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Connectivity-Preserving Formation Tracking for Multiple Double Integrators by a Self-Tuning Adaptive Distributed Observer

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***Abstract*— In this letter, we study the distributed formation tracking problem for multiple double-integrator systems with connectivity preservation over a state-dependent communication network. In particular, we employ an adaptive distributed observer for the leader system that can estimate both the state and the system matrix of the leader. As a result, unlike the existing results, we do not require all vehicles to know the system matrix of the leader. Furthermore, the adaptive distributed observer incorporates a self-tuning dynamic observer gain, which eliminates the need of computing the observer gain in advance. The effectiveness of our approach is illustrated by an example.**

I. Introduction

Communication plays a crucial role in the cooperative control of multi-agent systems. Due to the limited range of communications, the validity of the communication link between each pair of agents may depend on their distance. Thus, a communication network is inherently state-dependent. For this reason, it is of practical interest to study the cooperative control problems of multi-agent systems over state-dependent communication networks. Furthermore, it calls for the investigation of how to preserve the connectivity of a state-dependent communication network in cooperative control problems.

In fact, preserving the connectivity of a state-dependent communication network has been taken into account in various cooperative control problems, such as the rendezvous problem, which is also referred to as the flocking problem. The rendezvous problem for multiple double-integrator systems has been studied for both the leaderless case in [11] and the leader-following case in [2], [3], [13]. Specifically, edge-based strict Lyapunov functions were proposed in [11] to achieve velocity synchronization. A position feedback control law was designed in [13] to synchronize the velocities of multiple double-integrator systems while maintaining the connectivity of the network. A dynamic distributed state feedback control law was proposed in [2] to achieve leader-following rendezvous with connectivity preservation. Later, the result in [2] was enhanced in [3] from full-state feedback to position-only feedback.

In addition to the rendezvous problem, connectivity preservation has also been considered in both the leaderless [6], [8], [9], [12] and the leader-following [10], [15], [16] formation control problems. In particular, a distributed control law was developed in [9] such that multiple nonholonomic agents can form a desired formation configuration while preserving the connectivity of the communication network. A class of Lyapunov-like barrier functions were proposed in [8] to make a group of unicycles converge to their desired destinations while preserving the connectivity of the network. Considering uncertainties in the communication network, the leaderless formation control problem for multiple nonholonomic agents was addressed by a distributed gradient-based controller in [6]. The connectivity-preserving leaderless consensus-based formation control for high-order nonlinear multi-agent systems with limited-range measurements and output constraints was investigated in [12]. The connectivity-preserving leader-following formation control problem of multiple nonholonomic mobile robots with a time-varying leader was investigated in [15]. The result in [15] was further extended in [16] to handle agents with non-uniform communication ranges. The leader-following formation tracking problem for multiple n-dimensional double-integrator systems with connectivity preservation was studied in [10], where a distributed position feedback control law was proposed to steer the vehicles to track a leader with prescribed offsets.

However, the result in [10] still has the following limitations. First, the design of the distributed observer for the leader system in [10] relies on the knowledge of the system matrix of the leader. In other words, all followers are assumed to know the system matrix of the leader. Second, the observer gain of the distributed observer in [10] needs to be larger than some threshold that depends on the Laplacian matrices of all possible connected graphs associated with the multi-agent system. As a result, it is time-consuming to compute such a threshold for the observer gain when the number of agents is large. Furthermore, the observer gain needs to be recalculated once the number of agents in the network changes.

To overcome the aforementioned limitations, we further study the connectivity-preserving distributed formation tracking problem for multiple double-integrator systems by a self-tuning adaptive distributed observer. In comparison with the distributed observer in [10] which requires all followers to know the system matrix of the leader system, the self-tuning adaptive distributed observer eliminates such a requirement. Unlike the observer gain of the distributed observer in [10] that is computed offline based on the knowledge of the Laplacian matrices of all possible connected graphs associated with the multi-agent system, the dynamic observer gain of the self-tuning adaptive distributed observer is governed by

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a self-tuning law and does not rely on the knowledge of the Laplacian matrices. The main contributions of this letter are summarized as follows:

1) We establish a distributed position feedback control law for multiple double-integrator systems to asymptotically track a moving target with desired offsets while preserving the connectivity of the communication network.
2) By employing the self-tuning adaptive distributed observer, we remove the assumption that all followers know the system matrix of the leader system and eliminate the need of computing the observer gain offline.

Compared with the existing results on the cooperative formation control problem in the literature, our result has the following distinct features. i) Unlike [10], [15], [16] that assumed all followers have the exact knowledge of the leader, we use an adaptive distributed observer to estimate the system matrix and the state of the leader for each follower. ii) Unlike the result in [10] that required the offline computation of the observer gain, the adaptive distributed observer in this letter uses a self-tuning dynamic gain. iii) The leader in this letter can generate a large class of reference trajectories, including polynomial functions with any coefficients and sinusoidal functions with any amplitudes, frequencies, and initial phases, and their finite combinations. In particular, the leader can generate a reference trajectory with an unbounded velocity that cannot be handled by the approaches in [15], [16].

The rest of this letter is organized as follows. We first provide some preliminaries and formulate our problem in Section II. We then present our main result in Section III. In Section IV, we use an example to illustrate our approach. The letter is concluded in Section V with some remarks.

Notation. $\otimes$ denotes the Kronecker product of matrices. $\|x\|$ denotes the Euclidean norm of a vector x. For column vectors a_i, $i = 1, \ldots, s$, $\mathrm{col}(a_1, \ldots, a_s) = [a_1^\top \ldots a_s^\top]^\top$. $\mathbf{1}_N$ denotes an N-dimensional column vector with all elements being 1.

II. Preliminaries and Problem Formulation

We consider a multi-agent system with N followers and a leader. The dynamics of the followers are described by

$$\ddot{q}_i = u_i + d_i, \ i = 1, \ldots, N \tag{1}$$

where, for $i = 1, \ldots, N$, $q_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^n$, and $d_i \in \mathbb{R}^n$ are the position, the input, and the external disturbance of the ith follower. It is assumed that, for $i = 1, \ldots, N$, d_i is generated by following exosystem:

$$\dot{\omega}_i = Q_i \omega_i, \quad d_i = D_i \omega_i \tag{2}$$

where $\omega_i \in \mathbb{R}^{s_i}$, and $Q_i \in \mathbb{R}^{s_i \times s_i}$ and $D_i \in \mathbb{R}^{n \times s_i}$ are constant matrices. Without loss of generality, we assume that the pair (D_i, Q_i) is detectable [5].

