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A Peaking Free Time-varying High-gain Observer with Reduced Sensitivity to Measurement Noise

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***Abstract*— This study is concerned with observer design for a class of Lipschitz nonlinear systems. A high-gain observer with a straightforward structure is proposed. As opposed to the well-known high gain observers, dynamic gains obtained are used to reduce the effect of peaking. In addition, the injection term of the observer is passed through a linear filter to reduce its sensitivity to noise. It is shown that the suggested observer is peaking free with respect to the initial conditions, while achieving the input to state stability with respect to measurement noise, as a HGO. The analysis of the steady-state response also shows that the proposed observer performs better in the presence of high-frequency noise. The simulation results compare the performance of the proposed method with some existing observers.**

I. INTRODUCTION

High-gain observers have been widely used in the nonlinear dynamical control systems literature due to their simple structure, simplicity in tuning, and robustness to model uncertainties. In fact, it has been demonstrated that they are able to provide a reliable estimate even when the system's nonlinearity is unknown [1]. For these observers, a nonlinear separation principle is also provided [2] which allows the implementation of stabilizing state feedback controllers for nonlinear systems using output feedback. One caveat is the requirement for high observer gains which results in the so-called peaking phenomena. Even with a stabilizing controller, peaking can destabilize the closed loop. The large gains also affect the region of attraction. The use of saturation functions in the control input (see section 14.5.1 in [3]) has been proposed to avoid the problems associated with the peaking. However, a certain pre-knowledge is required about the system's behaviour to choose a suitable saturation level that saturates the estimates outside a region of interest. An alternative approach is the use of time-varying gains as proposed in [4]–[7] which is shown to effectively eliminate the peaking [8]. The simultaneous use of multiple observers has been investigated in [9], [10] where a weighted average of the individual estimates is used to produce state estimates with reduced peaking and improved transient behaviour.

The aforementioned methods, as well as the HGO, remain vulnerable to measurement noise. In each case, even small noise can have a large effect on the estimates due to the use of large gains (see Figure 6 in [4]). To reduce the sensitivity to noise, the combination of the HGO with the extended Kalman filters is performed in [11]. A re-design of the HGO observers with gains powered up to 2 is proposed in [12] that reduces the steady-state bounds on the estimates by increasing the relative degree between the system's output and the estimations. Different low-pass filters have also been employed in [13]–[15] to achieve similar goals. Only a few works in the literature address both the peaking and sensitivity to noise. We note the method proposed in [16] where saturation functions are used in low-power observers [12] to avoid the peaking. As discussed above for the saturation of control inputs, this approach requires the knowledge of the upper-bound of the system's states. In [17], [18], the MHGO is combined with low-power and filtered HGOs to improve the results. However, these observers require the tuning of a number of parameters which can be difficult in practice.

In this manuscript, we propose an HGO observer implemented with two additional subsystems. The first subsystem is a linear filter, as suggested in [14], that reduces the observer's sensitivity to noise. The second subsystem is a differential Riccati equation that determines dynamic observer gains. The proposed observer has a simple structure which requires no tuning, except for the choice of the initial conditions of the observer states and a large gain parameter that is used to arbitrarily adjust the convergence speed of the observer.

This paper is organized as follows. Section II presents some preliminaries on HGOs. Section III introduces the dynamics of the observer, and investigates the convergence, peaking, and the sensitivity to noise of the observer. In section IV, a numerical example is used to compare the estimations provided by the observer, a conventional HGO, a HGO with a linear filter, and a TV HGO. Finally, we conclude the results in section V.

II. PROBLEM FORMULATION AND PRELIMINARIES

In this paper, we consider nonlinear dynamical systems expressed in the canonical observable form [19]:

$$\begin{aligned} \dot{x}(t) &= A_n x(t) + B_n \phi(x(t), u(t)) \\ y(t) &= C_n x(t) + v(t) \end{aligned} \tag{1}$$

where $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$ is the vector of state variables, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the system's input and output variables, respectively. The variable $v(t)$ denotes the measurement noise. It is assumed to be bounded, i.e., for some positive μ, we have $|v(t)| \leq \mu$ for all $t \geq 0$. The matrices $A_n \in \mathbb{R}^{n\times n}$, $B_n \in \mathbb{R}^{n\times 1}$ and $C_n \in \mathbb{R}^{1\times n}$ are given as follows

$$\begin{aligned} A_n &= \begin{bmatrix} 0_{(n-1)\times 1} & I_{n-1} \\ 0 & 0_{1\times(n-1)} \end{bmatrix} \\ B_n &= \begin{bmatrix} 0_{1\times(n-1)} & 1 \end{bmatrix}^T, C_n = \begin{bmatrix} 1 & 0_{1\times(n-1)} \end{bmatrix} \end{aligned} \tag{2}$$

where I_n shows the $n \times n$ identity matrix. The function $\phi(x, u)$, with $\phi(0,0) = 0$, is assumed to be locally Lipschitz in (x, u) for all $x \in \mathcal{X} \subseteq \mathbb{R}^n$, where $\mathcal{X}$ is a bounded set, uniformly in $u \in \mathcal{U} \subseteq \mathbb{R}$. That is, the following inequality holds for $x \in \mathcal{X}, u \in \mathcal{U}$, and a positive constant $\bar{\phi}$:

$$\|\phi(x,u) - \phi(\bar{x},u)\| \leq \bar{\phi}\|x - \bar{x}\|. \tag{3}$$

Note that $\|.\|$ indicates the Euclidean norm throughout the paper. Given the triangular form (1), it follows that the designed observer for this system also performs as a numerical differentiator since it provides estimates of the derivatives of the first state variable. A conventional HGO for (1) is given by

$$\dot{\hat{x}}(t) = A_n\hat{x}(t) + B_n\phi_s(\hat{x}(t), u(t)) + \frac{1}{\epsilon}\Gamma_\epsilon^{-1}\bar{K}(y(t) - C\hat{x}(t)) \tag{4}$$

