IEEE Control Systems Letters paper presented at
2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

# Hybrid cost function distributed MPC for vehicle platoons with stability and string stability properties

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***Abstract*— Distributed MPC schemes for control of vehicle platoons typically employ a p-norms-based cost function to achieve stability and string stability. Quadratic cost functions yield smoother trajectories, but they are not aligned with string stability conditions. Hence, in this paper, we develop distributed MPC controllers for vehicle platoons based on a hybrid cost function, which combines infinity norms and quadratic forms. Sufficient conditions for global platoon stability and leader-follower string stability are derived for the developed hybrid cost function and applied to lateral dynamics control. Simulation results show that the hybrid cost function yields lateral position errors that are 10 times smaller (for the maximum error) compared with an infinity norms-based cost function.**

## I. Introduction

The vehicle platooning strategy ensures more efficient use of roads, improves the operational quality of traffic flow, and, most importantly, targets people's safety [1]. A critical demand for a vehicle platoon is the string stability property, which ensures that the platoon does not amplify disturbances along the platoon. There are two main types of string stability [2]: *i)* leader-follower string stability, which requires that the maximum deviation from the reference of the leader vehicle is above the maximum deviation of the follower vehicles and *ii)* predecessor-follower string stability, which requires that the maximum deviation of a follower vehicle is under the maximum deviation of the vehicle in front.

The longitudinal dynamics describes the velocity of vehicles and the distance between them. The longitudinal control has as a target to minimise the velocity and distance errors and mitigate them along the platoon [3], [4]. The direction of the vehicles is described by lateral dynamics. The leader vehicle determines the path of the platoon using information from sensors (radar, camera), and the follower vehicles track it so that the lateral deviation is minimised [5]. Classical string stable control design for linear dynamics may yield high-gain aggressive controllers that violate the saturation of actuators. Due to this, distributed model predictive control (MPC) is a method of interest for achieving string stability in platoons. Among the advantages of distributed MPC are the guarantee of constraints, anticipative control actions, and optimal control. A fundamental work that developed a solution for vehicle platoon string stability based on distributed MPC is [2]. In this paper, the authors propose an MPC controller for the longitudinal dynamics of a platoon. The method uses a cost function based on p-norms and gives the conditions required for global stability, predecessor-follower string stability, and leader-follower string stability. The disadvantage of this method is that the same proof of stability does not apply to quadratic cost functions. Also, p-norms-based cost functions yield aggressive control actions and are not ideal for platooning. For the lateral dynamics, [6] developed a distributed MPC scheme based on quadratic cost functions, without providing a global stability analysis. Therein, a constraint that promotes string stability inspired by [2] was added to the MPC optimization problem.

This paper proposes a new, hybrid cost function for distributed MPC for vehicle platoons, which combines infinity norms and quadratic forms. First, the conditions required by a hybrid cost function to be a Lyapunov function are established. Second, starting from [2], the conditions required for leader-follower string stability are derived. With respect to [2], [6], the contributions of this paper are: *i)* sufficient conditions for closed-loop global stability of an interconnected system with a chain architecture and distributed MPC with hybrid cost function; $(iii)$ the constraint imposed for the terminal cost of the cost function is less restrictive compared to the constraints imposed in [2]; $(iv)$ application of the developed string stable distributed MPC scheme to lateral vehicle dynamics. It is also worth mentioning that all results in [2] are for continuous-time dynamics, while we provide the corresponding results in the discrete-time setting. With respect to more recent works in the transportation field, the novelty of this paper is given by the development of the distributed MPC algorithm for a large class of interconnected systems compared to the works from [7]–[9] where the algorithm and stability profs are focused on a specific system, i.e., longitudinal dynamics of a platoon. Simulation results illustrate significant improvement in the lateral position error for the developed hybrid cost function compared to an infinity norms cost function.

## II. Preliminaries

This section introduces notation, the interconnected system dynamics in a chain architecture, and the definition of string stability. The infinity norm of a real vector is defined as $||X||_\infty = \max_{i\in\{1,...,n\}} |x_i|$, $X = [x_1, ..., x_n]^T$, $x_i \in \mathbb{R}$, $n \in \mathbb{Z}_{\geq 1}$. The symbol $Q \succ 0$ denotes that the matrix $Q$ is positive definite and the symbol $Q \succeq 0$ denotes that the matrix $Q$ is semi-positive definite, $Q \in \mathbb{R}^{n\times n}$. The identity matrix of dimensions $n \times n$ is represented by $I_n$, the square matrix with all elements equal to zero is represented by $O_n$,

*This work was not supported by any organization

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and the symbol $diag\{x_1, ..., x_n\}$ represents a diagonal matrix with $x_1, .., x_n$ on the main diagonal.

Interconnected systems in a chain architecture can be described by the following equations:

$$\begin{cases} x_j(k+1) &= f_j(x_j(k), u_j(k), d_j(k)), \\ y_j(k) &= h_j(x_j(k)), \end{cases} \tag{1}$$

where $x_j \in \mathbb{R}^n$, $y_j \in \mathbb{R}^p$, $u_j \in \mathbb{R}^m$ are the states, outputs and inputs of the subsystem $j$, $f_j$ and $h_j$ are nonlinear functions, $n, p, m \in \mathbb{Z}_{\geq 1}$, $k \in \mathbb{Z}_{\geq 1}$ is the discrete time, $d_1(k) = d_{ref}(k)$, $d_{j+1}(k) = x_j(k)$, $j = 1, ..., M-1$, $M$ is the number of subsystems and $d_{ref}$ represents the imposed reference.

