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# Cooperative Nonlinear Aircraft Defense using Super Twisting Algorithm with State Constraints

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*Abstract*— **This paper addresses the problem of cooperative guidance design for aircraft defense systems involving a target, a defender, and an attacker, using a super-twisting algorithm and barrier Lyapunov function. The system dynamics involve unknown functions, which are treated like disturbances and estimated using an observer. Imposing practical assumptions on the lateral accelerations of the vehicles, we propose a robust guidance design also to constrain the system states by definite bounds. We derived the controller using generalized triangle guidance concepts that provide these bounds for the states. Guidance commands are derived without any kind of linearizations that enable a wider domain of applicability of the proposed approach. The convergence is analyzed using a Lyapunov function. Simulation results are presented to study the efficacy of the proposed approach.**

## I. Introduction

Protecting civilian and military assets against highly sophisticated missile attackers is of utmost importance and concern. The required defense mechanisms need to counteract advanced attacker vehicles, which are intelligent with high maneuverability. Existing guidance and control schemes to combat such unforeseen attacks using active defenses can be broadly categorized as line-of-sight (LOS) obeying guidance laws, interactive game theoretic approaches, and non-linear control strategies, like sliding modes and barrier Lyapunov function (BLF) based designs. The sliding mode control is known for its robust performance in systems (refer [1]) but introduces a high-frequency chattering in the actuation circuitry. To overcome this issue, the higher-order sliding mode theory has been developed; see [2]. In this paper, we design a guidance strategy for the three-body engagement mechanism involving a target, a defender, and an attacker using a higher-order sliding mode algorithm; the super twisting algorithm (STA). The novelty lies in constraining the system states using a barrier Lyapunov function and proving this combined approach's finite time convergence.

The initial works to address target defense using a guided interceptor can be traced to [3]. These initial proportional guidance schemes using the LOS strategy work well for noise and disturbance-free scenarios and require various acceleration restrictions. To address these issues, non-linear LOS guidance laws have been investigated, [4], [5]. The three-body engagement mechanism involving a target, a defender, and an attacker was introduced in [6]. It has been proven for its efficacy via different control strategies like the linear quadratic differential game theory in [7] and [8], optimal control in [9] and [10] and sliding mode control in [11]. The work in [12] presents a detailed study of these control strategies for cooperative nonlinear aircraft defense systems. We design a robust STA-based guidance control using the generalized triangle guidance rule developed in [12]. These rules constrain the flight path angle between the target, defender, and attacker by a permissible range. To achieve this constraint along with a robust yet continuous control law, we propose the BLF-based STA design.

A BLF is a non-linear, scalar function defined on an open region and strictly constrains the trajectories by its bounds [13]. It thus provides an elegant approach to restrict the system states. The function itself can be used to design non-linear control for a large class of systems (see [14] and [12]), or it can be coupled with other control methods like sliding modes to complement its features, for example, [15], [16], and [17]. In [14], the non-linear control is designed for attitude regulation in spacecraft and [12] addresses guidance control for three body engagement mechanisms. It is observed that BLF-based control is not robust to system perturbations, and hence as seen in [14] and [12], a disturbance observer is used to estimate and compensate for the disturbance via the control. Coupling BLF and SMC overcomes this problem, and robust control can be designed as seen in [15], [16] and [17]. The approach in [15] is a novel adaptive gain formulation using BLF wherein the bounds of disturbance are not required to be known, as is the case in conventional SMC design. A similar approach for adaptive STA is presented in [18]. We are designing STA with variable gains imposing state constraints via the BLF for the problem under consideration. To the best of our knowledge, such a design is not reported in the literature yet.

It is observed that guidance control for the three-agent setup was addressed for the first time using BLF in [12]. The non-linear control developed in it faces vulnerability to high control amplitude and ensures asymptotic convergence of its trajectories. In this paper, we build upon the methods from [12] for a robust control design to achieve finite time convergence of the states. The main contribution of this paper is STA-based control design to achieve finite time convergence along with constrained states using BLF. The gains of STA are time-varying and finitely bounded. The resulting control is simple, with no possibility of unbounded shoot-up. A disturbance term which is a function of the attacker vehicle's acceleration, is estimated using a finite time disturbance observer. The closed loop convergence is

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proved using Lyapunov analysis, and simulation results are presented to validate our theory.

