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Classical Bouc-Wen hysteresis modeling and force control of a piezoelectric robotic hand manipulating a deformable object

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***Abstract*— This letter focuses on the modeling and control of a piezoelectric actuator that is designed to manipulate objects. This research considers both the non-linearity caused by the hysteresis of the actuator and the deformation of the object being manipulated. To approximate the hysteresis, a classical Bouc-Wen model is used. To stabilize the force-tracking error, we propose a novel control approach combining three advanced methodologies: an output-feedback method based on a nonlinear observer, a Barrier-Lyapunov function design, and bounded control based on saturation functions. Combining these three powerful techniques produces a bounded and highly robust controller that can effectively reject aggressive disturbances while maintaining the tracking error inside a predefined set. Under such a scenario, it is demonstrated that the equilibrium point of the closed-loop system is asymptotically stable. The effectiveness of the proposed control method is validated through extensive numerical simulations.**

I. Introduction

Thanks to their high resolution, high bandwidth, and force density, lead-zirconate-titanate (PZT) piezoelectric materials are widely used for actuators in high-precision and rapidity applications. They are used in applications such as diesel injection, miniaturized drones, atomic force microscopy, and medical micro-robotics [1]. Recently, we have used three piezoelectric actuators to construct a robotic hand devoted to manipulating sensitive objects [2], as displayed in Fig.1-a.

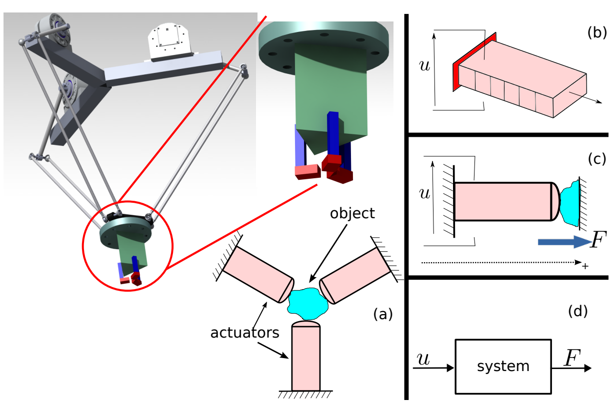

Fig. 1: (a): robotic hand and its three piezoelectric actuators. (b) and (c): one piezoelectric actuator. (d): the system.

Although the above features, PZT piezoelectric actuators exhibit strong hysteresis that can substantially degrade the overall performances of the tasks. Such hysteresis can even lead to instability if not adequately accounted for. Therefore, modeling and controlling hysteresis in these actuators has raised numerous works in the literature, which can be categorized as feed-forward control and feedback control. Feed-forward control is only used when it is impossible to use sensors for feedback. Its main limitation is the lack of robustness against model uncertainties and external disturbances. Regarding feedback control, they can be sub-categorized as model-based and non-model-based techniques. Model-based techniques explicitly use a hysteresis model to synthesize the controller. Its advantage is that we have more knowledge of the hysteresis non-linearity, allowing us to demonstrate the stability of the closed-loop formally and to potentially ensure prescribed performances. In [2], the classical Bouc-Wen hysteresis model was used to design a robust output feedback controller for piezoelectric actuators in robotic hands. A generalized Bouc-Wen model was afterward employed for synthesizing a model predictive control in [3] and then a finite-time stabilization controller in [4]. However, all these controllers were devoted to the position control of the actuator, and no manipulation force was considered.

This letter proposes to model and control the force while manipulating objects with a piezoelectric actuator in the robotic hand. It complements the above works, which focused on position control. In fact, for better object manipulation, one (resp. two) of the three actuators can be controlled in position while two (resp. one) are controlled in force. In the past, force control of piezoelectric actuators has raised a few works [5], [6]. In these, the force controller was based on linear models and did not fully consider the hysteresis non-linearity. The features of this letter are, therefore: i) the force control during the manipulation of an object by a piezoelectric actuator, ii) the explicit consideration of the hysteresis non-linearity in the controller design by using the classical Bouc-Wen model, and iii) the consideration of the deformation of the object during manipulation. Furthermore, the designed controller has the important property of rejecting aggressive disturbances while maintaining the tracking error inside a predefined set, as long as the initial conditions lie on that set. The robustness is achieved thanks to the combination of saturated control terms consisting of a proportional-error term, a barrier term, and a feed-forward term obtained through an exponential observer. They are combined to accomplish the results above.

The remainder of this letter is as follows. In Section II, the modeling and the problem statement are presented. In Section III, we expose the design of 1) a nonlinear observer,

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2) a Barrier-Lyapunov-function and saturation-based control, and 3) an output feedback control. Section IV concludes this letter by including supplementary material with a set of simulations.

II. Problem Setting

Let Fig.1-b be one of the three piezoelectric actuators of the robotic hand. It furnishes a displacement ϱ when subjected to a driving voltage. However, during the manipulation task, the actuator is in contact with the object, and an interaction force F is involved, Fig.1-c. Our target is, therefore: first to derive a model that links the voltage with the manipulation force (Fig.1-d), and then to design the force controller.

