2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Passivity-Based Conditions for Asymptotic Stability of Speed Control for Three-Phase and Dual Three-Phase Permanent Magnet Synchronous Motors

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***Abstract*— In this paper, we present sufficient conditions for guaranteeing global asymptotic speed regulation for three-phase and dual three-phase permanent magnet synchronous motors. Contrary to standard cascade control design approaches, where the inner and outer controllers cannot be tuned separately, we derive conditions on the controllers gains that are independent of each other. This tuning method is possible thanks to decomposing the machine dynamics as a negative feedback interconnection of two subsystems. The provided conditions then ensure the passivity of each subsystem, which in turn guarantees the asymptotic stability of the closed-loop equilibrium.**

I. Introduction

Permanent magnet synchronous motors (PMSMs) offer significant advantages, such as high efficiency, high power density, small size, and high dynamic performance. These properties have made the PMSMs to be widely used in applications such as electric vehicles, wind generators, ship propulsion systems, and electric aircraft [1]. Among the PMSMs, multi-phase PMSMs have additional benefits, such as smaller torque ripple, stronger fault tolerance, and high output power [2]. From them, dual three-phase PMSMs (DT-PMSMs) have two sets of three-phase stator windings with isolated neutral points and a phase-shift of 30 electrical degrees. While an inverter controls each set of windings, the control design for DT-PMSMs becomes complex due to the coupling between the windings. In this setting, vector space decomposition (VSD) becomes essential since it provides a method for studying DT-PMSMs in two orthogonal planes: the α-β and z_1-z_2 [3]. In an ideal DT-PMSM, the electrical torque is thus effectively regulated by means of controllers in the plane of the fundamental components α and β, while the rest of the dynamics is projected into the z_1-z_2 plane [4], [5]. Applying Park's transformation to the fundamental α-β plane results in the d-q [3] orthogonal reference system, analogous to that of a three-phase PMSM, allowing the application of the controllers of such machines also in DT-PMSMs.

In the literature of electrical drives, various control techniques have been proposed for speed/position tracking/regulation of PMSMs (see, for instance [6], [7], [8]). At present, proportional-integral (PI) control is the dominant technique among industrial applications, due to its simplicity and it is often used in a cascade scheme. There, an inner controller is employed to ensure the tracking of the current, while the outer controller ensures the speed/position regulation and provides a reference for the inner control. In this way, inner and outer controllers achieve the desired speed or position regulation [9]. Due to the wide use of this scheme, it is of interest to investigate stability conditions for this cascade control scheme, as well as suitable methods for tuning the controller gains. To guarantee the stability of electrical machines controlled with a cascade scheme, often the time-scale separation principle is employed. This principle imposes a faster response to inner loop control with respect to the outer loop [10].

In contrast, passivity-based control provides a framework for designing controllers for linear or nonlinear systems that exploits structural properties of the systems rather than behaviors conditioned by their parameters [11]. In [12], it is shown that a class of PMSMs possesses an inherent passive structure, which is exploited for designing a speed control based on the interconnection and damping assignment passivity-based control (IDA-PBC) method. Other related works are [9], where it is shown that PMSMs can be globally regulated around a desired equilibrium point (EP) using a PI controller in combination with a state estimator while providing tuning conditions for the PI controller, and [13], where a cascade PI controller is implemented for non-salient PMSMs and global stability of the cascade approach is demonstrated by setting the proportional gains sufficiently large, saturating the integrators, and imposing a persistency of excitation condition on the signals.

An important feature of passivity-based control design methods is that complex systems can be decomposed into simpler interconnected subsystems that preserve the passivity property. Then, time-scale separation arguments are not needed, and stability conditions can be established from the passivity properties of the systems [12]. In this context, the main contribution in this note is a passivity-based design method for the cascade control of the speed in DT-PMSMs. More precisely, we provide the following:

1) A novel decomposition of the DT-PMSM dynamics as the interconnection of two passive systems. The subsystems can be related to the electrical dynamics and the mechanical dynamics of the machine, respectively.
2) Sufficient conditions over the controller gains that ensure the passivity property of each subsystem and,

This work is partially supported by the Bundesministerium für Wirtschaft und Klima (BMWK), Germany project ETHAN (project number: 20L2103F1).