where $\Gamma_\epsilon = \operatorname{diag}(1, \epsilon, \epsilon^2, \ldots, \epsilon^{n-1})$, and $\bar{K} = [k_1, \ldots, k_n]^T \in \mathbb{R}^n$ is chosen such that the matrix $\bar{A}_n = A_n - \bar{K}C_n$ is Hurwitz. If ϕ is globally Lipschitz, then ϕ_s can be equal to ϕ. If it is locally Lipschitz, then ϕ_s agrees with ϕ on the domain of interest. It is also saturated outside this domain, i.e., we let $\phi_s = \operatorname{sat}_R(\phi)$ where R indicates the saturation level that is chosen according to the set in which (3) holds. For the observer (4) and the system (1), it is shown in [1] that the following bounds hold for $x(0) \in \mathcal{X}_0, u \in \mathcal{U}, x \in \mathcal{X}$, where $\mathcal{X}_0$ a compact set inside $\mathcal{X}$, a small ϵ, and some positive constants $\kappa_1, \kappa_2, \kappa_3$ independent of ϵ:

$$\|x_i(t) - \hat{x}_i(t)\| \leq \frac{\kappa_1}{\epsilon^{i-1}}\exp(-\frac{\kappa_2}{\epsilon}t)\|x(0) - \hat{x}(0)\| + \frac{\kappa_3}{\epsilon^{i-1}}\mu. \tag{5}$$

In the absence of noise ($\mu = 0$), the convergence rate of the observer can be assigned by adjusting the design parameter ϵ. Moreover, the HGO (4) is ISS with respect to measurement noise. Equation (5) shows that the estimation error peaks to the order of ϵ^{1-n} for the n^{th} state variable, x_n. However, it does not provide a clear assessment of the sensitivity of the HGO to noise since the analysis only considers an upper bound for the noise. The steady-state behaviour of a HGO is studied in more details in [20]. It is assumed that the measurement noise is the output of an autonomous system that generates n_v harmonics corresponding to the frequencies $\frac{\omega_i}{\delta}$. The system is given by:

$$\begin{aligned} \delta\dot{w} &= \mathcal{S}w, \; w \in \mathbb{R}^{2n_v} \\ v &= \mathcal{P}w \end{aligned} \tag{6}$$

where the matrices $\mathcal{S} \in \mathbb{R}^{2n_v \times 2n_v}, \mathcal{P} \in \mathbb{R}^{1 \times 2n_v}$ are $\mathcal{S} := \operatorname{blkdiag}(\mathcal{S}_1, \ldots, \mathcal{S}_{n_v})$, with $\mathcal{S}_i = \begin{bmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{bmatrix}$, and $\mathcal{P} := \begin{bmatrix} 0 & 1 & 0 & 1 & \cdots & 0 & 1 \end{bmatrix}$. The analysis captures the low-pass filtering feature of an HGO leading to the bound:

$$\lim_{t\to\infty}\sup|x_i(t) - \hat{x}_i(t)| \leq \frac{\rho_i\delta}{\epsilon^i}\mu \tag{7}$$

for some positive constants ρ_i. Equation (7) shows how the ultimate value of the estimates is affected by the relative degree between the output measurement and the estimates for a standard HGO. There have been several attempts (*e.g.*, in [12], [14], [15]) to augment the relative degree and increase the effect of δ at steady-state. Using different filtering methods, it can be shown that higher powers of δ can appear in bounds such as (7) which indicate improvements in the estimation since $\delta \in (0,1)$.

III. The Filtered time-varying Observer design

In this section, we present the proposed observer. We also provide an analysis of the performance of the observer in the presence of high-frequency noise. We consider the TVHGO initially proposed in [4], [8] that is implemented using the filtering technique proposed in [14]. The dynamics of the resulting filtered time-varying high-gain observer (FTV HGO) are given as follows:

$$\begin{aligned} \dot{\hat{x}}(t) =& A_n\hat{x}(t) + B_n\phi_s(\hat{x}(t), u(t)) + \Gamma_\epsilon G(t)z(t) \\ \dot{z}(t) =& -\frac{1}{\epsilon}Dz(t) + \frac{1}{\epsilon^2}A_n^T z(t) + \frac{1}{\epsilon}C_n^T(y - C_n\hat{x}(t)) \\ \dot{L}(t,\epsilon) =& -\frac{1}{\epsilon}L(t,\epsilon) - A_n^T L(t,\epsilon) - L(t,\epsilon)A_n + C_n^T C_n \end{aligned} \tag{8}$$

where the positive definite solution of the matrix $L(t)$ in (8) is explicitly expressed as [4]:

$$\begin{aligned} L(t,\epsilon) = & e^{-\frac{1}{\epsilon}t}e^{-A_n^T t}L(0)e^{-A_n t} \\ & + \int_0^t e^{-\frac{1}{\epsilon}(t-\tau)}e^{-A_n^T(t-\tau)}C_n^T C_n e^{-A_n(t-\tau)}d\tau. \end{aligned} \tag{9}$$

and $G(t) = \operatorname{diag}(L^{-1}(t)C_n^T)$, $x(0) = x_0 \in \mathcal{X}_0, z(0) = 0, L(0) = I_n$ and $D = 2I_n$.

Theorem 1: Let the trajectories of (1) be such that $x \in \mathcal{X}$ for all $u \in \mathcal{U}$, and let the noise satisfy $|v(t)| \leq \mu \; \forall t \geq 0$. For the observer (8), choose $x(0) = x_0 \in \mathcal{X}_0 \subseteq \mathbb{R}^n, z(0) = 0, L(0) = I_n$ and $D = 2I_n$. Then, for a small enough ϵ and some positive constants $\hat{\kappa}_1, \hat{\kappa}_2, \hat{\kappa}_3, \alpha_1, \alpha_2, \gamma_2$, θ, ζ and σ independent of ϵ, the estimation error obtained from the observer satisfies

$$\begin{aligned} \|x_i(t) - \hat{x}_i(t)\| \leq & \frac{1}{\epsilon^{i-1}}\hat{\kappa}_1\exp(-\frac{\alpha_1}{\epsilon}t)\|e(0)\| + \frac{1}{\epsilon^{i-1}}\hat{\kappa}_2\mu + \\ & + \frac{1}{\epsilon^{i-1}}\hat{\kappa}_3(\exp(-\frac{\gamma_2}{\epsilon}t) - \exp(-\frac{\alpha_2}{\epsilon}t)) \end{aligned} \tag{10}$$

and

$$\|x(t) - \hat{x}(t)\| \leq \theta f(t,\epsilon)\|e(0)\| + \frac{\zeta\epsilon}{\lambda_{min}(L(t,\epsilon))}\mu + \sigma g(t,\epsilon) \tag{11}$$

for all $t \geq 0$, where $f(t,\epsilon)$ and $g(t,\epsilon)$ satisfy $\limsup_{\epsilon\to 0} f(t,\epsilon) = 1$, $\limsup_{\epsilon\to 0} g(t,\epsilon) = 0$, $\lim_{t\to\infty} f(t,\epsilon) = 0$, $\lim_{t\to\infty} g(t,\epsilon) = 0$, and $\lambda_{min}(L(t,\epsilon))$ denotes the smallest eigenvalue of the matrix $L(t,\epsilon)$.

