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A Passivity Based Integral Sliding Mode Controller for Mechanical Port-Hamiltonian Systems*

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Abstract— **This paper proposes a passivity based integral sliding mode controller for mechanical port-Hamiltonian systems. Recently, passivity based sliding mode control (PBSMC) has been proposed for mechanical and electro-mechanical systems. This method has properties of both sliding mode control (SMC) and passivity based control. However, the robustness of the closed-loop system is not guaranteed in the reaching phase. For this problem, integral sliding mode control (ISMC), which eliminates the reaching phase, has been proposed. This paper proposes a unified control method of passivity based control and integral sliding mode control based on the idea of PBSMC. In order to achieve ISMC in the port-Hamiltonian form, an integral term of the sliding variable of PBSMC is firstly added to the system equation. Next, by adding an appropriate potential function to the Hamiltonian function, the dynamics of ISMC is obtained. The proposed method is more robust than PBSMC and ensures Lyapunov stability even if the resulting feedback controller is replaced by its continuous approximation to alleviate the chattering phenomena. The effectiveness of the proposed method is demonstrated by a numerical example.**

I. INTRODUCTION

Passivity based control is a method to find Lyapunov function candidates using the physical energy and the related conserved quantities of the plant system. One of the standard models for this control is a port-Hamiltonian system [1]. Port-Hamiltonian systems are represented by Hamilton's canonical equation, and many physical systems are described in this form, for example, mechanical systems, electric circuits, electromechanical systems, nonholonomic systems, and so on [2]–[5]. In this method, an appropriate control input is added to the system so that the closed-loop system has a desired Hamiltonian function, which represents physical energy of the system and serves as a Lyapunov function candidate. This method is called energy shaping. Recently, Kinetic-potential energy shaping (KPES) [6]–[8] is proposed for mechanical port-Hamiltonian systems. It allows us to design a special class of potential functions whose arguments are both configuration and momentum.

On the other hand, sliding mode control (SMC) [9]–[11] is a nonlinear control method belonging to variable structure control, and is known to be robust control against modeling errors and external disturbances. A sliding mode control usually has two phases, a reaching phase and a sliding phase. In the reaching phase, the system states are forced to converge to a subspace called the sliding surface within a finite time transient. Once the system states reach the sliding surface, the states slide toward the origin of the state space along the sliding surface. This motion is called the sliding phase. In the sliding phase, the system response is invariant for modeling errors and external disturbances. However, during the reaching phase, the invariance of SMC is not guaranteed and the system response is sensitive to them. For this problem, integral sliding mode control (ISMC) has been proposed [12], [13]. ISMC eliminates the reaching phase by enforcing the sliding mode in the entire system response so that the invariance of SMC is guaranteed throughout entire system response from the initial time instance.

Recently, for mechanical and electro-mechanical port-Hamiltonian systems, passivity based sliding mode control (PBSMC) is proposed [14]–[16]. This method designs a sliding mode controller by selecting a non-smooth function as an artificial function based on KPES for port-Hamiltonian systems. Since SMC is achieved in the framework of passivity based control, Lyapunov stability is ensured even if the input is replaced by a continuous approximation of the sliding mode control law to alleviate chattering phenomena. However, the invariance of PBSMC in the reaching phase is not guaranteed as the conventional SMC.

The scope of this paper is to design unified control of passivity based control and ISMC based on the idea of PBSMC. We consider an integral term of the sliding variable of PBSMC as a state. By appropriate feedback, the dynamics of ISMC is represented in the port-Hamiltonian form. It is expected to result in a more robust controller compared to PBSMC by eliminating the reaching phase and forcing the states into the sliding surface from the initial time. It also ensures Lyapunov stability when the chattering phenomenon is mitigated, compared to the conventional ISMC.

The remainder of the paper is organized as follows. Section II briefly refers to the background of the proposed method. Section III gives the main result of the present paper. Integral sliding mode control with an energy based Lyapunov function is proposed. Section IV gives a numerical example to show the effectiveness of the proposed method. Finally, Section V concludes the paper.

Throughout this paper, the symbol ∇_x denotes the gradient with respect to x, that is,$\nabla_x f \equiv \frac{\partial f}{\partial x}^\top = \left(\frac{\partial f}{\partial x_1}, \cdots, \frac{\partial f}{\partial x_n}\right)^\top$ with $x = (x_1, \cdots, x_n)$.

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II. BACKGROUND

This section introduces the background of kinetic-potential energy shaping and passivity based sliding mode control. The basic idea of sliding mode control is also introduced.