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simultaneously, the passivity of the interconnection.

3) These conditions result in independent criteria for the tuning of the inner and outer controllers that ensure the global asymptotic regulation of the speed, making it unnecessary to appeal to a time-separation argument.

The structure of the paper is as follows. In Section II, the model of the DT-PMSM is introduced. In Section III, the cascade control scheme for speed regulation is explained, and the problem formulation is given. In Section IV, the error dynamics induced by the speed control is analyzed in a passivity-based framework, and conditions for ensuring the passivity of the system that ensure the global asymptotic regulation of the speed are given. Finally, in Section V, the speed control is illustrated via numerical simulations.

II. DT-PMSM Model

As is done in standard PMSMs, the dynamics of the DT-PMSM is investigated in a reference frame synchronized with the motion of the machine's rotor. In this way, a system with stationary steady states is obtained, leaving aside the need to analyze periodic trajectories. For the DT-PMSM, this is obtained with the method of vector space decomposition introduced by [14], which results in the following set of equations describing the electrical and mechanical behavior of the machine:

$$\begin{aligned} L_d \frac{\mathrm{d}}{\mathrm{d}t} i_d &= u_d - R_s i_d + \omega_e L_q i_q, \\ L_q \frac{\mathrm{d}}{\mathrm{d}t} i_q &= u_q - R_s i_q - \omega_e (L_d i_d + \phi_f), \\ J_m \frac{\mathrm{d}}{\mathrm{d}t} \omega_m &= T_e - T_l - R_m \omega_m, \\ T_e &= 3p((L_d - L_q) i_d i_q + \phi_f i_q), \end{aligned} \tag{1a}$$

$$\begin{aligned} L_{z_1} \frac{\mathrm{d}}{\mathrm{d}t} i_{z_1} &= u_{z_1} - R_s i_{z_1}, \\ L_{z_2} \frac{\mathrm{d}}{\mathrm{d}t} i_{z_2} &= u_{z_2} - R_s i_{z_2}. \end{aligned} \tag{1b}$$

Here, i_d, i_q, i_{z_1}, and i_{z_2} are the machine's currents, u_d, u_q, u_{z_1}, and u_{z_2} are the input voltages to the motor, ω_e and ω_m are the electrical and mechanical angular speed of the machine ($\omega_e = p\omega_m$). The parameters of the model are the inductances $L_d > 0$, $L_q > 0$, $L_{z_1} > 0$, and $L_{z_2} > 0$, the electrical resistance of the stator $R_s > 0$, the flux established by the permanent magnets ϕ_f, the rotor inertia $J_m > 0$, the viscous friction factor $R_m > 0$, and the number of pole-pairs of the machine $p \geq 1$. T_e represents the electrical torque, and T_l is the unknown but constant load torque. The definitions of L_d, L_q, and L_z as well as the 6-phase transformation matrix to model (1) can be found in Appendix A of [5].

III. DT-PMSM Speed Regulation and Problem Statement

To regulate the speed of a DT-PMSM, represented by ω_e in (1a), it is common practice to implement a cascade control scheme, where the inner loop is meant to achieve torque tracking by controlling the d and q currents, whereas the outer loop regulates the machine's speed by generating

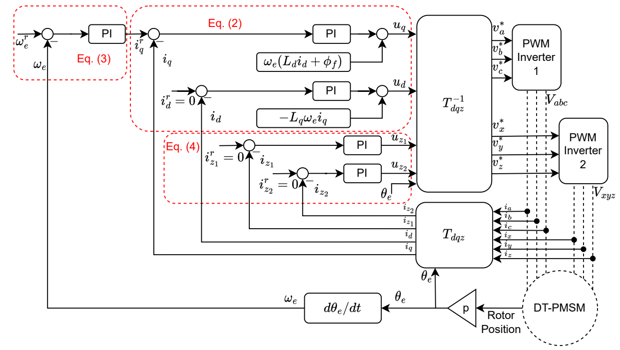

Fig. 1. Block diagram of the control scheme for speed regulation in DT-PMSMs.