A. Governing equations

The model that links the driving voltage $u(t)$ and the output displacement $\varrho(t)$ of the piezoelectric actuator is:

$$\begin{cases} \alpha_2 \frac{d\varrho}{dt} + \alpha_1 \varrho = d_p u - h - pF \\ \frac{dh}{dt} = A_{bw}\frac{du}{dt} - B_{bw}\left|\frac{du}{dt}\right| h - \Gamma_{bw}\frac{du}{dt}\left|h\right| \end{cases} \tag{1}$$

where $F(t)$ is the force that is applied by the actuator to any external object, $h(t)$ is the hysteresis state variable, and α_i ($i \in \{1,2\}$) are dynamics parameters. On the other hand, d_p is a piezoelectric coefficient, p is the compliance coefficient, while A_{bw}, B_{bw} and Γ_{bw} are the parameters that define the hysteresis shape and amplitude.

Regarding the manipulated object, we assume that it is deformable under a first-order system's behavior,

$$c_{ob}\frac{d\varrho}{dt} + k_{ob}\varrho = F \tag{2}$$

where F is the same force as in (1) when the object is in contact with the actuator. Parameters c_{ob} and k_{ob} are the damping and stiffness coefficients of the object deformation.

B. Model for control design and problem statement

Combining the actuator's model in (1) and the object deformation model in (2), we yield a nonlinear force model:

$$\begin{cases} e_2\frac{dF}{dt} + e_1 F = f_2\frac{du}{dt} + f_1 u - g_2\frac{dh}{dt} - g_1 h + e_2\delta(t) \\ \frac{dh}{dt} = A_{bw}\frac{du}{dt} - B_{bw}\left|\frac{du}{dt}\right| h - \Gamma_{bw}\frac{du}{dt}\left|h\right| \end{cases} \tag{3}$$

where $e_2 = \alpha_2 + pc_{ob}$, $e_1 = \alpha_1 + pk_{ob}$, $f_2 = d_p c_{ob}$, $f_1 = d_p k_{ob}$, $g_2 = c_{ob}$ and $g_1 = k_{ob}$. We always have: $e_i > 0$, $f_i > 0$ and $g_i > 0$. Notice that we added an extra signal $\delta(t)$ in the above model, which lumps any disturbances during the object manipulation, for instance, those produced when the other actuators move the object. For simplicity, we rewrite system (3) as follows,

$$\Sigma : \begin{cases} \dot{f} = -a_1 f + a_2\dot{u} + a_3 u - a_4\dot{h} - a_5 h + \delta(t) & \text{(4a)} \\ \dot{h} = A_{bw}\dot{u} - B_{bw}|\dot{u}|h - \Gamma_{bw}\dot{u}|h| & \text{(4b)} \\ \dot{\delta} = f_\delta(t) & \text{(4c)} \\ y = f & \text{(4d)} \end{cases}$$

where $f = F$, $\dot{f} = \frac{dF}{dt}$, $\dot{u} = \frac{du}{dt}$, $\dot{h} = \frac{dh}{dt}$, and parameters are: $a_1 = \frac{e_1}{e_2}$, $a_2 = \frac{f_2}{e_2}$, $a_3 = \frac{f_1}{e_2}$, $a_4 = \frac{g_2}{e_2}$, and $a_5 = \frac{g_1}{e_2}$.

Assumption 1. *Functions $\delta(t)$ in (4a) and $f_\delta(t)$ in (4c) are unknown. Besides, $\delta(t)$ is differentiable with bounded derivative.*

Problem 1. *Consider system Σ in (4). We need to solve the tracking problem for the force f, i.e. $f \to f^d$ considering that:*

1. *The desired trajectory $f^d(t) : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is of class C^1.*
2. *The hysteresis $h(t)$ and unknown disturbance $\delta(t)$ are given by the solutions of (4b) and (4c), respectively.*
3. *The only available state is the force f; see (4d).*
4. *All the system parameters are known.*
5. *The control input u must be bounded and smooth.*
6. *Force $f(t)$ must respond smoothly. Moreover, the tracking error $e = f - f^d$ must be enclosed in a predefined set given by $|e(t)| < l$ with small $l \in \mathbb{R}_{>0}$ for all $t \geq t_0$.*

III. Nonlinear Output Feedback Strategy

The control strategy consists of three steps: 1) designing a high-gain observer responsible for estimating the hysteresis and disturbances to use as feedback elements, 2) designing a robust barrier-Lyapunov-function and saturation-based controller to ensure that the control is bounded and effectively rejects disturbances and maintains the tracking error enclosed in the predefined set exposed in point 6 of the Problem 1, and 3) combining the above two points in an output feedback scheme where the disturbance is directly rejected.

A. Observer design

To simplify the observer design and study the system's observability, let us represent system Σ as,

$$\begin{aligned} \dot{x} &= \Lambda x + g(x, \dot{u}, u) + B\dot{u} + \Delta(t) \\ y &= Cx \end{aligned} \tag{5}$$

where,

$$\Lambda = \begin{pmatrix} 0 & l_o & 0 \\ 0 & 0 & 1 \\ k_o & 0 & 0 \end{pmatrix}, \quad \Delta(t) = \begin{pmatrix} 0 \\ 0 \\ f_\delta(t) \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix},$$
$$B = \begin{pmatrix} a_2 \\ A_{bw} \\ 0 \end{pmatrix}, \quad g(x,\dot{u},u) = \begin{pmatrix} -a_1 f + a_3 u - a_4\frac{d}{dt}h - a_5 h + \delta - l_o h \\ -B_{bw}|\dot{u}|h - \Gamma_{bw}\dot{u}|h| - \delta \\ -k_o f \end{pmatrix} \tag{6}$$

with $k_o, l_o \in \mathbb{R}_{>0}$ to be defined in the observer design. Notice that now the extended state vector is $x = \begin{pmatrix} f & h & \delta \end{pmatrix}^\intercal$.

