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Data-driven Self-triggering Mechanism for State Feedback Control

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Abstract— **This paper presents a novel approach for data-driven self-triggered state feedback control of unknown linear systems using noisy data gathered offline. The self-triggering mechanism determines the next triggering time by checking whether the difference between the predicted state and the current state is significant or not. However, when the system matrices are unknown, the challenge lies in characterizing the distance between future states and the current state using only data. To address this, we put forth a data-driven online optimization problem for trajectory prediction by using noisy input-state data. Its optimal solution, together with another unknown parameter that reflects the open-loop divergence rate, is shown sufficient for explicitly quantifying the distance. Moreover, a data-driven set-based over-approximating algorithm using matrix zonotopes is subsequently proposed to upper-bound the open-loop divergence rate. Leveraging the optimal solution and the upper bound, a self-triggering mechanism is devised for state feedback control systems, which is proven to ensure input-to-state stability. Numerical examples are presented to validate the effectiveness of the proposed method.**

I. Introduction

In recent years, data-driven control has experienced prosperous development, wherein control laws are designed directly from data [1]–[4]. This literature contains a multitude of publications, a majority of which were inspired by the *Willems et al.*'s fundamental lemma in the seminal contribution [5]. This lemma provides an effective way to describe the trajectory of a linear time-invariant (LTI) system using a linear combination of some sufficiently rich past trajectories. Several control problems have been addressed using this lemma, including stabilization and optimization in [1], [6], linear quadratic regulation in [7], robust control in [8]–[10], quantized control in [11], model predictive control (MPC) in [12]–[15], and control of complex and multi-agent networks in [16]–[18], stochastic systems in [19].

However, most of these aforementioned works employ periodic transmission protocols, which may be resource-inefficient for real-world systems in terms of processor usage, communication bandwidth, and energy. To address these limitations, a resource-efficient scheduling approach for data transmissions, known as event-based control, has been thoroughly studied in the context of model-based control.

There are two event-based approaches that have been proven effective, that is event-triggered control and self-triggered control [20]. In event-triggered control, a data transmission event is triggered only after some triggering conditions, in terms of the state/output, are met. These conditions should be examined continuously or periodically at the sensor side, which means that the sensor should be activated frequently, leading to a waste of resources. However, this operating manner improves the system's robustness against uncertainties and unmodeled dynamics; see e.g., [21]–[24].

Self-triggered control is an approach to determining the next sampling and transmission time once a sampled measurement is received, eliminating the need for continuous or periodic sampling. This method saves energy and prolongs the service life of sensors, as sensors can be completely shut off between sampling times. Self-triggered control has been studied extensively in the context of model-based control, with several studies focusing on its resource-efficient properties; see, for example, [25]–[27]. Recently, self-triggered control scheme has been designed using input-state data in [28], in which only offline data are corrupted by process noise. In this paper, we aim to design a data-driven self-triggering mechanism for unknown linear state feedback systems, accounting for both offline and online noise.

To achieve this goal, we first consider measurement noise, and then provide modification suggestion accounting for both process and measurement noise. Inspired by the previous works on data-driven MPC, we formulate a data-driven online optimization problem for trajectory prediction, which is reminiscent of the data-driven MPC in [12], [15] but does not optimize and thus output the control inputs. Upon solving this problem, we show that the distance between the predicted state and the state from the most recent triggering time depends only on the optimal solution and an unknown parameter relating to the system open-loop divergence rate. By open-loop, we refer to the system with a zero control input. In addition, an algorithm is proposed which returns an upper bound on the open-loop divergence rate, using the data-driven set-based technique in [29]. Finally, we combine the optimal solution with the upper bound to develop a data-driven self-triggering mechanism, followed by stability guarantees.

In succinct form, the contribution of this work is threefold, summarized as follows.

c1) A data-driven online optimization problem is formulated

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to predict future states using only noisy input-state data;

c2) A data-driven set-based algorithm using matrix zonotopes for estimating the system open-loop divergence rate is proposed; and,

c3) Leveraging the optimal solution of the optimization problem and the upper bound on the open-loop divergence rate, a data-driven self-triggering mechanism is designed, for which input-to-state stability (ISS) is established.

Notation: Denote the set of real numbers, integers, and positive integers by $\mathbb{R}$, $\mathbb{N}$, and $\mathbb{N}_+$, respectively. For a full row-rank matrix M, its right pseudo-inverse is denoted by $M^\dagger$. Let $\mathbb{N}_{[t_1,t_2]} := [t_1, t_2] \cap \mathbb{N}$. Given a vector $x \in \mathbb{R}^{n_x}$, $\|x\|$, $\|x\|_1$, and $\|x\|_\infty$ denote its Euclidean, ℓ_1-, and ℓ_∞-norm, respectively, and for a positive definite matrix $P = P' \succ 0$, define the weighted norm $\|x\|_P = \sqrt{x'Px}$. If the argument is matrix-valued, then these mean the corresponding induced norms. The contraction norm for matrix G of vector x in [30, Lemma 2] is defined by $|x|_G$ [1].