The dynamics of the leader system is as follows:

$$\dot{x}_0 = S_0 x_0 \tag{3}$$

where $x_0 = \mathrm{col}(q_0, p_0)$ with $q_0, p_0 = \dot{q}_0 \in \mathbb{R}^n$ and $S_0 = \begin{bmatrix} 0_{n\times n} & I_n \\ S_{021} & S_{022} \end{bmatrix}$ with $S_{021}, S_{022} \in \mathbb{R}^{n \times n}$.

Like in [10], we also introduce the following design parameters:

1) $h_i \in \mathbb{R}^n$, $i = 1, \ldots, N$, is the desired offset between the ith follower and the leader. Let $h_0 = 0$ and $\bar{h} = \max_{i,j=0,1,\ldots,N,\ i\neq j} \|h_i - h_j\|$.
2) $r \in \mathbb{R}$ is the maximum sensing range of the followers.
3) $\epsilon \in (0, r)$ is to introduce the effect of hysteresis.

Given the multi-agent system composed of (1) and (3), let us define a digraph $\bar{\mathcal{G}}(t) = (\bar{\mathcal{V}}, \bar{\mathcal{E}}(t))$ where $\bar{\mathcal{V}} = \{0, 1, \ldots, N\}$ is the node set with node 0 associated with the leader (3) and node i, $i = 1, \ldots, N$, associated with the ith follower of (1), and $\bar{\mathcal{E}}(t) \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ is the edge set.

In what follows, we define the state-dependent communication graph as in [2], [10]. Given any $r > 0$ and $\epsilon \in (0, r)$, for any $t \geq 0$, $\bar{\mathcal{E}}(t) = \{(i,j) \mid i, j \in \bar{\mathcal{V}}, i \neq j\}$ is such that

1) $\bar{\mathcal{E}}(0) = \{(i,j) \mid \|q_i(0) - q_j(0)\| < (r - \epsilon), i, j = 1, \ldots, N\} \bigcup \{(0,j) \mid \|q_0(0) - q_j(0)\| < (r-\epsilon), j = 1, \ldots, N\}$;
2) if $\|q_i(t) - q_j(t)\| \geq r$, then $(i,j) \notin \bar{\mathcal{E}}(t)$;
3) for $i = 1, \ldots, N$, $(i, 0) \notin \bar{\mathcal{E}}(t)$;
4) for $i = 0, 1, \ldots, N, j = 1, \ldots, N$, if $(i,j) \notin \bar{\mathcal{E}}(t^-)$ and $\|q_i(t) - q_j(t)\| < (r - \epsilon)$, then $(i,j) \in \bar{\mathcal{E}}(t)$;
5) for $i = 0, 1, \ldots, N, j = 1, \ldots, N$, if $(i,j) \in \bar{\mathcal{E}}(t^-)$ and $\|q_i(t) - q_j(t)\| < r$, then $(i,j) \in \bar{\mathcal{E}}(t)$.

Let $\bar{\mathcal{A}}(t) = [a_{ij}(t)]_{i,j=0}^N \in \mathbb{R}^{(N+1)\times(N+1)}$ be the adjacency matrix of $\bar{\mathcal{G}}(t)$. For $i = 1, \ldots, N$, $j = 0, 1, \ldots, N$, and $i \neq j$, let $a_{ij}(t) = 1$ whenever $(j, i) \in \bar{\mathcal{E}}(t)$ and $a_{ij}(t) = 0$ otherwise. At time t, the graph $\bar{\mathcal{G}}(t)$ is connected if every node i, $i = 1, \ldots, N$, is reachable from node 0. For $i = 1, \ldots, N$, let $\bar{\mathcal{N}}_i(t)$ denote the neighbor set of the ith follower at time t. Let $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ be a subgraph of $\bar{\mathcal{G}}(t)$ where $\mathcal{V} = \{1, \ldots, N\}$ and $\mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V}$ is derived from $\bar{\mathcal{E}}(t)$ by removing all the edges between node 0 and the nodes in $\mathcal{V}$. Hence, $\mathcal{G}(t)$ is undirected. For $i = 1, \ldots, N$, we denote the neighbor set of the ith follower at time t with respect to $\mathcal{V}$ by $\mathcal{N}_i(t) = \bar{\mathcal{N}}_i(t) \bigcap \mathcal{V}$. Let the Laplacian matrix of $\mathcal{G}(t)$ be $\mathcal{L}(t)$ and let $H(t) = \mathcal{L}(t) + \Delta(t)$ where $\Delta(t) = \mathrm{diag}(a_{10}(t), \ldots, a_{N0}(t))$. By [14, Lemma 1], for any time instant $t \geq 0$, if $\bar{\mathcal{G}}(t)$ is connected, then $H(t)$ is positive definite. We consider a control law of the following form:

$$\begin{aligned} u_i &= l_i\left(q_i - q_j, \zeta_i, \zeta_j, j \in \bar{\mathcal{N}}_i(t)\right) \\ \dot{\zeta}_i &= g_i\left(q_i, \zeta_i, \zeta_j, j \in \bar{\mathcal{N}}_i(t)\right), \ i = 1, \ldots, N \end{aligned} \tag{4}$$

where $\zeta_0 = (S_0, q_0, \dot{q}_0)$, and, for $i = 1, \ldots, N$, $l_i(\cdot)$ and $g_i(\cdot)$ are some sufficiently smooth functions to be specified, ζ_i is the state of the distributed dynamic compensator.

We describe the leader-following formation tracking problem with connectivity preservation as follows.

Problem 1: Given the multi-agent system composed of (1) and (3). For any $r > 0$, $\epsilon \in (0, r)$, and $h_i \in \mathbb{R}^n$, $i = 0, 1, \ldots, N$, such that $\bar{h} < \frac{\epsilon}{2}$, design a distributed control law of the form (4) such that, for any initial conditions $q_0(0)$, $\dot{q}_0(0)$, $\omega_i(0)$, $q_i(0)$, $\dot{q}_i(0)$, $\zeta_i(0)$, $i = 1, \ldots, N$, that make

$\bar{\mathcal{G}}(0)$ connected, the closed-loop system has the following properties:

1) $\bar{\mathcal{G}}(t)$ is connected for all $t \geq 0$.
2) $\lim_{t\to\infty}(q_i(t) - (q_0(t) + h_i)) = 0,\ i = 1, \ldots, N$.
3) $\lim_{t\to\infty}(\dot{q}_i(t) - \dot{q}_0(t)) = 0,\ i = 1, \ldots, N$.

