{E}[p_{j_1}p_{j_2}]=\mathbb{E}[p_{j_1}]\mathbb{E}[p_{j_2}]=0,\ \forall\, j_1\neq j_2$, $\mathbb{E}[w_k p_j]=\mathbb{E}[w_k]\mathbb{E}[p_j]=0$, and

$$\mathbb{E}[x_0 p_{j_1}\dots p_{j_\ell}]=\mu_0\mathbb{E}[p_{j_1}\dots p_{j_\ell}], \tag{2a}$$

$$\begin{aligned}&\bar{\Sigma}[x_0\bar{p}_{j_\ell},x_0\bar{p}_{i_\iota}]\\&=(\Sigma_0+\mu_0\mu_0^\top)\mathbb{E}[\bar{p}_{j_\ell},\bar{p}_{i_\iota}]-\mu_0\mu_0^\top\mathbb{E}[\bar{p}_{j_\ell}]\mathbb{E}[\bar{p}_{i_\iota}],\end{aligned} \tag{2b}$$

$$\bar{\Sigma}[x_0\bar{p}_{j_\ell},\bar{p}_{i_\iota}]=\mu_0\mathbb{E}[\bar{p}_{j_\ell},\bar{p}_{i_\iota}]-\mu_0\mathbb{E}[\bar{p}_{j_\ell}]\mathbb{E}[\bar{p}_{i_\iota}], \tag{2c}$$

where $\ell=1,2,\dots$ and $\iota=1,2,\dots$, and where, for brevity, we write $\bar{p}_{j_\ell}=p_{j_1}p_{j_2}\dots p_{j_\ell}$, $\tilde{\Sigma}[x\bar{p}_{j_\ell}]=(x\bar{p}_{j_\ell})-\mathbb{E}[x\bar{p}_{j_\ell}]$, and $\bar{\Sigma}[x\bar{p}_{j_\ell},y\bar{p}_{i_\iota}]=\mathbb{E}[\sigma[x\bar{p}_{j_\ell}]\sigma[y\bar{p}_{i_\iota}]^\top]$. Finally, we assume that all moments of p_j are known (e.g., as is the case if p_j is Gaussian distributed with known variance or uniformly distributed with known bounds).

Contrary to most works on stochastic control of linear systems (e.g., [11], [12]), we can no longer make the assumption that the state and disturbance realization at a given time-step are independent. That is, $\mathbb{E}[x_k p_j]\neq\mathbb{E}[x_k]\mathbb{E}[p_j]=0$, for $k=0,1,\dots,N-1$ and $j=1,\dots,n_p$. We will derive an expression for $\mathbb{E}[x_k p_j]$ in Section III.

The state and control inputs in (1) are subject to a collection of linear chance constraints given by

$$\Pr(\alpha_{x,i_x}^\top x_k\le\beta_{x,i_x})\ge 1-\delta_{x,i_x},\quad i_x=1,\dots,N_x, \tag{3a}$$

$$\Pr(\alpha_{u,i_u}^\top u_k\le\beta_{u,i_u})\ge 1-\delta_{u,i_u},\quad i_u=1,\dots,N_u, \tag{3b}$$

for all $k=0,1,\dots,N-1$, where $\alpha_{x,i_x}\in\mathbb{R}^{n_x}$ and $\alpha_{u,i_u}\in\mathbb{R}^{n_u}$ are constant vectors, $\beta_{x,i_x}\ge 0$ and $\beta_{u,i_u}\ge 0$ are constant scalars, and $\delta_{x,i_x},\delta_{u,i_u}>0$ are given maximal probabilities of constraint violation. We impose a chance constraint on the control action u_k given by (3b) because, as will be seen in Section III, we consider the control action to be a function of the state x_k, making u_k a random variable as well.

We wish to steer (1) to a given final mean μ_F and covariance $\Sigma_F\succ 0$ at time N, such that $\mathbb{E}[x_N]=\mu_F$, $\mathbb{E}[(x_N-\mathbb{E}[x_N])(x_N-\mathbb{E}[x_N])^\top]=\Sigma_F$, while minimizing the cost function $J(\mu_0,\Sigma_0;u_0,\dots,u_{N-1})=\mathbb{E}\left[\sum_{k=0}^{N-1}\ell(x_k,u_k)\right]$. In particular, we will investigate the case where $\ell(\cdot,\cdot)$ has the form $\ell(x,u)=x^\top Qx+u^\top Ru$, where $Q\in\mathbb{R}^{n_x\times n_x}$, $R\in\mathbb{R}^{n_u\times n_u}$, $Q\succeq 0$, and $R\succ 0$. The problem may thus be summarized as follows: given $\mu_0,\Sigma_0,\mu_F,\Sigma_F$, determine the control sequence $\mathbf{u}=\{u_0,\dots,u_{N-1}\}$ which solves the following finite-time, optimal covariance steering problem

$$\min_{\mathbf{u}}\ J(\mu_0,\Sigma_0;\mathbf{u})=\mathbb{E}\left[\sum_{k=0}^{N-1}x_k^\top Qx_k+u_k^\top Ru_k\right], \tag{4a}$$

subject to

$$\mathbb{E}[x_0]=\mu_0, \tag{4b}$$

$$\mathbb{E}[(x_0-\mathbb{E}[x_0])(x_0-\mathbb{E}[x_0])^\top]=\Sigma_0, \tag{4c}$$

$$\begin{aligned}x_{k+1}&=(\bar{A}+\sum_{j=1}^{n_p}\tilde{A}_j p_j)x_k+(\bar{B}+\sum_{j=1}^{n_p}\tilde{B}_j p_j)u_k\\&\quad+(\bar{D}+\sum_{j=1}^{n_p}\tilde{D}_j p_j)w_k,\end{aligned} \tag{4d}$$

$$\Pr(\alpha_{x,i_x}^\top x_k\le\beta_{x,i_x})\ge 1-\delta_{x,i_x},\quad i_x=1,\dots,N_x, \tag{4e}$$

$$\Pr(\alpha_{u,i_u}^\top u_k\le\beta_{u,i_u})\ge 1-\delta_{u,i_u},\quad i_u=1,\dots,N_u, \tag{4f}$$

$$\mathbb{E}[x_N]=\mu_F, \tag{4g}$$

$$\mathbb{E}[(x_N-\mathbb{E}[x_N])(x_N-\mathbb{E}[x_N])^\top]=\Sigma_F, \tag{4h}$$

for $k=0,\dots,N-1$.