Given a signal $x : \mathbb{N} \to \mathbb{R}^{n_x}$, for $N \in \mathbb{N}_+$, let $x_{[0,N-1]} := [x_0' \; x_1' \; \cdots \; x_{N-1}']'$ denote its vectorized form, and let $\|x_{[0,N-1]}\|_\infty := \max_{s\in[0,N-1]} \|x_t\|_\infty$. Denote the Hankel matrix of signal $x_{[0,N-1]}$ by

$$H_L(x) := \begin{bmatrix} x_0 & x_1 & \dots & x_{N-L} \\ \vdots & \vdots & \ddots & \vdots \\ x_{L-1} & x_L & \dots & x_{N-1} \end{bmatrix}$$

where x is used to represent $x_{[0,N-1]}$ for brevity. The definition of persistency of excitation as in [5] is given below.

Definition 1.1 (Persistency of excitation): Given $L \in \mathbb{N}_+$, a signal $x_{[0,N-1]} \in \mathbb{R}^{n_x}$ with $N \geq (n_x + 1)L + 1$ is persistently exciting of order L if $\text{rank}(H_L(x)) = n_x L$.

II. Preliminaries and Problem Formulation

A. State feedback control systems

Consider the following discrete-time state feedback control system

$$x_{t+1} = Ax_t + Bu_t, \qquad t \in \mathbb{N} \tag{1a}$$
$$\zeta_{t_\ell} = x_{t_\ell} + n_{t_\ell}, \qquad t_\ell \in \mathbb{N} \tag{1b}$$
$$u_t = K\zeta_{t_\ell}, \qquad t_\ell \leq t < t_{\ell+1} \tag{1c}$$

where $x_t \in \mathbb{R}^{n_x}$, $u_t \in \mathbb{R}^{n_u}$, $\zeta_{t_\ell} \in \mathbb{R}^{n_x}$ and $n_{t_\ell} \in \mathbb{R}^{n_x}$ are the state, control input, sensor measurement and measurement noise, respectively. The feedback gain matrix $K \in \mathbb{R}^{n_u \times n_x}$ is arbitrary given such that matrix $A + BK$ is Schur stable. Let $t_\ell \in \mathbb{N}$ denote the time at which the ℓ-th event is triggered, i.e., the state sampling and transmission occur. Assume without loss of generality that $t_0 = 0$, and the subsequent times t_ℓ for all $\ell \in \mathbb{N}_+$ are dictated by a self-triggering mechanism, which is to be designed. In this paper, we make the following assumptions.

[1]Given a matrix $G \in \mathbb{R}^{n_x \times n_x}$ and constants $c \geq 1$, $\phi > 0$ such that $\|G^t\|_\infty \leq c\phi^t$ holds for all $t \in \mathbb{N}$, then the function $|\cdot|_G : \mathbb{R}^{n_x} \mapsto \mathbb{R}$ defined by $x \mapsto |x|_G := \sup_{t\in\mathbb{N}} \|\phi^{-t} G^t x\|_\infty$ is the contraction norm of x, satisfying all the properties of a norm on $\mathbb{R}^{n_x}$. In addition, for every $x \in \mathbb{R}^{n_x}$ and $t \in \mathbb{N}$, it holds that $\|x\|_\infty \leq \|x\|_G \leq c\|x\|_\infty$ and $|G^t x|_G \leq \phi^t |x|_G$.

Assumption 2.1 (Controllability): The pair (A, B) in (1) is unknown but assumed stabilizable.

Assumption 2.2 (System trajectory): Given $N, L \in \mathbb{N}_+$, under an input sequence $u^p_{[0,N-1]}$ that is persistently exciting of order $L + n_x$, data $\zeta^p_{[0,N]}$, $x^p_{[0,N]}$, and $n^p_{[0,N]}$ are generated from the open-loop system (1a)-(1b) through offline experiments. We only have access to data $(u^p_{[0,N-1]}, \zeta^p_{[0,N]})$.

Assumption 2.3 (Bounded noise): There exists a known constant $\bar{n} \geq 0$ such that $n^p_t \in \mathbb{B}_{\bar{n}} := \{n | \|n\|_\infty \leq \bar{n}\}$ for all $t \in \mathbb{N}_{[0,N]}$, and $n_t \in \mathbb{B}_{\bar{n}}$ for all $t \in \mathbb{N}$.

Regarding the assumptions, we have the next observation.

Remark 2.1 (Matrix K): Assumptions 2.2 and 2.3 indicate that $\zeta^p_{[0,N]} = x^p_{[0,N]} + n^p_{[0,N]}$ with bounded noise $n^p_{[0,N]}$. Hence, several methods can be used to design a stabilizing matrix K from data $(u^p_{[0,N-1]}, \zeta^p_{[0,N]})$; see, e.g. [1], [7], [10].

Fig. 1 depicts the networked system structure. Specifically, we consider the plant is connected to a remote controller through an ideal network, i.e., the network phenomena such as delay and packet loss are not taken into account here but leave it for future research. At time t_ℓ, the ℓ-th event is triggered at the controller side, and the sensor samples the current state x_{t_ℓ} and transmits its noisy version ζ_{t_ℓ} to the controller over the network. The self-triggering module computes the next triggering time $t_{\ell+1}$ by evaluating a triggering function on a predicted trajectory using the received noisy state ζ_{t_ℓ} and then feeds it back to the plant. Meanwhile, the control input is updated using ζ_{t_ℓ} and sent to the actuator at the plant side, which implements a sample and hold controller as in (1c).