Proof: Let us define the scaled estimation error as $\eta = \begin{bmatrix} \Gamma_\epsilon e \\ \Gamma_\epsilon z \end{bmatrix}$, where $e = x - \hat{x}$ is the vector of state estimation error. Considering the properties $\Gamma_\epsilon A_n \Gamma_\epsilon^{-1} = \frac{1}{\epsilon}A_n, \Gamma_\epsilon A_n^T \Gamma_\epsilon^{-1} = \epsilon A_n^T, \Gamma_\epsilon G(t)\Gamma_\epsilon^{-1} = G(t), \Gamma_\epsilon C_n^T C_n \Gamma_\epsilon^{-1} = C_n^T C_n, \Gamma_\epsilon C_n^T = C_n^T, \Gamma_\epsilon D \Gamma_\epsilon^{-1} = D, \Gamma_\epsilon B_n = \epsilon^{n-1}B_n$, and using (1) and (8) we can write

$$\dot{\eta} = \frac{1}{\epsilon}M(t)\eta + \epsilon^{n-1}\bar{B} + \frac{1}{\epsilon}\bar{C}v(t) \tag{12}$$

where

$$M(t) = \begin{bmatrix} A_n & -\epsilon\Gamma_\epsilon G(t) \\ C_n^T C_n & -D + A_n^T \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B_n \Delta_\phi(x, \hat{x}, u) \\ 0_{n\times 1} \end{bmatrix}$$

$$\Delta_\phi(x, \hat{x}, u) = \phi(x, u) - \phi_s(\hat{x}, u), \quad \bar{C} = \begin{bmatrix} 0_{n\times 1} \\ C_n^T \end{bmatrix}$$

From this point forward, unless otherwise specified, we will use L (or $L(t)$) to denote $L(t, \epsilon)$ for simplicity. We denote the steady-state solution of (8) (*i.e.,* $\dot{L}(t) = 0$) L_∞. We also denote by $\hat{x}_\infty(t), z_\infty(t)$ the trajectories of system (8) for the same initial conditions and by replacing G with G_∞ (and $\dot{L} = 0$). One can write $\eta = \eta_1 + \eta_2$ where $\eta_1 = \begin{bmatrix} \Gamma_\epsilon(x - \hat{x}_\infty) \\ \Gamma_\epsilon z_\infty \end{bmatrix}$, and $\eta_2 = \begin{bmatrix} \Gamma_\epsilon(\hat{x}_\infty - \hat{x}) \\ \Gamma_\epsilon(z - z_\infty) \end{bmatrix}$. Let g_i show the ith element of the vector $L_\infty^{-1} C^T$ for $i = 1, \ldots, n$. As discussed in [8], one gets from (8) that $L_\infty A_n - C_n^T C_n = -\frac{1}{\epsilon} L_\infty - A_n^T L_\infty$, which yields $L_\infty(A_n - L_\infty^{-1} C_n^T C_n) L_\infty^{-1} = -\frac{1}{\epsilon} I_n - A_n^T$. Let the notation $\Xi(.)$ denote the characteristic polynomial, then, following the properties of similar matrices we get $\Xi(A_n - L_\infty^{-1} C_n^T C_n) = \Xi(-\frac{1}{\epsilon} I_n - A_n^T)$. One has that $\Xi(-\frac{1}{\epsilon} I_n - A_n^T) = (s + \frac{1}{\epsilon})^n = s^n + \sum_{i=1}^n \frac{\mathcal{C}_i^n}{\epsilon^i} s^{n-i}$ where $\mathcal{C}_i^n = \binom{n}{i} = \frac{n!}{i!(n-i)!}$. Similarly, we can easily show that $\Xi(A_n - L_\infty^{-1} C_n^T C_n) = s^n + \sum_{i=1}^n g_i s^{n-i}$. As a result, we get $g_i = \frac{\mathcal{C}_i^n}{\epsilon^i}$, which shows the diagonal elements of the matrix G_∞. Next, one has

$$M_\infty = \begin{bmatrix} A_n & -\tilde{\mathcal{C}} \\ C_n^T C_n & -D + A_n^T \end{bmatrix} \tag{13}$$

where $\tilde{\mathcal{C}} = \text{diag}[\mathcal{C}_i^n]$ for $i = 1, \ldots, n$. To find the eigenvalues of M_∞, we use the equation $M_\infty X = sX$ where $X = [\xi_1, \ldots, \xi_n, \xi_{n+1}, \ldots, \xi_{2n}]^T$, along with (13) to write

$$\xi_2 - \mathcal{C}_1^n \xi_{n+1} = s\xi_1 \tag{14a}$$
$$\xi_3 - \mathcal{C}_2^n \xi_{n+2} = s\xi_2 \tag{14b}$$
$$\vdots$$
$$\xi_{n-1} - \mathcal{C}_{n-2}^n \xi_{2n-2} = s\xi_{n-2} \tag{14c}$$
$$\xi_n - \mathcal{C}_{n-1}^n \xi_{2n-1} = s\xi_{n-1} \tag{14d}$$
$$-\mathcal{C}_n^n \xi_{2n} = s\xi_n \tag{14e}$$
$$\xi_1 - 2\xi_{n+1} = s\xi_{n+1} \tag{14f}$$
$$\xi_{n+1} - 2\xi_{n+2} = s\xi_{n+2} \tag{14g}$$
$$\vdots$$
$$\xi_{2n-2} - 2\xi_{2n-1} = s\xi_{2n-1} \tag{14h}$$
$$\xi_{2n-1} - 2\xi_{2n} = s\xi_{2n} \tag{14i}$$