III. Covariance Steering Controller Design

A. Moment Formulation

We introduce the state-feedback control policy $u_k=L_k x_k+v_k$, where $L_k\in\mathbb{R}^{n_u\times n_x}$ is the feedback gain and $v_k\in\mathbb{R}^{n_u}$ is the open-loop control action at time-step k. Consequently, notice that the system (1) can be written as

$$\begin{aligned}x_{k+1}&=(\bar{A}+\bar{B}L_k)x_k+\bar{B}v_k+\bar{D}w_k\\&\quad+\sum_{j=1}^{n_p}\Big((\tilde{A}_j+\tilde{B}_jL_k)x_k+\tilde{B}_jv_k+\tilde{D}_jw_k\Big)p_j.\end{aligned} \tag{5}$$

From (5), straightforward calculations show that the expected state at time $k=0,\dots,N-1$, may be succinctly described by the difference equation

$$\begin{aligned}&\mathbb{E}[x_{k+1-\ell}\bar{p}_{j_\ell}]\\&=f(\{\mathbb{E}[x_{k-\ell}\bar{p}_{j_\eta}],\mathbb{E}[\bar{p}_{j_\eta}]\}_{\eta=\ell}^{\ell+1},L_{k-\ell},v_{k-\ell})\\&=(\bar{A}+\bar{B}L_{k-\ell})\mathbb{E}[x_{k-\ell}\bar{p}_{j_\ell}]+\bar{B}v_{k-\ell}\mathbb{E}[\bar{p}_{j_\ell}]\\&+\sum_{j_{\ell+1}=1}^{n_p}(\tilde{A}_{j_{\ell+1}}+\tilde{B}_{j_{\ell+1}}L_{k-\ell})\mathbb{E}[x_{k-\ell}\bar{p}_{j_{\ell+1}}]\\&+\tilde{B}_{j_{\ell+1}}v_{k-\ell}\mathbb{E}[\bar{p}_{j_{\ell+1}}],\end{aligned} \tag{6}$$

where $\mathbb{E}[x_0\bar{p}_{j_\ell}]=\mu_0\mathbb{E}[\bar{p}_{j_\ell}]$, where $\ell=0,\dots,k$ and $j=1,\dots,n_p$. The derivation of (6) is given in Appendix A. Note that (6) depends on the moments of increasing order of p_j and the previous state. Therefore, (6) can be evaluated using (2a).

Likewise, the state covariance can be described by a similar (albeit more extensive) set of difference equations. We can compute

$$\begin{aligned}&\bar{\Sigma}[x_{k+1-\kappa}\bar{p}_{i_\iota},x_{k+1-\kappa}\bar{p}_{j_\ell}]\\&=g(\{\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_\upsilon},x_{k-\kappa}\bar{p}_{j_\eta}],\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_\upsilon},\bar{p}_{j_\eta}],\\&\bar{\Sigma}[\bar{p}_{i_\upsilon},\bar{p}_{j_\eta}],\mathbb{E}[\bar{p}_{i_\upsilon},\bar{p}_{j_\eta}]\}_{\upsilon=\iota,\eta=\ell}^{\iota+1,\ell+1},L_{k-\kappa},v_{k-\kappa}),\end{aligned} \tag{7}$$

where $\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_\iota},\bar{p}_{j_\ell}]$

$$=h(\{\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_\upsilon},\bar{p}_{j_\ell}],\bar{\Sigma}[\bar{p}_{i_\upsilon},\bar{p}_{j_\ell}]\}_{\upsilon=\iota}^{\iota+1},v_{k-\kappa}), \tag{8}$$

and where $k = 0, \dots, N-1$, $(\iota, \ell) = 0, \dots k$, $\kappa = \max[\iota, \ell]$ and $i, j = 0, \dots, n_p$. $\bar{\Sigma}[x_0 \bar{p}_{j_\ell}, x_0 \bar{p}_{i_\iota}]$ and $\bar{\Sigma}[x_0 \bar{p}_{j_\ell}, \bar{p}_{i_\iota}]$ are given by (2b) and (2c), respectively, and $g(\cdot)$ and $h(\cdot)$ are given in Appendix B. Note that $g(\cdot)$ and $h(\cdot)$ depend on the previous covariances and increasing moments of p_j and therefore can be evaluated using (2b) and (2c) as the initialization.

In conclusion, the mean and covariance of the state at time k can be succinctly described in terms of the control parameters $\mathbf{v} = \{v_0, \dots, v_{N-1}\}$ and $\mathbf{L} = \{L_0, \dots, L_{N-1}\}$, the initial conditions (2a)-(2c), and by the three difference equations (6), (7), and (8). Therefore, we may reformulate Problem (4) as the deterministic optimization problem

$$\min_{\mathbf{v}, \mathbf{L}} \quad J(\mu_0, \Sigma_0; \mathbf{v}, \mathbf{L}) = \sum_{k=0}^{N-1} \mathbb{E}[x_k]^\top Q \mathbb{E}[x_k] + (v_k + L_k \mathbb{E}[x_k])^\top R (v_k + L_k \mathbb{E}[x_k]) + \mathtt{tr}(\bar{\Sigma}[x_k, x_k] Q) + \mathtt{tr}(L_k \bar{\Sigma}[x_k, x_k] L_k^\top R), \tag{9a}$$

subject to

$$\mathbb{E}[x_0 \bar{p}_{j_\ell}] = \mu_0 \mathbb{E}[\bar{p}_{j_\ell}], \tag{9b}$$