A. Port-Hamiltonian systems

Let us consider a fully-actuated mechanical system described by the following port-Hamiltonian form [1]

$$\begin{aligned}\begin{pmatrix}\dot q\\ \dot p\end{pmatrix} &= \begin{pmatrix}0 & I\\ -I & -D_0(x)\end{pmatrix}\begin{pmatrix}\nabla_q H_0(x)\\ \nabla_p H_0(x)\end{pmatrix} + \begin{pmatrix}0\\ G_0(x)\end{pmatrix}u,\\ H_0(x) &= \frac{1}{2}p^\top M^{-1}(q)p. \end{aligned}\tag{1}$$

Here the state vector $x = (q^\top, p^\top)^\top \in \mathbb{R}^{2m}$ consists of the configuration $q \in \mathbb{R}^m$ and momentum $p \in \mathbb{R}^m$, and $u \in \mathbb{R}^m$ denotes the input vector. The positive definite matrix $M(q) \succ 0 \in \mathbb{R}^{m\times m}$ denotes the inertia matrix and the positive semi-definite matrix $D_0(q,p) \in \mathbb{R}^{m\times m}$ denotes the damping matrix. The matrix $G_0(q,p) \in \mathbb{R}^{m\times m}$ denotes the full rank input mapping matrix. The symbol $H_0(x) \in \mathbb{R}$ is called the Hamiltonian function which represents the physical energy of the system. For such a class of systems, passivity based control is often employed. Let us consider the following control input for the plant system (1)

$$u = -G_0^{-1}(x)\left(Kq + C\dot q\right), \quad K \succ 0,\ C \succ 0. \tag{2}$$

The closed-loop system becomes the following port-Hamiltonian system

$$\begin{aligned}\begin{pmatrix}\dot q\\ \dot p\end{pmatrix} &= \underbrace{\begin{pmatrix}0 & I\\ -I & -(D_0(x)+C)\end{pmatrix}}_{J_{\rm d}(x)}\begin{pmatrix}\nabla_q H_{\rm d}\\ \nabla_p H_{\rm d}\end{pmatrix},\\ H_{\rm d}(x) &= \underbrace{\frac{1}{2}p^\top M^{-1}(q)p}_{H_0(x)} + \underbrace{\frac{1}{2}q^\top Kq}_{U(q)} \succ 0.\end{aligned}\tag{3}$$

The Hamiltonian function of the closed-loop system satisfies $H_{\rm d}(x) \succ 0$[1] and

$$\dot H_{\rm d}(x) = \nabla_x H_{\rm d}(x)^\top J_{\rm d}(x)\nabla_x H_{\rm d}(x) \preceq 0.$$

It follows from La Salle's invariance principle that the origin is asymptotically stable with a Lyapunov function $H_{\rm d}(x)$. The second term of the Hamiltonian function $(1/2)q^\top Kq$ can be replaced with any function $U(q)$ positive definite with respect to q. The function $U(q)$ is called an artificial potential function.

B. Kinetic-potential energy shaping

Kinetic-potential energy shaping [8], [14] is one of energy shaping methods and it allows us to select a wider class of potential functions. The following coordinate transformation for the plant system (1)

$$\begin{pmatrix}q\\ p\end{pmatrix} \mapsto \begin{pmatrix}q\\ \eta\end{pmatrix} \equiv \begin{pmatrix}q\\ T^\top(q)p\end{pmatrix}$$

where $T(q) \in \mathbb{R}^{m\times m}$ is a nonsingular matrix satisfying

$$T(q)T^\top(q) = M^{-1}(q) \tag{4}$$

converts the system (1) into the port-Hamiltonian system

$$\begin{pmatrix}\dot q\\ \dot \eta\end{pmatrix} = \underbrace{\begin{pmatrix}0 & T(q)\\ -T^\top(q) & -D(x)\end{pmatrix}}_{J(x)}\begin{pmatrix}\nabla_q H\\ \nabla_\eta H\end{pmatrix} + \begin{pmatrix}0\\ G(x)\end{pmatrix}u \tag{5}$$

with $H(x) = (1/2)\|\eta\|^2$ where $G(x) = T^\top(q)G_0(x)$ and $D(x)$ is an appropriate matrix satisfying $D(x) + D^\top(x) \succeq 0$. The new Hamiltonian function $H(x)$ depends only on η, and no longer depends on q. By modifying the upper left block of the structure matrix $J(x)$ defined in Eq. (5) appropriately, we can select potential function which depends on both configuration q and momentum η.

C. Sliding mode control

This subsection briefly refers to sliding mode control [9]–[11]. In sliding mode control, the system states reach a subspace of the state space called the sliding surface within a finite time, which is called the reaching phase. After the reaching phase, the states evolve along the desired dynamics on the surface, which is called the sliding phase. Here we consider a general input-affine nonlinear system

$$\dot x = f(x) + g(x)u \tag{6}$$

where $x \in \mathbb{R}^n$ is the state vector and $u \in \mathbb{R}^m$ is the input vector. We select a switching function called a sliding variable $\sigma(x) \in \mathbb{R}^m$. To enforce the sliding variable σ to zero within a finite time, a discontinuous feedback is employed so that the closed-loop system contains the following dynamics

$$\dot\sigma_i = -k_i\mathrm{sgn}(\sigma_i),\ k_i > 0, \quad i = 1, 2, \ldots m. \tag{7}$$

Here the signum function sgn is defined by

$$\mathrm{sgn}(z)\begin{cases} =1 & (z>0)\\ \in[-1,1] & (z=0)\\ =-1 & (z<0)\end{cases}.$$

If its argument is a vector $x = (x_1, \cdots, x_n)^\top \in \mathbb{R}^n$, then

$$\mathrm{sgn}(x) = (\mathrm{sgn}(x_1), \cdots, \mathrm{sgn}(x_n))^\top.$$

The input u is selected to include the function sgn so that the dynamics in Eq. (7) is achieved. Throughout this paper, by solutions on the sliding surface we mean Filippov solutions.