The string stability property ensures that the output error $e_{yj}$ is attenuated along the upstream direction, as formally defined next.

*Definition 1:* The interconnected system (1) is leader-follower string stable if for each system $j = 2, ..., M$ there exists a constant $\alpha_j \in (0, 1)$ such that:

$$\max_{k\geq 0} |e_{yj}(k)| \leq \alpha_j \max_{k\geq 0} |e_{y1}(k)|. \tag{2}$$

Next, the distributed MPC problem with a hybrid cost function is introduced along with sufficient conditions for global stability of the interconnected system (1).

## III. Main results

For each system and MPC controller $j$, the cost function is defined as:

$$\begin{aligned} J_j(x_j(k), U_j(k)) = &F_P(x_j(N|k)) + F_G(x_j(N|k), \tilde{x}_j(N|k)) \\ &+ F_H(x_j(N|k), \tilde{x}_{j-1}(N|k)) \\ &+ \sum_{i=0}^{N-1} [l_Q(x_j(i|k)) + l_R(u_j(i|k))] + \\ &+ \sum_{i=1}^{N-1} [l_G(x_j(i|k), \tilde{x}_j(i|k)) \\ &\quad + l_H(x_j(i|k), \tilde{x}_{j-1}(i|k))], \end{aligned} \tag{3}$$

where

$$\begin{cases} F_P(x_j(N|k)) = x_j^T(N|k) P_j x_j(N|k); \\ F_G(x_j(N|k), \tilde{x}_j(N|k)) = ||G_j(x_j(N|k) - \tilde{x}_j(N|k))||_\infty; \\ F_H(x_j(N|k), \tilde{x}_{j-1}(N|k)) = \\ \qquad ||H_j(x_j(N|k) - \tilde{x}_{j-1}(N|k))||_\infty; \\ l_Q(x_j(i|k)) = x_j^T(i|k) Q_j x_j(i|k); \\ l_R(u_j(i|k)) = u_j^T(i|k) R_j u_j(i|k); \\ l_G(x_j(i|k), \tilde{x}_j(i|k)) = ||G_j(x_j(i|k) - \tilde{x}_j(i|k))||_\infty; \\ l_H(x_j(i|k), \tilde{x}_{j-1}(i|k)) = ||H_j(x_j(i|k) - \tilde{x}_{j-1}(i|k))||_\infty. \end{cases} \tag{4}$$

Notice that, by definition of the terminal costs $F_G(\cdot)$, $H_G(\cdot)$ and stage costs $l_G(\cdot)$, $l_H(\cdot)$ we have $l_G(x_j^*(N|k), \tilde{x}_j(N|k)) = F_G(x_j^*(N|k), \tilde{x}_j(N|k))$ and $l_H(x_j^*(N|k), \tilde{x}_{j-1}(N|k)) = F_H(x_j^*(N|k), \tilde{x}_{j-1}(N|k))$. Moreover, for the cost function of each MPC controller, different weighting matrices $P_j, G_j, H_j$, $Q_j$, $R_j$, are allowed, which should satisfy:

- $Q_1 \succ 0$, $P_1 \succ 0$, $G_1 \succeq 0$, $H_1 = 0$, $R_1 \succ 0$ for system $j = 1$, i.e., the leader;
- $Q_j \succeq 0$, $P_j \succ 0$, $G_j \succeq 0$, $H_j \succeq 0$, $R_j \succ 0$ for systems $j = 2, ..., M$, i.e., the followers.

With respect to [2] where the cost function contains only infinity norms (or p-norms), in this paper, the cost function (3) is a hybrid one, i.e., it contains both quadratic terms and terms based on infinity norms. The quadratic terms minimise the tracking error and the infinity norm terms penalise deviation of predicted state trajectories from the previous own and predecessor's state trajectories, respectively. This new way of building the cost function ensures a smooth variation of the system states (via the quadratic terms), while still promoting string stability (via the infinity norm terms).

*Problem 1:* For each subsystem $j \in \{1, ..., M\}$, at each discrete step $k$, given $x_j(k)$, $\tilde{X}_j(k)$, $\tilde{X}_{j-1}(k)$ solve:

$$\min_{U_j(k)} J_j(x_j(k), U_j(k)) \tag{5}$$

subject to constraints:

$$\begin{aligned} x_j(i+1|k) = f_j(x_j(i|k), u_j(i|k), d_j(i|k)), \\ i = 0, ..., N-1; \end{aligned} \tag{6}$$

$$u_j(i|k) \in \mathbb{U}_j, i = 0, ..., N-1; \tag{7}$$

$$x_j(i|k) \in \mathbb{X}_j, i = 1, ..., N, \tag{8}$$

where $U_j(k) = [u_j(i|k)^T, ..., u_j(N-1|k)^T]^T$.