a torque/current reference for the inner loop [10]. A PI controller is combined with a feedforward term for the inner loop to achieve the current control. The feedforward term is meant to cancel the cross-couplings between the d and q currents [15], [16], whereas the PI controller achieves the reference tracking. For the inner loop control with a feedforward term, one has then the following:

$$\begin{aligned} u_{dq} &= -K_{P,1}\big((i_{dq} - i_{dq}^r) + T_{I,1} z_1\big) \\ &\quad + \big[-L_q \omega_e i_q \quad L_d \omega_e i_d + \omega_e \phi_f\big]^\top, \\ \dot{z}_1 &= i_{dq} - i_{dq}^r, \end{aligned} \tag{2}$$

with $u_{dq} = [u_d \ \ u_q]^\top$, $i_{dq} = [i_d \ \ i_q]^\top$, integral state represented by $z_1 = [z_{11} \ z_{12}]^\top$, and reference given by $i_{dq}^r = [i_d^r \ \ i_q^r]^\top$. The controller gains are represented by $K_{P,1} = \mathrm{diag}\{k_{p,11}, k_{p,12}\}$ and $T_{I,1} = \mathrm{diag}\{1/T_{i,11}, 1/T_{i,12}\}$, i.e., the proportional and integral gains.

For a non-salient machine, where $L_d = L_q = L$, the direct current i_d does not contribute to the torque generation (see (1a)), and thus its reference is set to zero for reducing the energy consumption of the motor. In such a situation, only the reference for the quadrature current i_q is generated by the outer control, which is also conventionally implemented as a PI controller. This results in the following outer loop control:

$$\begin{aligned} i_q^r &= -k_{p,2}\Big((\omega_e - \omega_e^r) + \frac{1}{T_{i,2}} z_2\Big), \\ \dot{z}_2 &= \omega_e - \omega_e^r. \end{aligned} \tag{3}$$

In (3) $k_{p,2} \in \mathbb{R}$ and $T_{i,2} \in \mathbb{R}$ are the proportional and integral gains, respectively, and ω_e^r is the constant reference for the angular electrical speed of the machine.
In the case of the currents i_{z_1} and i_{z_2} in (1b), their references are also set to zero since they do not contribute to the torque generation, but only to the ohmic losses of the machine. To regulate them, it is common practice to implement the following PI controller:

$$\begin{aligned} u_{z_{12}} &= -K_{P,3}\big(i_{z_{12}} + T_{I,3} z_3\big), \\ \dot{z}_3 &= i_{z_{12}}. \end{aligned} \tag{4}$$

Here, $u_{z_{12}} = [u_{z_1} \ \ u_{z_2}]^\top$, $i_{z_{12}} = [i_{z_1} \ \ i_{z_2}]^\top$, $z_3 = [z_{31} \ \ z_{32}]^\top$, and the controller gains are $K_{P,3} = \mathrm{diag}\{k_{p,31}, k_{p,32}\}$ and $T_{I,3} = \mathrm{diag}\{1/T_{i,31}, 1/T_{i,32}\}$.

The full implementation of the controllers (2), (3), and (4) for the DT-PMSM is illustrated in the block diagram in Figure 1, where T_{dqz} (see [5]), represents the transformation matrix used to obtain the signals in the dqz-frame, which model corresponds to (1). After the substitution of the controllers (2), (3), and (4) in (1), one obtains the following subsystems:

$$\dot{\chi}_1 = A_1\chi_1,\ \chi_1 = \begin{bmatrix} i_d & i_{z_1} & i_{z_2} & z_{11} & z_{31} & z_{32}\end{bmatrix}^\top, \tag{5a}$$

$$A_1 = \begin{bmatrix} -\frac{R_s+k_{p,11}}{L} & 0 & 0 & -\frac{k_{p,11}}{LT_{i,11}} & 0 & 0 \\ 0 & -\frac{R_s+k_{p,31}}{L} & 0 & 0 & -\frac{k_{p,31}}{LT_{i,31}} & 0 \\ 0 & 0 & -\frac{R_s+k_{p,32}}{L} & 0 & 0 & -\frac{k_{p,32}}{LT_{i,32}} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned} \dot{\chi}_2 &= A_2\chi_2 + B_2 i_q^r, \qquad \chi_2 = \begin{bmatrix} i_q & z_{12}\end{bmatrix}^\top, \\ A_2 &= \begin{bmatrix} -\frac{R_s+k_{p,12}}{L} & -\frac{k_{p,12}}{T_{i,12}L} \\ 1 & 0 \end{bmatrix},\ B_2 = \begin{bmatrix} \frac{k_{p,12}}{L} \\ -1 \end{bmatrix}, \end{aligned} \tag{5b}$$