$$\bar{\Sigma}[x_0 \bar{p}_{j_\ell}, x_0 \bar{p}_{i_\iota}] = (\Sigma_0 + \mu_0 \mu_0^\top) \mathbb{E}[\bar{p}_{j_\ell}, \bar{p}_{i_\iota}] - \mu_0 \mu_0^\top \mathbb{E}[\bar{p}_{j_\ell}] \mathbb{E}[\bar{p}_{i_\iota}], \tag{9c}$$

$$\mathbb{E}[x_{k+1-\ell} \bar{p}_{j_\ell}] = f(\{\mathbb{E}[x_{k-\ell} \bar{p}_{j_\eta}], \mathbb{E}[\bar{p}_{j_\eta}]\}_{\eta=\ell}^{\ell+1}, L_{k-\ell}, v_{k-\ell}), \tag{9d}$$

$$\bar{\Sigma}[x_{k+1-\kappa} \bar{p}_{i_\iota}, x_{k+1-\kappa} \bar{p}_{j_\ell}] = g(\{\bar{\Sigma}[x_{k-\kappa} \bar{p}_{i_\upsilon}, x_{k-\kappa} \bar{p}_{j_\eta}], \bar{\Sigma}[x_{k-\kappa} \bar{p}_{i_\upsilon}, \bar{p}_{j_\eta}], \bar{\Sigma}[\bar{p}_{i_\upsilon}, \bar{p}_{j_\eta}], \mathbb{E}[\bar{p}_{i_\upsilon}, \bar{p}_{j_\eta}]\}_{\upsilon=\iota, \eta=\ell}^{\iota+1, \ell+1}, L_{k-\kappa}, v_{k-\kappa}), \tag{9e}$$

$$\bar{\Sigma}[x_{k+1-\kappa} \bar{p}_{i_\iota}, \bar{p}_{j_\ell}] = h(\{\bar{\Sigma}[x_{k-\kappa} \bar{p}_{i_\upsilon}, \bar{p}_{j_\ell}], \bar{\Sigma}[\bar{p}_{i_\upsilon}, \bar{p}_{j_\ell}]\}_{\upsilon=\iota}^{\iota+1}, L_{k-\kappa}, v_{k-\kappa}), \tag{9f}$$

$$\sqrt{\alpha_{x,i_x}^\top \bar{\Sigma}[x_k, x_k] \alpha_{x,i_x}} \sqrt{\frac{1 - \delta_{x,i_x}}{\delta_{x,i_x}}} + \alpha_{x,i_x}^\top \mathbb{E}[x_k] - \beta_{x,i_x} \leq 0, \tag{9g}$$

$$\sqrt{\alpha_{u,i_u}^\top L_k \bar{\Sigma}[x_k, x_k] L_k^\top \alpha_{u,i_u}} \sqrt{\frac{1 - \delta_{u,i_u}}{\delta_{u,i_u}}} + \alpha_{u,i_u}^\top (v_k + L_k \mathbb{E}[x_k]) - \beta_{u,i_u} \leq 0, \tag{9h}$$

$$\mathbb{E}[x_N] = \mu_F, \tag{9i}$$

$$\bar{\Sigma}[x_N, x_N] = \Sigma_F, \tag{9j}$$

where $(j_n, i_n) = 1, \dots, n_p$, $(\ell, \iota) = 0, \dots, N-1$ and where we have applied Cantelli's inequality [27] to the chance constraints (3).

B. Solution Methodology

Problem (9) is nonconvex owing to the multiplication between L_k and $\mathbb{E}[x_k]$ in (6), and similar bilinearities in (7) and (8). Previous work has overcome these issues by proposing an alternative control policy (e.g., $u_k = L_k(x_k - \mathbb{E}[x_k]) + v_k$ or $u_k = \sum_{t=0}^{k} K_t w_t + v_k$) in order to ensure that only the additive term v_k appears in the mean dynamics and overcome the bilinearities in the covariance constraint by utilizing symmetry and performing a change of variables to create a semidefinite program (SDP) [9], [11], [21]. However, due to the state-dependent nature of the multiplicative disturbances, it is not possible to remove the feedback policy from (6) because the state mean is not independent of the disturbances. Moreover, the structure of (7) and (8) does not admit a straightforward conversion to a SDP. Instead, we solve Problem (9) using sequential convex programming (SCP) [23]–[25].

We define a convex approximation of (9) by

$$\min_{\mathbf{v}, \mathbf{L}} \psi(\mu_0, \Sigma_0, \mathbf{v}, \mathbf{L}, \hat{\chi}, \hat{\epsilon}, \hat{\mathbf{v}}, \hat{\mathbf{L}}), \tag{10a}$$

subject to

$$0 = \phi(\mu_0, \Sigma_0, \mathbf{v}, \mathbf{L}, \hat{\chi}, \hat{\epsilon}, \hat{\mathbf{v}}, \hat{\mathbf{L}}) \tag{10b}$$

where $\hat{\chi} = \{\hat{\mu}[x_k \bar{p}_{j_\ell}]\}_{k=0, j=1, \ell=0}^{N-1, n_p, N-k}$, $\hat{\xi} = \{\hat{\Sigma}[x_k], \hat{\Sigma}[x_k p_{i_1}, p_{j_1}], \dots, \hat{\Sigma}[x_k p_{i_1} \dots p_{i_\iota}, p_{j_1} \dots p_{j_\ell}], \hat{\Sigma}[x_k p_{i_1}, x_k p_{j_1}], \dots, \hat{\Sigma}[x_k p_{i_1} \dots p_{i_\iota}, x_k p_{j_1} \dots p_{j_\ell}]\}_{k=0, i=1, \iota=0, j=1, \ell=0}^{N-1, n_p, N-k, n_p, N-k}$, $\hat{\mathbf{L}} = \{\hat{L}_k\}_{k=0}^{N-1}$, and $\hat{\mathbf{v}} = \{\hat{v}_k\}_{k=0}^{N-1}$, and where $\hat{L}_k$, $\hat{\mu}[x_{k-\ell} \bar{p}_{j_\ell}]$, $\hat{v}_k$, $\hat{\Sigma}[x_{k-\kappa} \bar{p}_{i_\iota}, \bar{p}_{j_\ell}]$, and $\hat{\Sigma}[x_{k-\kappa} \bar{p}_{i_\iota}, x_{k-\kappa} \bar{p}_{j_\ell}]$ are the linearization points about the decision variables $L_{k-\ell}$, $\mathbb{E}[x_{k-\ell} \bar{p}_{j_\ell}]$, $v_{k-\kappa}$, $\bar{\Sigma}[x_{k-\kappa} \bar{p}_{i_\iota}, \bar{p}_{j_\ell}]$, and $\Sigma[x_{k-\kappa} \bar{p}_{i_\iota}, x_{k-\kappa} \bar{p}_{j_\ell}]$, respectively. The definitions of $\psi(\cdot)$ and $\phi(\cdot)$ are given in [28]. Note that, in practice, (10) can be computed automatically using automatic differentiation software to linearize the nonconvex terms in the cost function and constraints.