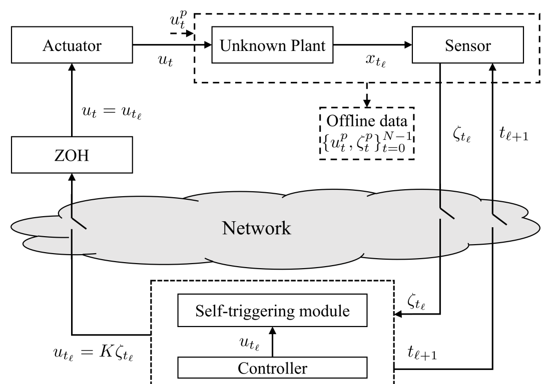

Fig. 1. Data-driven self-triggered state feedback control.

Due to the presence of measurement noise n_t, instead of pursing asymptotic stability, a weaker notion of stability, known as input-to-state stability (ISS), is discussed.

Definition 2.1 ([31, Definition 2.2]): System (1) is ISS if, for any $x_0 \in \mathbb{R}^{n_x}$ and measurable essentially bounded n_t for $t \in \mathbb{N}$, the solution satisfies

$$\|x_t\| \leq \alpha(\|x_t\|, t) + \pi_n(\|n_{[0,t-1]}\|_\infty), \quad \forall t \in \mathbb{N} \tag{2}$$

where α is a $\mathcal{KL}$-function and π_n is a $\mathcal{K}$-function[2].

[2]A function $\pi_n : [0, \infty) \to [0, \infty)$ is said to be of class $\mathcal{K}$ if it is continuous, strictly increasing, and $\pi_n(0) = 0$. A function $\alpha : [0, \infty) \times [0, \infty) \to [0, \infty)$ is a $\mathcal{KL}$-function if $\alpha(\cdot, t)$ is of class $\mathcal{K}$ for each fixed $k \geq 0$ and $\alpha(s, t)$ decreases to 0 as $t \to \infty$ for any fixed $s \in \mathbb{N}$.

With the preliminaries above, the problem to be addressed is formally stated as follows.

Problem 1: For the state feedback control system (1), under Assumptions 2.1-2.3, design a self-triggering mechanism to save transmissions while ensuring ISS of the system.

B. Fundamental lemma

In this section, we briefly review the data-driven system representation based on the *Willems et al.'s* fundamental lemma in [5], which is crucial to derive our data-driven self-triggering mechanism.

According to Definition 1.1, it has been shown in [5] that any finite trajectory of noise-free system (1a) can be expressed as a linear combination of some pre-collected input-state data $(u^p_{[0,N-1]}, x^p_{[0,N]})$, provided that the input sequence $u^p_{[0,N-1]}$ is persistently exciting of sufficient order. This result is formally summarized below.

Lemma 2.1 ([5, Theorem 1]): For input sequence $u^p_{[0,N-1]}$ as in Assumption 2.2, if noise-free data $(u^p_{[0,N-1]}, x^p_{[0,N]})$ are available, then $(\bar{u}_{[0,L-1]}, \bar{x}_{[0,L-1]})$ is a trajectory of the noise-free system (1a) if and only if there exists a vector $g \in \mathbb{R}^{N-L+1}$ such that the following holds

$$\begin{bmatrix} H_L(u^p) \\ H_L(x^p) \end{bmatrix} g = \begin{bmatrix} \bar{u}_{[0,L-1]} \\ \bar{x}_{[0,L-1]} \end{bmatrix}. \tag{3}$$

III. Data-Driven Self-Triggering Mechanism

This section advocates a data-driven approach to designing a self-triggering mechanism for the unknown linear feedback system (1). Typically, a self-triggering module determines the next transmission by sequentially comparing the current state with predicted future states at each triggering time, and if they differ considerably, the associated time index of the future state is the next triggering time. The challenge of addressing Problem 1 lies in how to predict future states and quantify its distance with the current state without knowing the system matrices but only using data. In the following, inspired by Lemma (Lem.) 2.1 and robust data-driven MPC schemes in e.g. [12], [15], a data-driven online optimization problem is developed to predict future states using noisy input-state data, whose solution facilitates the design of a data-based self-triggering mechanism. Moreover, describing the distance between the future states and the current one requires also the open-loop divergence rate of the system, i.e., $\|A^i\|_\infty$. To tackle this issue, the data-driven set-based technique in [29] is recalled and an algorithm computing an upper bound of the system open-loop divergence rate is put forth. Finally, standard conditions are derived, under which the system with the data-driven self-triggering mechanism achieves ISS.