$$\begin{aligned} \dot{\chi}_3 &= A_3\chi_3 + B_3 i_q - D_3, \qquad \chi_3 = \begin{bmatrix} \omega_e & z_2\end{bmatrix}^\top, \\ A_3 &= \begin{bmatrix} -\frac{R_m}{J_m} & 0 \\ 1 & 0 \end{bmatrix},\ B_3 = \begin{bmatrix} \frac{3\phi_f}{J_m} \\ 0 \end{bmatrix},\ D_3 = \begin{bmatrix} \frac{T_l}{pJ_m} \\ \omega_e^r \end{bmatrix}. \end{aligned} \tag{5c}$$

Note that (5a) is autonomous, whereas (5b) and (5c) are interconnected by i_q^r and i_q. Furthermore, it is easy to see that the origin of (5a) is asymptotically stable as long as the respective controller gains are positive. This follows from the structure of the matrix A_1 in (5a) and by recognizing that the subsystem consists of three decoupled second-order systems. Thus, the analysis problem can be focused on analyzing the stability of the interconnected subsystems (5b) and (5c), and showing that $\omega_e \to \omega_e^r$ as $t \to \infty$. Formally, we address the following problem.

Problem 1: Consider a DT-PMSM described by (1) with controllers (2), (3), and (4). Assume that the motor does not have saliency, i.e., $L_d = L_q = L$. Derive sufficient conditions over the controller gains $k_{p,11}$, $k_{p,12}$, $T_{i,11}$, $T_{i,12}$, $k_{p,2}$, $T_{i,2}$, $k_{p,31}$, $k_{p,32}$, $T_{i,31}$ and $T_{i,32}$, such that $\omega_e \to \omega_e^r$ as $t \to \infty$ despite the presence of a constant load torque T_l.

Not only sufficient conditions over the controller gains are required, but it is also desirable that such conditions be accessible. We have partially answered Problem 1 with the argumentation given above for (5a). Finding such conditions for the subsystems (5b) and (5c), however, is not as trivial. Fortunately, we can exploit the fact that the subsystems (5b) and (5c) are interconnected, and that their input/output maps possess some passive qualities. For establishing this, in the next section, the error dynamics induced by the subsystems (5b) and (5c) are investigated.

IV. Passivity-based tuning conditions

A. Error dynamics of the controlled DT-PMSM

After the preliminary analysis carried out in the previous section, the problem has been reduced to investigate if the subsystems (5b) and (5c) have an asymptotically stable equilibrium point such that ω_e^r results in the equilibrium value for ω_e. Denote by $\chi_2^\star$ and $\chi_3^\star$ the equilibrium points of the subsystems (5b) and (5c). By considering the equations in (3), (5b), and (5c), and solving for the equilibrium we obtain

$$\begin{aligned} \chi_2^\star &= \begin{bmatrix} \frac{T_l + R_m p\omega_e^r}{3p\phi_f} \\ \frac{-R_s T_{i,12}(T_l + R_m p\omega_e^r)}{3p\phi_f k_{p,12}} \end{bmatrix}, \\ \chi_3^\star &= \begin{bmatrix} \omega_e^r \\ \frac{-T_{i,2}(T_l + R_m p\omega_e^r)}{3p\phi_f k_{p,2}} \end{bmatrix}. \end{aligned} \tag{6}$$