Assumption 2 ([7]–[9]). *The disturbance vector $\Delta(t)$ in (6) is bounded as follows,*

$$\|\Delta(t)\| = |f_\delta(t)| \leq \rho\|\varepsilon\|, \text{ with } \rho \in \mathbb{R}_{>0} \tag{7}$$

and ε is the error observer defined in (20).

Lemma 1. *The system (5) [and hence Σ] is observable.*

Proof. The observability matrix

$$\mathcal{O} = \begin{pmatrix} C \\ C\Lambda \\ C\Lambda^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & l_o & 0 \\ 0 & 0 & k_o \end{pmatrix} \tag{8}$$

has a full rank for l_o and k_o different of zero. □

Lemma 2. *The function $f(h,\dot{u}) = -B_{bw}|\dot{u}|h - \Gamma_{bw}\dot{u}|h|$ in* (6) *is global Lipschitz with respect to h uniformly in $\dot{u}$ for all reals B_{bw}, Γ_{bw} except both zero.*

Proof. The function is $f(h,\dot{u}) = -B_{bw}|\dot{u}|h - \Gamma_{bw}\dot{u}|h|$ and we want to prove that $|f(h_1,\dot{u}) - f(h_2,\dot{u}) \le L|h_1 - h_2|$ for a Lipschitz constant $K > 0$. Then, computing the left-hand side of the previous inequality, it follows that,

$$\begin{aligned} \Big| - B_{bw}|\dot{u}|h_1 - \Gamma_{bw}\dot{u}|h_1| + B_{bw}|\dot{u}|h_2 + \Gamma_{bw}\dot{u}|h_2|\Big| = \\ \Big| - B_{bw}|\dot{u}|\left(h_1 - h_2\right) - \Gamma_{bw}\dot{u}\left(|h_1| - |h_2|\right)\Big| \end{aligned} \tag{9}$$

and since $|ab| = |a||b|$ for $a, b \in \mathbb{R}$, (9) is reduced to,

$$\begin{aligned} \Big| - 1\Big|\Big|B_{bw}|\dot{u}|\left(h_1 - h_2\right) + \Gamma_{bw}\dot{u}\left(|h_1| - |h_2|\right)\Big| \le \\ \Big|B_{bw}|\dot{u}|\left(h_1 - h_2\right)\Big| + \Big|\Gamma_{bw}\dot{u}\left(|h_1| - |h_2|\right)\Big| \end{aligned} \tag{10}$$

where we have used the triangle inequality $|a+b| \le |a|+|b|$. Since $\Big||a|-|b|\Big| \le |a-b|$, the previous inequality is expressed as,

$$\begin{aligned} \Big|B_{bw}|\dot{u}|\Big|\Big|\left(h_1 - h_2\right)\Big| + \Big|\Gamma_{bw}\dot{u}\Big|\Big|h_1 - h_2\Big| \le \\ \underbrace{\left(|B_{bw}| + |\Gamma_{bw}|\right)}_{K}|\dot{u}|\,|h_1 - h_2| \end{aligned} \tag{11}$$

and then $K = |B_{bw}| + |\Gamma_{bw}| > 0$ is positive as long as $B_{bw} \neq 0$ or $\Gamma_{bw} \neq 0$. ☐

Assumption 3. *The vector function $g(x,\dot{u},u)$ in* (6) *is global Lipschitz with respect to x uniformly in $\dot{u}$ and u.*

Assumption 3 makes sense when we consider Assumption 1, Lemma 2, and conditions 1, 2, 5, and 6 of Problem 1.

Lemma 3. *Let matrices Λ and C be defined as in* (6), *and $I \in \mathbb{R}^{3\times 3}$ is the identity matrix. Also, let,*

$$\theta > 0,\; k_o > 0,\; \text{and } 0 < l_0 < \frac{\theta^3}{k_o}, \tag{12}$$

then, the solution of the Lyapunov-like equation,

$$\theta S + \Lambda^\intercal S + S\Lambda - C^\intercal C = 0, \tag{13}$$

is given by $S \succ 0 \in \mathbb{R}^{3\times 3}$, i.e., it is positive definite.

Proof. A simple computation follows that,

$$\mathrm{S}^{-1} = \begin{pmatrix} 3\theta & 3\frac{\theta^2}{l_o} & \frac{1}{l_o}\left(\theta^3 + 2k_o l_o\right) \\ 3\frac{\theta^2}{l_o} & \frac{1}{l_o^2}\left(5\theta^3 - 2k_o l_o\right) & \frac{1}{l_o^2}\left(2\theta^4 + k_o l_o\theta\right) \\ \frac{1}{l_o}\left(\theta^3 + 2k_o l_o\right) & \frac{1}{l_o^2}\left(2\theta^4 + k_o l_o\theta\right) & \frac{1}{l_o^2}\left(2\theta^4 + k_o l_o\theta\right)\frac{\theta^4 + 2k_o l_o\theta}{2\theta^3 + k_o l_o} \end{pmatrix}. \tag{14}$$