Over a prediction interval $[k, k+N]$, the following trajectories are defined:

- $X_j(k)$ – the predicted state trajectory;
- $X_j^*(k)$ – the optimal predicted state trajectory;
- $\tilde{X}_j(k)$ – the assumed state trajectory;
- $U_j^*(k)$ – the optimal sequence of control inputs;
- $\tilde{U}_j(k)$ – the assumed input trajectory;
- $\overline{X}_j(k)$ – the state trajectory obtained considering as input $\tilde{U}_j(k-1)$.

The distributed MPC scheme requires that subsystem $j$ sends to subsystem $j+1$ assumed state trajectories. The assumed state trajectory is constructed as a shifted sequence based on the optimal sequence $X_j^*$ computed at time $k-1$:

$$\tilde{X}_j(k) = [x_j^*(2|k-1)^T, ..., x_j^*(N|k-1)^T, x_j^*(N|k-1)^T]^T. \tag{9}$$

The assumed input trajectory, $\tilde{U}_j(k)$, is similarly constructed based on the optimal sequence $U_j^*$ computed at time $k-1$:

$$\tilde{U}_j(k) = \\ [u_j^*(1|k-1)^T, ..., u_j^*(N-1|k-1)^T, \kappa_j(x_j^*(N|k-1))^T]^T.$$

Notice that, the state trajectory obtained using $\tilde{U}_j(k-1)$ is $\overline{X}_j(k) = [x_j^*(2|k-1)^T, ..., x_j^*(N|k-1)^T, \overline{x}_j(N|k-1)^T]^T$.

### A. System global stability

First, we denote the concatenated arrays $x = [x_1, ..., x_M]$, $x^* = [x_1^*, ..., x_M^*]$, $\tilde{x} = [\tilde{x}_1, ..., \tilde{x}_M]$, $u = [u_1, ..., u_M]$, $\tilde{u} = [\tilde{u}_1, ..., \tilde{u}_M]$. Then, the global optimal cost of system (1) can be defined as:

$$J_\Sigma^*(x(k)) = \sum_{j=1}^{M} J_j^*(x_j(k), U_j^*(k)). \tag{10}$$

Let us define the following terms:

$$L_1 \triangleq \sum_{j=1}^{M}[F_P(\overline{x}_j(N|k+1)) - (l_Q(x_j^*(0|k)) + l_R(u_j^*(0|k))) \\ +(l_Q(x_j^*(N|k)) + l_R(\kappa_j(x_j^*(N|k)))) - F_P(x_j^*(N|k))]; \tag{11}$$

$$L_2 \triangleq \sum_{j=1}^{M}[L_{2aj} + L_{2bj}]; \tag{12}$$

$$L_{2aj} \triangleq F_G(\overline{x}_j(N|k+1), \tilde{x}_j(N|k+1)) \\ + F_H(\overline{x}_j(N|k+1), \tilde{x}_{j-1}(N|k+1)) \\ - [l_G(x_j^*(1|k), \tilde{x}_j(1|k)) + l_H(x_j^*(1|k), \tilde{x}_{j-1}(1|k))]; \tag{13}$$

$$L_{2bj} \triangleq F_H(x_j^*(N|k), x_{j-1}^*(N|k)) \\ - [F_G(x_j^*(N|k), \tilde{x}_j(N|k)) + F_H(x_j^*(N|k), \tilde{x}_{j-1}(N|k))] \\ + \sum_{i=2}^{N-1}[l_H(x_j^*(i|k), x_{j-1}^*(i|k)) \\ - l_G(x_j^*(i|k), \tilde{x}_j(i|k)) - l_H(x_j^*(i|k), \tilde{x}_{j-1}(i|k))]. \tag{14}$$

*Assumption 1: (Terminal cost property)* The terminal costs $F_P(\cdot)$ and the auxiliary control laws $\kappa_j(\cdot)$, for $j = 1, ..., M$, are such that $L_1 \leq -x(k)^T diag\{Q_1, ..., Q_M\}x(k)$.

*Assumption 2:* The weighting matrices, $G_j$ and $H_j$, and prediction horizon $N$ are chosen such that $L_{2aj} \leq 0$, for all $j = 1, ..., M$.

In what follows, we will show that the cost function (10) is a Lyapunov function for the global dynamics of system (1). Since it is a positive definite function and zero at zero by construction, we only need to prove the decrease along closed-loop trajectories.

*Lemma 1:* Supposes that Assumptions 1 and 2 hold, and

$$G_j \geq H_{j+1}, \quad \forall j = 1, ..., M-1. \tag{15}$$

Then it holds that

$$J_\Sigma^*(x(k+1)) - J_\Sigma^*(x(k)) \leq -x(k)^T diag\{Q_1, ..., Q_M\}x(k), \tag{16}$$

for all $k \in \mathbb{Z}_{\geq 0}$.