The sequential convex programming algorithm used to solve Problem (9) using Problem (10) is given by Algorithm 1. Let $\mathbf{v}^*, \mathbf{L}^*$ be the solution returned by Algorithm 1

Algorithm 1 Sequential Convex Programming

Require: Initial moments: μ_0, Σ_0
Require: Initial control guesses: $\hat{\mathbf{v}}, \hat{\mathbf{L}}$
Require: Convergence tolerance: ϵ
loop
  $\hat{\chi} \leftarrow$ Evaluate (6) using $\{\mu_0, \hat{\mathbf{v}}, \hat{\mathbf{L}}\}$
  $\hat{\xi} \leftarrow$ Evaluate (7) and (8) using $\{\Sigma_0, \hat{\mathbf{v}}, \hat{\mathbf{L}}\}$
  $\{\mathbf{v}, \mathbf{L}\} \leftarrow$ Solve Problem (10) using $\{\hat{\chi}, \hat{\xi}, \hat{\mathbf{v}}, \hat{\mathbf{L}}\}$
  if $\sum_{k=0}^{N-1} \|v_k - \hat{v}_k\|_2 + \|\mathtt{vec}(L_k - \hat{L}_k)\|_2 < \epsilon$ **then**
    return $\{\mathbf{v}, \mathbf{L}\}$
  end if
  $\hat{\mathbf{v}} \leftarrow \mathbf{v}$, $\hat{\mathbf{L}} \leftarrow \mathbf{L}$
end loop

with corresponding moments χ^*, ξ^*, where χ^* is the result of evaluating (6) using μ_0, $\mathbf{v}^*$, and $\mathbf{L}^*$, and where ξ^* is the result of evaluating (7) and (8) using Σ_0, $\mathbf{v}^*$, and $\mathbf{L}^*$. Note that $\mathbf{v}^*, \mathbf{L}^*$ is therefore a solution of Problem (10) for a particular linearization point, which we denote by $\hat{\mathbf{v}}^*, \hat{\mathbf{L}}^*$. We introduce the following theorem regarding the validity of this solution, which is found using the convex local approximation (10), in relation to the original covariance steering problem (4).

Theorem 1: If $\hat{\mathbf{v}}^*, \hat{\mathbf{L}}^*$ is a stationary point of Problem (10) (that is, $\mathbf{v}^* = \hat{\mathbf{v}}^*$ and $\mathbf{L}^* = \hat{\mathbf{L}}^*$) then $\mathbf{v}^*, \mathbf{L}^*$ is a stationary point of Problem (9), and, furthermore, $\mathbf{v}^*, \mathbf{L}^*$ is a feasible solution of Problem (4).

Proof: Note that when $\mathbf{v}^* = \hat{\mathbf{v}}^*$ and $\mathbf{L}^* = \hat{\mathbf{L}}^*$, Problem (10) collapses to Problem (9), since the convex approximation is exact at the linearization points. Therefore, if $\hat{\mathbf{v}}^*, \hat{\mathbf{L}}^*$ is a stationary point of Problem (10), it is also a stationary point of Problem (9). For the second statement, Problem (9) is equivalent to Problem (4) except for the chance constraints (9g)-(9h), which are conservative approximations of (4e)-(4f) due to the use of Cantelli's inequality in (9g)-(9h). Therefore, a solution satisfying (9b)-(9j) is guaranteed to also satisfy (4b)-(4h). ■

Therefore, if Algorithm 1 converges, it yields a feasible solution to the original covariance steering problem (4). For guarantees on the rate of convergence of SCP algorithms, see, for example, [23], in which it is shown that SCP converges linearly under mild assumptions, in particular, given an initial guess for $\hat{\mathbf{v}}, \hat{\mathbf{L}}$ which is sufficiently close to a stationary point.

IV. Numerical Results

A. Spacecraft Control Example

The proposed approach is verified on a spacecraft control task using Monte Carlo simulations, and the results are compared to a naïve solution of a stochastic problem formulation which assumes that the noise realizations are time-varying and i.i.d and a robust problem formulation that assumes the realizations belong to an ellipsoidal set. The spacecraft is considered to move in a plane and is shown in Fig. 1(a). The equations of motion are given by

$$\dot{X} = \nu_x \cos\Psi - \nu_y \sin\Psi, \tag{11a}$$
$$\dot{Y} = \nu_x \sin\Psi + \nu_y \cos\Psi, \tag{11b}$$
$$\dot{\nu}_x = \tau_x/m, \quad \dot{\nu}_y = \tau_y/m, \tag{11c}$$

where X and Y are the positions of the spacecraft in an inertial Cartesian frame, ν_x and ν_y are the longitudinal and lateral velocity of the spacecraft in the spacecraft body frame, τ_x and τ_y are the longitudinal and lateral forces, respectively, applied by the spacecraft's thrusters in the body frame, m is the mass of the spacecraft, and Ψ is the heading angle of the spacecraft body with respect to the inertial X axis. Additionally, we assume the forces are given by $\tau_x = \bar{\tau}_x + \tilde{\tau}_x$ and $\tau_y = \bar{\tau}_y + \tilde{\tau}_y$ and consist of controlled components $\bar{\tau}_x, \bar{\tau}_y$ and uncontrolled stochastic components $\tilde{\tau}_x, \tilde{\tau}_y$, representing actuation error. Assuming the spacecraft is stabilized around $\Psi \approx 0$, Ψ becomes an uncertain parameter of the system, and a small angle approximation may be used to write the system as