Thus the equilibrium value for ω_e is indeed ω_e^r. To proceed, consider the error variables $\tilde{\chi}_2 = \chi_2 - \chi_2^\star$ and $\tilde{\chi}_3 = \chi_3 - \chi_3^\star$. The dynamics of the error variables result in

$$\begin{aligned} \dot{\tilde{\chi}}_2 &= A_2\tilde{\chi}_2 + B_2\mu_2, \\ y_2 &= C_2\tilde{\chi}_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}\tilde{\chi}_2, \end{aligned} \tag{7a}$$

$$\begin{aligned} \dot{\tilde{\chi}}_3 &= A_3\tilde{\chi}_3 + B_3\mu_3, \\ y_3 &= C_3\tilde{\chi}_3 = \begin{bmatrix} k_{p,2} & \frac{k_{p,2}}{T_{i,2}} \end{bmatrix}\tilde{\chi}_3, \end{aligned} \tag{7b}$$

with $\mu_2 = -y_3$ and $\mu_3 = y_2$. Note that the equilibrium point of (7) is the origin. Furthermore, the subsystems (7a) and (7b) are in negative feedback interconnection. Describing the subsystems (5b) and (5c) as (7) has two advantages. First, finding conditions under which the origin of (7) is globally asymptotically stable will answer Problem 1, and second, this can be investigated by looking at the problem as the stability of two interconnected systems.

B. Two preliminary results

Now, with the systems (7a) and (7b) at hand, we are in a position to provide our first two results.

Lemma 1: Consider the system (7a) with input μ_2 and output y_2. Fix $T_{i,12} > 0$ and choose $k_{p,12}$ such that

$$k_{p,12} > \frac{(L(L+1) - R_s T_{i,12})^2}{4L^2 T_{i,12}}, \tag{8}$$

then the system (7a) is strictly passive, with storage function

$$V_2(\tilde{\chi}_2) = \frac{1}{2}\tilde{\chi}_2^\top P_2\tilde{\chi}_2,\ P_2 = \begin{bmatrix} \frac{L+1}{k_{p,12}} & \frac{1}{L} \\ \frac{1}{L} & \frac{k_{p,12}}{L^2} \end{bmatrix}. \tag{9}$$

Lemma 2: Consider the system (7b) with input μ_3 and output y_3. The system (7b) is passive if and only if

$$T_{i,2} > \frac{J_m}{R_m} \quad \text{and} \quad k_{p,2} > 0, \tag{10}$$

with storage function

$$V_3(\tilde{\chi}_3) = \frac{1}{2}\tilde{\chi}_3^\top P_3\tilde{\chi}_3,\ P_3 = \begin{bmatrix} \frac{J_m k_{p,2}}{3\phi_f} & \frac{J_m k_{p,2}}{3\phi_f T_{i,2}} \\ \frac{J_m k_{p,2}}{3\phi_f T_{i,2}} & \frac{R_m k_{p,2}}{3\phi_f T_{i,2}} \end{bmatrix}. \tag{11}$$

The proofs of Lemma 1 and Lemma 2 can be found in the Appendix.

With Lemma 1 and Lemma 2, we can next state the result that enables answering Problem 1.

C. Main Result

Based on the passivity analysis done in Section IV-A, we provide a solution to Problem 1, i.e., we give sufficient conditions over the controller gains to ensure the internal stability of the DT-PMSM and the correct regulation of its speed. These conditions are given in the next theorem.

Theorem 1: Consider the Problem 1. Choose the controller gains $k_{p,_{11}} > 0$, $T_{i,_{11}} > 0$, $k_{p,_{31}} > 0$, $T_{i,_{31}} > 0$, $k_{p,_{32}} > 0$, $T_{i,_{32}} > 0$, and $k_{p,_{12}} > 0$, $T_{i,_{12}} > 0$, $k_{p,_2} > 0$, $T_{i,_2} > 0$, such that

$$\begin{aligned} T_{i,_2} &> \frac{J_m}{R_m}, \\ k_{p,_{12}} &> \frac{(L(L+1) - R_s T_{i,_{12}})^2}{4L^2 T_{i,_{12}}}. \end{aligned} \tag{12}$$

Then, the origin of the interconnected system (7) with $\mu_2 = -y_3$ and $\mu_3 = y_2$ is asymptotically stable. Consequently, the desired speed regulation of the DT-PMSM is achieved, i.e., $\omega_e \to \omega_e^r$ as $t \to \infty$.
The proof of Theorem 1 is given in the Appendix.