D. Passivity based sliding mode control

Recently, passivity based sliding mode control has been proposed, which ensures Lyapunov stability even if the resulting feedback is replaced by its continuous approximation. Here let us consider the system (5). Then, the following theorem gives a passivity based sliding mode controller.

Theorem 1: [15] Consider the system (5). Suppose that there exists a diffeomorphism $\phi : \mathbb{R}^m \to \mathbb{R}^m$ satisfying $\phi(0) = 0$ and

$$\Lambda(q) \equiv \frac{\partial\phi(q)}{\partial q}T(q) + T^\top(q)\frac{\partial\phi(q)}{\partial q}^\top \succ \epsilon I,\ \epsilon > 0,\ \forall q. \tag{8}$$

[1]The notation $f(x) \succ 0 \in \mathbb{R}$ implies positive definiteness of a scalar function f, while the same notation $P \succ 0 \in \mathbb{R}^{n\times n}$ implies positive definiteness of a symmetric square matrix P.

Then the feedback control

$$\begin{aligned} u =& - kG^{-1}(x)\Lambda(q)\mathrm{sgn}(\phi(q)+\eta) \\ &+ G^{-1}(x)\left(D(x) - \frac{\partial\phi(q)}{\partial q}T(q)\right)\eta, \quad k>0 \end{aligned} \tag{9}$$

achieves the following properties.
(i) The feedback (9) converts the system (5) into the following closed-loop port-Hamiltonian system

$$\begin{aligned} \begin{pmatrix} \dot{q} \\ \dot{\eta} \end{pmatrix} &= \begin{pmatrix} -T(q)\frac{\partial\phi(q)}{\partial q}^{-\top} & T(q) \\ -T^\top(q) & -\frac{\partial\phi(q)}{\partial q}T(q) \end{pmatrix} \begin{pmatrix} \nabla_q H_{\mathrm{smc}} \\ \nabla_\eta H_{\mathrm{smc}} \end{pmatrix} \\ H_{\mathrm{smc}}(x) &= \frac{1}{2}\|\eta\|^2 + k\|\phi(q)+\eta\|_1. \end{aligned} \tag{10}$$

(ii) Along the closed-loop system (10), the sliding variable

$$\sigma \equiv \phi(q) + \eta$$

is enforced to converge to zero within a finite time.
(iii) The origin of the closed-loop system (10) is asymptotically stable with the Lyapunov function $H_{\mathrm{smc}}(x)$.

This theorem gives the unified method of passivity based control and sliding mode control. Also asymptotic stability is ensured by the Lyapunov function $H_{\mathrm{smc}}(x)$ even if the discontinuous feedback is replaced by its continuous approximation to alleviate chattering phenomena.

However, this method has the reaching phase like conventional sliding mode control. In the reaching phase, the system response is sensitive to modeling errors and external disturbances. In the next section, we propose passivity based integral sliding mode control to solve this problem.

III. PASSIVITY BASED INTEGRAL SLIDING MODE CONTROL

This section gives the main result of this paper. A new integral sliding mode controller for port-Hamiltonian systems is proposed based on the framework of passivity based control.

A. Integral sliding mode control

This subsection briefly refers to integral sliding mode control [12], [13]. Let us consider a class of perturbed uncertain input-affine nonlinear systems such as

$$\dot{x} = f(x) + g(x)(u + h(x,t)) \tag{11}$$

where $x \in \mathbb{R}^n$ is the state vector and $u \in \mathbb{R}^m$ is the input vector. It is assumed that function h is uniformly bounded, that is, there exists a constant $\bar{h}_i > 0$ such that $|h_i(x,t)| \leq \bar{h}_i,\ 1 \leq i \leq m$. For the system (11), choose the following control law

$$u = u_0 + u_1$$

where $u_0 \in \mathbb{R}$ is the nominal control for (11), and u_1 is designed to be discontinuous for the rejection of $h(x,t)$. Next, we design a sliding variable ξ as

$$\xi = \sigma + z, \quad \xi, \sigma, z \in \mathbb{R}^m. \tag{12}$$

Here σ is selected as the linear combination of the system states like the sliding variable of the conventional first order sliding mode control, and z is an integral term given by

$$\dot{z} = -\frac{\partial\sigma}{\partial x}(f(x) + g(x)u_0), \quad z(0) = -\sigma(0) \tag{13}$$

where $z(0)$ is selected to be $-\sigma(0)$ such that $\xi(0) = 0$. Then the dynamics of ξ is