## II. Problem formulation and preliminaries

Consider a three-body air combat situation involving the attacker (interceptor), defender, and target (an aircraft) as shown in Fig. 1. The target is a protected entity and is to be safeguarded from the attacker via the defender's deployment to neutralize the attacker before it reaches the vicinity of the target. Assume that these three entities are point-mass vehicles in the inertial coordinate system $X_I O Y_I$. We denote

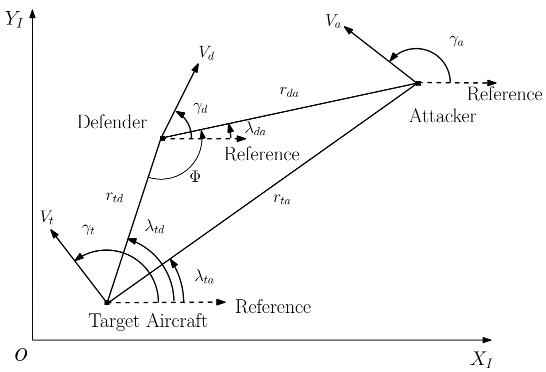



Fig. 1: Engagement geometry for three-body problem.

the flight path angles for the target, defender, and attacker by $\gamma_t$, $\gamma_d$, $\gamma_a$, respectively, and their respective velocities by $V_t$, $V_d$, $V_a$. Here and in all other notations further, we use the subscripts $t$, $d$, $a$ to represent the target, the defender, and the attacker, respectively. The line-of-sight (LOS) angle and the relative separation between these vehicles are denoted by $\lambda_{ij}$, and $r_{ij}$, respectively, where the subscript $ij = ta$, $da$, $td$ corresponds to the respective pairs. Fig. 1 represents all these variables geometrically demonstrating their physical meanings.

We consider the following kinematic equations of relative engagement among adversaries to formulate the mathematical model of the engagements. For the three relative engagements, the range rates are given by

$$\dot{r}_{ij} = V_j \cos\left(\gamma_j - \lambda_{ij}\right) - V_i \cos\left(\gamma_i - \lambda_{ij}\right),$$

and for the corresponding LOS rates, we have

$$r_{ij}\dot{\lambda}_{ij} = V_j \sin\left(\gamma_j - \lambda_{ij}\right) - V_i \sin\left(\gamma_i - \lambda_{ij}\right).$$

The heading angles are governed by

$$\dot{\gamma}_i = \frac{a_i}{V_o}, \ |a_i| \le a_i^{\max},$$

where $a_i$ denotes the lateral accelerations of the $i^{\text{th}}$ adversary, which is assumed to be bounded by $a_i^{\max}$.

Using these dynamics, we can derive the dynamics of the LOS angles $\lambda_{da}$, $\lambda_{td}$ and $\lambda_{ta}$ as follows:

$$\ddot{\lambda}_{da} = -\frac{2\dot{r}_{da}\dot{\lambda}_{da}}{r_{da}} - \frac{\cos\left(\gamma_d - \lambda_{da}\right)}{r_{da}} a_d + \frac{\cos\left(\gamma_a - \lambda_{da}\right)}{r_{da}} a_a. \tag{1a}$$

$$\ddot{\lambda}_{ta} = -\frac{2\dot{r}_{ta}\dot{\lambda}_{ta}}{r_{ta}} - \frac{\cos\left(\gamma_t - \lambda_{ta}\right)}{r_{ta}} a_t + \frac{\cos\left(\gamma_a - \lambda_{ta}\right)}{r_{ta}} a_a \tag{1b}$$

$$\ddot{\lambda}_{td} = -\frac{2\dot{r}_{td}\dot{\lambda}_{td}}{r_{td}} - \frac{\cos\left(\gamma_t - \lambda_{td}\right)}{r_{td}} a_t + \frac{\cos\left(\gamma_d - \lambda_{td}\right)}{r_{td}} a_d. \tag{1c}$$

The reader can refer [12] for the detailed derivations of these equations. We begin with the following assumption to formulate the problem addressed in this paper.

**Assumption II.1.** *The target's speed is assumed to be constant and satisfy $V_d \approx V_a \gg V_t$.*

The problem under consideration is stated as follows:

**Problem 1.** *Consider the generic geometric rule; Generalized Triangle Guidance, stated in [12], employing the team of the target, the defender, and the attacker, for successful interception of the attacker before its arrival within the target's proximity. Design a robust and continuous guidance law for the target and defender team to enforce this rule, with certain angle constraints, that ensures the capture of the attacker by the defender.*

We state the following Lemmas, which will be used in further analysis.