We make the following assumption for the solvability of Problem 1.

Assumption 1: S_0 has no eigenvalues with positive real parts.

Remark 2.1: Under Assumption 1, the leader system (3) can generate a large class of reference trajectories, including arbitrary polynomial functions, multi-tone sinusoidal functions, and their combinations.

III. Main Result

Like in [3], [10], we make use of the output regulation theory to deal with external disturbances. For this purpose, we rewrite the system (1) into the following form:

$$\dot{x}_i = Ax_i + Bu_i + E_i\omega_i \tag{5a}$$
$$y_i = Cx_i \tag{5b}$$
$$e_i = x_i - \left(x_0 + \begin{bmatrix} h_i \\ 0_{n\times 1}\end{bmatrix}\right),\ i = 1, \ldots, N \tag{5c}$$

where, for $i = 1, \ldots, N$, $x_i = \text{col}\,(q_i, p_i)$ with $p_i = \dot{q}_i$, $y_i \in \mathbb{R}^n$, and $e_i \in \mathbb{R}^{2n}$ are the state, the measurement output, and the regulated output of the ith follower, respectively; and $A = \left[\begin{smallmatrix} 0 & 1 \\ 0 & 0\end{smallmatrix}\right] \otimes I_n$, $B = \left[\begin{smallmatrix} 0 \\ 1\end{smallmatrix}\right] \otimes I_n$, $E_i = \begin{bmatrix} 0_{n\times s_i} \\ D_i\end{bmatrix}$, and $C = \begin{bmatrix} 1 & 0\end{bmatrix} \otimes I_n$.

Remark 3.1: Let $\hat{A}_i = \begin{bmatrix} A & E_i \\ 0_{s_i\times 2n} & Q_i\end{bmatrix}$, $\hat{C}_i = \begin{bmatrix} C & 0_{n\times s_i}\end{bmatrix}$. Then we can assume without loss of generality that the pair $(\hat{C}_i, \hat{A}_i)$ is detectable since (C, A) is observable [5]. Hence, there exists a $L_i = \text{col}(L_{i1}, L_{i2})$ with $L_{i1} \in \mathbb{R}^{2n\times n}$ and $L_{i2} \in \mathbb{R}^{s_i\times n}$ such that $\hat{A}_i + L_i\hat{C}_i$ is Hurwitz. Furthermore, there exists a positive definite matrix $\bar{P}_i$ such that $(\hat{A}_i + L_i\hat{C}_i)^\top \bar{P}_i + \bar{P}_i(\hat{A}_i + L_i\hat{C}_i) = -I_{2n+s_i}$.

We perform the following coordinate transformations:

$$\bar{x}_i = \begin{bmatrix} \bar{q}_i \\ \bar{p}_i\end{bmatrix} = x_i - \left(x_0 + \begin{bmatrix} h_i \\ 0_{n\times 1}\end{bmatrix}\right),\ i = 0, 1, \ldots, N \tag{6a}$$
$$\bar{u}_i = u_i - S_{02}x_0 + D_i\omega_i,\ i = 1, \ldots, N \tag{6b}$$

where $S_{02} = \begin{bmatrix} S_{021} & S_{022}\end{bmatrix}$. Then, system (5) becomes the following double-integrator system without disturbance:

$$\dot{\bar{q}}_i = \bar{p}_i, \quad \dot{\bar{p}}_i = \bar{u}_i,\ i = 1, \ldots, N. \tag{7}$$

Let us define the potential function as follows:

$$\psi(s) = \frac{1}{2\left(\left(r - \bar{h}\right)^2 - s^2\right)},\ 0 \leq s < r - \bar{h}. \tag{8}$$

Motivated by [4], [10], we propose a distributed position feedback control law as follows:

$$u_i = -\alpha \sum_{j\in\bar{\mathcal{N}}_i(t)} \nabla_{\bar{q}_i}\psi\left(\|\bar{q}_i - \bar{q}_j\|\right) - \sum_{j\in\bar{\mathcal{N}}_i(t)} a_{ij}(t)\left(\xi_{2i} - \xi_{2j}\right) + S_{i2}\eta_i - D_i\hat{w}_i \tag{9a}$$
$$\dot{\xi}_i = A\xi_i + Bu_i + E_i\hat{w}_i + L_{i1}\left(C\xi_i - y_i\right) \tag{9b}$$
$$\dot{\hat{w}}_i = Q_i\hat{w}_i + L_{i2}\left(C\xi_i - y_i\right) \tag{9c}$$
$$\dot{S}_i = \beta \sum_{j\in\bar{\mathcal{N}}_i(t)} \left(S_j - S_i\right) \tag{9d}$$
$$\dot{\eta}_i = S_i\eta_i + \sum_{j\in\bar{\mathcal{N}}_i(t)} a_{ij}(t)\gamma_{ij}(t)\left(\eta_j - \eta_i\right) \tag{9e}$$
$$\dot{\gamma}_{ij} = k_{ij}a_{ij}(t)\left(\eta_i - \eta_j\right)^\top\left(\eta_i - \eta_j\right), \tag{9f}$$
$$i = 1, \ldots, N,\ j = 0, 1, \ldots, N,\ i \neq j$$

where $\alpha, \beta > 0$; $k_{ij} = k_{ji} > 0, i, j = 1, \ldots, N, i \neq j, k_{i0} > 0,\ i = 1, \ldots, N$; $S_i = \begin{bmatrix} 0_{n\times n} & I_n \\ S_{i21} & S_{i22}\end{bmatrix}$ with $S_{i21}, S_{i22} \in \mathbb{R}^{n\times n}$, and $S_{i2} = \begin{bmatrix} S_{i21} & S_{i22}\end{bmatrix}$; for $i = 1, \ldots, N$, $\eta_i \in \mathbb{R}^{2n}$, $\hat{w}_i \in \mathbb{R}^{s_i}$, $\xi_i = \text{col}(\xi_{1i}, \xi_{2i})$ with $\xi_{1i}, \xi_{2i} \in \mathbb{R}^n$, $L_i = \text{col}(L_{i1}, L_{i2})$ is as defined in Remark 3.1; $\eta_0 = x_0$ and $\xi_{20} = p_0$; for $i = 1, \ldots, N,\ j = 0, 1, \ldots, N,\ i \neq j,\ \gamma_{ij} \in \mathbb{R}$. Note that $\gamma_{ij} = \gamma_{ji}$ for $i, j = 1, \ldots, N,\ i \neq j$. Thus, the control law (9) is in the form of (4) with $\zeta_i = (\xi_i, \hat{\omega}_i, S_i, \eta_i, \gamma_{ij}, j \in \bar{\mathcal{V}}, i \neq j)$.