Using (14e), and given that $\mathcal{C}_n^n = 1$, we get $\xi_{2n} = -s\xi_n$. Substituting in (14i) gives $\xi_{2n-1} = -s(s+2)\xi_n$. One can recursively use (14h) to (14g) to obtain

$$\xi_{n+1} = -s(s+2)^{n-1}\xi_n \tag{15}$$

Also, from (15) and (14f) we have

$$\xi_1 = -s(s+2)^n \xi_n \tag{16}$$

Now, one can employ (14d) to get

$$\xi_{n-1} = \frac{1}{s}[1 + \mathcal{C}_{n-1}^n(s(s+2))]\xi_n$$

Similarly, from (14c) one has

$$\xi_{n-2} = \frac{1}{s}[\frac{1}{s}(1 + \mathcal{C}_{n-1}^n(s(s+2))) + \mathcal{C}_{n-2}^n(s(s+2)^2)]\xi_n$$

Using the rest of the equations recursively down to (14b) yields

$$\xi_2 = [\frac{1}{s^{n-2}} + \sum_{i=1}^{n-2} \mathcal{C}_{n-i}^n \frac{(s+2)^i}{s^{n-i-2}}]\xi_n \tag{17}$$

Finally, we can substitute (15), (16), and (17) in (14a), and multiply by $\frac{s^{n-2}}{\xi_n}$ to get

$$1 + \mathcal{C}_1^n s^{n-1}(s+2)^{n-1} + s^n(s+2)^n + \sum_{i=1}^{n-2} \mathcal{C}_{n-i}^n s^i (s+2)^i = 0$$

which is equivalent to $[s(s+2)+1]^n = (s+1)^{2n} = 0$. This shows the characteristic polynomial of M_∞ and shows that the eigenvalues of this matrix are placed at -1. Therefore, we can claim the existence of a positive definite matrix P_1 such that $M_\infty^T P_1 + P_1 M_\infty = -I_{2n}$. Let $\bar{p}_1$ and $\underline{p}_1$ denote the largest and the smallest eigenvalues of P_1 (which are independent of ϵ), respectively. Since η_1 satisfies (12) for M_∞ and $\Delta_\phi(x, \hat{x}_\infty, u)$, we can pose $V_1 = \eta_1^T P_1 \eta_1$ as a Lyapunov candidate and use $\|\Delta_\phi(x, \hat{x}_\infty, u)\| \leq \bar{\phi}\epsilon^{1-n}\|\eta_1\|$ to write

$$\begin{aligned} \dot{V}_1 &= -\frac{1}{\epsilon}\|\eta_1\|^2 + 2\epsilon^{n-1}\bar{B}^T P_1 \eta_1 + \frac{2}{\epsilon}\bar{C}^T P_1 \eta_1 v(t) \\ &\leq -\frac{2\alpha_1}{\epsilon} V_1 + \frac{2\sqrt{\bar{p}_1}}{\epsilon}\sqrt{V_1}\|v(t)\| \end{aligned}$$

where $\alpha_1 > 0$ satisfies $\frac{1}{\bar{p}_1} - 2\epsilon\bar{\phi} \geq 2\alpha_1$ for a small enough ϵ. Then, by choosing $W_1 = \sqrt{V_1}$, we can write $\dot{W}_1 \leq -\frac{\alpha_1}{\epsilon} W_1 + \frac{\sqrt{\bar{p}_1}}{\epsilon}\|v(t)\|$ away from the origin. By using the comparison principle, we get

$$\begin{aligned} W_1(\eta_1(t)) &\leq \exp(-\frac{\alpha_1}{\epsilon}t) W_1(\eta_1(0)) \\ &\quad + \frac{\sqrt{\bar{p}_1}}{\epsilon}\int_0^t \exp(-\frac{\alpha_1}{\epsilon}(t-\tau))|v(\tau)|d\tau \\ &\leq \exp(-\frac{\alpha_1}{\epsilon}t) W_1(\eta_1(0)) + \frac{\sqrt{\bar{p}_1}}{\alpha_1}\mu. \end{aligned} \tag{18}$$

Taking into account the fact that $\hat{x}_\infty(0) = \hat{x}(0)$ and $\eta_1(0) = \eta(0)$, we get

$$\|\eta_1(t)\| \leq \sqrt{\frac{\bar{p}_1}{\underline{p}_1}} \exp(-\frac{\alpha_1}{\epsilon}t)\|\eta(0)\| + \frac{1}{\alpha_1}\sqrt{\frac{\bar{p}_1}{\underline{p}_1}}\mu. \tag{19}$$

For η_2, we can use (8) to obtain

$$\begin{aligned} \dot{\eta}_2 = \frac{1}{\epsilon} M_\infty \eta_2 + \epsilon^{n-1} &\begin{bmatrix} B_n \Delta_\phi(\hat{x}_\infty, \hat{x}, u) \\ 0_{n\times 1} \end{bmatrix} \\ &+ \begin{bmatrix} \Gamma_\epsilon^2 (G_\infty - G(t)) z(t) \\ 0_{n\times 1} \end{bmatrix}. \end{aligned} \tag{20}$$

Since $G_\infty - G(t)$ is diagonal, the expression $\Gamma_\epsilon^2(G_\infty - G(t))z(t)$ is equivalent to $\Gamma_\epsilon(G_\infty - G(t))\Gamma_\epsilon(z(t) - z_\infty(t)) +$

$\Gamma_\epsilon(G_\infty - G(t))\Gamma_\epsilon z_\infty(t)$. From the definition of η_2 we have $\|\Gamma_\epsilon(z(t) - z_\infty(t))\| \leq \|\eta_2\|$. Also, it follows from (19) that

$$\|\Gamma_\epsilon z_\infty(t)\| \leq \sqrt{\frac{\bar{p}_1}{\underline{p}_1}}\exp(-\frac{\alpha_1}{\epsilon}t)\|\eta(0)\| + \frac{1}{\alpha_1}\sqrt{\frac{\bar{p}_1}{\underline{p}_1}}\mu \quad (21)$$
$$\triangleq c_1(t) \leq \bar{c}_1.$$

where $\bar{c}_1 = \sqrt{\frac{\bar{p}_1}{\underline{p}_1}}\|\eta(0)\| + \frac{1}{\alpha_1}\sqrt{\frac{\bar{p}_1}{\underline{p}_1}}\mu$. Using a Lyapunov candidate $V_2 = \eta_2^T P_1 \eta_2$ and the equation $\|\Delta_\phi(\hat{x}_\infty, \hat{x}, u)\| \leq \bar{\phi}\epsilon^{1-n}\|\eta_2\|$, we have

$$\dot{V}_2 \leq -\frac{1}{\epsilon}\|\eta_2\|^2 + 2\bar{\phi}\bar{p}_1\|\eta_2\|^2$$
$$+ 2\|\eta_2^T P_1\|\|\Gamma_\epsilon(G_\infty - G(t))\|(\|\eta_2\| + \bar{c}_1)$$