*Proof:* From (10) we have that:

$$J_\Sigma^*(x(k)) = F_\Sigma(x^*(N|k), \tilde{x}(N|k), \tilde{x}(N|k)) + \\ + \sum_{i=0}^{N-1} L_{\Sigma QR}(x^*(i|k), \tilde{x}(i|k), u^*(i|k)) + \\ + \sum_{i=1}^{N-1} L_{\Sigma GH}(x^*(i|k), \tilde{x}(i|k)), \tag{17}$$

where

$$\begin{cases} F_\Sigma(x^*(N|k), \tilde{x}(N|k), \tilde{x}(N|k)) \triangleq \sum_{j=1}^{M}[F_P(x_j^*(N|k)) + \\ +F_G(x_j^*(N|k), \tilde{x}_j(N|k)) + F_H(x_j^*(N|k), \tilde{x}_{j-1}(N|k))]; \\ L_{\Sigma QR}(x^*(i|k), \tilde{x}(i|k), u^*(i|k)) \triangleq \\ \quad \sum_{j=1}^{M}[l_Q(x_j^*(i|k)) + l_R(u_j(i|k))]; \\ L_{\Sigma GH}(x^*(i|k), \tilde{x}(i|k)) \triangleq \sum_{j=1}^{M} l_G(x_j^*(i|k), \tilde{x}_j(i|k)) \\ \quad +l_H(x_j^*(i|k), \tilde{x}_{j-1}(i|k))]. \end{cases} \tag{18}$$

At the discrete time $k+1$, a feasible solution for Problem 1 is $\tilde{U}(k+1)$.

Applying $\tilde{U}(k+1)$ yields:

$$J_\Sigma^*(x(k+1)) \leq F_\Sigma(\overline{x}(N|k+1), \tilde{x}(N|k+1), \tilde{x}(N|k+1)) \\ + \sum_{i=0}^{N-1} L_{\Sigma QR}(\overline{x}(i|k+1), \tilde{x}(i|k+1), \overline{u}) \\ + \sum_{i=1}^{N-1} L_{\Sigma GH}(\overline{x}(i|k), \tilde{x}(i|k)) \tag{19}$$

where $\overline{x}$ is the state trajectory of the global system considering as input $\tilde{U}$. From (17)-(19) it results that:

$$J_\Sigma^*(x(k+1)) - J_\Sigma^*(x(k)) \leq \\ \leq \sum_{j=1}^{M}[F_P(\overline{x}_j(N|k+1)) - (l_Q(x_j^*(0|k)) \\ + l_R(u_j^*(0|k))) + (l_Q(x_j^*(N|k)) + l_R(K_j x_j^*(N|k))) \\ - F_P(x_j^*(N|k))] + \sum_{j=1}^{M}[F_G(\overline{x}_j(N|k+1), \tilde{x}_j(N|k+1)) \\ + F_H(\overline{x}_j(N|k+1), \tilde{x}_{j-1}(N|k+1)) \\ + \sum_{i=2}^{N} l_H(x_j^*(i|k), x_{j-1}^*(i|k)) \\ - F_G(x_j^*(N|k), \tilde{x}_j(N|k)) - F_H(x_j^*(N|k), \tilde{x}_{j-1}(N|k)) \\ - \sum_{i=1}^{N-1}[l_G(x_j^*(i|k), \tilde{x}_j(i|k)) + l_H(x_j^*(i|k), \tilde{x}_{j-1}(i|k))]] = \\ = L_1 + L_2. \tag{20}$$

Since $L_1 \leq -x(k)^T diag\{Q_1, ..., Q_M\}x(k)$ by Assumption 1, it suffices to prove that $L_2 \leq 0$. By Assumption 2, for $L_2 \leq 0$ to hold, it suffices that $L_{2bj} \leq 0$, i.e.,

$$||H_j(x_j^*(i|k) - x_{j-1}^*(i|k))||_\infty - ||G_j(x_j^*(i|k) - \tilde{x}_j(i|k))||_\infty \\ - ||H_j(x_j^*(i|k) - \tilde{x}_{j-1}(i|k))||_\infty \leq 0, \quad \forall j = 1, ...M. \tag{21}$$

Adding and substracting $H_j\tilde{x}_{j-1}(i|k)$ inside the norm $||H_j(x_j^*(i|k) - x_{j-1}^*(i|k))||_\infty$ and applying the triangle inequality gives:

$$||H_j(x_j^*(i|k) - x_{j-1}^*(i|k))||_\infty \leq \\ \leq ||H_j(x_j^*(i|k) - \tilde{x}_{j-1}(i|k))||_\infty + \\ + ||H_j(x_{j-1}^*(i|k) - \tilde{x}_{j-1}(i|k))||_\infty. \tag{22}$$

Combining (21), (22) and considering $H_1 = 0$ gives [2]:

$$\sum_{j=1}^{M-1} ||H_{j+1}(x_j^*(i|k) - \tilde{x}_j(i|k))||_\infty - \\ -||G_j(x_j^*(i|k) - \tilde{x}_j(i|k))||_\infty \leq 0, \tag{23}$$

which holds due to $H_{j+1} \leq G_j$. ■

*Remark 1:* The terminal costs $F_P(\cdot)$ can be computed using a linearization of the dynamics and a local linear LQR control law $\kappa_j(\cdot)$ as done in linear MPC if the interconnection term $d_j$ is neglected. Future work will consider more advanced methods for terminal cost computation for interconnected systems, as in, e.g., [10].