**Lemma II.2.** *[13, Lemma 2]: An asymmetric barrier Lyapunov function given by*

$$V_o(x) = \frac{1 - q(x)}{2} \log\left(\frac{\gamma_1^2}{\gamma_1^2 - x^2}\right) + \frac{q(x)}{2} \log\left(\frac{\gamma_2^2}{\gamma_2^2 - x^2}\right), \tag{2}$$

*for a continuous function $x(t) \in \mathbb{R}$ with decision rule defined by*

$$q(x) = \begin{cases} 1, & \text{if } x(t) > 0, \\ 0, & \text{if } x(t) \le 0, \end{cases} \tag{3}$$

*is positive definite for $-\gamma_1 < x(t) < \gamma_2$, $V_o(x) = 0$ for $x = 0$ and satisfies $V_o(x) \in \mathbb{C}^1$. The constant $\gamma_i'$s, $i = 1, 2$ are the imposed bounds on $x$.*

**Lemma II.3.** *For all $a > b$, $a$, $b \in \mathbb{R}$, the following inequality is satisfied:*

$$-\left(\frac{b^2}{a^2 - b^2}\right) \le -\log\left(\frac{a^2}{a^2 - b^2}\right).$$

For proof of the Lemma, please refer [19, Lemma 2].

## III. Proposed STA based control

This section presents the guidance design for the target and the defender. The angle that the defender-target team aims to perpetuate for a successful interception of the attacker is defined by $\phi(t) = \Phi(t) - \Phi_0$ where $\Phi(t) = \pi + \lambda_{da} - \lambda_{td}$

and $\Phi_0$ is the reference angle. Its dynamics can be obtained using (1a) and (1c) as follows:

$$\begin{aligned}\ddot{\phi}(t) =& \frac{-2\dot{r}_{da}(t)\dot{\lambda}_{da}(t)}{r_{da}(t)} + \frac{2\dot{r}_{td}(t)\dot{\lambda}_{td}(t)}{r_{td}(t)} \\ &+ \frac{cos(\gamma_t(t) - \lambda_{td}(t))}{r_{td}(t)} a_t(t) \\ &- \left( \frac{cos(\gamma_d(t) - \lambda_{td}(t))}{r_{td}(t)} + \frac{cos(\gamma_d(t) - \lambda_{da}(t))}{r_{da}(t)} \right) a_d(t) \\ &+ d(t), \end{aligned} \tag{4}$$

where $d(t) = \dfrac{cos(\gamma_a(t) - \lambda_{da}(t))}{r_{da}(t)} a_a(t)$. This term is unknown as the attackers acceleration is not measurable. Hence, assuming it is bounded, a higher order sliding mode disturbance observer is used to estimate disturbance term, $d$ using (1a). The observer dynamics are given by

$$\left.\begin{aligned} &\dot{z}_0 = -\frac{2\dot{r}_{da}\dot{\lambda}_{da}}{r_{da}} - \frac{\cos(\gamma_d - \lambda_{da})}{r_{da}} a_d + v_0, \\ &v_0 = -\lambda_2 L_{da}^{(1/3)} |z_0 - \dot{\lambda}_{da}|^{(2/3)} \text{sign}(z_0 - \dot{\lambda}_{da}) \\ &-\mu_2(z_0 - \dot{\lambda}_{da}) + z_1, \\ &\dot{z}_1 = v_1, \\ &v_1 = -\lambda_1 L_{da}^{(1/2)} |z_1 - v_0|^{(1/2)} \text{sign}(z_1 - v_0) \\ &-\mu_1(z_1 - v_0) + z_2, \\ &\dot{z}_2 = -\lambda_0 L_{da}\, \text{sign}(z_2 - v_1) - \mu_0(z_2 - v_1), \\ &\hat{d} = z_1, \end{aligned}\right\} \tag{5}$$

where the coefficients $\lambda_2 > \lambda_1 > \lambda_0 > 0$, $\mu_2 > \mu_1 > \mu_0 > 0$, and $L_{da}$ is a Lipschitz constant such that $|\dot{d}| \leq L_{da}$. The sign function is defined as $\text{sign}(x) = \frac{|x|}{x}$ for all $x \neq 0$ and $\text{sign}(x) = [1, -1]$ for $x = 0$. This disturbance observer is considered from [20] wherein it is proved that $\hat{d} \to d$ in a finite time.