$$\begin{bmatrix} \nu_{x_{k+1}} \\ \nu_{y_{k+1}} \\ X_{k+1} \\ Y_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \Delta t & 0 & 1 & 0 \\ 0 & \Delta t & 0 & 1 \end{bmatrix} \begin{bmatrix} \nu_{x_k} \\ \nu_{y_k} \\ X_k \\ Y_k \end{bmatrix} + \begin{bmatrix} \frac{\Delta t}{m} & 0 \\ 0 & \frac{\Delta t}{m} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \end{bmatrix}$$
$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -\theta_x \Delta t & 0 & 0 \\ \theta_x \Delta t & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \nu_{x_k} \\ \nu_{y_k} \\ X_k \\ Y_k \end{bmatrix} \Psi + \begin{bmatrix} \frac{\theta_w \Delta t}{m} & 0 \\ 0 & \frac{\theta_w \Delta t}{m} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{\tau}_x \\ \tilde{\tau}_y, \end{bmatrix} \tag{12}$$

where $\Delta t = 0.2$s is the time-step, and $\theta_x, \theta_w \geq 0 \in \mathbb{R}$ are the noise intensities. The initial condition is given by $\mu_0 = [1.0, -1.0, 1.5, 1.5]^\top$, $\Sigma_0 = 0.001 I_4$, and the terminal constraints are given as $\mu_F = [0, 0, 0, 0]^\top$, $\Sigma_F = \mathtt{diag}(1.2, 1.0, 0.12, 0.12)$. The trajectory is planned over $N = 10$ time steps. As the mean and covariance dynamics are coupled, the terminal covariance equality constraint (9j) is relaxed to the inequality $\bar{\Sigma}[x_N, x_N] \preceq \Sigma_F$ to avoid infeasibility.

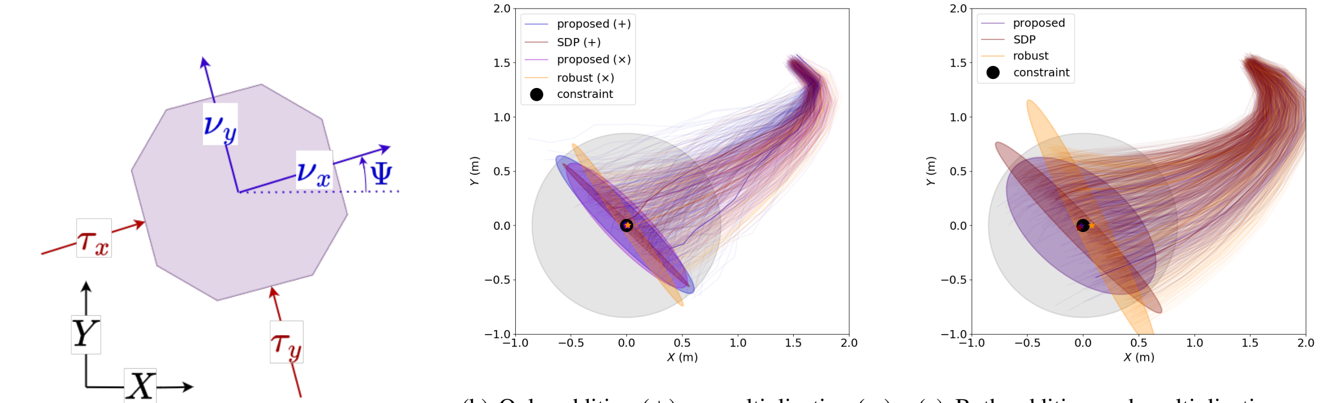
First, we consider the case where $\theta_x = 0$ and $\theta_w = 1.2$ so that the system is subject only to i.i.d. additive disturbances, denoted by $(+)$ in Fig. 1(b). In this case, we observe, as expected, that the proposed approach performs comparably to the semidefinite programming approach which relies on an i.i.d. noise assumption [12].

Next, let $\theta_w = 0$ and $\theta_x = 0.3$, so that system is subject only to multiplicative disturbances, denoted by $(\times)$ in Fig. 1(b), and let $\Psi \sim \mathcal{U}(-1, 1)$, so that the disturbances are sampled from a bounded set. In this case, we compare the proposed approach with a robust approach utilizing ellipsoidal sets [13], and find that the two perform comparably, as expected.

Finally, let $\theta_x = 0.5, \theta_w = 0.2$, so that the system is subject to both additive and multiplicative uncertainties drawn from distributions with unbounded and bounded support, given by $\tilde{F}_{x_k}, \tilde{F}_{y_k} \sim \mathcal{N}(0, 1)$ and $\Psi \sim \mathcal{U}(-1, 1)$. The results are shown in Fig. 1(c). The proposed approach outperforms both baselines. The SDP-based and robust ellipsoid-based approaches fail to control the dispersion of the trajectories and do not meet the terminal mean and covariance constraints because the SDP-based stochastic approach assumes that the noise is i.i.d., which is violated in the case of multiplicative disturbances Ψ, and the robust approach assumes that the noise is drawn from a bounded set, which is violated by $\tilde{F}_{x_k}, \tilde{F}_{y_k}$.

V. Conclusion

This work has investigated the optimal covariance steering problem for systems subject to unknown parameters, represented by constant random variables sampled from a distribution with known moments. The proposed covariance steering problem is solved using sequential convex programming, and it was shown that if the sequential convex programming algorithm converges, then a stationary point has been found that solves the nonconvex covariance steering problem. The proposed approach was compared with a stochastic semidefinite programming-based approach and a robust set-based approach which impose more restrictive assumptions on the characteristics of the noise. It was shown that the proposed approach effectively controls the terminal distribution, whereas these baselines fail to meet the terminal constraints in a spacecraft control example.