*Remark 2:* The condition of Assumption 2 will be imposed as a constraint in the MPC Problem 1:

$$\begin{aligned} L_{2aj} &= F_G(x_j^*(N|k+1), \tilde{x}_j(N|k+1)) \\ &+ F_H(x_j^*(N|k+1), \tilde{x}_{j-1}(N|k+1)) \\ &- [l_G(x_j^*(1|k), \tilde{x}_j(1|k)) \\ &+ l_H(x_j^*(1|k), \tilde{x}_{j-1}(1|k))] \leq 0, j = 1, ..., M. \end{aligned} \tag{24}$$

Notice that, condition (24) is less restrictive compared to the condition from [2] where the terminal cost is constrained to be equal to zero.

### B. Leader-follower string stability

In this section, the leader-follower string stability conditions for the discrete-time case are derived based on the solution from [2] for the continuous-time case. Over a prediction interval $[k, k+N]$, the following trajectories are defined:

- $Y_j(k)$ – the predicted output trajectory;
- $Y_j^*(k)$ – the optimal predicted output trajectory;
- $\tilde{Y}_j(k)$ – the assumed output trajectory.

*Definition 2:* For a step change in $x_1$ at time $k = 0$, the interconnected system (1) in closed-loop with the distributed MPC controller is string stable if for all $j = 2, ..., M$:

$$||Y_j^*(k)||_\infty \leq \alpha_j \max_{l \in \{0,...,k\}} ||Y_1^*(k)||_\infty, \forall k \geq 0, \tag{25}$$

where $Y_j^*(k) = [y_j^*(1|k), ..., y_j^*(N|k)]^T$, $\alpha_j \in (0, 1)$.

*Algorithm 1: Distributed MPC + string stability condition Initialisation:*

- Subsystem $j = 1$ solves Problem 1 setting $G_1 = H_1 = 0$ and then sends $Y_1(0)$ to all systems;
- Each subystem, $j = 2, ..., M$, receives $Y_1(0)$ and solves Problem 1 with the additional constraint:

$$||Y_j^*(0)||_\infty \leq \gamma_j ||Y_1^*(0)||_\infty, \tag{26}$$

where $\gamma_j \in (0, 1)$, $G_j = H_j = 0$.

*Controller:* At each discrete time $k \in \mathbb{Z}_{\geq 1}$ :

- Compute $\tilde{Y}_j(k)$;
- Send $\tilde{Y}_j(k)$ to subsystem $j + 1$ and for $j \geq 2$ receive $\tilde{Y}_{j-1}(k)$ from subsystem $j - 1$;
- All subsystems $j \geq 2$, receive $\tilde{Y}_1(k)$;
- Solve Problem 1 with the additional constraint:

$$||Y_j(k) - \tilde{Y}_j(k)||_\infty \leq \epsilon_j(k) ||\tilde{Y}_1(k)||_\infty, \tag{27}$$

where $\epsilon_j(k) \in (0, 1)$.

*Lemma 2:* The string stability condition (25) is satisfied for the interconnected system (1) in closed-loop with the distributed MPC controllers if the following condition holds:

$$\gamma_j + \sum_{k=1}^{\infty} \epsilon_j(k)(1 + \epsilon_1(k)) < 1, \tag{28}$$

where $\gamma$, $\epsilon \in (0, 1)$.

*Proof:* From (26) at $k = 0$ it results:

$$||Y_j^*(0)||_\infty \leq \gamma_j ||Y_1^*(0)||_\infty. \tag{29}$$

From the triangle inequality, (27) and using the inequality $||Y_j^*(k-1)||_\infty \geq ||\tilde{Y}_j(k)||_\infty$ (from (9)) and it results:

$$||Y_j^*(k)||_\infty \leq \epsilon_j(k) ||\tilde{Y}_1(k)||_\infty + ||Y_j^*(k-1)||_\infty. \tag{30}$$

Moreover, from the triangle inequality, (27) and using the inequality $||Y_1^*(k-1)||_\infty \geq ||\tilde{Y}_1(k)||_\infty$ for $j = 1$ it results:

$$||\tilde{Y}_1(k)||_\infty \leq \epsilon_1(k) ||Y_1^*(k-1)||_\infty + ||Y_1^*(k)||_\infty. \tag{31}$$

Combining (30) with (31) it gives:

$$\begin{aligned} ||Y_j^*(k)||_\infty \leq \epsilon_j(k)(1 + \epsilon_1(k)) \max_{l=k-1,k} ||Y_1^*(l)||_\infty \\ + ||Y_j^*(k-1)||_\infty. \end{aligned} \tag{32}$$

For $k = 1$ and from (29) and (32) it results:

$$||Y_j^*(1)||_\infty \leq [\epsilon_j(1)(1 + \epsilon_1(1)) + \gamma_j] \max_{l=0,1} ||Y_1^*(l)||_\infty. \tag{33}$$

For $k = 2$ and from (32) and (33) it results:

$$\begin{aligned} &||Y_j^*(2)||_\infty \leq \\ &[\epsilon_j(2)(1 + \epsilon_1(2)) + \epsilon_j(1)(1 + \epsilon_1(1)) + \gamma_j] \max_{l=0,1,2} ||Y_1^*(l)||_\infty. \end{aligned} \tag{34}$$