Recall that a symmetric matrix is positive definite if and only if all its leading principal minors are positive [10]. Thus, by computing the principal minors of S^{-1} one can deduce if S^{-1} is positive definite. Now, notice that the first leading principal minor is positive when $\theta > 0$, while the second leading principal minor is positive when $\frac{3\theta}{l_o^2}\left(5\theta^3 - 2k_o l_o\right) - \frac{9\theta^4}{l_o^2} > 0$, which holds when (12) is fulfilled. One arrives at the same conditions by computing the third leading principal minor. Therefore, all the leading principal minors are positive as long as inequalities (12) hold, and thus S^{-1} is positive definite. One concludes the proof by recalling that the inverse of a positive definite matrix is also a positive definite. ☐

Lemma 4. *The matrix $\left(\Lambda - S^{-1}C^\intercal C\right)$ is Hurwitz for $k_o > 0$, $l_o > 0$ and $\theta > \frac{1}{2}\sqrt[3]{k_o l_o}$.*

Proof. We know that all the eigenvalues of a Hurwitz matrix have strictly negative real parts. Thus, to verify that, we compute the eigenvalues of,

$$\Lambda - S^{-1}C^\intercal C = \begin{pmatrix} 3\theta & l_o & 0 \\ -3\frac{\theta^2}{l_o} & 0 & 1 \\ k_o - \frac{1}{l_o}\left(\theta^3 + 2k_o l_o\right) & 0 & 0 \end{pmatrix} \tag{15}$$

given by,

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} -\theta + \sqrt[3]{-k_o l_o} \\ -\theta - \frac{1}{2}\sqrt[3]{-k_o l_o} + i\frac{\sqrt{3}}{2}\sqrt[3]{-k_o l_o} \\ -\theta - \frac{1}{2}\sqrt[3]{-k_o l_o} - i\frac{\sqrt{3}}{2}\sqrt[3]{-k_o l_o} \end{pmatrix} \tag{16}$$

where $i = \sqrt{-1}$. Notice that the real part of eigenvalues λ_2 and λ_3 are always negative, but this is not the case with the real part of λ_1. In that case, using Euler's formula,

$$\lambda_1 = -\theta + \sqrt[3]{-k_o l_o} = -\theta + \frac{1}{2}\sqrt[3]{k_o l_o} + i\frac{\sqrt{3}}{2}\sqrt[3]{k_o l_o}, \tag{17}$$

and then, the real part of λ_1 is negative when $k_o > 0$, $l_o > 0$ and $\theta > \frac{1}{2}\sqrt[3]{k_o l_o}$ and the proof is complete. ☐

Remark 1. *In the proof of Lemma 4, we got the inequality for θ given by $\theta > \frac{1}{2}\sqrt[3]{k_o l_o}$ whereas in Lemma 3 we get $\theta > \sqrt[3]{k_o l_o}$ which is a condition to S be positive definite. Both results are in accordance, and hereafter we maintain the condition of θ given in* (12), *i.e., $\theta > \sqrt[3]{k_o lo}$.*

Corollary 1. *The matrix $S\Lambda - C^\intercal C$ is Hurwitz under conditions of Lemma 4.*

Proof. From Lemma 4, $\Lambda - S^{-1}C^\intercal C$ is Hurwitz. When we premultiply such a matrix by the positive definite matrix S, the result is $S\Lambda - C^\intercal C$, [1]. From (13) it follows that each side of

$$-\theta I = (\Lambda - S^{-1}C^\intercal C) + S^{-1}\Lambda^\intercal S \tag{18}$$

is negative definite. Thus, $-\theta S = (S\Lambda - C^\intercal C) + \Lambda^\intercal S$ is also negative definite, where $\Lambda^\intercal$ described in (6) has a positive eigenvalue, and then matrix above $\Lambda^\intercal S$ is not negative definite nor Hurwitz. From the previous discussion, and especially since $\left(\Lambda - S^{-1}C^\intercal C\right)$ is Hurwitz, the matrix $(S)\left(\Lambda - S^{-1}C^\intercal C\right) = S\Lambda - C^\intercal C$ is also Hurwitz as long as inequalities of Lemma 4 hold. ☐

We are ready to present the first main result.

Theorem 1 (Nonlinear state and disturbance observer)**.** *Consider that Assumptions 1, 2, and 3 are satisfied. Let us define the system,*

$$\dot{\hat{x}} = \Lambda\hat{x} + g(\hat{x},\dot{u},u) + B\dot{u} - S^{-1}C^\intercal(C\hat{x} - y), \quad \hat{x} \in \mathbb{R}^3 \tag{19}$$

[1] Generally, a Hurwitz matrix that is pre or post-multiplied by a positive definite matrix is not Hurwitz.

with elements as in (6)*, and S the solution of the Lyapunov equation* (13) *satisfying inequalities* (12) *with parameter $\theta \geq \max\left\{\sqrt[3]{k_o l_o}, [r+L_g+\rho]\frac{\lambda_{max}\{S\}}{\lambda_{min}\{S\}}\right\}$ for $r, L_g \in \mathbb{R}_{>0}$ defined in the proof. Then,* (19) *is a global exponential observer for system* (5)*, and its dynamics can be made arbitrarily fast.*

Proof. Let us begin by defining the error observer as

$$\varepsilon = \hat{x} - x \tag{20}$$

whose dynamics are given by,

$$\dot{\varepsilon} = \left(\Lambda - S^{-1}C^{\intercal}C\right)\varepsilon + g(\hat{x}, \dot{u}, u) - g(x, \dot{u}, u) - \Delta(t). \tag{21}$$