A. Self-triggering mechanism

For a given prediction horizon $L \in \mathbb{N}_+$, we consider, at each triggering time t_ℓ, the following optimization problem to predict the states for its subsequent L times

$$J^*_L(\zeta_{t_\ell}) := \min_{\substack{g(t_\ell), h(t_\ell) \\ \bar{x}_i(t_\ell)}} \sum_{i=0}^{L-1} \|\bar{x}_i(t_\ell)\|_Q + (\lambda_h/\bar{n})\|h(t_\ell)\|^2 + \lambda_g \bar{n}\|g(t_\ell)\|^2 \tag{4a}$$

$$\text{s.t.} \quad \begin{bmatrix} u(t_\ell) \\ \bar{x}(t_\ell) + h(t_\ell) \end{bmatrix} = \begin{bmatrix} H_L(u^p) \\ H_L(\zeta^p) \end{bmatrix} g(t_\ell) \tag{4b}$$

$$\bar{x}_0(t_\ell) = \zeta_{t_\ell} \tag{4c}$$

where $u(t_\ell) := [u'_{t_\ell} \cdots u'_{t_\ell}]' \in \mathbb{R}^{n_u L}$ copies u_{t_ℓ} a number of L times, and $(\bar{x}(t_\ell), h(t_\ell), g(t_\ell))$ are the optimization variables, i.e., $\bar{x}(t_\ell) = [\bar{x}'_0(t_\ell) \cdots \bar{x}'_{L-1}(t_\ell)]' \in \mathbb{R}^{n_x L}$ collects the predicted L–1 states from t_ℓ, $g(t_\ell) \in \mathbb{R}^{N-L+1}$ as in Lemma 2.1, and $h(t_\ell) := [h'_0(t_\ell) \cdots h'_{L-1}(t_\ell)]' \in \mathbb{R}^{n_x L}$ is a slack vector compensating the influence of measurement noise in both offline data n^p_t and online data n_t. Penalties imposed on $g(t_\ell)$ and $h(t_\ell)$ in the objective function $J^*_L(\zeta_{t_\ell})$ reflect the constraint on the noise in Assumption 2.3. Parameters $Q \succ 0$, $\lambda_h > 0$ and $\lambda_g > 0$ are appropriate weighting parameters, which can be chosen similar as in the data-driven MPC literature e.g. [12], [15]. Based on (4c), if $L = 1$, then $\bar{x}(t_\ell) = \zeta_{t_\ell}$ and no future state is returned rendering solving Problem (4) trivial. To avoid this situation, a lower bound on L is required.

Assumption 3.1 (Prediction horizon): The prediction horizon L of the data-driven problem (4) satisfies $L \geq 2$.

It is worth noticing that the optimization problem in (4) is similar to the robust data-driven MPC formulation in [12], [15], except that the subsequent control inputs are not optimized but replaced with that at the current triggering time t_ℓ. Hence, we refer to Problem (4) also as the trajectory prediction problem in the rest of this paper.

The inter-triggering time between two consecutive self-triggered times is defined as $\tau_\ell := t_{\ell+1} - t_\ell$ for each $\ell \in \mathbb{N}$. Observe from (4b) that at most $L-1$ future states can be predicted at time t_ℓ. Therefore, the inter-triggering time between any two consecutive self-triggering times obeys $1 \leq \tau_\ell \leq L-1$. With these definitions, the following lemma indicates that the error between the predicted state and the actual state is bounded and can be rigorously quantified using the optimizer of (4).

Lemma 3.1: Under Assumptions 2.1–3.1, for any $\ell \in \mathbb{N}$, Problem (4) is feasible at time t_ℓ, whose optimal solution is denoted by $(\bar{x}^*(t_\ell), g^*(t_\ell), h^*(t_\ell))$. Moreover, for any $\ell \in \mathbb{N}$, the prediction error between $x_{t_{\ell+1}}$ and $\bar{x}^*_{\tau_\ell}(t_\ell)$ satisfies

$$\begin{aligned} \|e(t_{\ell+1})\|_\infty &:= \|x_{t_{\ell+1}} - \bar{x}^*_{\tau_\ell}(t_\ell)\|_\infty \\ &\leq \rho^{\tau_\ell}\big(\bar{n} + \bar{n}\|g^*(t_\ell)\|_1 + \|h^*_0(t_\ell)\|_\infty\big) \\ &\quad + \|h^*_{\tau_\ell}(t_\ell)\|_\infty + \bar{n}\|g^*(t_\ell)\|_1 \end{aligned} \tag{5}$$

where $\rho^{\tau_\ell} := \|A^{\tau_\ell}\|_\infty$ is the system divergence rate when $u_t = 0$, and we refer to it as the open-loop divergence rate. The proof follows from [12, Lem. 2] and is omitted here.

B. Over-approximating the open-loop divergence rate

According to Lem. 3.1, an upper bound on the error between the predicted state and the actual state is characterized in terms of known or computable parameters $\bar{x}^*(t_\ell)$, $g^*(t_\ell)$, $h^*(t_\ell)$, $\bar{n}$, as well as the unknown parameter ρ^{τ_ℓ} that depends on the open-loop divergence rate. To design the self-triggering mechanism, upper bounds on the parameters ρ^i, $i = 1, \cdots, L-1$ should be prepared offline. When noise-free data $(u^p_{[0,N-1]}, x^p_{[0,N-1]})$ are available, upper bounds on ρ^i can be easily obtained from solving the optimization problem in [14, Sec. V.B (27)]. However, the requirement on clean data can be stringent in real-world settings. Inspired by the set-based method proposed in [29], we advocate an algorithm for approximating ρ^i from noisy input-state data. It is worth stressing that the proposed algorithm runs offline and thus will not affect the real-time performance of the self-triggering mechanism.