In the classical framework of designing cascade control systems, a well-established practice is first tuning the inner loop controller before addressing the outer loop. This approach is driven by the inherent differences in the dynamics of these loops, typically with the inner loop exhibiting faster dynamics than the outer loop. Departing from this conventional sequence can result in a precarious state of system instability, as documented in [17]. It is essential to emphasize that in this classical approach, any reconfiguration of the inner loop controller necessitates corresponding adjustments to the gain parameters of the outer loop controller.
Unlike the traditional cascade control tuning approach, the passivity-based method outlined in this paper provides a clear advantage by ensuring the stability of the overall closed-loop system for a PI-based cascade-speed control in a PMSM. This is facilitated by decomposing the closed-loop dynamics into a feedback interconnection of two subsystems and deriving conditions under which the input-output maps of both systems are passive. These conditions allow for an independent tuning of the inner and outer loop controllers. It should be emphasized that the stability criterion depends only on the inherent parameters of the motor, which can be particularly advantageous in a variety of applications and practical scenarios.

V. Simulation Example

To illustrate the applicability of the proposed method for choosing the controller gains given in Theorem 1, we simulate the cascade control scheme for a DT-PMSM using `Matlab/Simulink`. The parameters used for the DT-PMSM model are given in Table I, which are adapted from the ones used in [9]. During the simulation, both the speed reference ω_e^r and the load torque T_l are generated as piecewise-constant functions defined as

TABLE I
Parameters of the DT-PMSM used in simulation

Motor Parameters	Value
Number of pole pairs p	3
Inductance L [mH]	55.0
Stator resistance R_s [Ω]	6.0
Moment of inertia J_m [kgm^2]	3.61×10^{-4}
Viscous friction coefficient R_m [kgm/s^2]	0.2
Permanent magnet flux ϕ_f [Wb]	0.236

$$\omega_e^r \text{ [rad/s]} = \begin{cases} 0 & t \in [0, 0.5)\ [s] \\ 100 & t \in [0.5, 1.5)\ [s] \\ -50 & t \in [1.5, \infty)\ [s] \end{cases},$$

$$T_l \text{ [Nm]} = \begin{cases} 0 & t \in [0, 1.25)\ [s] \\ -2 & t \in [1.25, 2.25)\ [s] \\ 2 & t \in [2.25, \infty)\ [s] \end{cases}.$$

The reference for i_d is set to zero, as explained in Section III. In the case of i_q, its reference follows (3). The dynamics of i_{z_1} and i_{z_2} is not included in the simulation, since those states are decoupled from the dynamics of the motor's speed. For the gains of the controller (2), we chose $k_{p,_{11}} = k_{p,_{12}} = 184$ and $T_{i,_{11}} = T_{i,_{12}} = 0.08$. For the controller (3), the gains used in the simulation are $k_{p,_2} = 0.049$ and $T_{i,_2} = 0.002$. In both cases, the used gains satisfy the constraints in (12).

The simulation results are shown in Figure 2 and Figure 3. In Figure 2, it is shown, how the controllers with the proposed gains achieve the regulation of the motor speed even in the presence of a changing reference. Additionally, the tracking of the current reference is also illustrated, and, as can be seen, it is also correctly done. Finally, in Figure 3, the DC and quadrature currents are illustrated alongside the external load torque applied to the motor. Here it is shown how the DC current is kept at zero, while the quadrature current is the one used to generate the electric torque.
Finally, we would like to emphasize once again that the gains of each controller are chosen independently of the gains of the other, thanks to the passivity-based approach derived in Section IV-A to guarantee asymptotic stability of the closed-loop system. This is the main comparative advantage of the proposed method over the standard cascade control design, where the outer loop gains depend on the inner loop gains.

VI. Conclusion

In conclusion, this analysis has addressed the challenge of achieving global asymptotic speed regulation in DT-PMSMs with non-salient poles. By applying the VSD transformation and adopting a passivity-based approach, the presented method offers a novel solution that guarantees stable and efficient speed regulation, while enabling independent tuning of each controller.