Consider the auxiliary input $u(t) = a_1 a_t - a_2 a_d$, where

$$a_1 = \frac{cos(\gamma_t(t) - \lambda_{td}(t))}{r_{td}(t)}, \tag{6}$$

$$a_2 = \left( \frac{cos(\gamma_d(t) - \lambda_{td}(t))}{r_{td}(t)} + \frac{cos(\gamma_d(t) - \lambda_{da}(t))}{r_{da}(t)} \right). \tag{7}$$

Define the states $x_1(t) = \phi(t)$ and $x_2(t) = \dot{\phi}(t)$. The dynamics of angle $\phi$, given by (4), can be represented as

$$\left.\begin{aligned} &\dot{x}_1(t) = x_2(t), \\ &\dot{x}_2(t) = \tfrac{-2\dot{\gamma}_{da}(t)\dot{\lambda}_{da}(t)}{r_{da}(t)} + \tfrac{2\dot{\gamma}_{td}(t)\dot{\lambda}_{td}(t)}{r_{td}(t)} + u(t) + d(t). \end{aligned}\right\} \tag{8}$$

The state $x_1(t)$ is required to be bounded as follows:

$$-\gamma_1 < x_1(t) < \gamma_2, \ \forall\, t \geq 0, \tag{9}$$

where $\gamma_1 = 3\pi/2 - \Phi_0$ and $\gamma_2 = \Phi_0 - \pi/2$. We impose the following assumption on the system dynamics.

**Assumption III.1.** *Initial state $x_1(0)$ satisfies bounds (9), i.e. $-\gamma_1 < x_1(0) < \gamma_2$.*

Further, we also consider measured $x_1(t)$ and $x_2(t)$ and exact estimation of $d(t)$ using (5) for the purpose of control design. The following Theorem III.2 develops the main result for robust control of the three-body planar engagement system with the state constraints.

**Theorem III.2.** *Consider dynamics of the three body planar engagement given by (8) satisfying Assumptions II.1 and III.1 with measured states $x_1(t)$ and $x_2(t)$ and the state constraint, given by (9). Let the estimation $\hat{d}(t)$ be obtained using the observer (5) ensuring $d(t) - \hat{d}(t) = 0$ within a finite time. Consider the control input $u(t)$ given by*

$$\begin{aligned} u(t) =& \frac{2\dot{r}_{da}(t)\dot{\lambda}_{da}(t)}{r_{da}(t)} - \frac{2\dot{r}_{td}(t)\dot{\lambda}_{td}(t)}{r_{td}(t)} \\ &- k_1(t)|s(t)|^{1/2} sign(s(t)) - k_2(t) \int_0^t sign(s(\tau)) d\tau \\ &- k_3 s(t) - k_4(t) \int_0^t s(\tau) d\tau - x_2 - \hat{d}(t), \end{aligned} \tag{10}$$

*with variable gains $k_1(t)$ and $k_2(t) = k_4(t)$ satisfying*

$$k_1(t) = k_{11} \left( \frac{1 - q(x_1)}{\delta_1^2 - s(t)^2} + \frac{q(x_1)}{\delta_2^2 - s(t)^2} \right)^{-1/4}, \tag{11}$$

$$k_2(t) = k_4(t) = k_{22} \left( \frac{1 - q(x_1)}{\delta_1^2 - s(t)^2} + \frac{q(x_1)}{\delta_2^2 - s(t)^2} \right)^{-1}, \tag{12}$$

*for positive constant gains $k_{11}$, $k_{22}$, $k_3$ and*

$$\delta_i = \frac{\gamma_i}{\sqrt{2}}, \tag{13}$$

*for $i = 1, 2$. The sliding manifold is defined as*

$$s(t) = x_1(t) + x_2(t). \tag{14}$$

*Function $q(x_1)$ is as given in (3). Then, switching deviations $s(t)$ will converge to zero within a finite time and satisfy the bounds $-\delta_1 < s(t) < \delta_2$ for all $t \in [0, \infty)$. This implies that $x_1(t)$ will satisfy the bounds (9) for all $t \in [0, \infty)$ and in turn, the control $u(t)$ ensures that the defender will maintain an angle of $\Phi_0$ with respect to attacker and target, and subsequent interception of the attacker is guaranteed before it captures the target.*

*Proof:* We begin by taking derivative of (14) with respect to time, and substituting from (8), to get

$$\dot{s} = x_2 + \frac{-2\dot{\gamma}_{da}\dot{\lambda}_{da}}{r_{da}} + \frac{2\dot{\gamma}_{td}\dot{\lambda}_{td}}{r_{td}} + u + d. \tag{15}$$