$$\dot{\xi} = \frac{\partial\sigma}{\partial x}g(x)u_1 + h(x,t). \tag{14}$$

We assume that $(\partial\sigma/\partial x)g(x) + g^\top(x)(\partial\sigma/\partial x)^\top$ is positive definite without loss of generality. If we use $u_1 = -k\mathrm{sgn}(\xi)$ and k is large enough, $\xi(t) = 0, \forall t \geq 0$ is achieved. Then the reaching phase is eliminated and the sliding phase occurs immediately after the initial time instance. The motion equation of the system in the sliding phase will be

$$\dot{x} = f(x) + g(x)u_0. \tag{15}$$

Once the states are forced to the sliding surface $\xi = 0$, the equivalent control $u_{1\mathrm{eq}}$ is obtained by setting $\dot{\xi} = 0$ as

$$u_{1\mathrm{eq}} = -((\partial\sigma/\partial x)g(x))^{-1}h(x,t). \tag{16}$$

Remark 1: In the existing results [12], [13], to alleviate chattering phenomena, it is suggested that the discontinuous control u_1 is filtered by a first order linear low pass filter

$$\mu\dot{u}_{1\mathrm{av}} = -u_{1\mathrm{av}} + u_1, \quad u_{1\mathrm{av}}(0) = 0 \tag{17}$$

where $u_{1\mathrm{av}}$ can be used instead of u_1, and $\mu > 0$ is a filtering constant that is small enough to avoid distorting the slow component of the switched action which is equal to $u_{1\mathrm{eq}}$.

B. Passivity based integral sliding mode control

This subsection gives the main result of this paper. Here we consider the following system (18) in which a matched disturbance $d(t) \in \mathbb{R}^m$ is added to the system (5)

$$\begin{aligned} \begin{pmatrix} \dot{q} \\ \dot{\eta} \end{pmatrix} &= \begin{pmatrix} 0 & T(q) \\ -T^\top(q) & -D(x) \end{pmatrix} \begin{pmatrix} \nabla_q H \\ \nabla_\eta H \end{pmatrix} + \begin{pmatrix} 0 \\ G(x) \end{pmatrix}(u+d) \\ H(x) &= \frac{1}{2}\|\eta\|^2 \end{aligned} \tag{18}$$

where it is assumed that there exists a constant $\bar{d}_i$ satisfying $|(G(x)d(t))_i| \leq \bar{d}_i, i = 1, 2, \ldots m$.

The following feedback control based on Theorem 1

$$\begin{aligned} u =& - G^{-1}(x)\Lambda(q)\nabla_\sigma U(\phi(q)+\eta) \\ &+ G^{-1}(x)\left(D(x) - \frac{\partial\phi(q)}{\partial q}T(q)\right)\eta + u_1 \\ \equiv& \ u_0 + u_1 \end{aligned} \tag{19}$$

converts the system (18) into the following port-Hamiltonian system

$$\begin{aligned} \begin{pmatrix} \dot{q} \\ \dot{\eta} \end{pmatrix} &= \begin{pmatrix} -T(q)\frac{\partial\phi(q)}{\partial q}^{-\top} & T(q) \\ -T^\top(q) & -\frac{\partial\phi(q)}{\partial q}T(q) \end{pmatrix} \begin{pmatrix} \nabla_q \bar{H} \\ \nabla_\eta \bar{H} \end{pmatrix} \\ &\quad + \begin{pmatrix} 0 \\ G(x) \end{pmatrix}(u_1 + d), \\ \bar{H}(x) &= \frac{1}{2}\|\eta\|^2 + U(\phi(q)+\eta). \end{aligned} \tag{20}$$

The system (20) is equivalent to the system (10) if $U(\phi(q)+\eta) = k\|\phi(q)+\eta\|_1$ and $u_1 = d = 0$. If the assumption (8) holds and $d = 0$, asymptotic stability is ensured with the Lyapunov function $\bar{H}(x)$ as stated in Theorem 1. Here let us apply the following coordinate transformation

$$\begin{pmatrix} q \\ \eta \end{pmatrix} \mapsto \begin{pmatrix} \sigma \\ \eta \end{pmatrix} = \begin{pmatrix} \phi(q)+\eta \\ \eta \end{pmatrix}.$$

Then the system (20) is transformed into the following port-Hamiltonian system

$$\begin{pmatrix} \dot{\sigma} \\ \dot{\eta} \end{pmatrix} = \underbrace{\begin{pmatrix} -\Lambda(q) & 0 \\ -\Lambda(q) & -\frac{\partial \phi(q)}{\partial q}T(q) \end{pmatrix}}_{\bar{J}(x)} \begin{pmatrix} \nabla_\sigma \bar{H} \\ \nabla_\eta \bar{H} \end{pmatrix} + \begin{pmatrix} G(x) \\ G(x) \end{pmatrix} (u_1+d),$$
$$\bar{H}(x) = \frac{1}{2}\|\eta\|^2 + U(\sigma). \tag{21}$$