(a) Diagram of holonomic spacecraft model

(b) Only additive (+) or multiplicative (×) uncertainty

(c) Both additive and multiplicative uncertainty

Fig. 1: Spacecraft system and comparison of the proposed approach against baselines.

Appendix A

Derivation of Mean Propagation

From (5), it follows that the expected state is given by the equation

$$
\begin{aligned}
&\mathbb{E}[x_{k+1}] = (\bar{A} + \bar{B}L_k)\mathbb{E}[x_k] + \bar{B}v_k \\
&\quad + \sum_{j=1}^{n_p} (\tilde{A}_j + \tilde{B}_j L_k)\mathbb{E}[x_k p_j],
\end{aligned}
$$

where

$$
\begin{aligned}
&\mathbb{E}[x_k p_{j_1}] = (\bar{A} + \bar{B}L_{k-1})\mathbb{E}[x_{k-1} p_{j_1}] \\
&\quad + \sum_{j_2=1}^{n_p} (\tilde{A}_{j_2} + \tilde{B}_{j_2} L_{k-1})\mathbb{E}[x_{k-1} p_{j_1} p_{j_2}] + \tilde{B}_{j_2} v_{k-1}\mathbb{E}[p_{j_1} p_{j_2}], \\
&\mathbb{E}[x_{k-1} p_{j_1} p_{j_2}] = (\bar{A} + \bar{B}L_{k-2})\mathbb{E}[x_{k-2} p_{j_1} p_{j_2}] \\
&\quad + \bar{B}v_{k-2}\mathbb{E}[p_{j_1} p_{j_2}] \\
&\quad + \sum_{j_3=1}^{n_p} (\tilde{A}_{j_3} + \tilde{B}_{j_3} L_{k-2})\mathbb{E}[x_{k-2} p_{j_1} p_{j_2} p_{j_3}] \\
&\quad + \tilde{B}_{j_3} v_{k-2}\mathbb{E}[p_{j_1} p_{j_2} p_{j_3}], \\
&\quad \vdots \\
&\mathbb{E}[x_{k-n} p_{j_1} \dots p_{j_{n+1}}] = (\bar{A} + \bar{B}L_{k-n-1})\mathbb{E}[x_{k-n-1} p_{j_1} \dots p_{j_{n+1}}] \\
&\quad + \bar{B}v_{k-n-1}\mathbb{E}[p_{j_1} \dots p_{j_{n+1}}] \\
&\quad + \sum_{j_{n+2}=1}^{n_p} (\tilde{A}_{j_{n+2}} + \tilde{B}_{j_{n+2}} L_{k-n-1})\mathbb{E}[x_{k-n-1} p_{j_1} \dots p_{j_{n+2}}] \\
&\quad + \tilde{B}_j v_{k-n-1}\mathbb{E}[p_{j_1} \dots p_{j_{n+1}} p_{j_{n+2}}],
\end{aligned}
$$

where $k - n - 1 = 0$ and $\mathbb{E}[x_0 \bar{p}_{j_\ell}] = \mu_0 \mathbb{E}[\bar{p}_{j_\ell}]$, and where $\ell = 0, \dots, n+2$, $j = 1, \dots, n_p$.

Therefore, the expected state at time $k = 0, \dots, N-1$, may be succinctly described in terms of $f(\mathbb{E}[x_{k-\ell}\bar{p}_{j_\ell}], \mathbb{E}[x_{k-\ell}\bar{p}_{j_{\ell+1}}], v_{k-\ell}, \mathbb{E}[\bar{p}_{j_\ell}], \mathbb{E}[\bar{p}_{j_{\ell+1}}])$, given by (6).