Here and further, we denote the functions by the first argument for simplicity. Consider the Lyapunov function candidate, given by $V = V_1 + V_2$, where

$$V_1 = \frac{1 - q}{2} \log\left( \frac{\delta_1^2}{\delta_1^2 - s^2} \right) + \frac{q}{2} \log\left( \frac{\delta_2^2}{\delta_2^2 - s^2} \right), \tag{16}$$

$$V_2 = 2k_2|s| + k_4 s^2 + \frac{z^2}{2} + \frac{1}{2}(k_1|s|^{1/2}\text{sign}(s) + k_3 s - z)^2, \tag{17}$$

where $\dot{z} = -k_2 \text{sign}(s) - k_4 s$. The function $V_1$ is the asymmetric barrier Lyapunov function from Lemma II.2 and $V_2$ is as defined for the modified super-twisting algorithm in [21]. Hence $V$ is positive definite for $-\delta_1 < s(t) < \delta_2$ and continuously differentiable for all $\{s \in (-\delta_1, \delta_2) | s \neq 0\}$. We assume that initial value of $s$ given by $s(0) = s_0$ satisfies $-\delta_1 < s_0 < \delta_2$. Taking time derivative of $V$, and substituting $\dot{s}$ from (15) and $u$ from (10), we get

$$\begin{aligned}\dot{V} &= \left(\frac{1-q}{\delta_1^2 - s^2} + \frac{q}{\delta_2^2 - s^2}\right) s(-k_1|s|^{1/2}\text{sign}(s) \\ &\quad - k_2 \int_0^t \text{sign}(s(\tau))d\tau - k_3 s - k_4(t)\int_0^t s(\tau)d\tau) \\ &\quad - \frac{1}{|s|^{1/2}}\xi^T Q_1 \xi - \xi^T Q_2 \xi,\end{aligned}$$

where $\xi = \begin{bmatrix} |s|^{1/2}\text{sign}(s) & s & z \end{bmatrix}^T$,
$Q_1 = \frac{k_1}{2}\begin{bmatrix} 2k_2 + k_1^2 & 0 & -k_1 \\ 0 & 2k_4 + 5k_3^2 & -3k_3 \\ -k_1 & -3k_3 & 1 \end{bmatrix}$,
$Q_2 = k_3 \begin{bmatrix} k_2 + 2k_1^2 & 0 & 0 \\ 0 & k_4 + k_3^2 & -k_3 \\ 0 & -k_3 & 1 \end{bmatrix}$. Here we have used $d - \hat{d} = 0$. Substituting $k_2 = k_4$ from (12) results in

$$\begin{aligned}\dot{V} &= \left(\frac{1-q}{\delta_1^2 - s^2} + \frac{q}{\delta_2^2 - s^2}\right) s(-k_1|s|^{1/2}\text{sign}(s) \\ &\quad - k_3 s) - \frac{1}{|s|^{1/2}}\xi^T Q_1 \xi - \xi^T \bar{Q}_2 \xi, \qquad (18)\end{aligned}$$

where $\bar{Q}_2 = k_3 \begin{bmatrix} k_2 + 2k_1^2 & 0 & 0 \\ 0 & k_4 + k_3^2 & -k_3 - \frac{k_{22}}{2k_2k_3} \\ 0 & -k_3 - \frac{k_{22}}{2k_2k_3} & 1 \end{bmatrix}$

Assume that the trajectory of $s$, beginning at $|s_0|$, converges to a value $\bar{s} < s_0$. Thus we will satisfy

$$k_{22}(\delta_i^2 - s_0^2) < k_2 = k_4 < k_{22}(\delta_i^2 - \bar{s}^2), \qquad (19)$$

where $\delta_i = \{\delta_1, \delta_2\}$ depending on value of $q$. The matrices $Q_1$ and $\bar{Q}_2$ are positive definite if the conditions

$$4k_2k_4 > (8k_2 + 9k_1^2)k_3^2, \qquad (20)$$

$$k_{22} > \frac{1}{k_3^2(\delta_i^2 - \bar{s}^2)^3} + \frac{2}{(\delta_i^2 - s_0^2)^2}, \qquad (21)$$

and $k_i > 0$ $(i = 1, .., 4)$ are satisfied. Refer to Remark 1 for design of these gains. Thus, with the validity of the Lyapunov function $V_1$ according to Lemma II.2, we can infer that $\dot{V} \leq 0$. We now prove the finite time convergence of $s$ to zero.