For the system (21), we introduce the controller state z based on Eq. (13) as

$$\begin{aligned} \dot{z} &= -\frac{\partial \sigma}{\partial x} \begin{pmatrix} -\Lambda(q) & 0 \\ -\Lambda(q) & -\frac{\partial \phi(q)}{\partial q}T(q) \end{pmatrix} \begin{pmatrix} \nabla_\sigma \bar{H} \\ \nabla_\eta \bar{H} \end{pmatrix} \\ &= \Lambda(q)\nabla_\sigma U(\sigma), \quad z(0) = -\sigma(0). \end{aligned} \tag{22}$$

Adding the new state z to the system (21) converts the system (21) into the port-Hamiltonian system

$$\begin{aligned} \begin{pmatrix} \dot{z} \\ \dot{\sigma} \\ \dot{\eta} \end{pmatrix} &= \begin{pmatrix} 0 & \Lambda(q) & 0 \\ 0 & -\Lambda(q) & 0 \\ 0 & -\Lambda(q) & -\frac{\partial \phi(q)}{\partial q}T(q) \end{pmatrix} \begin{pmatrix} \nabla_z \bar{H} \\ \nabla_\sigma \bar{H} \\ \nabla_\eta \bar{H} \end{pmatrix} \\ &\quad + \begin{pmatrix} 0 \\ G(x) \\ G(x) \end{pmatrix} (u_1+d), \\ \bar{H}(x) &= \frac{1}{2}\|\eta\|^2 + U(\sigma). \end{aligned} \tag{23}$$

Then the next theorem holds.

Theorem 2: Consider the system (23) with any scalar positive definite functions $U(\sigma)$ and $V(\xi)$, and any diffeomorphism $\phi : \mathbb{R}^m \to \mathbb{R}^m$. The feedback

$$u_1 = -G^{-1}(x)\Lambda(q)\nabla_\xi V(\sigma+z), \quad \xi = \sigma + z \tag{24}$$

with Λ defined in Eq. (8) achieves the following properties.
(i) The feedback (24) converts the system (21) into the following closed-loop port-Hamiltonian system

$$\begin{aligned} \begin{pmatrix} \dot{z} \\ \dot{\sigma} \\ \dot{\eta} \end{pmatrix} &= \begin{pmatrix} -\Lambda(q) & \Lambda(q) & 0 \\ 0 & -\Lambda(q) & 0 \\ 0 & -\Lambda(q) & -\frac{\partial \phi(q)}{\partial q}T(q) \end{pmatrix} \begin{pmatrix} \nabla_z H_{\mathrm{ismc}} \\ \nabla_\sigma H_{\mathrm{ismc}} \\ \nabla_\eta H_{\mathrm{ismc}} \end{pmatrix} \\ &\quad + \begin{pmatrix} 0 \\ G(x) \\ G(x) \end{pmatrix} d, \\ H_{\mathrm{ismc}}(x) &= \frac{1}{2}\|\eta\|^2 + U(\sigma) + V(\sigma+z). \end{aligned} \tag{25}$$

(ii) Suppose that there exists a constant $\epsilon > 0$ satisfying Eq. (8). Assume moreover that there exists a constant $\bar{d}_i$ satisfying $|(G(x)d(t))_i| \le \bar{d}_i, i = 1, 2, \ldots m$. Select $V(\xi) = k_1\|\xi\|_1$ with $k_1 > 0$. If k_1 is large enough, the system states are forced to stay in the sliding surface $\xi = 0$ from the initial time instance.
(iii) Suppose that all assumptions in (ii) hold. Then, asymptotic stability of the origin of the closed-loop system (25) is ensured with the Lyapunov function $H_{\mathrm{ismc}}(x)$ in the presence of any disturbance d satisfying the assumptions.

Proof: The property (i) follows from the direct calculation as follows. In fact, the first element of Eq. (23) is

$$\begin{aligned} \dot{z} &= \Lambda(q)\nabla_\sigma U(\sigma) \\ &= -\Lambda(q)\nabla_\xi V(\sigma+z) + \Lambda(q)(\nabla_\xi V(\sigma+z) + \nabla_\sigma U(\sigma)) \\ &= -\Lambda(q)\nabla_z H_{\mathrm{ismc}}(x) + \Lambda(q)\nabla_\sigma H_{\mathrm{ismc}}(x) \end{aligned}$$

which is equivalent to the first element of Eq. (25). Similarly, substituting u_1 in (24) into Eq. (23), the second and third elements of Eq. (25) are obtained.

The proof of the properties (ii) and (iii) needs an additional coordinate change as follows

$$\begin{pmatrix} z \\ \sigma \\ \eta \end{pmatrix} \mapsto \begin{pmatrix} \xi \\ \sigma \\ \eta \end{pmatrix} = \begin{pmatrix} \sigma+z \\ \sigma \\ \eta \end{pmatrix}.$$