Remark 3.2: Suppose $\bar{\mathcal{G}}(t)$ is connected for all $t \geq 0$. Under Assumption 1, by [1, Lemma 1], for any $S_i(0)$, $i = 1, \ldots, N$, the solution $S_i(t)$ of (9d) is bounded for all $t \geq 0$, and satisfies $\lim_{t\to\infty}(S_i(t) - S_0) = 0$ exponentially.

Remark 3.3: The control law in [10] used a conventional distributed observer of the following form:

$$\dot{\eta}_i = S_0\eta_i + \gamma \sum_{j\in\bar{\mathcal{N}}_i(t)} a_{ij}(t)\left(\eta_j - \eta_i\right),\ i = 1, \ldots, N \tag{10}$$

where γ is the constant observer gain that needs to be greater than some threshold depending on the Laplacian matrices of all possible connected graphs with $N+1$ nodes. By contrast, the control law (9) uses an adaptive distributed observer (9d)–(9f) with a self-tuning dynamic gain $\gamma_{ij}(t)$. Unlike the distributed observer (10) that requires all followers to know the system matrix S_0 of the leader system, the self-tuning adaptive distributed observer (9d)–(9f) does not impose this requirement on all followers. Furthermore, in contrast to the observer gain γ of the distributed observer (10), which is computed offline based on the knowledge of the Laplacian matrices of all possible connected graphs associated with the multi-agent system, the dynamic observer gain $\gamma_{ij}(t)$ of (9d)–(9f) is governed by the self-tuning law (9f) and does not rely on the knowledge of the Laplacian matrices.

Remark 3.4: Suppose a new edge (i, j) is added to the graph $\bar{\mathcal{G}}(t)$ at some time instant $t_* > 0$, that is, $(i, j) \notin \bar{\mathcal{E}}(t_*^-)$ and $(i, j) \in \bar{\mathcal{E}}(t_*)$. By the definition of $\bar{\mathcal{G}}(t)$, we have $\|\bar{q}_i(t_*) - \bar{q}_j(t_*)\| = \|q_i(t_*) - q_j(t_*) - h_i + h_j\| \leq \|q_i(t_*) - q_j(t_*)\| + \|h_j - h_i\| \leq r - \epsilon + \bar{h}$. Since $\psi(\cdot)$ is strictly increasing on $(0, r - \bar{h})$, noting that $\bar{h} < \frac{\epsilon}{2}$, we have $0 \leq \psi(\|\bar{q}_i(t_*) - \bar{q}_j(t_*)\|) \leq \psi(r - \epsilon + \bar{h}) = \frac{1}{2\left(\left(r-\bar{h}\right)^2 - \left(r-\epsilon+\bar{h}\right)^2\right)} = \frac{1}{2(2r-\epsilon)\left(\epsilon-2\bar{h}\right)} < \infty$.

For $i = 1, \ldots, N$, $j = 0, 1, \ldots, N$, $i \neq j$, let $\bar{\gamma}_{ij} = \gamma_{ij} - \gamma$ with some unknown $\gamma > 0$. Let $\bar{S}_{i2} = S_{i2} - S_{02}$, $\bar{\xi}_i = \xi_i - x_i$, $\bar{\omega}_i = \hat{\omega}_i - \omega_i$, $i = 1, \ldots, N$, and $\bar{\eta}_i = \eta_i - x_0$, $\bar{S}_i = S_i - S_0$, $\bar{\xi}_{2i} = \xi_{2i} - p_i$, $i = 0, 1, \ldots, N$. The closed-loop system composed of (7) and (9) is as follows:

$$\dot{\bar{q}}_i = \bar{p}_i \tag{11a}$$

$$\begin{aligned}\dot{\bar{p}}_i = & -\alpha \sum_{j\in\bar{\mathcal{N}}_i(t)} \nabla_{\bar{q}_i}\psi\left(\|\bar{q}_i - \bar{q}_j\|\right) - \sum_{j\in\bar{\mathcal{N}}_i(t)} \left(\bar{p}_i - \bar{p}_j\right) \\ & - \left(\sum_{j\in\mathcal{N}_i(t)} \left(\bar{\xi}_{2i} - \bar{\xi}_{2j}\right) + a_{i0}(t)\bar{\xi}_{2i} + D_i\bar{w}_i\right) \\ & + \bar{S}_{i2}\bar{\eta}_i + S_{02}\bar{\eta}_i + \bar{S}_{i2}x_0\end{aligned} \tag{11b}$$

$$\begin{bmatrix}\dot{\bar{\xi}}_i \\ \dot{\bar{w}}_i\end{bmatrix} = \left(\hat{A}_i + L_i\hat{C}_i\right)\begin{bmatrix}\bar{\xi}_i \\ \bar{w}_i\end{bmatrix} \tag{11c}$$

$$\dot{\bar{S}}_i = \beta \sum_{j\in\bar{\mathcal{N}}_i(t)} \left(\bar{S}_j - \bar{S}_i\right) \tag{11d}$$

$$\dot{\bar{\eta}}_i = \bar{S}_i\bar{\eta}_i + S_0\bar{\eta}_i + \bar{S}_ix_0 + \sum_{j\in\bar{\mathcal{N}}_i(t)} \gamma_{ij}\left(\bar{\eta}_j - \bar{\eta}_i\right) \tag{11e}$$

$$\dot{\bar{\gamma}}_{ij} = k_{ij}a_{ij}(t)\left(\bar{\eta}_i - \bar{\eta}_j\right)^\top\left(\bar{\eta}_i - \bar{\eta}_j\right), \tag{11f}$$

$$i = 1,\dots,N,\ j = 0,1,\dots,N,\ i \neq j.$$

For $t \geq 0$, we let $P(t) = \begin{bmatrix} H(t)\otimes I_n & \frac{\Lambda(t)}{2} & Z_1 \\ \frac{\Lambda^\top(t)}{2} & \theta I_\iota & 0_{\iota\times 2Nn} \\ Z_1^\top & 0_{2Nn\times\iota} & Y(t)\end{bmatrix}$ where $\Lambda(t) = \left[0_{Nn\times Nn}\ \ H(t)\otimes I_n\ \ \text{block diag}(D_1,\dots,D_N)\right]$, $\iota = 2Nn + s_1 + \dots + s_N$, $Z_1 = -\frac{1}{2}I_N \otimes S_{02}$, θ is some positive real number, and $Y(t) = \gamma H(t)\otimes I_{2n} - I_N \otimes \frac{S_0 + S_0^\top}{2}$ with γ being some positive real number.