Doing this recursively, for $k = n$ we obtain:

$$||Y_j^*(n)||_\infty \leq [\gamma_j + \sum_{i=1}^{n} \epsilon_j(i)(1 + \epsilon_1(i))] \max_{l=0,...,n} ||Y_1^*(l)||_\infty, \tag{35}$$

where $\gamma_j = \gamma \in (0, 1)$, $\epsilon_j(i) = \epsilon^i$, $\epsilon \in (0, 1)$, $\forall\ j = 1, ..., M$ and $n \geq 1$. From (35), when $n$ tends to infinity, it holds that

$$\begin{aligned} \alpha_j = [\gamma_j + \sum_{i=1}^{n} \epsilon_j(i)(1 + \epsilon_1(i))] < 1 \Leftrightarrow \\ \Leftrightarrow \gamma + \frac{1}{1-\epsilon} + \frac{1}{1-\epsilon^2} < 3, \end{aligned} \tag{36}$$

where $\gamma$, $\epsilon \in (0, 1)$. Hence, if (28) holds, there exists an $\alpha_j \in (0, 1)$ such that (25) holds, which completes the proof. ■

The constraints imposed for leader-follower string stability will be imposed as soft constraints as follows:

$$\begin{cases} ||Y_j^*(0)||_\infty - \gamma_j ||Y_1^*(0)||_\infty \leq \rho(0); \\ ||Y_j(k) - \tilde{Y}_j(k)||_\infty - \epsilon_j(k) ||\tilde{Y}_1(k)||_\infty \leq \rho(k), k \geq 1. \end{cases} \tag{37}$$

Thus, we can formulate next the complete distributed MPC problem with stability and string stability constraints included.

*Problem 2:* For each subsystem $j \in \{1, ..., M\}$, at each discrete step $k$, given $x_j(k)$, $\tilde{X}_j(k)$, $\tilde{X}_{j-1}(k)$ solve:

$$\min_{[U_j(k), \rho(k)]} J_j(x_j(k), U_j(k)) + \lambda \rho(k)^2 \tag{38}$$

subject to constraints: (6), (7), (8), (15), (24), (37).

*Remark 3:* The stability proof considers the cost functions $J_j(\cdot)$ without the term $\lambda \rho(k)$. When the variable $\rho(k)$ is nonzero, only practical global stability is obtained. However, as $\rho(k)$ is minimized, the output error converges to very small values, as observed in the simulations.

TABLE I: Parameters of vehicles

| Symbol | Name | Value |
|---|---|---|
| $m$ | Vehicle mass | $1094\ Kg$ |
| $C_f$ | Front tire cornering stiffness coefficient | $63291\ N/rad$ |
| $C_r$ | Rear tire cornering stiffness coefficient | $50041\ N/rad$ |
| $l_f$ | Longitudinal distance from the center of gravity to the front tires | $1.108\ m$ |
| $l_r$ | Longitudinal distance from the center of gravity to the rear tires | $1.392\ m$ |
| $I$ | Vehicle's rotational inertia | $1608\ Kg \cdot m^2$ |
| $v_x$ | Longitudinal velocity | $13.89\ m/s$ |

TABLE II: Parameters of controllers

| Symbol | Hybrid cost MPC | Infinity norm cost MPC [2] |
|---|---|---|
| N | 50 | 50 |
| Q | diag{0.002,0.002,250} | diag{0.01,0.01, 2.5} |
| $\epsilon$ | 0.25 | 0.25 |
| $\gamma$ | 0.4 | 0.4 |
| $G_j$ | $I_3$ | $2I_3$ |
| $H_j$ | $O_3$ | $O_3$ |

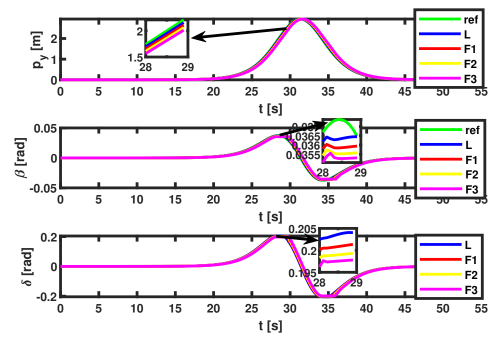


Fig. 1: Hybrid cost MPC: lateral position, slip angles, steering angles.

## IV. Simulation Results

### A. Platoon lateral dynamics

This subsection presents the model used to describe the lateral dynamics of the platoon. A vehicle platoon is formed by a leader vehicle, $j = 1$, followed by follower vehicles, $j = 2, 3, 4$. Assuming that the vehicles are moving with constant velocity, the lateral model can be described as [6]:

$$\dot{x}_j(t) = Ax_j(t) + Bu_j(t) + Ed_j(t), \tag{39}$$

where $x_j = [\beta_j, \dot{\psi}_j, e_{yj}]^T$, $u_j = \delta_j$, $d_1 = \beta_{ref}$, $d_{j+1} = \beta_j$, $j = 1, ..., M$, $B = [2C_f/mv_x, 2l_fC_f/I, 0]^T$, $A = \begin{bmatrix} -2(C_f + C_r)/mv_x & -1 - 2(l_fC_f - l_rC_r)/mv_x^2 & 0 \\ -2(l_fC_f - l_rC_r)/I & -2(l_f^2C_f + l_r^2C_r)/Iv_x & 0 \\ v_x & 0 & 0 \end{bmatrix}$, $E = [0, 0, -v_x]^T$, and where $m$ and $I$ denote the vehicle mass and inertia, $l_f$ and $l_r$ are the distances from the centre of gravity of the vehicle to the front and rear axles, $\dot{\psi}$ denotes the yaw rate, $e_{yj}$ represents the lateral position error, $v_x$ is the longitudinal velocity, $\beta$ is car slip angle and $\delta$ is the steering angle of the front tire. The model (39) is discretised using the zero-order hold method with the sample time $T_s = 0.1s$. The output is considered the lateral error (i.e., $y_j(t) = e_{yj}(t)$). The constraints imposed for the inputs and vehicle states are represented by: $-0.78\ rad \leq \delta_j \leq 0.78\ rad$, $-0.2\ rad \leq \beta_j \leq 0.2\ rad$, $-0.1\ m \leq e_{yj} \leq 0.1\ m$.

Remark 3.4 regarding the terminal cost $F_P(\cdot)$ applies also to the lateral dynamics due to the interconnection term, i.e., the slip angle, which is relatively small compared to the position error, which makes this approach feasible.

*Remark 4:* In the case of lateral dynamics, the position error, even if it is related to the lateral position of the vehicle in front, it represents the distance of the vehicle to the centre of the line or the error to the imposed trajectory. Therefore, it suffices that the lateral position error of the followers is smaller than the error of the leader. Based on this fact the solution considers leader-follower string stability, which is less restrictive than predecessor-follower string stability.

### B. Vehicle platoon lane changing manoeuvre

The study supposes that the leader obtains the target slip angle $\beta_{ref}$. Moreover, the followers receive from the vehicle in front the assumed trajectory state, $\tilde{X}_{j-1}(k)$, at each sample time. This information also contains the prediction of the slip angle of the vehicle in front $\beta_{j-1}$.

The lateral dynamics of the platoon are controlled by the MPC solution proposed in Section III (referred to as *Hybrid cost MPC*). Also, the solution is compared with the MPC controller proposed in [2] (referred to as *Infinity norm cost MPC*). The second method uses a cost function that differs from the cost function (3) because the latter uses only infinity norm terms. The parameters of the vehicles are contained in Table I, and the parameters of the controller for both methods are contained in Table II. The weighting matrices $P_j$, $Q_j$, and $R_j$ have the same values for all vehicles. The weighting matrices for the terminal cost are computed as a solution of a Riccati equation $P_j = \begin{bmatrix} 1.0829 & -0.048 & 1.457 \\ -0.048 & 0.0048 & -0.058 \\ 1.457 & -0.058 & 252.069 \end{bmatrix}$, for the method based on hybrid cost and $P_j = 10Q_j$ for the method based on infinity norms, $R_j = 0.1$, $\lambda = 1000$.

The manoeuvre used to test the proposed control solution represents a double-lane change. The obtained trajectory,

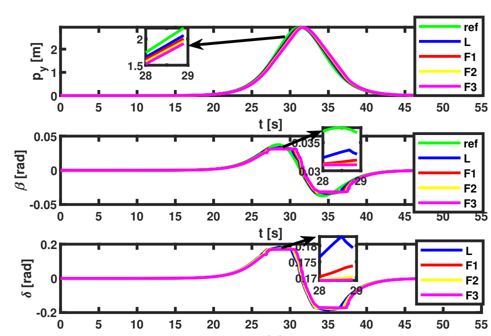


Fig. 2: Infinity norm cost MPC [2]: lateral position, slip angles, steering angles.

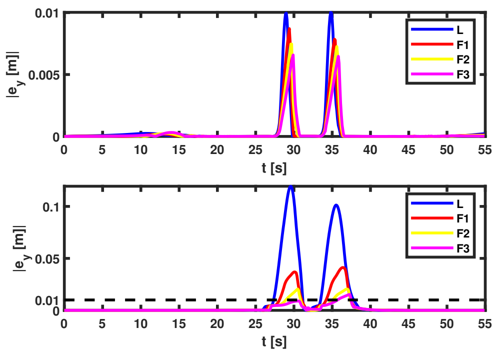


Fig. 3: Lateral position error: *a)* Hybrid cost MPC - top plot; *b)* Infinity norm cost MPC [2] - bottom plot (the dashed black line in this plot represents the maximum error for the hybrid cost).

the slip angles of the vehicles, and the commands (steering angles) are illustrated in Figs. 1 and 2. From these figures, it results that the controllers, designed using the method based on hybrid cost MPC or the method based on infinity norm cost MPC, succeeded in steering the platoon so that each vehicle follows its trajectory. Analysing the lateral errors (see Fig. 3) and slip angle errors (see Fig. 4), it can be seen that the vehicles are moving with small errors. Still, improved results (10 times better for the maximum error) are obtained by the solution based on the hybrid cost function. Moreover, all errors are decreasing along the upstream direction. However, for the lateral dynamics, these errors have to be as small as possible and without oscillations to ensure the safety and comfort of passengers. So, the solution based on hybrid cost accomplished these requirements better than the solution based on infinity norm. The constraints that imply the leader-follower string stability required by the method based on hybrid cost (24), (37), are illustrated in Fig. 5. These conditions were successfully respected, which implies that the proposed control solution attains the leader-follower string stability for the lateral dynamics of the platoon. For the method based on the infinity norm, the maximum values of the constraints (37) are $10^{-8} \cdot [-0.95, -5.46, -4.34, -4.59]$ for leader and follower vehicles. The maximum values of $\rho(k)$ are $[0.026, 0.022, 0.02, 0.018]$. As seen in Fig. 5, $\rho(k)$ is smaller for the method based on the hybrid cost compared to the method based on infinity norms.