We use the abbreviated form $\|\Gamma_G\|$ for $\|\Gamma_\epsilon(G_\infty - G(t))\|$. Then, we can write

$$\dot{V}_2 \leq -\frac{1}{\epsilon}(2\alpha_1 - 2\epsilon\|\Gamma_G\|)V_2 + 2\bar{c}_1\sqrt{\bar{p}_1}\|\Gamma_G\|\sqrt{V_2} \quad (22)$$

From equations (8) and (9), we can see that $L(t) \leq L(0)$, implying that $G(t) \geq G(0)$ and thus we can derive $\|\Gamma_G\| \leq \|\Gamma_\epsilon(G_\infty - G(0))\|$. Using the fact that $G(0) = \text{diag}(C_n^T)$, we can rewrite this as $\|\Gamma_G\| \leq \|\text{diag}([\frac{\mathcal{C}_1^n}{\epsilon} - 1, \frac{\mathcal{C}_2^n}{\epsilon}, \ldots, \frac{\mathcal{C}_n^n}{\epsilon}]^T)\|$. Furthermore, from equation (9), we can infer that the convergence rate of $G(t)$ is proportional to $\frac{1}{\epsilon}$, and, as t approaches infinity, $\|\Gamma_G\|$ becomes zero. So, for some positive constants γ_1, γ_2 we have $\|\Gamma_G\| \leq \frac{\gamma_1}{\epsilon}\exp(-\frac{\gamma_2}{\epsilon}t)$. Now, let $T = -\frac{\epsilon\ln(\frac{\alpha_1}{\gamma_1})}{\gamma_2}$. Then, define $W_2 = \sqrt{V_2}$ and employ (22) to obtain $\dot{W}_2 \leq -\frac{\alpha_2}{\epsilon}W_2 + \frac{\gamma_1}{\epsilon}\bar{c}_1\sqrt{\bar{p}_1}\exp(-\frac{\gamma_2}{\epsilon}t)$ $\forall t \geq T$ and some positive constant α_2 satisfying $0 < \alpha_2 < \alpha_1$. Note that $T \to 0$ as $\epsilon \to 0$. Now, by using the comparison lemma, with $\eta_2(0) = W_2(0) = 0$, we get

$$W_2(t) \leq \frac{\gamma_1}{\epsilon}\bar{c}_1\sqrt{\bar{p}_1}\int_0^t \exp(-\frac{\alpha_2}{\epsilon}(t-\tau))\exp(-\frac{\gamma_2}{\epsilon}\tau)d\tau$$
$$= \frac{\gamma_1\bar{c}_1\sqrt{\bar{p}_1}}{\alpha_2 - \gamma_2}(\exp(-\frac{\gamma_2}{\epsilon}t) - \exp(-\frac{\alpha_2}{\epsilon}t)). \quad (23)$$

One can consider $\|e_i(t)\| \leq \frac{1}{\epsilon^{i-1}}(\|\eta_1(t)\| + \|\eta_2(t)\|)$ and employ (19) and (23) to write

$$\|e_i(t)\| \leq \frac{1}{\epsilon^{i-1}}\sqrt{\frac{\bar{p}_1}{\underline{p}_1}}\exp(-\frac{\alpha_1}{\epsilon}t)\|e(0)\| + \frac{1}{\epsilon^{i-1}}\frac{1}{\alpha_1}\sqrt{\frac{\bar{p}_1}{\underline{p}_1}}\mu +$$
$$+ \frac{1}{\epsilon^{i-1}}\frac{\gamma_1\bar{c}_1}{\alpha_2 - \gamma_2}\sqrt{\frac{\bar{p}_1}{\underline{p}_1}}(\exp(-\frac{\gamma_2}{\epsilon}t) - \exp(-\frac{\alpha_2}{\epsilon}t)) \quad (24)$$

The first result of the theorem follows by choosing $\hat{\kappa}_1 = \sqrt{\frac{\bar{p}_1}{\underline{p}_1}}, \hat{\kappa}_2 = \frac{1}{\alpha_1}\sqrt{\frac{\bar{p}_1}{\underline{p}_1}}, \hat{\kappa}_3 = \frac{\gamma_1\bar{c}_1}{\alpha_2 - \gamma_2}\sqrt{\frac{\bar{p}_1}{\underline{p}_1}}$. This, completes the first part of the proof. To obtain (11), we define $\tilde{e} = Le$, and use (1) and (8) to get

$$\dot{\tilde{e}} = -\frac{1}{\epsilon}\tilde{e} - A_n^T\tilde{e} + C_n^T C_n e + LB_n\Delta_\phi - LG\Gamma_\epsilon z. \quad (25)$$

Note that Γ_ϵ and G are both diagonal and we have $\Gamma_\epsilon G = G\Gamma_\epsilon$. One can pose the Lyapunov candidate $V = \tilde{e}^T\tilde{e}$ for (25) to obtain

$$\dot{V} = -\frac{2}{\epsilon}\|\tilde{e}\|^2 - \tilde{e}^T(A_n + A_n^T)\tilde{e}$$
$$+ 2\tilde{e}^T(C_n^T C_n e + LB_n\Delta_\phi - LG\Gamma_\epsilon z). \quad (26)$$