Traditionally, the approach to speed regulation in this class of machines involves the use of two nested PI controllers in a cascade configuration. However, this method introduces a dependence between the gains of the inner and outer

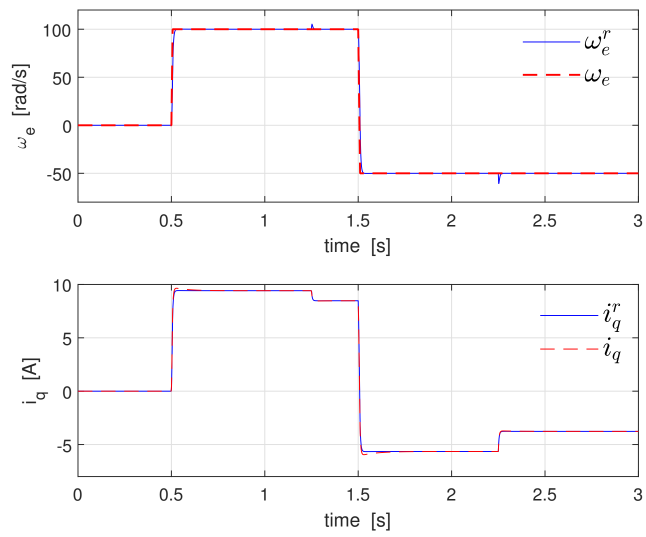

Fig. 2. Results of the speed regulation simulation. The graphs show the tracking of speed and quadrature current.

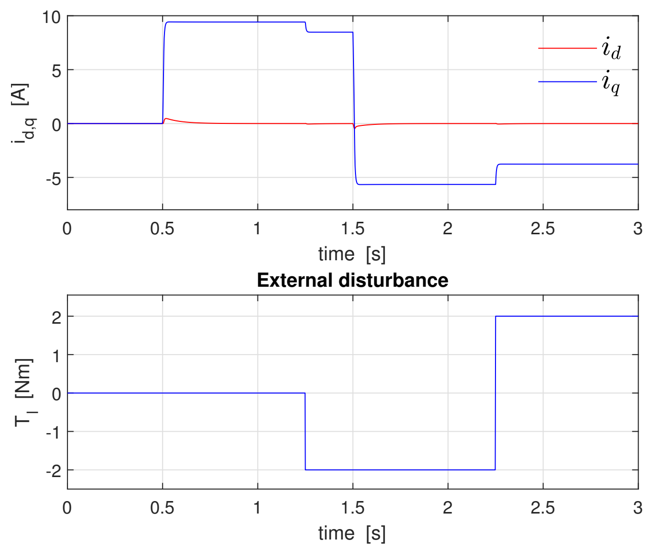

Fig. 3. Behavior of direct and quadrature currents and external load torque applied to the motor.

controllers, making it challenging to design their gains to ensure stable machine operation. In contrast, by adopting a passivity-based approach, this paper establishes sufficient conditions that guarantee global asymptotic regulation of the speed, while enabling independent tuning of each controller. This achievement is made possible through a novel decomposition of the closed-loop machine dynamics, resulting in two passive subsystems interconnected in negative feedback.

Possible directions for extending this work are the consideration of machines with saliency and cross-magnetization and the gain design for attenuation of disturbances.

Appendix

A. *Proof of Lemma 1*

Consider P_2 in (9). Since its trace is positive, P_2 is positive definite if $\det(P_2) > 0$. This results in the inequality

$$\frac{L+1}{L^2} - \frac{1}{L^2} = \frac{1}{L} > 0,$$

which trivially holds for any positive inductance value $L > 0$. Now, it is straightforward to verify that

$$P_2 B_2 = C_2^\top = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Thus, to show that (7a) is passive, it is only left to demonstrate that [18, Lem. 6.4]

$$Q_2 = P_2 A_2 + A_2^\top P_2 = \begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix} \leq 0, \tag{13}$$

with

$$q_{11} = -\frac{2(k_{p,12} L + R_s(L+1))}{L k_{p,12}}, \quad q_{22} = -\frac{2k_{p,12}}{L^2 T_{i,12}},$$
$$q_{12} = -\frac{R_s T_{i,12} + L(L+1)}{L^2 T_{i,12}}.$$

The trace of Q_2 is negative for $k_{p,12} > 0$ and $T_{i,12} > 0$. Thus, to show that Q_2 is negative definite, it is enough to prove that $\det(Q_2) > 0$. For $\det(Q_2)$, we have:

$$\det(Q_2) = -\frac{R_s^2}{L^4} - \frac{1}{(T_{i,12})^2} - \frac{1}{L^2 (T_{i,12})^2} - \frac{2}{L (T_{i,12})^2} + \frac{4k_{p,12}}{L^2 T_{i,12}} + \frac{2R_s}{L^3 T_{i,12}} + \frac{2R_s}{L^2 T_{i,12}}.$$

By isolating $k_{p,12}$, $\det(Q_2) > 0$ is equivalent to (8).