Substituting $k_1$ from (11), the first term of (18) simplifies to, $\left(\frac{1-q}{\delta_1^2 - s^2} + \frac{q}{\delta_2^2 - s^2}\right) s(-k_1|s|^{1/2}\text{sign}(s)) = -\left(\frac{(1-q)s^2}{\delta_1^2 - s^2} + \frac{qs^2}{\delta_2^2 - s^2}\right)^{3/4} k_{11}$. Using this and Lemma II.3, we can write

$$\dot{V} = -k_3 2V_1 - k_{11}2V_1^{3/4} - \alpha_1 V_2^{1/2} - \alpha_2 V_2, \qquad (22)$$

where $\alpha_1 = \frac{\lambda_{\min}^{1/2}(P)\lambda_{\min}(Q_1)}{\lambda_{\max}(P)}$, $\alpha_2 = \frac{\lambda_{\min}(\bar{Q}_2)}{\lambda_{\max}(P)}$ and
$P = \frac{1}{2}\begin{bmatrix} 4k_2 + 2k_1^2 & k_1k_3 & -k_1 \\ k_1k_3 & 2k_4 + k_3^2 & -k_3 \\ -k_1 & -k_3 & 2 \end{bmatrix}$.
Thus, we get

$$\dot{V} \leq -\kappa_1 V^{1/2} - \kappa_2 V,$$

where $\kappa_1 = \min\{2k_{11}V_1^{1/4}, \alpha_1\}$ and $\kappa_2 = \min\{-k_3, \alpha_2\}$. This inequality is valid as all the elements of $\kappa_1$ and $\kappa_2$ are finitely bounded and we have used the relation $(V_1 + V_2)^{1/2} \leq V_1^{1/2} + V_2^{1/2}$. Further, based on finite time convergence results of [22], we can imply that $s$ will converge to zero in finite time $T \leq \frac{2}{3\kappa_1}\frac{\ln(\kappa_1 V^{1/2}(s(0))) + \kappa_2}{\kappa_2}$. Also, the bounds $-\delta_1 < s < \delta_2$ are satisfied for all time $t$.

Consider the case $x_1 > 0$. It implies that the upper bound $s < \delta_2$ is satisfied. Substituting from (14), we get the constraint $x_1 + x_2 < \delta_2$. Using the system dynamics (8),

$$\dot{x}_1 < -x_1 + \delta_2. \qquad (23)$$

Consider a Lyapunov function candidate $V_3 = \frac{1}{2}x_1^2$. Taking time derivative and substituting (23), we get

$$\dot{V}_3 \leq -x_1^2 + \frac{\delta_2^2}{2} + \frac{x_1^2}{2}.$$

Here we used Young's inequality to simplify the term $\delta_2 x_1$. This gives us $\dot{V}_3 \leq -\frac{1}{2}V_3$ for all $V_3 > \theta := \delta_2^2$. This implies that the trajectory of $x_1$ starting in the set $-\gamma_1 < x_1(0) < \gamma_2$ remain in the set $\{x_1 \in \mathbb{R} : V_3 \leq \theta\}$, i.e. $x_1 < \gamma_2$, $\forall\, t \geq 0$. A similar analysis will prove the relation that $x_1 > -\gamma_1$, $\forall\, t \geq 0$. Thus, $\Phi$ follows the reference value of angle $\Phi_0$ and it implies interception of the attacker before it actually reaches near the target based on the results proven in [12, Theorem 1]. This completes the proof. □

Owing to the fact that the defender and the target lateral acceleration need to satisfy the equation (10), one can have infinitely many choices for these two. The individual expressions for defender and target lateral accelerations, which minimize instantaneous control effort, can be obtained in [23], and are given by

$$a_t = \frac{a_1\Gamma^2 u}{a_2^2 + a_1^2\Gamma^2}, \quad a_d = -\frac{a_2 u}{a_2^2 + a_1^2\Gamma^2}, \quad \Gamma = \frac{w_t}{w_d}. \qquad (24)$$

Here $w_t$ and $w_d$ denote the positive weights assigned to the $a_t$ and $a_d$, respectively, for optimization, depending on the vehicle capability.

**Remark 1.** *It is noted that $\bar{s} = 0$ and the gains $k_i$ ($i = 1, .., 4$) are always positive. A simple calculation verifies that $0 < k_{22}$ will satisfy* (21). *It can be further bounded by $k_{22} << 1$ to avoid high gains. Further, as $k_2 = k_4$ is large compared to $k_1$, it satisfies* (20) *for a small $k_3 \approx k_1$.*

The control gains in the resulting continuous control law, though variable, are finitely bounded with minimal variation.

## IV. Simulation results

In this section, we present the validation of the proposed guidance strategy for various three-body engagement scenarios. We consider the speeds of the attacker and defender as 200 m/s and the target has a speed of 100 m/s. The lateral accelerations of the defender and attacker are assumed to not exceed $20\,g$, while that of the target is bounded by $10\,g$. The attacker is assumed to employ a proportional navigation guidance strategy with a navigation constant of $N = 3$.