At this point, it is required to discuss the upperbound of the term $C_n^T C_n e + LB_n\Delta_\phi - LG\Gamma_\epsilon z$. For $\|C_n^T C_n e\|$ we have $C_n e = e_1$, where e_1 is the first element of the vector e, whose bound is given in (24) by letting $i = 1$. Next, we let $\epsilon \to 0$. As a result, $L(t) \approx L_\infty$ and $G(t) \approx G_\infty$. First, we have $\|L_\infty B_n\Delta_\phi\| \leq \|B_n^T L_\infty\|\|\Delta_\phi\| \leq \bar{\phi}\|B_n^T L_\infty\|\|e\|$. Let l_{ij} denote the elements of L_∞. Also, it is clear that $B_n^T L_\infty$ corresponds to the last row of L_∞. We can use $\frac{1}{\epsilon}L_\infty + L_\infty A_n + A_n^T L_\infty = C_n^T C_n$ to write $\frac{1}{\epsilon}l_{11} = 1, \frac{1}{\epsilon}l_{12} + l_{11} = 0, \frac{1}{\epsilon}l_{13} + l_{12} = 0, \ldots$. Solving these equations for l_{1i}, we get $l_{1i} = -(-\epsilon)^i$. Similar calculations for l_{2i} and l_{3i} can be performed to obtain $l_{2i} = (-1)^i\epsilon^{i+1}(i), l_{3i} = (-1)^{i+1}(\frac{i(i+1)}{2})\epsilon^{i+2}$. One can continue the calculations to show that l_{n1} is the dominant term in the last row of L_∞, and it is proportional to ϵ^n. On the other hand, from (24), $\|e\|$ is proportional to ϵ^{1-n}. Therefore, $\bar{\phi}\|B_n^T L_\infty\|\|e\|$ is proportional to ϵ and vanishes as $\epsilon \to 0$. For $\Gamma_\epsilon z$, we use the definition of η to write $\|\Gamma_\epsilon z\| \leq \|\eta\| \leq \|\eta_1\| + \|\eta_2\|$. This, along with (19), $\sqrt{\underline{p}_1}\|\eta_2\| \leq W_2$, $\|\eta(0)\| \leq \|e(0)\|$, and (23) shows that $\|\Gamma_\epsilon z\|$ is bounded by the right hand side of (24) for $i = 1$. For $\|L_\infty G_\infty\|$, previous deductions on g_i and l_{ij} can be used to show that the first row of $L_\infty G_\infty$ is $(-1)^{i+1}\mathcal{C}_1^n$. More generally, all the elements in the ith row of $L_\infty G_\infty$ are proportional to ϵ^{i-1}. Hence, the first row is dominant in $L_\infty G_\infty$, and the norm of this matrix is a constant independent on ϵ. Recalling the above arguments, and using (23), (24), and $\|\eta(0)\| \leq \|e(0)\|$, we can write:

$$\|C_n^T C_n e + LB_n\Delta_\phi - LG\Gamma_\epsilon z\| \leq$$
$$\tilde{k}_1 e^{-\frac{\alpha_1}{\epsilon}t}\|e(0)\| + \tilde{k}_2\mu + \tilde{k}_3 e^{-\frac{\gamma_2}{\epsilon}t} \triangleq \tilde{k}(t)$$

for some positive constants $\tilde{k}_1, \tilde{k}_2, \tilde{k}_3$ independent of $\epsilon, \|e(0)\|$ and μ. If one chooses $\epsilon < \epsilon^*$ such that $\epsilon^*\|A_n + A_n^T\| \leq 2(1 - \beta_1)$ is satisfied for a positive β_1, then we can define $W = \sqrt{V}$ and use $V = \|\tilde{e}\|^2$ to write (26) as $\dot{W} \leq -\frac{\beta_1}{\epsilon}W + \tilde{k}(t)$. Upon integration, and by employing $\lambda_{min}(L(t,\epsilon))\|e\| \leq W \leq \|e\|$, we obtain

$$\|e\| \leq \frac{1}{\lambda_{min}(L(t,\epsilon))}[e^{-\frac{\beta_1}{\epsilon}t}\|e(0)\| +$$
$$\frac{\tilde{k}_1\epsilon}{\beta_1 - \alpha_1}\|e(0)\|(e^{-\frac{\alpha_1}{\epsilon}t} - e^{-\frac{\beta_1}{\epsilon}t}) \quad (27)$$
$$+ \frac{\tilde{k}_2\epsilon}{\beta_1}(1 - e^{-\frac{\beta_1}{\epsilon}t})\mu + \frac{\tilde{k}_3\epsilon}{\beta_1 - \gamma_2}(e^{-\frac{\gamma_2}{\epsilon}t} - e^{-\frac{\beta_1}{\epsilon}t})]$$

One can briefly write (27) as

$$\|e\| \leq \frac{\theta}{\lambda_{min}(L(t,\epsilon))}\|e(0)\|e^{-\frac{\beta_1}{\epsilon}t} + \frac{\zeta\epsilon}{\lambda_{min}(L(t,\epsilon))}\mu$$
$$+ \frac{\sigma\epsilon}{\lambda_{min}(L(t,\epsilon))}e^{-\frac{\gamma_2}{\epsilon}t} \quad (28)$$

where θ, ζ and σ are some positive constants. Next, we need to investigate $\lambda_{min}(L(t,\epsilon))$ as discussed in [6]. From (9) we have:

$$\lambda_{min}(L(t,\epsilon)) \geq e^{-\frac{1}{\epsilon}t}\lambda_{min}(e^{-A_n^T t}e^{-A_n t}) \quad (29)$$
$$+\lambda_{min}(\int_0^t e^{-\frac{1}{\epsilon}(t-\tau)}e^{-A_n^T(t-\tau)}C_n^T C_n e^{-A_n(t-\tau)}d\tau).$$

Considering the observability of the pair (A_n, C_n), we have

$$\lambda_{min}\left(\int_0^t e^{-\frac{1}{\epsilon}(t-\tau)}e^{-A_n^T(t-\tau)}C_n^T C_n e^{-A_n(t-\tau)}d\tau\right) \geq p(\epsilon) \quad (30)$$

where $p(\epsilon) > 0$ is a polynomial in ϵ. We can also write $\lambda_{min}(e^{-A_n^T t}e^{-A_n t}) \geq e^{-t}$. Therefore, from (29) and (30) we have $\lambda_{min}(L(t,\epsilon)) \geq p(\epsilon)+e^{-t(\frac{1}{\epsilon}+1)}$. Next, we consider the function $f(t,\epsilon) = \frac{\exp(-\frac{\beta_1}{\epsilon}t)}{p(\epsilon)+\exp(-t(\frac{1}{\epsilon}+1))}$. It is clear that, since since $p(\epsilon) > 0$, $f(t,\epsilon) \to 0$ as $t \to \infty$. Taking the derivative of $f(t)$ with respect to time, we obtain $\frac{df(t)}{dt} = 0$ at $t_m = \frac{\ln(\frac{\epsilon+1-\beta_1}{\beta_1 p(\epsilon)})}{1+\frac{1}{\epsilon}}$. Note that, by definition, β_1 is the largest constant satisfying $\beta_1 \leq 1 - \frac{\epsilon}{2}\|A_n + A_n^T\|$. So, $\beta_1 \to 1$ as $\epsilon \to 0$. Using this fact, along with the obtained equation for t_m, we can calculate $\sup\|f(t,\epsilon)\|$ as

$$\lim_{\epsilon\to 0}\sup f(t,\epsilon) = \lim_{\epsilon\to 0}\frac{\exp(\frac{-\beta_1\ln(\frac{\epsilon+1-\beta_1}{\beta_1 p(\epsilon)})}{\epsilon+1})}{p(\epsilon)+\exp(-\ln(\frac{\epsilon+1-\beta_1}{\beta_1 p(\epsilon)}))} = 1$$