The solvability of Problem 1 is summarized as follows.

Theorem 3.1: Under Assumption 1, Problem 1 is solvable by the distributed position feedback control law (9).

Proof: The proof consists of four parts.

Part I: Let $\bar{\eta} = \text{col}\left(\bar{\eta}_1,\dots,\bar{\eta}_N\right)$, $\bar{q} = \text{col}\left(\bar{q}_1,\dots,\bar{q}_N\right)$, $\bar{p} = \text{col}\left(\bar{p}_1,\dots,\bar{p}_N\right)$, $\bar{\xi} = \text{col}\left(\bar{\xi}_1,\dots,\bar{\xi}_N\right)$, $\mu_i = \text{col}\left(\bar{\xi}_i, \bar{\omega}_i\right)$, $i = 1,\dots,N$, $\mu = \text{col}\left(\mu_1,\dots,\mu_N\right)$, $\bar{\gamma} = \text{col}\left(\bar{\gamma}_{10},\dots,\bar{\gamma}_{N0},\bar{\gamma}_{12},\dots,\bar{\gamma}_{1N},\bar{\gamma}_{1N},\dots,\bar{\gamma}_{(N-1)N}\right)$, and $\bar{\mu} = \text{col}\left(\bar{\xi}_{11},\dots,\bar{\xi}_{1N},\bar{\xi}_{21},\dots,\bar{\xi}_{2N},\bar{w}_1,\dots,\bar{w}_N\right) = T\mu$ with T being some orthogonal matrix satisfying $(T^{-1})^\top T^{-1} = I_\iota$.

Consider the following energy function:

$$\begin{aligned}& V\left(\bar{q},\bar{p},\mu,\bar{\eta},\bar{\gamma},t\right) \\ = & \frac{1}{2}\sum_{i=1}^{N}\left(\alpha \sum_{j\in\mathcal{N}_i(t)} \psi\left(\|\bar{q}_i - \bar{q}_j\|\right) + 2\alpha a_{i0}(t)\psi\left(\|\bar{q}_i\|\right)\right. \\ & \left. + \bar{p}_i^\top\bar{p}_i + 2\theta\mu_i^\top\bar{P}_i\mu_i\right) + \frac{1}{2}\sum_{i=1}^{N}\bar{\eta}_i^\top\bar{\eta}_i \\ & + \frac{1}{2}\sum_{i=1}^{N}\left(\sum_{j\in\mathcal{N}(t)} \frac{\bar{\gamma}_{ij}^2}{2k_{ij}} + a_{i0}(t)\frac{\bar{\gamma}_{i0}^2}{k_{i0}}\right)\end{aligned} \tag{12}$$

with $\bar{P}_i$, $i = 1,\dots,N$, being defined in Remark 3.1.

Let $\bar{S} = \text{block diag}(\bar{S}_1,\dots,\bar{S}_N)$ and $\hat{S} = \text{block diag}(\bar{S}_{12},\dots,\bar{S}_{N2})$. The time derivative of (12) along the trajectories of the closed-loop system (11) is

$$\begin{aligned}\dot{V} = & -\bar{p}^\top\left(H(t)\otimes I_n\right)\bar{p} - \bar{p}^\top\Lambda(t)\bar{\mu} - \theta\bar{\mu}^\top(T^{-1})^\top T^{-1}\bar{\mu} \\ & + \bar{p}^\top\hat{S}\bar{\eta} + \bar{p}^\top\left(I_N\otimes S_{02}\right)\bar{\eta} + \bar{p}^\top\hat{S}\left(\mathbf{1}_N\otimes x_0\right) \\ & + \bar{\eta}^\top\bar{S}\bar{\eta} + \bar{\eta}^\top\bar{S}\left(\mathbf{1}_N\otimes x_0\right) + \bar{\eta}^\top\left(I_N\otimes S_0\right)\bar{\eta} \\ & - \gamma\bar{\eta}^\top\left(H(t)\otimes I_{2n}\right)\bar{\eta} \\ = & -\begin{bmatrix}\bar{p}\\ \bar{\mu}\\ \bar{\eta}\end{bmatrix}^\top P(t)\begin{bmatrix}\bar{p}\\ \bar{\mu}\\ \bar{\eta}\end{bmatrix} + \bar{\eta}^\top\bar{S}\bar{\eta} \\ & + \bar{\eta}^\top\bar{S}\left(\mathbf{1}_N\otimes x_0\right) + \bar{p}^\top\hat{S}\bar{\eta} + \bar{p}^\top\hat{S}\left(\mathbf{1}_N\otimes x_0\right).\end{aligned} \tag{13}$$

For any $\epsilon_1 > 0$, let $\varrho = \frac{1+\epsilon_1}{\epsilon_1}$. Then, we have

$$\begin{aligned}& \bar{\eta}^\top\bar{S}\bar{\eta} + \bar{\eta}^\top\bar{S}\left(\mathbf{1}_N\otimes x_0\right) + \bar{p}^\top\hat{S}\bar{\eta} + \bar{p}^\top\hat{S}\left(\mathbf{1}_N\otimes x_0\right) \\ \leq & \frac{\epsilon_1}{2}\|\bar{p}\|^2 + \left(\frac{1}{2} + \varrho\left\|\hat{S}\right\|^2\right)\|\bar{\eta}\|^2 + \varrho\left\|\hat{S}\left(\mathbf{1}_N\otimes x_0\right)\right\|^2.\end{aligned} \tag{14}$$

Let $Q(t) = P(t) - \begin{bmatrix}\frac{\epsilon_1}{2}I_{Nn} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \left(\frac{1}{2} + \varrho\left\|\hat{S}\right\|^2\right)I_{2Nn}\end{bmatrix}$. Then, substituting (14) into (13) gives

$$\dot{V} \leq -\begin{bmatrix}\bar{p}\\ \bar{\mu}\\ \bar{\eta}\end{bmatrix}^\top Q(t)\begin{bmatrix}\bar{p}\\ \bar{\mu}\\ \bar{\eta}\end{bmatrix} + \varrho\left\|\hat{S}\left(\mathbf{1}_N\otimes x_0\right)\right\|^2. \tag{15}$$

By [4, Lemma 4.1], for any $t_a > 0$, if $\bar{\mathcal{G}}(t)$ is connected and $\mathcal{G}(t)$ is undirected for $t \in [0, t_a)$, then there exist positive constants θ, γ, and ϵ_1 such that $Q(t)$ is positive definite for all $t \in [0, t_a)$. Fix such θ, γ, and ϵ_1.