Consider the candidate Lyapunov function $W = \frac{1}{2}\varepsilon^{\intercal}S\varepsilon$ where $S = S^{\intercal}$ is computed from (13). The first time-derivative of W is computed as,

$$\begin{aligned}\dot{W} &= \varepsilon^{\intercal}S\left[\left(\Lambda - S^{-1}C^{\intercal}C\right)\varepsilon + g(\hat{x}, \dot{u}, u) - g(x, \dot{u}, u) - \Delta\right] \\ &= \varepsilon^{\intercal}S\left(\Lambda - S^{-1}C^{\intercal}C\right)\varepsilon + \varepsilon^{\intercal}S(g(\hat{x}, \dot{u}, u) - g(x, \dot{u}, u)) \\ &\quad - \varepsilon^{\intercal}S\Delta(t).\end{aligned} \tag{22}$$

After considering Assumption 2 and 3 with Lipschitz constant L_g as, $\|g(\hat{x}, \dot{u}, u) - g(x, \dot{u}, u)\| \leq L_g\|\varepsilon\|$, it follows that,

$$\begin{aligned}\dot{W} &\leq \varepsilon^{\intercal}\left(S\Lambda - C^{\intercal}C\right)\varepsilon + L_g\|S\varepsilon\|\|\varepsilon\| + \rho\|S\varepsilon\|\|\varepsilon\| \\ &\leq -\varepsilon^{\intercal}\left(\theta S + \Lambda^{\intercal}S\right)\varepsilon + L_g\|S\varepsilon\|\|\varepsilon\| + \rho\|S\varepsilon\|\|\varepsilon\|.\end{aligned} \tag{23}$$

Claiming Corollary 1 it follows that $S\Lambda - C^{\intercal}C$ is Hurwitz, and from (13) the matrix $Q = -\theta S - \Lambda^{\intercal}S$ is also Hurwitz. Notice that $-\Lambda^{\intercal}S$ *disturbs* the contribution of the negative definite matrix $-\theta S$ in Q but not as much to lose the property of Q of being a Hurwitz matrix in (23). Thus, we can assume that $-\varepsilon\Lambda^{\intercal}S\varepsilon \leq r\|S\varepsilon\|\|\varepsilon\|$ with $r > 0$. Then,

$$\dot{W} \leq -\theta\varepsilon^{\intercal}S\varepsilon + (r + L_g + \rho)\|S\varepsilon\|\|\varepsilon\|. \tag{24}$$

Since S is a symmetric positive definite matrix, it can be diagonalized by an orthogonal matrix D, such that $S = DLD^{\intercal}$, where L is a diagonal matrix with the eigenvalues of S on the diagonal. Let $v = D^{\intercal}\varepsilon$ be a change of coordinates that transforms ε into v. Then we have $\varepsilon = Dv$, and $\varepsilon^{\intercal} = v^{\intercal}Lv = \sum_{i=1}^{3}\lambda_i\{S\}v_i^2$ where $\lambda_i\{S\}$ is the i-th eigenvalue of S, and v_i is the ith component of v. Let the smallest and largest eigenvalues of S be represented by $\lambda_{\min}\{S\}$ and $\lambda_{\max}\{S\}$, respectively. After simple computations, it follows that $2|W| \leq \sum_{i=1}^{3}\lambda_i\{S\}|v_i^2| \leq \lambda_{\max}\{S\}\sum_{i=1}^{3}|v_i^2|$. Also note that $v^{\intercal}v = \|\varepsilon\|^2$, since $QQ^{\intercal} = I$ for all the orthogonal matrices such as D. On the other hand, $\sum_{i=1}^{3}\lambda_i\{S\}v_i^2 \geq \lambda_{\min}\{S\}\sum_{i=1}^{3}v_i^2 = \lambda_{\min}\{S\}v^{\intercal}v$. Thus, after simple computations and since $2|W| = \|S\varepsilon\|\|\varepsilon\|$, (24) is expressed as,

$$\dot{W} \leq -2\underbrace{\left(\theta - [r + L_g + \rho]\frac{\lambda_{\max}\{S\}}{\lambda_{\min}\{S\}}\right)}_{\varpi}W \tag{25}$$

with θ chosen as in Theorem 1. Finally, since W is radially unbounded, the exponential stability of the error observer $\varepsilon = 0$ is achieved, and the proof is complete. □

B. Barrier-Lyapunov-function and saturation-based controller

Lemma 5. *Let A_{bw}, B_{bw} and Γ_{bw} be constant parameters such that $B_{bw} > |\Gamma_{bw}|$ holds. Also, consider that the first-time derivative of the control input $\dot{u}$ is bounded. Then, the hysteresis subsystem* (4b) *is bounded.*

Proof. Let $\mathcal{H} = |h|$ a candidate Lyapunov function for the hysteresis system with time derivative evaluated in the trajectories of (4b) as follows,

$$\begin{aligned}\dot{\mathcal{H}} &= \operatorname{sgn}(h)\left(A_{bw}\dot{u} - B_{bw}|\dot{u}|h - \Gamma_{bw}\dot{u}|h|\right) \\ &\leq |A_{bw}||\dot{u}| - B_{bw}|\dot{u}||h| + |\Gamma_{bw}||h||\dot{u}| \\ &\leq -|\dot{u}|\left(B_{bw} - |\Gamma_{bw}|\right)|h| + |A_{bw}||\dot{u}|.\end{aligned} \tag{26}$$

when we choose $B_{bw} > |\Gamma_{bw}|$ as in the conditions of the present lemma, it follows that

$$\dot{\mathcal{H}} \leq -\varphi_1\mathcal{H} + \varphi_2, \tag{27}$$

where $\underbrace{|\dot{u}|\left(B_{bw} - |\Gamma_{bw}|\right)}_{\varphi_1} \geq 0$, and $|A_{bw}||\dot{u}| \leq \varphi_2$ by hypothesis, where $\varphi_1, \varphi_2 \in \mathbb{R}_{>0}$ when $\dot{u} \neq 0$. Notice that when $\dot{u} = 0$, $\dot{\mathcal{H}} = 0$ and the hysteresis $h(t)$ remains bounded as desired. Following a similar procedure as Lemma 1 in [11], the hysteresis h is bounded. □