Similar arguments can be used to show that the function $\frac{\exp(-\frac{\gamma_2}{\epsilon}t)}{\lambda_{min}(L(t,\epsilon))}$ converges to zero as $t \to \infty$, and remains bounded as $\epsilon \to 0$. Therefore, we can conclude that $g(t,\epsilon) = \frac{\epsilon\exp(-\frac{\gamma_2}{\epsilon}t)}{\lambda_{min}(L(t,\epsilon))} \to 0$ as $t \to \infty$ or $\epsilon \to 0$. This, along with (28) completes the proof. ■

Remark 1: One can compare (5) with (11) to observe that, in the absence of noise, the estimation error of the proposed observer converges to zero without exhibiting unbounded overshoot, regardless of the value of ϵ chosen.

In the following, we analyze the steady state behaviour of the proposed observer. Using the ideas from [20], it is shown that the use of linear filter improves the estimates. The next theorem summarizes the result of the analysis.

Theorem 2: Consider system (1) and the observer (8) in the linear case, that is, $\phi(x,u) = \phi_s(x,u) = \Phi x$ where $\Phi \in \mathbb{R}^{1\times n}$ is the vector of coefficients, with $|\Phi| \leq \bar{\phi}$. Let the assumptions of Theorem 1 be satisfied. Let the noise be generated by the system (6). Then, there exists some $\delta^*(\epsilon)$ and $\rho_i > 0$ such that, for all positive $\delta \leq \delta^*(\epsilon)$ we have

$$\lim_{t\to\infty}\sup|x_i(t) - \hat{x}_i(t)| \leq \frac{\rho_i\delta^{i+1}}{\epsilon^{2i}}\mu. \quad (31)$$

Proof: It was demonstrated in the proof of Theorem 1 that, since η_2 converges to zero, the trajectories of the observer (8) converge to the states of the following observer at steady-state:

$$\dot{\hat{x}}_\infty(t) = A_n\hat{x}_\infty(t) + B_n\Phi\hat{x}_\infty(t) + \Gamma_\epsilon G_\infty z_\infty(t)$$
$$\dot{z}_\infty(t) = -\frac{1}{\epsilon}Dz_\infty(t) + \frac{1}{\epsilon^2}A_n^T z_\infty(t) + \frac{1}{\epsilon}C_n^T(y(t) - C_n\hat{x}_\infty(t))$$
$$0 = -\frac{1}{\epsilon}L_\infty - A_n^T L_\infty - L_\infty A_n + C_n^T C_n$$

where $G_\infty = \text{diag}(L_\infty^{-1}C_n^T), D = 2I_n$. We define the change of coordinates $\bar{\eta} = \eta_1 - \Pi w$. We can then re-express the dynamics as

$$\dot{\bar{\eta}} = \tilde{M}\bar{\eta} + \tilde{M}\Pi w + \tilde{C}\mathcal{P}w - \frac{1}{\delta}\Pi\mathcal{S}w \quad (32)$$

where $\tilde{M} = \frac{1}{\epsilon}M_\infty + \epsilon^{n-1}\begin{bmatrix}B_n\Phi\Gamma_\epsilon^{-1}\\0\end{bmatrix}$, $\tilde{C} = \frac{1}{\epsilon}\begin{bmatrix}0\\C_n^T\end{bmatrix}$. If we can find $\Pi(\delta)$ that satisfies the Sylvester equation $\Pi\mathcal{S} = \delta(\tilde{M}\Pi + \tilde{C}\mathcal{P})$, then the system (32) can be transformed to the simple form $\dot{\bar{\eta}} = \tilde{M}\bar{\eta}$. On the other hand, for a sufficiently small ϵ, we have that $\tilde{M} \approx \frac{1}{\epsilon}M_\infty$, which was shown to be Hurwitz. As a result, $\bar{\eta}$ converges to zero, such that $\eta_{1(ss)} = \Pi w$ at steady-state. Since the eigenvalues of $\tilde{M}$ are negative and those of $\mathcal{S}$ are on the imaginary axis, the solution of the Sylvester equation is unique. In addition, one can check that the solution is given by:

$$\Pi(\delta) = \sum_{k=1}^{+\infty}\delta^k\bar{\Pi}_k, \quad \bar{\Pi}_k = \tilde{M}^{k-1}\tilde{C}\mathcal{P}\mathcal{S}^{-k}. \quad (33)$$

Therefore, we can write:

$$\eta_{1(ss)} = \delta\tilde{C}\mathcal{P}\mathcal{S}^{-1}w + \delta^2\tilde{M}\tilde{C}\mathcal{P}\mathcal{S}^{-2}w + \delta^3\tilde{M}^2\tilde{C}\mathcal{P}\mathcal{S}^{-3}w + \ldots \quad (34)$$

The ith element of $\frac{1}{\epsilon^{i-1}}\eta_{1(ss)}$ represents the ith estimation error at steady-state. Given that the first n row of $\tilde{C}$ are zero, it follows that the first term of (34) does not affect the steady state values of the error terms, e_i. For $i = 1$, the norm of the first row of $\tilde{M}\tilde{C}$ is $\frac{\mathcal{C}_1^n}{\epsilon^2}$. Since $\delta \in (0,1)$, then $\delta^2\tilde{M}\tilde{C}\mathcal{P}\mathcal{S}^{-2}w$ is the dominant term in the first row of $\eta_{1(ss)}$ in (34). Thus, we obtain the bound $\|e_1(ss)\| \leq \frac{\rho_1\delta^2}{\epsilon^2}\mu$ for some positive ρ_1 and some $\delta \leq \delta^*$ such that

$$\|(\delta^*)^2\tilde{M}\tilde{C}\mathcal{P}\mathcal{S}^{-2}w + (\delta^*)^3\tilde{M}^2\tilde{C}\mathcal{P}\mathcal{S}^{-3}w + \ldots\| \leq \rho_1\|(\delta^*)^2\tilde{M}\tilde{C}\|\|w\|.$$