Part II: Next, let us show that under the control law (9), the graph $\bar{\mathcal{G}}(t)$ is connected for all $t \geq 0$. Let $V(t) = V(\bar{q}(t), \bar{p}(t), \mu(t), \bar{\eta}(t), \bar{\gamma}(t), t)$. Note that, by the continuity of the solution of system (11), there exists a $0 < t_1 \leq \infty$ such that $\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(t_1)$ for all $t \in [0, t_1)$. If $t_1 = \infty$, $\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(0)$ is connected for all $t \geq 0$. If $t_1 < \infty$, $\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(0)$ does not hold for all $t \geq 0$. We assume without loss of generality that t_1 is such that

$$\begin{aligned}\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(0),\ t \in [0, t_1) \\ \bar{\mathcal{G}}(t_1) \neq \bar{\mathcal{G}}(0).\end{aligned} \tag{16}$$

We claim that $\bar{\mathcal{G}}(t_1) \supset \bar{\mathcal{G}}(0)$. Since $\bar{\mathcal{G}}(t)$ is connected for all $t \in [0, t_1)$, by [4, Lemma 4.1], the positive constants θ, γ, and ϵ_1 are such that $Q(t)$ is positive definite for all $t \in [0, t_1)$. Thus, (15) implies

$$\dot{V}(t) \leq \varrho\left\|\hat{S}(t)\left(\mathbf{1}_N\otimes x_0(t)\right)\right\|^2,\ \forall t \in [0, t_1). \tag{17}$$

Under Assumption 1, for all $t \geq 0$, $\|x_0(t)\| \leq \hat{p}(t)$ for some polynomial $\hat{p}(t)$ of t. Thus, Remark 3.2 implies that $\lim_{t\to\infty}\hat{S}(t)\left(\mathbf{1}_N\otimes x_0(t)\right) = 0$ exponentially, that is, for any $\hat{S}(0)$ and $x_0(0)$, there exist positive real numbers ϱ_1 and φ_1 such that, for all $t \geq 0$,

$$\left\|\hat{S}(t)\left(\mathbf{1}_N\otimes x_0(t)\right)\right\| \leq \varrho_1\left\|\hat{S}(0)\left(\mathbf{1}_N\otimes x_0(0)\right)\right\|e^{-\varphi_1 t}. \tag{18}$$

Then, for all $0 \leq t < t_1$, we have

$$\dot{V}(t) \leq \varrho\varrho_1^2\left\|\hat{S}(0)\left(\mathbf{1}_N\otimes x_0(0)\right)\right\|^2 e^{-2\varphi_1 t} = \varrho_2 e^{-\varphi_2 t} \tag{19}$$

where $\varrho_2 = \varrho\varrho_1^2 \left\|\hat{S}(0)\left(\mathbf{1}_N \otimes x_0(0)\right)\right\|^2$ and $\varphi_2 = 2\varphi_1$. Then, for all $0 \le t < t_1$,

$$V(t) \le V(0) + \frac{\varrho_2}{\varphi_2}\left(1 - e^{-\varphi_2 t}\right) \le V(0) + \frac{\varrho_2}{\varphi_2}. \quad (20)$$

Now, we show $\bar{\mathcal{G}}(t_1) \supset \bar{\mathcal{G}}(0)$ by contradiction. Suppose this is not the case, then we can find some edge (i,j) such that $(i,j) \in \bar{\mathcal{E}}(0)$ and $(i,j) \notin \bar{\mathcal{E}}(t_1)$. Hence, $\lim_{t\to t_1^-} \|q_i(t) - q_j(t)\| = r$ and $\lim_{t\to t_1^-} \|\bar{q}_i(t) - \bar{q}_j(t)\| \ge r - \bar{h}$. Note that $\|\bar{q}_i(t) - \bar{q}_j(t)\|$ is a continuous function of t. Thus, there exists a $\hat{t}_1 \in (0, t_1]$ such that $\lim_{t\to \hat{t}_1^-} \|\bar{q}_i(t) - \bar{q}_j(t)\| = r - \bar{h}$. Therefore, we can conclude that $\lim_{t\to \hat{t}_1^-} V(t) = \infty$, which, however, contradicts (20). The contradiction shows that $\bar{\mathcal{G}}(t_1) \supset \bar{\mathcal{G}}(0)$, and $\bar{\mathcal{G}}(t_1)$ is connected.

If there exists a $t_2 > t_1$ such that

$$\begin{aligned} \bar{\mathcal{G}}(t) &= \bar{\mathcal{G}}(t_1),\ t \in [t_1, t_2) \\ \bar{\mathcal{G}}(t_2) &\ne \bar{\mathcal{G}}(t_1) \end{aligned} \quad (21)$$

then we claim that $\bar{\mathcal{G}}(t_2) \supset \bar{\mathcal{G}}(t_1)$. In fact, since $\bar{\mathcal{G}}(t)$ is connected for all $t \in [t_1, t_2)$, by [4, Lemma 4.1], the positive constants θ, γ, and ϵ_1 are such that $Q(t)$ is positive definite for all $t \in [t_1, t_2)$. From (15), we have

$$\dot{V}(t) \le \varrho \left\|\hat{S}(t)\left(\mathbf{1}_N \otimes x_0(t)\right)\right\|^2,\ \forall t \in [t_1, t_2). \quad (22)$$

Then, applying (18) to (22) yields

$$\dot{V}(t) \le \varrho\varrho_1^2 \left\|\hat{S}(0)\left(\mathbf{1}_N \otimes x_0(0)\right)\right\|^2 e^{-2\varphi_1 t} = \varrho_2 e^{-\varphi_2 t}. \quad (23)$$