However, attacker dynamics are unknown to the defender and the target. The circle, diamond, and square markers in the trajectory plots denote the initial positions of the target, defender, and attacker, respectively. The guidance commands for target and defender are obtained using (24), where the term $u(t)$ is the same as defined in (10). The gains of the proposed controller are chosen as $k_{11} = k_2 = 1$, $k_{22} = 0.005$ and $\Gamma = 1$ based on the requirements of satisfying necessary conditions.

We assume that there is an initial separation between the target and defender after which we commence the engagement. When the defender is launched by the target itself, the initial LOS angle and relative distances of both defender and target with respect to the interceptor will be the same. However, this distance might be different when the defender is launched from another platform. We initiated these numerical simulations with $r_{ta} = 5$ km, $\lambda_{ta} = 0°$, $r_{td} = 300$ m and $\lambda_{td} = -70°$.

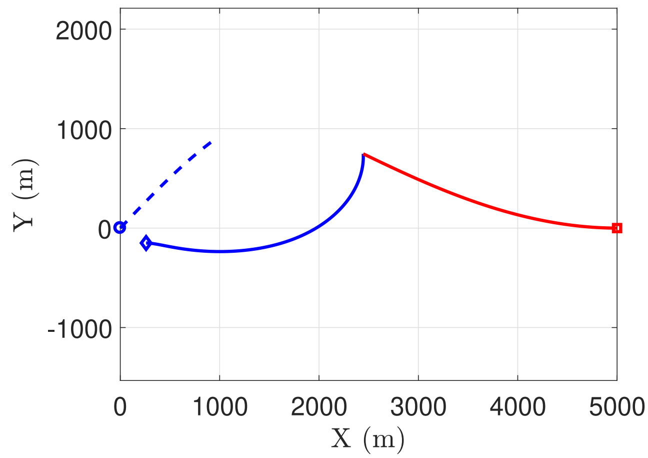
For the first case, the initial launch angles of the defender, the target, and the attacker are $0°$, $60°$, and $180°$, respectively, while the desired value of the angle to be maintained by the target and defender is chosen as $\Phi_0 = 220°$. The asymmetric constraint bounds for angle $\phi$ are $\phi \in [-50°, 130°]$.

Simulation results for this case are plotted in Fig. 2, which depicts the trajectories of all entities, their corresponding lateral acceleration profiles, and the variation of angle $\Phi$ throughout the engagement in Fig. 2c. It can be seen from Fig. 2a that the defender intercepts the attacker before it could achieve its mission of capturing the target. The lateral acceleration are within desired bounds as shown in Fig. 2b. The sliding surface converges to zero in finite time of less than 2 seconds.

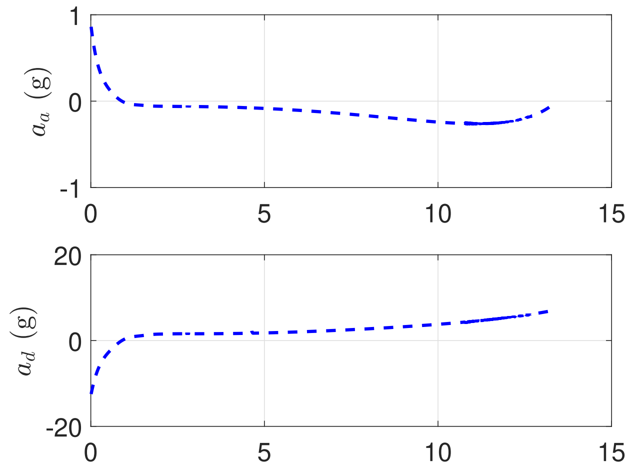
To evaluate the performance of the guidance strategy for the symmetric case, we also performed another numerical simulation with the attacker's initial separation and LOS angle with respect to the target to be the same as before. The results for this case are presented in Fig. 3. It can be seen from Fig. 3a that the defender can again capture the attacker, with lateral acceleration, as shown in Fig. 3b. The convergence of angle $\Phi$ to $\pi$ is shown in Fig. 3c.