Remark 2. *Lemma 5 is essential since the hysteresis h in* (4b) *is not directly controllable, and we aim to design a controller for the force equation* (4a) *without indirectly controlling the hysteresis behavior, in contrast to [2]–[4]. Thus, Lemma 5 gives us information about the hysteresis behavior under bounded control inputs as designed in this letter.*

Let us introduce a fundamental concept of saturation function proper to design bounded controls[2].

Definition 1. *Consider $L, M \in \mathbb{R}_{>0}$ with $L \leq M$. A linear saturation function for (L, M) is given by $\sigma : \mathbb{R} \to \mathbb{R}$ being continuous and non-decreasing satisfying: 1) $s\sigma(s) > 0$ for all $s \neq 0$; 2) $\sigma(s) = s$ when $|s| \leq L$; and 3) $|\sigma(s)| \leq M$ for all $s \in \mathbb{R}$.*

Before presenting our second main result, we define the tracking error by,

$$e = f - f^d(t) \tag{28}$$

where $f^d(t)$ is the desired force fulfilling the conditions of Problem 1. The dynamics of (28) is given by,

$$\dot{e} = -a_1(e + f^d(t)) + a_2\dot{u} + a_3u - a_4\dot{h} - a_5h + \delta(t) - \dot{f}^d(t). \tag{29}$$

In (29), there is the control u and its first-time derivative $\dot{u}$. For convenience, we chose the latter one for the control design. Then, one must integrate $\dot{u}$ to get the real control input u.

Assumption 4. *The initial conditions of the error force differential equation* (29) *lie in the set $e(t_0) = f(t_0) - f^d(t_0) \in (-l, l)$, where $l \in \mathbb{R}_{>0}$ and t_0 is the initial time.*

[2]For more details about these functions, refer to [12].

Theorem 2 (Barrier-Lyapunov-function and saturation-based controller)**.** *Consider that system Σ satisfies Assumption 4, and that $h(t)$ is bounded under bounded inputs as demonstrated in Lemma 5. Also assume that σ_1 and σ_2 are saturation functions as in Definition 1 with parameters (L_1, M_1) and (L_2, M_2); and where $k_1, k_2 \in \mathbb{R}_{>0}$. Besides, consider that $\delta(t)$ is available for feedback. Then, the control algorithm,*

$$\dot{u} = \frac{1}{a_2}\Big(-\sigma_1\left([l^2 - e^2][k_1 e]\right) - \sigma_2\left(\frac{k_2 e}{l^2 - e^2}\right) + a_1 f - a_3 u + a_4\dot{h} + a_5 h - \delta(t)\Big) \tag{30}$$

exponentially stabilizes the origin for all initial conditions $e(t_0) \in (-l, l)$, where $l \in \mathbb{R}_{>0}$. Moreover, l limits the maximum permissible error during all the evolution of the closed-loop system, i.e., $|e(t)| \leq l$ for all $t \geq t_0$.

Proof. Let $V = \frac{1}{2}\log\left(\frac{l^2}{l^2-e^2}\right)$ a Barrier Lyapunov function [13] with time-derivative along the trajectories of (29):

$$\dot{V} = \frac{e\dot{e}}{l^2 - e^2} = \left(\frac{e}{l^2 - e^2}\right)\Big(-a_1(e + f^d(t)) + a_2\dot{u} + a_3 u - a_4\dot{h} - a_5 h + \delta(t) - \dot{f}^d(t)\Big). \tag{31}$$

By substituting the control (30) in (31) the result is

$$\dot{V} = \left(\frac{e}{l^2 - e^2}\right)\Big(-\sigma_1\underbrace{\left([l^2 - e^2][k_1 e]\right)}_{\arg(\sigma_1)} - \sigma_2\underbrace{\left(\frac{k_2 e}{l^2 - e^2}\right)}_{\arg(\sigma_2)}\Big). \tag{32}$$

Let us define the following pair of sets, $\mathcal{S}_1 = \{e : |e| \leq L_1\}$, and $\mathcal{S}_2 = \{e : |e| \leq L_2\}$ where we assume that $L_1 \leq L_2$ and $M_1 \leq M_2$ without loss of generality. Now, consider that the arguments of the saturation functions are out of the linear region: $e \notin \mathcal{S}_2$ and $e \notin \mathcal{S}_1$. Also, for the particular saturation functions and their particular arguments, notice that $\operatorname{sgn}(\arg(\sigma_i)) = \operatorname{sgn}(e)$ for $i = \{1, 2\}$ holds as long as $e \in (-l, l)$. Then, in the above scenario, by definition of saturation function (Definition 1) and signum function definition, it follows that $L_i \operatorname{sgn}(\arg(\sigma_i)) \leq \sigma_i(\arg(\sigma_i))$ since, in the present scenario, the saturation function is greater than the bound L_i that delimits the linear region. And then, $-\sigma_i(\arg(\sigma_i)) \leq -L_i \operatorname{sgn}(\arg(\sigma_i)) = -L_i \operatorname{sgn} e$. With the above expression, (32) results in

$$\begin{aligned}\dot{V} &= \left(\frac{e}{l^2 - e^2}\right)\Big(-L_1 \operatorname{sgn} e - L_2 \operatorname{sgn} e\Big) \\ &\leq -\frac{(L_1 + L_2)}{l^2 - e^2}|e| < 0,\ \forall e \in (-l, l).\end{aligned} \tag{33}$$