□

B. *Proof of Lemma 2*

Consider P_3 in (11). Since its trace is positive, P_3 is positive definite if and only if $\det(P_3) > 0$. This results in the following inequality:

$$\frac{J_m R_m (k_{p,2})^2}{(3\phi_f)^2 T_{i,2}} - \frac{J_m^2 (k_{p,2})^2}{(3\phi_f)^2 (T_{i,2})^2} > 0.$$

The inequality above holds if and only if the conditions in (10) are satisfied. Thus, $V_3(\tilde{\chi}_3)$ in Lemma 2 is a proper storage function. Now, it is straightforward to verify that

$$P_3 B_3 = C_3^\top = \begin{bmatrix} k_{p,2} & \frac{k_{p,2}}{T_{i,2}} \end{bmatrix}^\top.$$

Thus, to show that (7b) is passive, it is only left to demonstrate that [18, Lem. 6.4]

$$\begin{aligned} Q_3 &= P_3 A_3 + A_3^\top P_3 \\ &= \begin{bmatrix} -\frac{1}{3\phi_f}\left(-J_m \frac{k_{p,2}}{T_{i,2}} + R_m k_{p,2} \right) & 0 \\ 0 & 0 \end{bmatrix} \leq 0. \end{aligned}$$

This is also the case if and only if the gains satisfy (10). Therefore, the subsystem (7b) is passive if and only if (10) is satisfied.

□

C. Proof of Theorem 1

The conditions over the first set of gains ($k_{p,_{11}} > 0$, $T_{i,_{11}} > 0$, $k_{p,_{31}} > 0$, $T_{i,_{31}} > 0$, $k_{p,_{32}} > 0$, $T_{i,_{32}} > 0$) follow from the arguments given in Section IV. The conditions for the second set of gains ($k_{p,_{12}} > 0$, $T_{i,_{12}} > 0$, $k_{p,_2} > 0$, $T_{i,_2} > 0$) in (12) are obtained from Theorem 1, following the next rationale.

To prove asymptotic stability of the origin of (7), we take as Lyapunov function candidate the storage function

$$V(\tilde{\chi}_2, \tilde{\chi}_3) = V_2(\tilde{\chi}_2) + V_3(\tilde{\chi}_3),$$

with V_2 and V_3 as in Lemma 1 and Lemma 2, respectively. Under the condition (12) (or equivalently (8) and (10)) V is positive definite and its derivative along the solutions of (7) satisfies

$$\dot{V} = \tilde{\chi}_2^\top Q_2 \tilde{\chi}_2 - \frac{1}{3\phi_f}\big(-J_m \frac{k_{p,_2}}{T_{i,_2}} + R_m k_{p,_2}\big)\tilde{\omega}_e^2, \tag{14}$$

with Q_2 in (13). Note that the interconnection between (7a) and (7b) cancels out due to the passivity of the subsystems and the negative feedback interconnection. Under condition (12), Q_2 is negative definite. Hence, there exists a $\varepsilon > 0$, such that

$$\dot{V} \leq -\varepsilon\, \tilde{\chi}_2^\top \tilde{\chi}_2 - \frac{1}{3\phi_f}\big(-J_m \frac{k_{p,_2}}{T_{i,_2}} + R_m k_{p,_2}\big)\tilde{\omega}_e^2 \leq 0.$$

This shows that the derivative of V is negative semidefinite. Thus, by invoking LaSalle's Invariance Principle [18] and inspecting the set $W = \{\tilde{\chi}_2 \in \mathbb{R}^2,\ \tilde{\chi}_3 \in \mathbb{R}^2 \mid \dot{V} = 0\}$, we conclude that $\tilde{\chi}_2 = 0$ and $\tilde{\omega}_e = 0$ on this set. Furthermore, from (7b) we see that $\tilde{\omega}_e = 0$ implies z_2 constant. Thus, any element contained in W is an equilibrium point. Since the origin is the only equilibrium point of the dynamics (7), we can conclude that the origin is asymptotically stable, thus completing the proof of the theorem.

□

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