Since $\bar{\mathcal{G}}(t_1) \supset \bar{\mathcal{G}}(0)$, there exists at least one edge (i,j) such that $(i,j) \in \bar{\mathcal{E}}(t_1)$ and $(i,j) \notin \bar{\mathcal{E}}(0)$. We can assume without loss of generality that there exist τ edges $(i_1, j_1), \ldots, (i_\tau, j_\tau)$ such that $(i_k, j_k) \in \bar{\mathcal{E}}(t_1)$ and $(i_k, j_k) \notin \bar{\mathcal{E}}(0)$ for $k \in \{1, \ldots, \tau\}$, where τ is a positive integer. Recalling (20), by Remark 3.4, $V(t_1)$ satisfies

$$V(t_1) \le V(0) + \frac{\varrho_2}{\varphi_2} + \tau\alpha\psi\left(r - \epsilon + \bar{h}\right). \quad (24)$$

Thus, from (23), for all $t \in [t_1, t_2)$,

$$\begin{aligned} V(t) &\le V(t_1) + \frac{\varrho_2}{\varphi_2}\left(e^{-\varphi_2 t_1} - e^{-\varphi_2 t}\right) \\ &\le V(0) + \frac{\varrho_2}{\varphi_2} + \tau\alpha\psi\left(r - \epsilon + \bar{h}\right) + \frac{\varrho_2}{\varphi_2} e^{-\varphi_2 t_1}. \end{aligned} \quad (25)$$

We also show $\bar{\mathcal{G}}(t_2) \supset \bar{\mathcal{G}}(t_1)$ by contradiction. Suppose $\bar{\mathcal{G}}(t_2) \not\supset \bar{\mathcal{G}}(t_1)$, that is, there exists some edge (i,j) such that $(i,j) \in \bar{\mathcal{E}}(t_1)$ and $(i,j) \notin \bar{\mathcal{E}}(t_2)$. Then, $\lim_{t\to t_2^-} \|q_i(t) - q_j(t)\| = r$, which implies $\lim_{t\to t_2^-} \|\bar{q}_i(t) - \bar{q}_j(t)\| \ge r - \bar{h}$. Note that there exists a $\hat{t}_2 \in (0, t_2]$ such that $\lim_{t\to \hat{t}_2^-} \|\bar{q}_i(t) - \bar{q}_j(t)\| = r - \bar{h}$ since $\|\bar{q}_i(t) - \bar{q}_j(t)\|$ is a continuous function of t. Then, $\lim_{t\to \hat{t}_2^-} V(t) = \infty$, which contradicts (25). Hence, $\bar{\mathcal{G}}(t_2) \supset \bar{\mathcal{G}}(t_1)$.

Moreover, note that $\bar{\mathcal{G}}(t)$ can only have a finite number of edges. Hence, by repeating the above arguments, we conclude that there exists a finite integer $k > 0$ such that

$$\begin{aligned} &\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(0),\ t \in [0, t_1) \\ &\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(t_i) \supset \bar{\mathcal{G}}(t_{i-1}),\ t \in [t_i, t_{i+1}),\ i = 1, \ldots, k-1 \\ &\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(t_k) \supset \bar{\mathcal{G}}(t_{k-1}),\ t \in [t_k, \infty). \end{aligned} \quad (26)$$

Thus, the graph $\bar{\mathcal{G}}(t)$ remains connected for all $t \ge 0$ under the control law (9). Moreover, for all $t \ge t_k$,

$$\dot{V}(t) \le \varrho_2 e^{-\varphi_2 t}. \quad (27)$$

Part III: In this part, we show that $\lim_{t\to\infty}(\dot{q}_i(t) - \dot{q}_0(t)) = 0$, $i = 1, \ldots, N$. From (27), we have

$$\begin{aligned} V(t) &\le V(t_k) + \frac{\varrho_2}{\varphi_2}\left(e^{-\varphi_2 t_k} - e^{-\varphi_2 t}\right) \\ &\le V(t_k) + \frac{\varrho_2}{\varphi_2} e^{-\varphi_2 t_k},\ \forall t \ge t_k. \end{aligned} \quad (28)$$

Thus, $V(t)$ is bounded for all $t \ge t_k$, which implies that $\bar{p}(t)$, $\bar{\eta}(t)$, $\bar{q}(t)$, $\mu(t)$, $\bar{\gamma}(t)$, and $\bar{q}_i(t) - \bar{q}_j(t)$, $i = 1, \ldots, N$, $j \in \bar{\mathcal{N}}_i(t_k)$ are bounded over $[t_k, \infty)$. Note that $\bar{S}(t)$ is bounded since $\lim_{t\to\infty} \bar{S}(t) = 0$ exponentially. From (11), for all $t \ge t_k$ and for $i = 1, \ldots, N$, $\dot{\bar{p}}_i$, $\dot{\mu}_i$, $\dot{\bar{\eta}}_i$, and $\dot{\bar{S}}_i$ are bounded. Hence, for all $t \ge t_k$, $\ddot{V}(t)$ is bounded, which implies that $\dot{V}(t)$ is uniformly continuous over $[t_k, \infty)$. By a direct calculation,

$$\begin{aligned} &\lim_{t\to\infty} \int_{t_k}^{t} \dot{V}(\tau) d\tau = \lim_{t\to\infty} \left(V(t) - V(t_k)\right) \\ &\le \lim_{t\to\infty} \int_{t_k}^{t} \varrho_2 e^{-\varphi_2 \tau} d\tau \le \frac{\varrho_2}{\varphi_2} e^{-\varphi_2 t_k}. \end{aligned} \quad (29)$$

Thus, $\lim_{t\to\infty} \int_{t_k}^{t} \dot{V}(\tau) d\tau$ exists and is finite. By Barbalat's lemma (see [7, Lemma 8.2]), we have $\lim_{t\to\infty} \dot{V}(t) = 0$. Using (18) in (13), for $i = 1, \ldots, N$, we further have $\lim_{t\to\infty} \bar{p}_i(t) = 0$, $\lim_{t\to\infty} \bar{\eta}_i(t) = 0$, and $\lim_{t\to\infty} \bar{\mu}_i(t) = 0$. Hence, $\lim_{t\to\infty}(\dot{q}_i(t) - \dot{q}_0(t)) = 0$, $i = 1, \ldots, N$.