A consequence of $\dot{V} < 0$ is that after a finite time, e enters in $\mathcal{S}_2$, and then,

$$\begin{aligned}\dot{V} &= \left(\frac{e}{l^2 - e^2}\right)\Big(-L_1 \operatorname{sgn} e - \frac{k_2 e}{l^2 - e^2}\Big) \\ &\leq -\frac{L_1}{l^2 - e^2}|e| - \frac{k_2 e^2}{(l^2 - e^2)^2} < 0.\end{aligned} \tag{34}$$

Again, since $\dot{V} < 0$, after a finite time $e \in \mathcal{S}_1$ and finally,

$$\dot{V} = -k_1 e^2 - \frac{k_2 e^2}{(l^2 - e^2)^2} \leq -V,\ \forall e \in (-l, l) \tag{35}$$

and one gets exponential stability as long as $e \in (-l, l)$. □

Remark 3. *Condition of Assumption 4 is easily achieved by choosing a proper initial condition of the reference trajectory $f^d(t)$, the positive value of l, and the initial condition of the system* (4a)*. Thus, we have at least two variables that we can play of it to accomplish Assumption 4. Notice that l can be selected equal to the maximal range of manipulation force $f^{d_{max}}$, generally.*

Remark 4. *The term $-\sigma_2(\frac{k_2 e}{e^2 - l^2})$ assures that $|e| < l$ since when $|e|$ approaches l, $-\sigma_2(\cdot) \to \pm\infty$ making that $e \to 0$ quickly. However, having unbounded terms in the controller is undesirable. For that, we implemented saturation functions in the controller, and thus, we can preset how much the effect of $-\sigma_2(\cdot)$ impacts the control effort.*

In Theorem 2, we assume we know disturbance $\delta(t)$, hysteresis h, and its first-time derivative. In reality, it is unfeasible to measure those variables. Then, we combine our previous results to get an output feedback scheme given next.

Theorem 3 (Output feedback barrier-Lyapunov-function and saturation-based controller)**.** *Let us consider error systems* (21) *and* (29) *with conditions given Theorems 1 and 2. Then, the output feedback law,*

$$\begin{aligned}\dot{u}(\hat{f}, \hat{h}, \hat{\delta}, \dot{\hat{h}}) = \frac{1}{a_2}\Big(&-\sigma_1\left([l^2 - (\hat{f} - f^d)^2][k_1(\hat{f} - f^d)]\right) \\ &- \sigma_2\left(\frac{k_2(\hat{f} - f^d)}{l^2 - (\hat{f} - f^d)^2}\right) + a_1\hat{f} - a_3 u + a_4\dot{\hat{h}} + a_5\hat{h} - \hat{\delta}(t)\Big)\end{aligned} \tag{36}$$

asymptotically stabilizes the origin $e = 0$, $\varepsilon = (0,\ 0,\ 0)^\intercal$ as long as $e(t_0) \in (-l, l)$. Furthermore, $|e(t)| \in (-l, l)$ $\forall t \geq t_0$.

Proof. The structure of the output feedback control (36) is the same as the full state feedback control (30). First, we define the change of coordinates by using the tracking and observer errors as follows,

$$\xi = e + \varepsilon_1 = \hat{f} - f^d \in \mathbb{R},\ \text{with}\ \varepsilon_1 = \begin{pmatrix}1 & 0 & 0\end{pmatrix}\varepsilon. \tag{37}$$

We rewrite the tracking error dynamics (29) and observer error dynamics (21) equations in closed-loop form with output feedback control (36) as follows,

$$\begin{aligned}\dot{e} &= G_t(e, \dot{u}(\xi), t) \in \mathbb{R} \\ \dot{\varepsilon} &= F_o(\varepsilon, \dot{u}(\xi), t) \in \mathbb{R}^3,\end{aligned} \tag{38}$$

where $G_t(\cdot)$ is the RHS of tracking error system (29), and $F_o(\cdot)$ is the RHS of observer error system (21). Considering the above notation and the change of coordinates (37), the system (38) can be rewritten as follows,

$$\begin{aligned}\dot{\xi} &= G_t\left(\xi - \varepsilon_1, \dot{u}(\xi), t\right) + F_{o,1}(\varepsilon, \dot{u}(\xi), t) \in \mathbb{R}^1 \\ \dot{\varepsilon} &= F_o(\varepsilon, \dot{u}(\xi), t) \in \mathbb{R}^3.\end{aligned} \tag{39}$$

where $F_{o,1} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} F_o$. To prove that the origin $(\xi, \varepsilon) = 0 \in \mathbb{R}^4$ of (39) converges to zero, let us propose the candidate Lyapunov function, $E(\xi, \varepsilon) = \kappa V(\xi) + W(\varepsilon)$, where $\kappa \in \mathbb{R}_{>0}$ and $W(\cdot), V(\cdot)$ are defined as in the proofs of Theorems 1 and 2, respectively. The time derivative of $E(\xi, \varepsilon)$ along the trajectories of (39) is given by,