Appendix B

Covariance Dynamics

The covariance dynamics are given by

$$
\begin{aligned}
&\bar{\Sigma}[x_{k+1-\kappa}\bar{p}_{i_\iota}, x_{k+1-\kappa}\bar{p}_{j_\ell}] \\
&= g(\{\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_\upsilon}, x_{k-\kappa}\bar{p}_{j_\eta}], \bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_\upsilon, \bar{p}_{j_\eta}}], \\
&\bar{\Sigma}[\bar{p}_{i_\upsilon}, \bar{p}_{j_\eta}], \mathbb{E}[\bar{p}_{i_\upsilon}, \bar{p}_{j_\eta}]\}_{\upsilon=\iota, \eta=\ell}^{\iota+1, \ell+1}, L_{k-\kappa}, v_{k-\kappa}) \\
&= (\bar{A} + \bar{B}L_{k-\kappa})\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_\iota}, x_{k-\kappa}\bar{p}_{j_\ell}](\bar{A} + \bar{B}L_{k-\kappa})^\top \\
&+ (\bar{A} + \bar{B}L_{k-\kappa})\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_\iota}, \bar{p}_{j_\ell}]v_{k-\kappa}^\top \bar{B}^\top \\
&+ \sum_{j_{\ell+1}=1}^{n_p} ((\bar{A} + \bar{B}L_{k-\kappa})\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_\iota}, x_{k-\kappa}\bar{p}_{j_{\ell+1}}](\tilde{A}_{j_{\ell+1}} \\
&+ \tilde{B}_{j_{\ell+1}} L_{k-\kappa})^\top \\
&+ (\bar{A} + \bar{B}L_{k-\kappa})\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_\iota}, \bar{p}_{j_{\ell+1}}]v_{k-\kappa}^\top \tilde{B}_{j_{\ell+1}}^\top) \\
&+ \bar{B}v_{k-\kappa}\bar{\Sigma}[\bar{p}_{i_\iota}, x_{k-\kappa}\bar{p}_{j_\ell}](\bar{A} + \bar{B}L_{k-\kappa})^\top \\
&+ \bar{B}v_{k-\kappa}\bar{\Sigma}[\bar{p}_{i_\iota}, \bar{p}_{j_\ell}]v_{k-\kappa}^\top \bar{B}^\top \\
&+ \sum_{j_{\ell+1}=1}^{n_p} (\bar{B}v_{k-\kappa}\bar{\Sigma}[\bar{p}_{i_\iota}, x_{k-\kappa}\bar{p}_{j_{\ell+1}}](\tilde{A}_{j_{\ell+1}} + \tilde{B}_{j_{\ell+1}} L_{k-\kappa})^\top \\
&+ \bar{B}v_{k-\kappa}\bar{\Sigma}[\bar{p}_{i_\iota}, \bar{p}_{j_{\ell+1}}]v_{k-\kappa}^\top \tilde{B}_{j_{\ell+1}}^\top) \\
&+ \bar{D}\mathbb{E}[\bar{p}_{i_\iota}, \bar{p}_{j_\ell}]\bar{D}^\top + \sum_{j_{\ell+1}=1}^{n_p} (\bar{D}\mathbb{E}[\bar{p}_{i_\iota}, \bar{p}_{j_{\ell+1}}]\tilde{D}_{j_{\ell+1}}^\top) \\
&+ \sum_{i_{\iota+1}=1}^{n_p} ((\tilde{A}_{i_{\iota+1}} + \tilde{B}_{i_{\iota+1}} L_{k-\kappa})\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_{\iota+1}}, x_{k-\kappa}\bar{p}_{j_\ell}](\bar{A} \\
&+ \bar{B}L_{k-\kappa})^\top \\
&+ (\tilde{A}_{i_{\iota+1}} + \tilde{B}_{i_{\iota+1}} L_{k-\kappa})\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_{\iota+1}}, \bar{p}_{j_\ell}]v_{k-\kappa}^\top \bar{B}^\top \\
&+ \sum_{j_{\ell+1}=1}^{n_p} ((\tilde{A}_{i_{\iota+1}} + \tilde{B}_{i_{\iota+1}} L_{k-\kappa})\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_{\iota+1}}, \\
&\quad x_{k-\kappa}\bar{p}_{j_{\ell+1}}](\tilde{A}_{j_{\ell+1}} + \tilde{B}_{j_{\ell+1}} L_{k-\kappa})^\top \\
&+ (\tilde{A}_{i_{\iota+1}} + \tilde{B}_{i_{\iota+1}} L_{k-\kappa})\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_{\iota+1}}, \bar{p}_{j_{\ell+1}}]v_{k-\kappa}^\top \tilde{B}_{j_{\ell+1}}^\top))
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{i_{\iota+1}=1}^{n_p}(\tilde{B}_{i_{\iota+1}}v_{k-\kappa}\bar{\Sigma}[\bar{p}_{i_{\iota+1}},x_{k-\kappa}\bar{p}_{j_\ell}](\bar{A}+\bar{B}L_{k-\kappa})^\top\\
&+\tilde{B}_{i_{\iota+1}}v_{k-\kappa}\bar{\Sigma}[\bar{p}_{i_{\iota+1}},\bar{p}_{j_\ell}]v_{k-\kappa}^\top\bar{B}^\top\\
&+\sum_{j_{\ell+1}=1}^{n_p}(\tilde{B}_{i_{\iota+1}}v_{k-\kappa}\bar{\Sigma}[\bar{p}_{i_{\iota+1}},x_{k-\kappa}\bar{p}_{j_{\ell+1}}](\tilde{A}_{j_{\ell+1}}\\
&+\tilde{B}_{j_{\ell+1}}L_{k-\kappa})^\top\\
&+\tilde{B}_{i_{\iota+1}}v_{k-\kappa}\bar{\Sigma}[\bar{p}_{i_{\iota+1}},\bar{p}_{j_{\ell+1}}]v_{k-\kappa}^\top\tilde{B}_{j_{\ell+1}}^\top))\\
&+\sum_{i_{\iota+1}=1}^{n_p}(\tilde{D}_{i_{\iota+1}}\mathbb{E}[\bar{p}_{i_{\iota+1}},\bar{p}_{j_\ell}]\bar{D}^\top\\
&+\sum_{j_{\ell+1}=1}^{n_p}(\tilde{D}_{i_{\iota+1}}\mathbb{E}[\bar{p}_{i_{\iota+1}},\bar{p}_{j_{\ell+1}}]\tilde{D}_{j_{\ell+1}}^\top)), \qquad (13)
\end{aligned}
$$

where

$$
\begin{aligned}
&\bar{\Sigma}[x_{k+1-\kappa}\bar{p}_{i_\iota},\bar{p}_{j_\ell}]\\
&=h(\{\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_\upsilon},\bar{p}_{j_\eta}],\bar{\Sigma}[\bar{p}_{i_\upsilon},\bar{p}_{j_\eta}]\}_{\upsilon=\iota,\eta=\ell}^{\iota+1,\ell+1},v_{k-\kappa})\\
&=(\bar{A}+\bar{B}L_{k-\kappa})\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_\iota},\bar{p}_{j_\ell}]+\bar{B}v_{k-\kappa}\bar{\Sigma}[\bar{p}_{i_\iota},\bar{p}_{j_\ell}]\\
&+\sum_{i_{\iota+1}=1}^{n_p}(\tilde{A}_{i_{\iota+1}}+\tilde{B}_{i_{\iota+1}}L_{k-\kappa})\bar{\Sigma}[x_{k-\kappa}\bar{p}_{i_{\iota+1}},\bar{p}_{j_\ell}]\\
&+\tilde{B}_{i_{\iota+1}}v_{k-\kappa}\bar{\Sigma}[\bar{p}_{i_{\iota+1}},\bar{p}_{j_\ell}]. \qquad (14)
\end{aligned}
$$

Due to space limitations, the full derivation of (13) and (14) may be found in [28].

References

[1] A. Hotz and R. E. Skelton, "Covariance control theory," *International Journal of Control*, vol. 46, no. 1, pp. 13–32, 1987.

[2] T. Iwasaki and R. E. Skelton, "Quadratic optimization for fixed order linear controllers via covariance control," in *American Control Conference*, Chicago, IL, Jun. 24–26, 1992, pp. 2866–2870.