Then the system (25) is transformed into the following port-Hamiltonian system

$$\begin{aligned} \begin{pmatrix} \dot{\xi} \\ \dot{\sigma} \\ \eta \end{pmatrix} &= \underbrace{\begin{pmatrix} -\Lambda(q) & 0 & 0 \\ -\Lambda(q) & -\Lambda(q) & 0 \\ -\Lambda(q) & -\Lambda(q) & -\frac{\partial \phi(q)}{\partial q}T(q) \end{pmatrix}}_{J_{\mathrm{ismc}}(x)} \begin{pmatrix} \nabla_\xi H_{\mathrm{ismc}} \\ \nabla_\sigma H_{\mathrm{ismc}} \\ \nabla_\eta H_{\mathrm{ismc}} \end{pmatrix} \\ &\quad + \begin{pmatrix} G(x) \\ G(x) \\ G(x) \end{pmatrix} d, \\ H_{\mathrm{ismc}}(x) &= \frac{1}{2}\|\eta\|^2 + U(\sigma) + k_1\|\xi\|_1. \end{aligned} \tag{26}$$

We can see that the dynamics of the sliding variable ξ is

$$\dot{\xi} = -\Lambda(q)\nabla_\xi H_{\mathrm{ismc}} + G(x)d = -k_1\Lambda(q)\mathrm{sgn}(\xi) + G(x)d.$$

Now, let us consider $V(\xi)$ as a Lyapunov function candidate. First of all, for the reaching phase, i.e., $\sigma_i \neq 0$ for $\forall i$, the following inequality holds from the assumption (8)

$$\begin{aligned} \dot{V}(\xi) &= k_1\mathrm{sgn}(\xi)^\top(-k_1\Lambda(q)\mathrm{sgn}(\xi) + G(x)d) \\ &\le -k_1^2\epsilon\|\mathrm{sgn}(\xi)\|^2 + k_1\mathrm{sgn}(\xi)^\top G(x)d \\ &\le -k_1\sum_{i=1}^{m}(k_1\epsilon - \bar{d}_i) \equiv -k_1 a(k_1). \end{aligned}$$

Thus, if $k_1 > \bar{d}_i/\epsilon, \forall i$ holds, $\dot{V}(\xi) < -k_1 a(k_1), a(k_1) > 0$. Next, for the sliding phases, i.e., there exist some i's for which $\xi_i = 0$. Suppose there are m_{sp} sub-sliding phases and $m - m_{\mathrm{sp}}$ sub-reaching phases, i.e., m_{sp} elements of ξ are enforced to be zero and $m - m_{\mathrm{sp}}$ elements of ξ are not yet zero for $1 \le m_{\mathrm{sp}} \le m-1$. Let us denote $\xi_{\mathrm{sp}} \equiv (\xi_1, \ldots, \xi_{m_{\mathrm{sp}}})^\top$, $\xi_{\mathrm{rp}} \equiv (\xi_{m_{\mathrm{sp}+1}}, \ldots, \xi_m)^\top$ and suppose $\xi_{\mathrm{sp}} = \dot{\xi}_{\mathrm{sp}} = 0$ (sliding phase) and $\xi_{\mathrm{rp}} \neq 0$ (reaching phase) for simplicity. Then the dynamics of ξ is given by

$$\begin{pmatrix} \dot{\xi}_{\mathrm{sp}} \\ \dot{\xi}_{\mathrm{rp}} \end{pmatrix} = -k_1 \begin{pmatrix} \Lambda_{11}(q) & \Lambda_{12}(q) \\ \Lambda_{12}^\top(q) & \Lambda_{22}(q) \end{pmatrix} \begin{pmatrix} \mathrm{sgn}(\xi_{\mathrm{sp}}) \\ \mathrm{sgn}(\xi_{\mathrm{rp}}) \end{pmatrix} + \begin{pmatrix} d_{\mathrm{sp}} \\ d_{\mathrm{rp}} \end{pmatrix} = \begin{pmatrix} 0 \\ * \end{pmatrix}$$

where $*$ is an arbitrary value and $(d_{\rm sp}^\top, d_{\rm rp}^\top)^\top = G(x)d$. Thus, the equivalent sliding mode control system is described as

$$\dot{\xi}_{\rm rp} = -k_1 \underbrace{(\Lambda_{22} - \Lambda_{12}^\top \Lambda_{11}^{-1}\Lambda_{12})}_{\Lambda/\Lambda_{11}(q)} \mathrm{sgn}(\xi_{\rm rp}) - \Lambda_{12}^\top \Lambda_{11}^{-1} d_{\rm sp} + d_{\rm rp}.$$

Calculating the time derivative of $V(\xi)$ along the above equation, we obtain

$$\begin{aligned}\dot{V}(\xi) &= k_1 \left(\mathrm{sgn}(\xi_{\rm sp})^\top, \mathrm{sgn}(\xi_{\rm rp})^\top\right) \begin{pmatrix}\dot{\xi}_{\rm sp}\\ \dot{\xi}_{\rm rp}\end{pmatrix}\\ &\le -k_1 \sum_{i=1}^{m-m_{\rm sp}} \left(k_1\epsilon - \bar{d}_{{\rm rp}i}\right) \equiv -k_1 a_{\rm rp}(k_1),\end{aligned}$$

where we define a constant $\bar{d}_{{\rm rp}i}$ satisfying

$$\left|\left(-\Lambda_{12}^\top(q)\Lambda_{11}^{-1}(q)d_{\rm sp} + d_{\rm rp}\right)_i\right| \le \bar{d}_{{\rm rp}i}, 1 \le i \le m - m_{\rm sp} \quad (27)$$

and use the fact that $\Lambda/\Lambda_{11}(q) \succ \epsilon I$ which is derived from the assumption (8) and the following decomposition of $\Lambda(q)$:

$$\Lambda = \begin{pmatrix} I & 0\\ \Lambda_{12}^\top\Lambda_{11}^{-1} & I\end{pmatrix}\begin{pmatrix}\Lambda_{11} & 0\\ 0 & \Lambda/\Lambda_{11}\end{pmatrix}\begin{pmatrix} I & \Lambda_{11}^{-1}\Lambda_{12}\\ 0 & I\end{pmatrix}.$$