For $i = 2$, the second row of $\tilde{C}$ and $\tilde{M}\tilde{C}$ are zero, and the norm of the second row of $\tilde{M}^2\tilde{C}$ is $\frac{\mathcal{C}_2^n}{\epsilon^3}$. So, we can write $\|e_2(ss)\| \leq \frac{\rho_2\delta^3}{\epsilon^4}\mu$. Similarly, for $i = 3$ the third row of $\tilde{C}, \tilde{M}\tilde{C}, \tilde{M}^2\tilde{C}$ are zero and the norm of the third row of $\tilde{M}^3\tilde{C}$ is $\frac{\mathcal{C}_3^n}{\epsilon^4}$. So, we get $\|e_3(ss)\| \leq \frac{\rho_3\delta^4}{\epsilon^6}\mu$. The proof follows similar steps for $i = 4, \ldots, n$ to obtain (31). ■

Remark 2: Comparing (31) with (7) shows that the steady-state bound is multiplied by $(\frac{\delta}{\epsilon})^i$ which provides an improvement as long as $\delta < \epsilon$.

Remark 3: The extension of the results of Theorem 2 to the nonlinear setting is straightforward by following the methodology of [20].

IV. Simulation Study

We consider a single-link flexible-joint robot arm [14], [21] to illustrate the performance of the proposed observer. We perform a coordinate change to describe the single-link robot arm dynamics in the canonical form (1) with $n = 4$ and

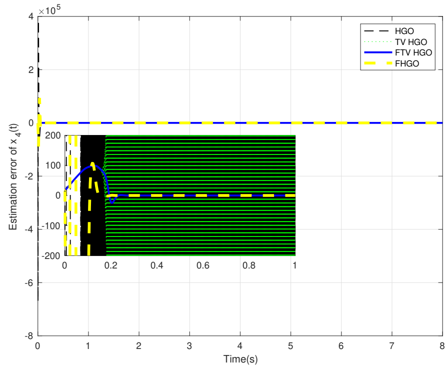

Fig. 1. Estimation error of x_4 for $\epsilon = 0.005$

$$
\begin{aligned}
\phi(x) = & \frac{\bar{k}}{J_1 J_2 N} u + \frac{\bar{k}}{J_2 N}\left(\frac{\bar{k}}{J_1 N} - \frac{\bar{k}}{J_2}\right) x_1 \\
& - \frac{\bar{k}}{J_2 N}\left(\frac{F_2}{J_2} + \frac{F_1 N}{J_1}\right) x_2 - \left(\frac{\bar{k}}{J_2 N}\left(1 + \frac{F_1 F_2 N}{J_1 \bar{k}}\right) + \frac{\bar{k}}{J_2}\right) x_3 \\
& - \left(\frac{F_1}{J_1} + \frac{F_2}{J_2}\right) x_4 + \frac{mgd}{J_2}\left(x_3 \sin(x_1) + x_2^2 \cos(x_1)\right) \\
& - \frac{\bar{k} mgd}{J_2^2 N}\left(\cos(x_1) - \frac{F_1 J_2 N}{J_1 \bar{k}} x_2 \sin(x_1)\right)
\end{aligned} \tag{35}
$$

where the physical parameters are chosen as $F_1 = 0.1$, $F_2 = 0.15$, $J_1 = 0.15$, $J_2 = 0.2$, $\bar{k} = 0.4$, $N = 2$, $m = 0.8$, $g = 9.81$ and, $d = 0.6$. A stabilizing state-feedback controller is proposed for (35). It is given by

$$
\begin{aligned}
u = sat_R \Bigg\{ & \frac{mgdJ_1}{J_2 N} - \frac{J_1 J_2 N}{\bar{k}}\Big[\Big(L^4 c_1 + L^2 c_3 \frac{\bar{k}}{J_2}\Big) x_1 \\
& + \Big(L^3 c_2 + L^2 c_3 \frac{F_2}{J_2} + L c_4 \frac{\bar{k}}{J_2}\Big) x_2 + \Big(L^2 c_3 + L c_4 \frac{F_2}{J_2}\Big) x_3 \\
& + L c_4 x_4 + L^2 c_3 \frac{mgd}{J_2}(cos(x_1) - 1)\Big] - L c_4 \frac{mgd}{J_2} x_2 sin(x_1) \Bigg\}
\end{aligned}
$$

where $c_1 = 4$, $c_2 = 7.91$, $c_3 = 6.026$, $c_4 = 1.716$, $L = 3$ and, $R = 200$. The initial conditions for the system (35) are $\begin{bmatrix} 0.5 & 0 & -22.6618 & 16.2464 \end{bmatrix}^T$. The measurement noise is chosen as $v(t) = 0.002 \sin(3000t)$. For the proposed observer (8) and the time-varying HGO proposed in [4], we only need to choose $\hat{x}(0) = 0$, $\epsilon = 0.005$. We choose the same parameters for the standard HGO and place the poles at -1. Figure 1 compares the estimation error of x_4 using the proposed observer, a standard HGO, the filtered HGO proposed in [14], and a TVHGO. As expected, the observer significantly reduces the sensitivity to noise on the estimates, similar to the FHGO of [14]. Moreover, while the standard HGO and the FHGO peak to 4×10^5 and 10^5, the overshoot of the proposed observer is significantly smaller, reaching 100 at $t_m \approx 0.1s$. One may achieve a faster convergence as well as a reduced overshoot by increasing the observer gain. However, as mentioned in remark 2, the reduced sensitivity to noise is lost if ϵ is chosen smaller than $\frac{1}{3000}$.

V. Conclusions

This study proposed a nonlinear observer that achieves the benefits of an HGO with reduced sensitivity to noise and improved transient response . The observer incorporates two subsystems. The first subsystem filters the estimation error, while the second subsystem is used to produce a dynamic observer gain. It was shown that the proposed observer improves the transient performance of the standard HGO. The observer's convergence speed can be adjusted using a single large gain parameter. Future work will focus on the design of an observer with dynamic gains that depends on the output error.

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