Rearranging the offline data $\zeta^p_{[0,N]}$, $u^p_{[0,N-1]}$, and $n^p_{[0,N]}$ into matrices, yields

$$Z^p_- := [\zeta^p_0\ \zeta^p_1\ \cdots\ \zeta^p_{N-1}],\ Z^p_+ := [\zeta^p_1\ \zeta^p_2\ \cdots\ \zeta^p_N], \tag{6a}$$
$$U^p_- := [u^p_0\ u^p_1\ \cdots\ u^p_{N-1}], \tag{6b}$$
$$N^p_- := [n^p_0\ n^p_1\ \cdots\ n^p_{N-1}],\ N^p_+ := [n^p_1\ n^p_2\ \cdots\ n^p_N] \tag{6c}$$

which satisfies

$$Z^p_+ - N^p_+ = [A \quad B]\begin{bmatrix} Z^p_- - N^p_- \\ U^p_- \end{bmatrix}. \tag{7}$$

Based on Assumption 2.3, there exists a zonotope $\mathcal{Z}_n = \langle c_n, G_n\rangle \subset \mathbb{R}^{n_x}$ such that $n^p_t, n_t \in \mathcal{Z}_n$, where $\langle c_n, G_n\rangle$ is used as a shorthand notation of the following zonotope

$$\mathcal{Z}_n = \left\{ n \in \mathbb{R}^{n_x} \middle| n = c_n + \sum_{i=1}^{\xi} \beta^{(i)} g_n^{(i)}, -1 \le \beta^{(i)} \le 1 \right\}$$

and $G_n = \left[g_n^{(1)}\ g_n^{(2)}\ \cdots\ g_n^{(\xi)}\right] \in \mathbb{R}^{n_x \times \xi}$. In fact, since the noise is upper bounded by $\bar{n}$, i.e., $\|n^p_t\|_\infty \le \bar{n}$, and $\|n_t\|_\infty \le \bar{n}$, zonotope $\mathcal{Z}_n$ can be constructed by letting $c_n = 0$, $\xi = n_x$, and the vector $g_n^{(i)} = [0 \cdots \bar{n} \cdots 0]'$ whose i-th element is $\bar{n}$ and remaining ones are zeros. Building on $\mathcal{Z}_n$, a matrix zonotope $\mathcal{M}_n = \langle C_{\mathcal{M},n}, G^{(1:\xi N)}_{\mathcal{M},n}\rangle$ can be constructed such that $N^p_-,\ N^p_+ \in \mathcal{M}_n$; see [29, Sec. III.A] for details. As has been shown in [29, Lem. 1], all the matrix pairs $[A, B]$ that are consistent with the data (U^p_-, Z^p_-, Z^p_+) and under Assumption 2.3 are contained in the following zonotope

$$\mathcal{M}_{(A,B)} = (Z^p_+ - \mathcal{M}_n)\begin{bmatrix} Z^p_- - \mathcal{M}_n \\ U^p_- \end{bmatrix}^{\dagger}. \tag{8}$$

For $i \in [0, L-1]$, let the set $\tilde{R}_i = \langle \tilde{c}_i, \tilde{G}_i\rangle$ be updated by $\tilde{R}_{i+1} = \mathcal{M}_{(A,B)}(\tilde{R}_i \times \mathcal{U})$ with $\mathcal{U} = \langle 0, 0\rangle$. The initial condition $\tilde{R}_0$ is given by $\tilde{c}_0 = 0$ and $\tilde{G}_0 = [\tilde{g}_0^{(1)}\ \tilde{g}_0^{(1)}\ \cdots\ \tilde{g}_0^{(\xi)}]$ with $\xi = n_x$, where the vector $\tilde{g}_0^{(i)} = [0\ \cdots\ 1\ \cdots\ 0]'$ whose i-th element is 1 and remaining ones are zeros. Let $B_i := \max_{r \in \tilde{R}_i} \|r\|_\infty$. It follows from [29, Thmorem 1] that $B_i \ge \|A^i\|_\infty = \rho^i$ which indicates that an upper bound on the prediction error $\|e(t_{\ell+1})\|_\infty$ can be expressed using only data. For reference, the over-approximation method is summarized in Algorithm (Alg.) 1 on the right column.

Leveraging the upper bound (5) in Lem. 3.1 and the optimal solution $(\bar{x}^*(t_\ell), g^*(t_\ell), h^*(t_\ell))$ of Problem (4), the next self-triggering time is determined by

$$t_{\ell+1} := t_\ell + \min\{L-1, \tau_\ell\}, \quad t_0 := 0 \tag{9}$$

with

$$\begin{aligned} \tau_\ell := \sup\big\{\tau_\ell \in\ \mathbb{N}_+ \,|\, &\|\bar{x}^*_\tau(t_\ell) - \zeta_{t_\ell}\|_\infty + \|h^*_{\tau_\ell}(t_\ell)\|_\infty \\ &+ \rho^{\tau_\ell}\big(\bar{n} + \bar{n}\|g^*(t_\ell)\|_1 + \|h^*_0(t_\ell)\|_\infty\big) \\ &+ \bar{n}\|g^*(t_\ell)\|_1 \le \sigma\|\zeta_\ell\|_\infty\big\} \end{aligned} \tag{10}$$

where the constant $\sigma \in (0, 1)$ balances between the system performance and the inter-triggering time.