Hence, if k_1 is large enough, that is, $k_1 > \max(\bar{d}_i, \bar{d}_{{\rm rp}i})/\epsilon$, then $\dot{V}(\xi) < -k_1 a_{\rm rp}(k_1)$ with $a_{\rm rp}(k_1) > 0$. The constant $\bar{d}_{{\rm rp}i}$ satisfying Eq. (27) exists at least locally because $\Lambda_{12}^\top(q)$ $\Lambda_{11}^{-1}(q)$ is evaluated along Eq. (20) with $d = 0$ whose origin is asymptotically stable with the Lyapunov function $\bar{H}(x)$.

It follows from the above discussion that $V(\xi)$ goes to zero within a finite time and so is ξ. The setting $\xi(0) = 0$ eliminates the reaching phase resulting in $\xi(t) = 0, \forall t \ge 0$.

Finally, the property (iii) follows from the inequality

$$\begin{aligned}\dot{H}_{\rm ismc} &= \nabla_x H_{\rm ismc}^\top \left(J_{\rm ismc}\nabla_x H_{\rm ismc} + (G(x), G(x), G(x))^\top d\right)\\ &= \nabla_x \bar{H}\bar{J}\nabla_x\bar{H} \prec 0.\end{aligned}$$

The second equality holds because $-k_1\Lambda(q)\mathrm{sgn}(\xi) + G(x)d = 0$ and the inequality holds because $\bar{J} + \bar{J}^\top \prec 0$ where $\bar{J}$ is defined in Eq. (21). ■

Theorem 2 provides a new integral sliding mode controller. The resulting ISMC dynamics is described in the port-Hamiltonian form (26). The design parameter k_1 should be selected satisfying $k_1 > \max(\bar{d}_i, \bar{d}_{{\rm rp}i})/\epsilon$ as explained in the proof.

Remark 2: According to [15], the proposed method can alleviate chattering phenomena by choosing $V(\xi) = k_1\|\xi\|_s^r, 1 \le s, 1 \le r < 2$. Lyapunov stability is still ensured, but ξ does not become strictly zero in this case.

IV. NUMERICAL EXAMPLE

In this section, we show the effectiveness of the proposed method through numerical simulations. The plant system is a fully-actuated two degrees of freedom planar manipulator arm shown in Fig 1. This system can be described in the port-Hamiltonian form

$$\begin{pmatrix}\dot{q}\\ \dot{p}\end{pmatrix} = \begin{pmatrix}0 & I\\ -I & -D\end{pmatrix}\begin{pmatrix}\nabla_q H\\ \nabla_p H\end{pmatrix} + \begin{pmatrix}0\\ I\end{pmatrix}(u + d),$$

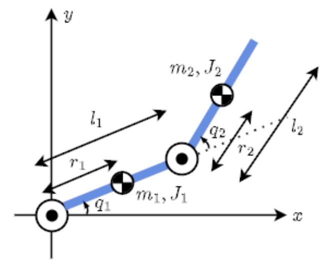


Fig. 1: A two degrees of freedom manipulator arm

TABLE I: Physical parameters

	Link 1	Link 2
Length of link	$l_1 = 1$	$l_2 = 1$
Mass of link	$m_1 = 1$	$m_2 = 1$
Center of mass of link	$r_1 = 1/2$	$r_2 = 1/2$
Moment of inertia of link	$J_1 = 1/12$	$J_2 = 1/2$
Friction coefficient	$\nu_1 = 1/2$	$\nu_2 = 1/2$

with $H = (1/2)p^\top M^{-1}(q)p$. System states are the angles of the links $q \in \mathbb{R}^2$ and the angular momenta $p = M(q)\dot{q} \in \mathbb{R}^2$. The inertia matrix $M(q)$ is

$$M(q) = \begin{pmatrix} M_1 + M_2 + 2M_3\cos q_2 & M_2 + M_3\cos q_2\\ M_2 + M_3\cos q_2 & M_2\end{pmatrix},$$
$$M_1 = m_1 r_1^2 + m_2 l_1^2 + J_1, M_2 = m_2 r_2^2 + J_2, M_3 = m_2 l_1 r_2.$$

Here m_i, J_i and l_i denote the mass of the i-th link, the moment of inertia of the i-th link and the length of the i-th link, respectively, and r_i denotes the length from the joint to the center of mass of the i-th link. The damping matrix $D = \mathrm{diag}(\nu_1, \nu_2)$ consists of the friction coefficients ν_1, ν_2. The physical parameters of the system are given in Table I.