## V. Conclusions

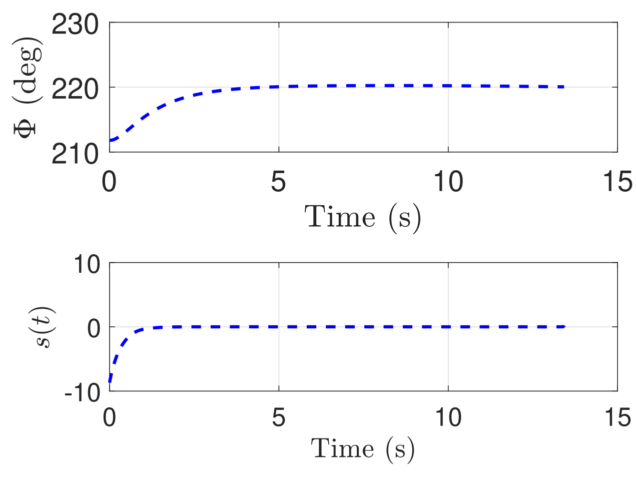
In this paper, we proposed a novel super-twisting algorithm based state-constrained control for a three-agent system comprising a target, a defender, and an attacker. The control design (guidance law) was developed on the basis of a barrier Lyapunov function and resulted in a simple, continuous, finitely bounded guidance law. The STA ensures finite-time convergence of the sliding manifold and exponential convergence of the states. The generalized triangle rule ensures successful interception of the attacker by the defender before the attacker hits the target. This barrier function based STA design is generic and can be implemented for a large class of linear and non-linear second-order mechanical systems. This guidance scheme can be investigated further to overcome the necessity of the disturbance observer.




(a) Trajectories of attacker, target, and defender.




(b) Lateral acceleration profiles of target and defender.




(c) Variation of angle $\Phi$ and switching surface deviation $s$.

Fig. 2: Interception of the attacker by a defender with proposed guidance strategy using $\Phi = 220°$.

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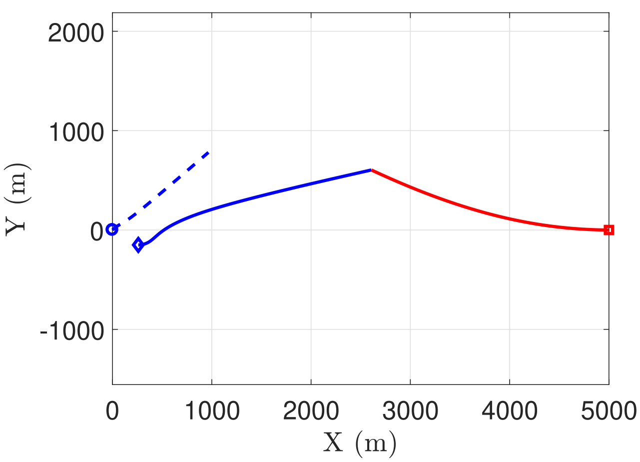
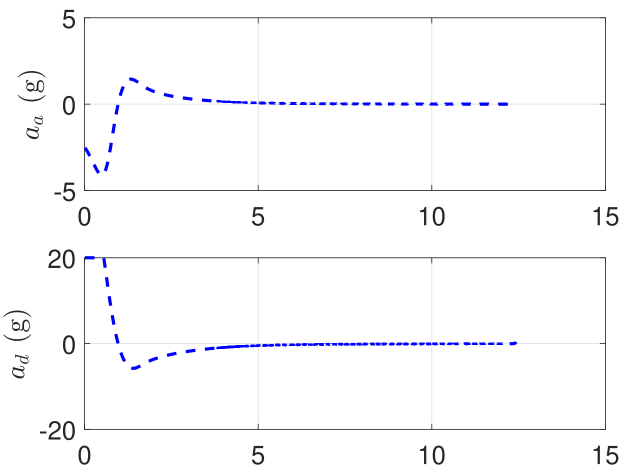
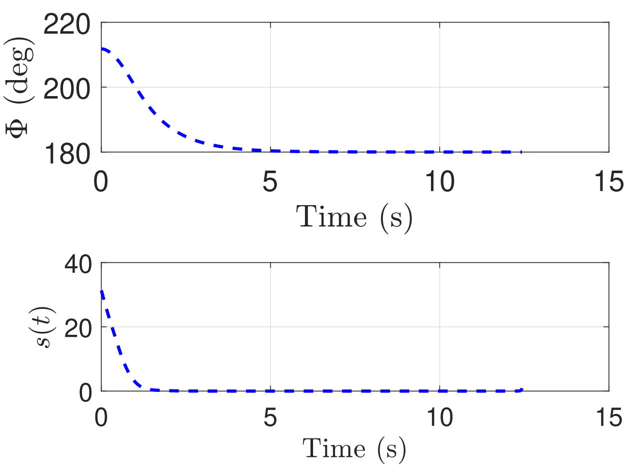



(a) Trajectories of attacker, target, and defender.




(b) Lateral acceleration profiles of target and defender.




(c) Variation of angle $\Phi$ and switching surface deviation $s$.

Fig. 3: Interception of the attacker by a defender with proposed guidance strategy using $\Phi = 180°$.

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