Algorithm 1 Over-approximation of ρ^i.

1: **Input:** Data $(u^p_{[0,N-1]}, \zeta^p_{[0,N-1]})$ from (1) with arbitrary x^p_0 and persistently exciting $u^p_{[0,N-1]}$ of order $L + n_x$.
2: **Construct** matrices Z^p_+, Z^p_-, U^p_-, and zonotopes $\mathcal{M}_n$ as in [29, Sec. III.A], $\mathcal{U} = \langle 0, 0\rangle$, $\tilde{R}_0 = \langle \tilde{c}_0, \tilde{G}_0\rangle$ with $\tilde{c}_0 = 0$, matrix $\tilde{G}_0 = [\tilde{g}_0^{(1)}\ \cdots\ \tilde{g}_0^{(\xi)}]$ with $\xi = n_x$ and $\tilde{g}_0^{(i)} = [0\ \cdots\ 1\ \cdots 0]'$.
3: **Compute** matrix zonotope $\mathcal{M}_{(A,B)}$ via (8), which contains all matrices $[A, B]$ consistent with the data Z^p_+, Z^p_- and U^p_-.
4: **For** $i = 0, \cdots, L-1$ do
5: Calculate the zonotope $\tilde{R}_{i+1} = \mathcal{M}_{(A,B)}(\tilde{R}_i \times \mathcal{U})$. Record $B_i := \max_{r_i \in \tilde{R}_i} \|r_i\|_\infty$. Set $\rho^i = B_i$.
6: **End for**

C. Stability analysis

Based on (9)–(10) with $(\bar{x}^*(t_\ell), g^*(t_\ell), h^*(t_\ell))$ from Problem (4) and ρ^{τ_ℓ} from Algorithm 1, the proposed data-driven self-triggering mechanism is presented in Algorithm 2, whose stability guarantees are provided below.

Algorithm 2 Data-driven self-triggering mechanism.

1: **Offline:** leftmargin=8mm,topsep = 0.5 pt
1) Input prediction horizon $L \ge 2$; coefficients $Q \succ 0$, $\lambda_g > 0$, $\lambda_h > 0$, $\sigma \in (0, 1)$; noise bound $\bar{n}$; data $(u^p_{[0,N-1]}, \zeta^p_{[0,N-1]})$ obeying Assumption 2.2.
1) Construct Hankel matrices $H_{L+1}(u^p)$ and $H_{L+1}(\zeta^p)$ for the input and noisy state data.
2) Design matrix K from data $(u^p_{[0,N-1]}, \zeta^p_{[0,N-1]})$ using the method in e.g., [1, Sec. V.A].
3) Compute ρ^i for $i = 1, 2, \ldots, L-1$ using Alg. 1.
2: **Online:** For $t = 0, 1, 2, \cdots$ do
If $t = t_\ell$, do
leftmargin=15mm,topsep = 0.5 pt
1) Solve Problem (4) with the current u_t and ζ_t.
2) Compute the next triggering time $t_{\ell+1}$ by (9).
3) Set $u_t = K\zeta_{t_\ell}$ and $t_\ell = t_{\ell+1}$.
Else if $t \ne t_\ell$, set $u_t = u_{t_\ell}$.

Theorem 3.1: Consider the state feedback control system (1) adopting the self-triggering mechanism in Alg. 2. Let Assumptions 2.1–3.1 hold. If i) the self-triggering times are generated by (9)–(10) with the parameters $g^*(t_\ell)$, $h^*(t_\ell)$, $\bar{x}^*(t_\ell)$ obtained from solving Problem (4) and ρ^i from Alg. 1, and ii) any $\sigma \in (0, \bar{\sigma})$ with $0 < \bar{\sigma} < (1-\phi_1)/(c_1^2\|BK\|)$, where the constants $c_1 \ge 1$ and $\phi_1 \in (1, 0)$ are such that $\|(A+BK)^i\|_\infty \le c_1\phi_1^i$ holds for all $i \in \mathbb{N}$, then system (1) achieves ISS under the proposed data-driven self-triggering mechanism in Algorithm 2.

Proof: For $k \in \mathbb{N}_{[0,\tau_\ell]}$, it follows from Lem. 3.1 that

$$\begin{aligned} \|\zeta_{t_\ell} - x_{t_\ell+k}\|_\infty &\le \|\zeta_{t_\ell} - \bar{x}^*_k(t_\ell)\|_\infty + \|x_{t_\ell+k} - \bar{x}^*_k(t_\ell)\|_\infty \\ &\overset{(5)}{\le} \|\zeta_{t_\ell} - \bar{x}^*_k(t_\ell)\|_\infty + \bar{n}\|g^*(t_\ell)\|_1 + \|h^*_k(t_\ell)\|_\infty \\ &\quad + \rho^k\big[\bar{n}(1 + \|g^*(t_\ell)\|_1) + \|h^*_0(t_\ell)\|_\infty\big] \qquad (11) \\ &\overset{(10)}{\le} \sigma\|\zeta_{t_\ell}\|_\infty. \qquad (12) \end{aligned}$$