The control objective is to stabilize the states at the desired equilibrium $q^* = (\pi/6, \pi/3)^\top$. We select σ as follows

$$\sigma(q, \eta) = \begin{pmatrix}4 & 0\\ 2 & 4\end{pmatrix}(q - q^*) + \eta.$$

In this simulation, we choose the function U and V as

$$U(\sigma) = \|\sigma\|^2, \quad V(\xi) = \|\xi\|_{1.35}^{1.35}$$

corresponding to $u_1 \propto (|\xi_1|^{0.35}\mathrm{sgn}(\xi_1), |\xi_2|^{0.35}\mathrm{sgn}(\xi_2))^\top$ as in Remark 2. Thus this choice of V gives a continuous approximation of the discontinuous controller given in Theorem 2 (ii), while it also ensures asymptotic stability. We compare the proposed method to conventional $u_{1\rm av}$, which is low-pass filtered u_1

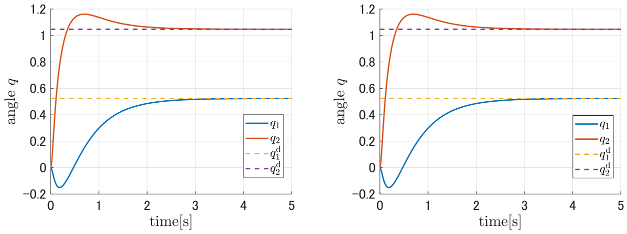
$$V(\xi) = \|\xi\|_1, \quad \text{filtering constant } \mu = 0.01.$$

The following matched disturbance $d(t)$ is considered.

$$d(t) = (\mathrm{sign}(\sin(6t)), \mathrm{sign}(\cos(6t)))^\top \quad (28)$$

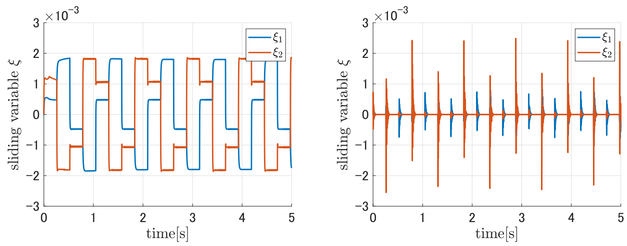
The initial condition of the state is given by $(q(0)^\top, p(0)^\top)^\top = (0, 0, 0, 0)^\top$.

Figures 2-5 show the results of the numerical simulations. Figure 2 shows the responses of the angles q. In Fig. 2, the solid lines denote the responses of the angles and the dashed-dotted lines denote the desired angles. This result shows the



(a) Proposed **(b)** Conventional

Fig. 2: Trajectories of the angles q with proposed and conventional controllers



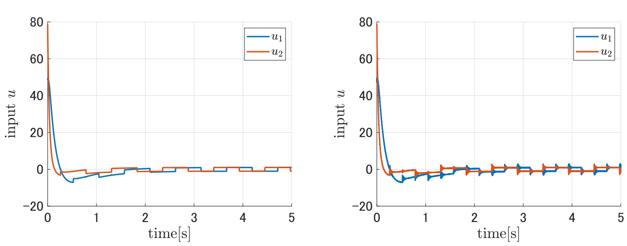
(a) Proposed **(b)** Conventional

Fig. 3: The sliding variables ξ with proposed and conventional controllers

angles converge to the desired angles with both proposed and conventional controllers. Figure 3 shows the responses of the integral sliding variable ξ. Since the disturbances are square waveform, ξ is not zero even with conventional method when the disturbances change stepwise. Also in the proposed method, ξ takes minute values when chattering is alleviated. Figure 4 shows the responses of input u. While the response $q(t)$ of the proposed method and that of the conventional method are very similar as illustrated in Fig. 2, the corresponding inputs are different as depicted in Fig. 4. Figure 4 shows that the conventional method causes very oscillatory chattering phenomena for the disturbance $d(t)$ in Eq. (28) while the proposed method gives a less oscillatory control input. Figure 5 shows the response of the Hamiltonian function $H_{\rm ismc}(x)$. It plays the role of the Lyapunov function of the closed-loop system.

V. CONCLUSIONS

This paper has proposed a new integral sliding mode controller using the passivity based approach. It provides a family of controllers smoothly connecting integral sliding mode control and passivity based control. Through the numerical example, we have confirmed that the proposed method ensures Lyapunov stability even if the discontinuous sliding mode controller is replaced by its continuous approximation to alleviate chattering phenomena.



(a) Proposed **(b)** Conventional

Fig. 4: The inputs u with proposed and conventional controllers

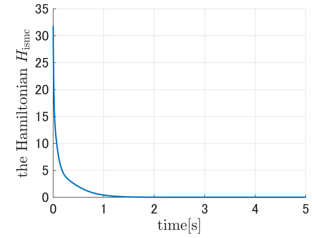


Fig. 5: The Hamiltonian function $H_{\rm ismc}$

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