## V. Conclusions

This paper derived a new hybrid cost function for distributed MPC of vehicle platoons that guarantees global platoon stability and, in combination with an explicit constraint, leader-follower string stability. The solution was tested in simulation to control the lateral dynamics of a vehicle platoon. The results show that the vehicles using the hybrid cost function MPC controller succeeded in following the imposed reference trajectory, also respecting the stability conditions, with significantly smaller lateral errors compared to an infinity norm cost function MPC scheme.

## References


[1] S. Maiti, S. Winter, L. Kulik, and S. Sarkar, "The impact of flexible platoon formation operations," *IEEE Transactions on Intelligent Vehicles*, vol. 5, no. 2, pp. 229–239, 2020.

[2] W. B. Dunbar and D. S. Caveney, "Distributed receding horizon control of vehicle platoons: Stability and string stability," *IEEE Transactions on Automatic Control*, vol. 57, no. 3, pp. 620–633, 2012.

[3] A. M. I. Mahbub and A. A. Malikopoulos, "A platoon formation framework in a mixed traffic environment," *IEEE Control Systems Letters*, vol. 6, pp. 1370–1375, 2022.

[4] S. Sadraddini, S. Sivaranjani, V. Gupta, and C. Belta, "Provably safe cruise control of vehicular platoons," *IEEE Control Systems Letters*, vol. 1, no. 2, pp. 262–267, 2017.

[5] Y. Wang, N. Bian, L. Zhang, and H. Chen, "Coordinated lateral and longitudinal vehicle-following control of connected and automated vehicles considering nonlinear dynamics," *IEEE Control Systems Letters*, vol. 4, no. 4, pp. 1054–1059, 2020.

[6] R. Kianfar, P. Falcone, and J. Fredriksson, "A distributed model predictive control approach to active steering control of string stable cooperative vehicle platoon," *IFAC Proceedings Volumes*, vol. 46, no. 21, pp. 750–755, 2013, 7th IFAC Symposium on Advances in Automotive Control.

[7] Y. Zheng, S. E. Li, K. Li, F. Borrelli, and J. K. Hedrick, "Distributed model predictive control for heterogeneous vehicle platoons under unidirectional topologies," *IEEE Transactions on Control Systems Technology*, vol. 25, no. 3, pp. 899–910, 05 2017.

[8] Y. Wu, Z. Zuo, Y. Wang, Q. Han, and C. Hu, "Distributed data-driven model predictive control for heterogeneous vehicular platoon with uncertain dynamics," *IEEE Transactions on Vehicular Technology*, pp. 1–14, 2023.

[9] J. Wang, X. Li, J. H. Park, and G. Guo, "Distributed MPC-based string stable platoon control of networked vehicle systems," *IEEE Transactions on Intelligent Transportation Systems*, vol. 24, no. 3, pp. 3078–3090, 2023.

[10] C. Conte, C. N. Jones, M. Morari, and M. N. Zeilinger, "Distributed synthesis and stability of cooperative distributed model predictive control for linear systems," *Automatica*, vol. 69, pp. 117–125, 2016.


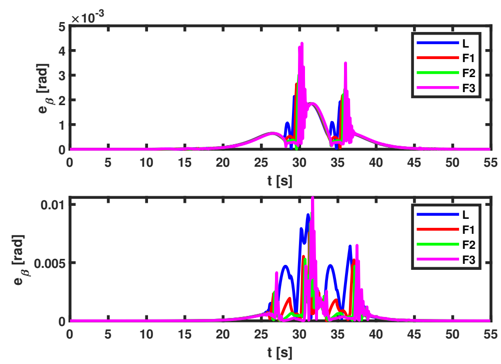


Fig. 4: Slip angle error: *a)* Hybrid cost MPC - top plot; *b)* Infinity norm cost MPC [2] - bottom plot.

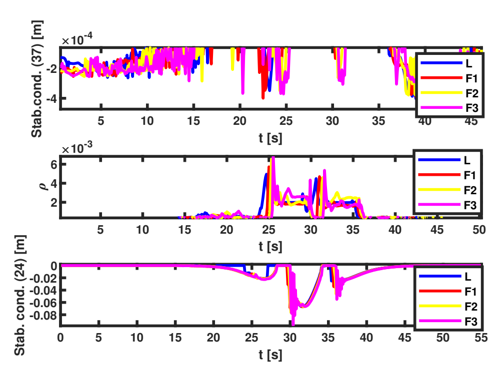


Fig. 5: Hybrid cost MPC: stability conditions and $\rho(k)$