Notice that matrix K is such that $\mathcal{A} := A + BK$ is Schur stable, meaning that there exist constants $c_1 \ge 1$

and $\phi_1 \in (0,1)$ such that $\|\mathcal{A}^i\|_\infty \leq c_1\phi_1^i$ holds for all $i \in \mathbb{N}$. Recalling the contraction norm that $|\mathcal{A}^i x|_\mathcal{A} \leq \phi_1^i |x|_\mathcal{A}$. Hence, it follows recursively from (1) that for $k \in \mathbb{N}_{[0,\tau_\ell]}$, the state obeys

$$|x_{t_\ell+k}|_\mathcal{A} \leq \left[\phi_1^k((1-\gamma\sigma)+\gamma\sigma)\right]|x_{t_\ell}|_\mathcal{A} + \sum_{i=0}^{k} c_1^2\phi_1^i\|BK\|\sigma\|n_{[0,t_\ell+k-1]}\|_\infty \tag{13}$$

where $\gamma := c_1^2\|BK\|/(1-\phi_1)$. If $\sigma < 1/\gamma$, then it follows from [30, Thmorem 3.4] that $\phi_1^k((1-\gamma\sigma)+\gamma\sigma) < 1$ and $(\phi_1^k(1-\gamma\sigma)+\gamma\sigma)^{1/k}$ is strictly increasing on $k \in [1,\infty)$. Letting $\phi_2 := (\phi_1^{L-1}(1-\gamma\sigma)+\gamma\sigma)^{1/(L-1)}$, the inequality (13) becomes $|x_{t_\ell+k}|_\mathcal{A} \leq \phi_2^k|x_{t_\ell}|_\mathcal{A} + \gamma\sigma\|n_{[0,t_\ell+k-1]}\|_\infty$. Recursively, one gets that $|x_{t_\ell+k}|_\mathcal{A} \leq \phi_2^{t_\ell+k}|x_0|_\mathcal{A} + (1+\sum_{i=0}^{\ell-1}\phi_2^{\tau_i})\gamma\sigma\|n_{[0,t_\ell+k-1]}\|_\infty$.

Furthermore, it follows from the fact $\|x\|/\sqrt{n_x} \leq \|x\|_\infty \leq |x|_\mathcal{A} \leq c_1\|x\|_\infty \leq c_1\|x\|$ that $\|x_{t_\ell+k}\| \leq c_1\sqrt{n_x}\phi_2^{t_\ell+k}\|x_0\| + (1+\sum_{i=0}^{\ell-1}\phi_2^{\tau_i})\gamma\sigma\sqrt{n_x}\|n_{[0,t_\ell+k-1]}\|_\infty$ which completes the proof according to Definition 2.1. ■

Remark 3.1 (Extension): Consider systems with bounded process noise, i.e., rather than (1a), the state obeys $x_{t+1} = Ax_t + Bu_t + w_t$ with $\|w_t\|_\infty \leq \bar{w}$ for all $t \in \mathbb{N}$. In this case, additional terms consisting of $\sum_{i=0}^{j} A^i w_{j-i}^p$ for all $j \in \mathbb{N}_{[0,N-1]}$ will be involved in the prediction error (5). Since $\|\sum_{i=0}^{j} A^i w_{j-i}^p\|_\infty \leq \sum_{i=0}^{j}\|A^i\|_\infty\bar{w}$ and $\|A^i\|_\infty$ can be bounded following a similar procedure as in Alg. 1, for any $j \in \mathbb{N}_{[0,N-1]}$, an upper bound on $\|\sum_{i=0}^{j} A^i w_{j-i}^p\|_\infty$ can be obtained and a more complex self-triggering mechanism can be designed. This method can be also used to extend the result in [32] by considering noisy offline data.

Remark 3.2 (Trade-off): It can be observed from (10) and (13) that as σ decreases, both the transmission frequency and the state convergence rate increase, which is similar to the effect of the performance parameter in multi-step MPC, see, e.g., [33]. Upper bounds of σ defined in Theorem 3.1 can be approximated using e.g., the set-based method in [29], which is left for future investigation. In addition, for a fixed L, it has been shown in [12] that larger N is beneficial for decreasing the impact of measurement noise on the prediction performance of Problem (4), which may consequently affect the system performance. Such an improvement becomes significant when N is small, and becomes marginal when N is large [34]. On the other hand, increasing the value of N leads to an increasing online complexity of (4), since $g(t) \in \mathbb{R}^{N-L+1}$. In conclusion, the proposed self-triggering mechanism allows a trade-off among transmission frequency, state convergence, and computational complexity by appropriately selecting N, L and σ.

IV. Numerical Examples

To examine the correctness and numerical effectiveness of the proposed data-driven self-triggering mechanism and associated state feedback controller, two examples are provided in this section.