$$\begin{aligned}
\dot{E} &= \kappa \frac{\partial V(\xi)}{\partial \xi} \dot{\xi} + \dot{W}(\varepsilon) \\
&= \kappa \frac{\partial V(\xi)}{\partial \xi} \Big[G_t\left(\xi - \varepsilon_1, \dot{u}(\xi), t\right) + F_{o,1}(\varepsilon, \dot{u}(\xi), t) \Big] + \dot{W}(\varepsilon) \\
&= \kappa \frac{\partial V(\xi)}{\partial \xi} G_t\left(\xi - \varepsilon_1, \dot{u}(\xi), t\right) + \kappa \frac{\partial V(\xi)}{\partial \xi} F_{o,1}(\varepsilon, u(\xi), t) \\
&\quad \underbrace{+ \kappa \frac{\partial V(\xi)}{\partial \xi} G_t\left(\xi, \dot{u}(\xi), t\right) - \kappa \frac{\partial V(\xi)}{\partial \xi} G_t\left(\xi, \dot{u}(\xi), t\right)}_{0} + \dot{W}(\varepsilon) \\
&= \kappa \frac{\partial V(\xi)}{\partial \xi} G_t\left(\xi, \dot{u}(\xi), t\right) + \kappa \frac{\partial V(\xi)}{\partial \xi} \Big[F_{o,1}(\varepsilon, \dot{u}(\xi), t) \\
&\qquad + G_t\left(\xi - \varepsilon_1, \dot{u}(\xi), t\right) - G_t\left(\xi, \dot{u}(\xi), t\right) \Big] + \dot{W}(\varepsilon).
\end{aligned} \tag{40}$$

Due to Assumption 3, it follows that $\| G_t\left(\xi - \varepsilon_1, \dot{u}(\xi), t\right) - G_t\left(\xi, \dot{u}(\xi), t\right) \| \leq L_G \|\varepsilon_1\|$ with $L_G \in \mathbb{R}_{>0}$. Also, $\|F_{o,1}(\varepsilon, \dot{u}(\xi), t) - F_{o,1}(0, \dot{u}(\xi), t)\| \leq L_F \|\varepsilon_1\|$ with $L_F \in \mathbb{R}_{>0}$ since $\dot{\varepsilon}_1 = F_{o,1}(0, \dot{u}(\xi), t) = 0$ when $\varepsilon_1 = 0$. Therefore (40) is expressed as,

$$\begin{aligned}
\dot{E} &\leq \kappa \frac{\partial V(\xi)}{\partial \xi} G_t\left(\xi, \dot{u}(\xi), t\right) + \dot{W}(\varepsilon) \\
&\qquad + \kappa \left\| \frac{\partial V(\xi)}{\partial \xi} \right\| \Big[(L_F + L_G) \|\varepsilon_1\| \Big].
\end{aligned} \tag{41}$$

Furthermore, $\frac{\partial V(\xi)}{\partial \xi} G_t\left(\xi, \dot{u}(\xi), t\right) \leq -\varrho_4 \|\xi\|^2$ follows from (35), with $\varrho_4 \in \mathbb{R}_{>0}$. Besides, according to [14], it follows that, $\left\| \frac{\partial V(\xi)}{\partial \xi} \right\| \leq \varrho_5 \|\xi\|$ with $\varrho_5 \in \mathbb{R}_{>0}$. Therefore, considering the above, (41) can be simplified to,

$$\dot{E}(\xi, \varepsilon) \leq -\kappa \varrho_4 \|\xi\|^2 - \varpi \varepsilon^\intercal S \varepsilon + \kappa \varrho_5 (L_F + L_G) \|\xi\| \|\varepsilon_1\|, \tag{42}$$

where ϖ is defined in (25). Then, by choosing $\kappa < \frac{4\varrho_4 \varpi}{[(\varrho_5)(L_F + L_G)]^2}$ it follows that $\dot{E}(\xi, \varepsilon)$ is negative definite. Thus, following the procedure similar to [15] and [16], it follows that the output feedback control $\dot{u}(\hat{f}, \hat{h}, \hat{\delta}, \dot{\hat{h}})$ asymptotically stabilizes the origin $e = 0$, $\varepsilon = (0,\ 0,\ 0)^\intercal$ as long as $e(t_0) \in (-l, l)$ assuring that $|e(t)| \in (-l, l)\ \forall t \geq t_0$. $\square$

IV. CONCLUSIONS

A. Supplementary material

This letter has a comprehensive set of simulation experiments with the Simulink Matlab files, which are available through the link: `https://github.com/gfloresc/Force_Barrier_Bouc-Wen`

B. Final remarks

We have presented a robust force control strategy for a piezoelectric actuator in a robotic hand manipulating an object. Using a model that links the driving voltage to the manipulation force, our approach accurately accounts for the actuator's hysteresis and the object's deformation. We have demonstrated the effectiveness of our output feedback barrier-Lyapunov-function and saturation-based controller in ensuring the closed-loop system's asymptotic stability while rejecting aggressive and high-magnitude disturbances. Moreover, our controller guarantees that the tracking error is enclosed in a predefined small set. The results of our simulations confirm the theoretical aspects of our work. We plan to conduct experimental benchmarks for future work using piezoelectric stacks and the self-sensing technique [17]. These experiments will allow us to validate the efficacy of our force control strategy in a practical setting and further improve its performance.

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