[3] J.-H. Xu and R. E. Skelton, "An improved covariance assignment theory for discrete systems," *IEEE Transactions on Automatic Control*, vol. 37, no. 10, pp. 1588–1591, 1992.

[4] M. Goldshtein and P. Tsiotras, "Finite-horizon covariance control of linear time-varying systems," in *56th IEEE Annual Conference on Decision and Control*, Melbourne, Australia, Dec. 12–15, 2017, pp. 3606–3611.

[5] Y. Chen, T. T. Georgiou, and M. Pavon, "Optimal steering of a linear stochastic system to a final probability distribution, part I," *IEEE Transactions on Automatic Control*, vol. 61, no. 5, pp. 1158–1169, 2015.

[6] ——, "Optimal steering of a linear stochastic system to a final probability distribution, part II," *IEEE Transactions on Automatic Control*, vol. 61, no. 5, pp. 1170–1180, 2015.

[7] ——, "Optimal steering of a linear stochastic system to a final probability distribution—part III," *IEEE Transactions on Automatic Control*, vol. 63, no. 9, pp. 3112–3118, 2018.

[8] E. Bakolas, "Optimal covariance control for discrete-time stochastic linear systems subject to constraints," in *55th IEEE Conference on Decision and Control*, Las Vegas, NV, Dec. 12–14, 2016, pp. 1153–1158.

[9] K. Okamoto, M. Goldshtein, and P. Tsiotras, "Optimal covariance control for stochastic systems under chance constraints," *IEEE Control Systems Letters*, vol. 2, no. 2, pp. 266–271, 2018.

[10] K. Okamoto and P. Tsiotras, "Input hard constrained optimal covariance steering," in *58th IEEE Conference on Decision and Control*, Nice, France, Dec. 11–13, 2019, pp. 3497–3502.

[11] I. M. Balci and E. Bakolas, "Covariance steering of discrete-time linear systems with mixed multiplicative and additive noise," *arXiv preprint arXiv:2210.01743*, 2022.

[12] "Computationally efficient covariance steering for systems subject to parametric disturbances and chance constraints," in *62nd IEEE Conference on Decision and Control*, Marina Bay Sands, Singapore, Dec. 13–15, 2023.

[13] B. Houska, A. Mohammadi, and M. Diehl, "A short note on constrained linear control systems with multiplicative ellipsoidal uncertainty," *IEEE Transactions on Automatic Control*, vol. 61, no. 12, pp. 4106–4111, 2016.

[14] J. Fleming, B. Kouvaritakis, and M. Cannon, "Robust tube MPC for linear systems with multiplicative uncertainty," *IEEE Transactions on Automatic Control*, vol. 60, no. 4, pp. 1087–1092, 2014.

[15] E. K. Kostousova, "On polyhedral estimates for trajectory tubes of dynamical discrete-time systems with multiplicative uncertainty," in *Conference Publications*, vol. 2011 (Special). American Institute of Mathematical Sciences, 2011, pp. 864–873.

[16] M. Evans, M. Cannon, and B. Kouvaritakis, "Robust MPC for linear systems with bounded multiplicative uncertainty," in *51st IEEE Conference on Decision and Control (CDC)*, Maui, HI, Dec. 10–13, 2012, pp. 248–253.

[17] T. Peschke and D. Görges, "Tube-based anticipative robust MPC for systems with multiplicative uncertainty," *IFAC-PapersOnLine*, vol. 53, no. 2, pp. 7091–7096, 2020.

[18] M. B. Saltık, L. Özkan, J. H. Ludlage, S. Weiland, and P. M. Van den Hof, "An outlook on robust model predictive control algorithms: Reflections on performance and computational aspects," *Journal of Process Control*, vol. 61, pp. 77–102, 2018.

[19] A. Mesbah, "Stochastic model predictive control: An overview and perspectives for future research," *IEEE Control Systems Magazine*, vol. 36, no. 6, pp. 30–44, 2016.

[20] E. Bakolas, "Finite-horizon covariance control for discrete-time stochastic linear systems subject to input constraints," *Automatica*, vol. 91, pp. 61–68, 2018.

[21] B. Benedikter, A. Zavoli, Z. Wang, S. Pizzurro, and E. Cavallini, "Convex approach to covariance control with application to stochastic low-thrust trajectory optimization," *Journal of Guidance, Control, and Dynamics*, vol. 45, no. 11, pp. 1–16, 2022.

[22] I. M. Balci and E. Bakolas, "Exact SDP formulation for discrete-time covariance steering with Wasserstein terminal cost," *arXiv preprint arXiv:2205.10740*, 2022.

[23] Q. T. Dinh and M. Diehl, "Local convergence of sequential convex programming for nonconvex optimization," in *Recent Advances in Optimization and its Applications in Engineering: The 14th Belgian-French-German Conference on Optimization*. Leuven, Belgium: Springer, Sep. 14–18, 2010, pp. 93–102.

[24] R. Bonalli, T. Lew, and M. Pavone, "Sequential convex programming for non-linear stochastic optimal control," *ESAIM: Control, Optimisation and Calculus of Variations*, vol. 28, p. 64, 2022.

[25] R. Dyro, J. Harrison, A. Sharma, and M. Pavone, "Particle MPC for uncertain and learning-based control," in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Prague, Czech Republic, Sep. 27–Oct. 1, 2021, pp. 7127–7134.

[26] R. Bonalli, A. Cauligi, A. Bylard, and M. Pavone, "Gusto: Guaranteed sequential trajectory optimization via sequential convex programming," in *IEEE International conference on robotics and automation (ICRA)*, Montreal, Canada, May 20–24, 2019, pp. 6741–6747.

[27] A. W. Marshall and I. Olkin, "Multivariate Chebyshev inequalities," *The Annals of Mathematical Statistics*, pp. 1001–1014, 1960.

[28] J. Knaup and P. Tsiotras, "Covariance steering for systems subject to unknown parameters," *arXiv preprint arXiv:2303.10293*, 2023 (submitted).