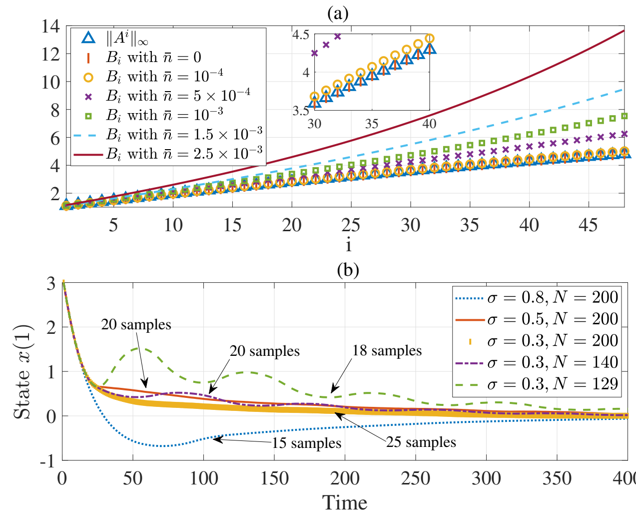

Fig. 2. (a): Estimated ρ^i by Alg. 1 under different levels of noise; (b): State trajectories under different N and σ.

1) Example 1: Consider the discrete-time version of the system in [28] with a sampling period of $0.1s$ as follows

$$\dot{x}(t) = \begin{bmatrix}0 & 1\\ 0 & -0.1\end{bmatrix}x(t) + \begin{bmatrix}0\\ 0.1\end{bmatrix}u(t), \quad t \geq 0. \tag{14}$$

Letting $N = 200$, input-state trajectories under different noise bounds were first collected by applying a sequence of inputs u uniformly and independently sampled in $[-1,1]$, and the divergence parameters ρ^i were computed using Alg. 1. For $i = 1,\cdots,L-1$ with $L = 41$, it has been shown in Fig. 2 (a) that $\rho^i \leq \|A^i\|_\infty$ holds, and moreover, ρ^i approaches $\|A^i\|_\infty$ as then noise vanishes, which verifies the effectiveness of Alg. 1 in upper-bounding $\|A^i\|_\infty$.

Letting $\bar{n} = 0.0015$, $Q = 3I_2$, $\lambda_g\bar{n} = 10^{-6}$, $\lambda_h/\bar{n} = 500$, $\sigma = 0.5$, and the feedback controller gain $K = [-0.2908\ -4.0340]$, which are same as in [28]. Over the interval $t \in [0,400]$ and with the initial state $x_0 = [3\ \ -2]'$, we first compare the system performances under different parameters N and σ. Fig. 2 (b) verifies the discussion in Remark 3.2.

In addition, using the same setting as in [28], i.e., only offline data subject to process noise with $\bar{n} = 0.001$, 3 (a1)-(a2) compares the proposed Alg. 2 (solid line) and the method in [28] (dashed line). Although both methods require the same number of transmissions (i.e., 15 out of 400 samples), the state converges faster under the proposed Alg. 1.

2) Example 2: Consider the discretized inverted pendulum control problem as in [35]

$$\dot{x}(t) = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & \frac{m_1 g}{-m_2} & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & \frac{g}{\ell} & 0\end{bmatrix}x(t) + \begin{bmatrix}0\\ \frac{1}{m_2}\\ 0\\ \frac{-1}{m_2\ell}\end{bmatrix}u(t) \tag{15}$$

where $m_1 = 1$, $m_2 = 10$, $\ell = 3$, and $g = 10$. A collection of $N = 100$ input-state data was obtained following the same procedure described in Example 1. Consider $\bar{n} = 15\times10^{-4}$, $Q = 3I_4$, $\lambda_g\bar{n} = 10^{-6}$, $\lambda_h/\bar{n} = 500$, $\sigma = 0.33$, $L = 11$, matrix $K = [2\ 12\ 378\ 210]$, and the initial state

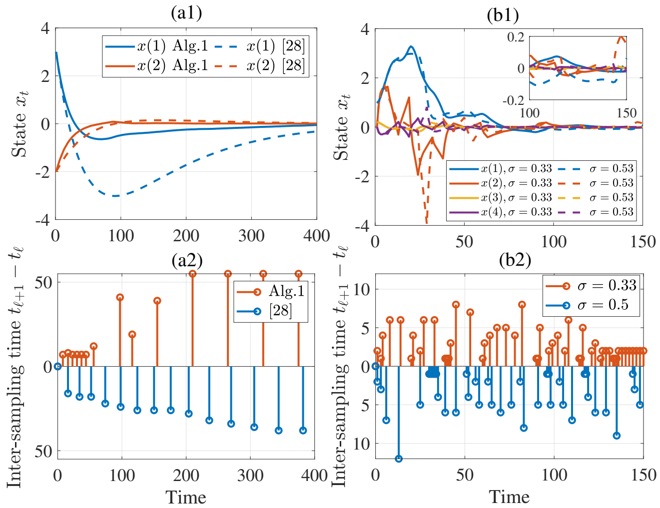

Fig. 3. (a1)-(a2): State trajectory in Example IV-.1; (b1)-(b2): State trajectory in Example IV-.2.

$x_0 = [0.98\ 0\ 0.2\ 0]'$. It can be seen from Fig. 3 (b1) and (b2) that the system converges to zero with 49 ($\sigma = 0.33$) and 39 ($\sigma = 0.5$) samples over a simulation horizon of 150 time steps.

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