Part IV: Finally, we show that $\lim_{t\to\infty}(q_i(t) - (q_0(t) + h_i)) = 0$, $i = 1, \ldots, N$. It can be verified that $\ddot{\bar{p}}_i(t)$ is bounded over $[t_k, \infty)$. Hence, $\dot{\bar{p}}(t)$ is uniformly continuous over $[t_k, \infty)$. By Barbalat's lemma again,

$$\begin{aligned} &\lim_{t\to\infty} \dot{\bar{p}}_i(t) \\ &= -\alpha \sum_{j\in\bar{\mathcal{N}}_i(t)} \frac{\bar{q}_i(t) - \bar{q}_j(t)}{\left(\left(r - \bar{h}\right)^2 - \|\bar{q}_i(t) - \bar{q}_j(t)\|^2\right)^2} = 0. \end{aligned} \quad (30)$$

We let

$$w_{ij}(t) = \begin{cases} \frac{1}{\left(\left(r-\bar{h}\right)^2 - \|\bar{q}_i(t) - \bar{q}_j(t)\|^2\right)^2}, & (j,i) \in \bar{\mathcal{E}}(t) \\ 0, & \text{otherwise.} \end{cases} \quad (31)$$

Thus, $w_{ij}(t)$ is nonnegative and bounded for all $t \ge t_k$. Let

$$H_1(t) = \begin{bmatrix} \hat{\beta}_1(t) & -w_{12}(t) & \cdots & -w_{1N}(t) \\ -w_{12}(t) & \hat{\beta}_2(t) & \cdots & -w_{2N}(t) \\ \vdots & \vdots & \ddots & \vdots \\ -w_{1N}(t) & -w_{2N}(t) & \cdots & \hat{\beta}_N(t) \end{bmatrix} \quad (32)$$

where $\hat{\beta}_i(t) = \sum_{j=0,\ j\ne i}^{N} w_{ij}(t)$. From (30), we have

$$\lim_{t\to\infty} \left(H_1(t) \otimes I_n\right) \bar{q}(t) = 0 \quad (33)$$

As shown in the proof of [10, Theorem 3.1], $H_1(t)$ is positive definite for all $t \ge t_k$. Thus, (33) implies that

$\lim_{t\to\infty} \bar{q}_i(t) = \lim_{t\to\infty}(q_i(t) - (q_0(t) + h_i)) = 0$, $i = 1, \ldots, N$, which completes the proof. □

Remark 3.5: The distributed control law (9) is more complex than the one in [10] since (9) incorporates the self-tuning adaptive distributed observer (9d)–(9f), while the control law in [10] used the conventional distributed observer (10). To overcome the additional technical challenges, we have modified the energy function V in [10] to its current form (12) and employed a different approach to study the convergence property of the closed-loop system.

IV. Example

In this section, we provide an example to illustrate our result. Consider system (1) with $N = 3$ with $q_i \in \mathbb{R}^2$ and the leader (3) with $S_0 = \left[\begin{smallmatrix} 0 & 1 \\ -0.4 & -1 \end{smallmatrix}\right] \otimes I_2$. For $i = 1, 2, 3$, the external disturbance $d_i \in \mathbb{R}^2$ is generated by (2) with $Q_1 = \left[\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right]$, $Q_2 = 1$, $Q_3 = \left[\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right]$, $D_1 = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right]$, $D_2 = \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right]$, and $D_3 = \left[\begin{smallmatrix} 1 & 0 \\ -1 & 1 \end{smallmatrix}\right]$. Let $r = 10$ be the maximum sensing range and $\epsilon = 4$. Let $h_1 = \text{col}(0, 0.8603)$, $h_2 = \text{col}(-0.7450, -0.4301)$, $h_3 = \text{col}(0.7450, -0.4301)$. Let $\alpha = 10000$, $\beta = 1$, and $k_{ij} = 1$, $i = 1, 2, 3$, $j = 0, 1, 2, 3$. We choose a L_i that makes $\hat{A}_i + L_i \hat{C}_i$ Hurwitz.

In the simulation, we let $q_0(0) = \text{col}(-1, 2)$, $q_1(0) = \text{col}(-1, -2)$, $q_2(0) = \text{col}(-2, -7)$, $q_3(0) = \text{col}(4, -7.5)$, and other initial conditions are randomly generated. Then, it can be verified that a connected graph $\bar{\mathcal{G}}(0)$ is formed with edge set $\bar{\mathcal{E}}(0) = \{(0,1), (1,2), (2,1), (2,3), (3,2)\}$. Under the above conditions, the simulation results for the closed-loop system are shown in Figs. 1 and 2. Specifically, Fig. 1 shows that the followers approach their desired positions and their velocities synchronize with the leader's velocity asymptotically, that is, formation tracking is achieved. Fig. 2 shows that the distances between any two initially connected agents remain smaller than the maximum sensing range $r = 10$. Thus, the connectivity of the network is preserved.

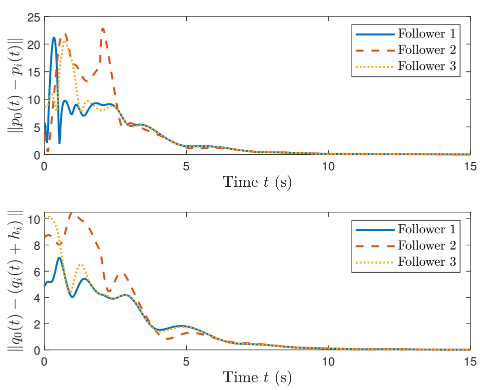

Fig. 1. Velocity and position tracking errors of followers

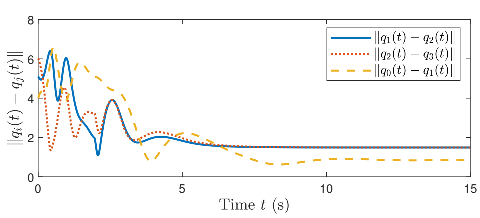

Fig. 2. Distances between any two vehicles with initially connected edge

V. Conclusion

In this letter, we have studied the distributed formation tracking problem with connectivity preservation for multiple double-integrator systems by a self-tuning adaptive distributed observer. By employing the self-tuning adaptive distributed observer, we have removed the restrictive assumption in [10] that all followers know the system matrix of the leader and eliminated the need of computing the observer gain offline, which makes our approach more practical.

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