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Controller Synthesis of Signal Temporal Logical Tasks for Cyber-Physical Production Systems via Acyclic Decomposition

Shuling Wang, Shanying Zhu, Cailian Chen and Liang Xu

Abstract— **Modularization facilitates the adaptability of cyber-physical production systems (CPPSs) with a variety of collaborative tasks. Various production rules can be captured by signal temporal logic (STL) specifications imposed on interconnected multi-agent systems (MASs). In this paper, we focus on the controller synthesis of STL tasks for interconnected MASs to accomplish collaborative tasks in modular CPPSs. Firstly, a class of STL specifications characterizing tasks by the combination of fixed-time reachability and finite-time persistence tasks is proposed, which encompasses a large class of production specifications for the MAS. Secondly, the acyclic decomposition of the global STL formula is constructed to enable conflict-free collaborative tasks and unidirectional couplings between subsystems. By establishing the equivalence between the proposition and the state set of the MAS, necessary and sufficient conditions are respectively proposed for the satisfaction of reachability and persistence tasks. In addition, an algorithm is presented to synthesize controllers for the MAS with the global STL specification based on local controllers of subsystems. An illustrative example is given to show the effectiveness of the proposed method.**

I. INTRODUCTION

As the core of Industrie 4.0, the world has witnessed rapid developments of cyber-physical production systems (CPPSs) [1]. By modular and reusable cyber-physical components, called production modules and optional behaviors they exhibit, CPPSs adapt quickly and efficiently to new production requirements [2], [3]. Operations and production rules required by various products can be achieved by enforcing different timing constraints between behaviors collaboratively performed by agents such as fixed-time reachability and finite-time persistence [4]. This spawns the need of synthesizing multi-agent systems (MASs) to complete collaborative tasks under given timing constraints [5].

Signal temporal logic (STL) is capable of capturing the execution order of production operations and temporal distance, which enables one to translate the timing constraints in CPPSs to syntactically correct formulas [6]–[8]. Existing methods on the synthesis of MASs with STL specifications suffer from the curse of dimensionality when encountering large-scale CPPSs with tight interactions between computational components and physical entities [9]. Divide and conquer technique is a feasible solution which breaks down large design problems into smaller pieces [10], [11].

Recently, several results have been proposed for the STL controller synthesis of interconnected MASs via synthesizing smaller subsystems with interconnection. These results can be subsumed under two categories: one is where all agents are subject to a global task and the other is where each agent is subject to a local task. These local tasks can be obtained in two ways, either a global task is decomposed into local ones as in [12], or each agent is assigned a local task regardless of whatever the others are assigned. Within the first category, using assume-guarantee contracts, Ref. [13], [14] designed decentralized controllers for interconnected MASs subject to STL constraints. However, the results were derived under the assumption that the global STL formula is separable with respect to subsystems. It is obvious that this kind of formula cannot well capture collaborative tasks that are ubiquitous in CPPSs. Within the second category, a challenge is that local tasks may be in conflict, that is, the satisfaction of each local task does not imply that of the conjunction of all local tasks. In view of this, Ref. [15], [16] found least violating solutions in these conflicting situations by firstly finding a solution for the case when local tasks are conflict-free, and then resolving the violation of the local task from online collaboration with other agents. Note that these results are established based on a trivial aggregation from local task for each agent and the assumption that the dynamical couplings between different blocks are bounded. How to find an aggregation to reduce couplings between subsystems and carefully design couplings to contribute to the satisfaction of local tasks are still unknown.

In this paper, by finding a decomposition of the global specification conflict-free for collaborative tasks and containing only unidirectional couplings, we synthesize local-based controllers for interconnected MASs with STL tasks characterizing production rules of CPPSs. Firstly, we provide a class of STL specifications, which characterizes each task as a combination of ubiquitous fixed-time reachability and finite-time persistence tasks in CPPSs [6], [12], [14]. Compared with [12], [14], the considered STL specification contains disjunction operator, which can capture more complex production rules such as different behaviors completing a certain operation. By analyzing the one-step reachability matrix for the MAS, criteria are respectively established for the satisfaction of reachability and persistence tasks. Secondly, a framework is proposed to decompose the global STL

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S. Wang, S. Zhu and C. Chen are with the Department of Automation, Shanghai Jiao Tong University, Shanghai 200240, China; Key Laboratory of System Control and Information Processing, Ministry of Education of China, Shanghai 200240, China, and also Shanghai Engineering Research Center of Intelligent Control and Management, Shanghai 200240, China e-mail: {shulingwang2021_sjtu, shyzhu, cailianchen}@sjtu.edu.cn

L. Xu is with the Institute of Artificial Intelligence, Shanghai University, Shanghai 200444, China e-mail: liang-xu@shu.edu.cn

TABLE I

NOTATIONS.

Notations	Definitions
$[a,b]_{\mathbb{N}}$	Set $\{a, a+1, \cdots, b\}$, $a, b \in \mathbb{N}$, $a \leq b$
$\mathscr{D}_s$	Logic domain $\{0, 1, \cdots, s-1\}$
I_s	s-dimensional identity matrix
δ_s^i	i-th column of I_s
$[F]_{i,:}$ ($[F]_{:,j}$)	i-th row (column) of matrix F
$[F]_{i,j}$	(i,j)-th entry of matrix F
$\mathbb{R}^{m\times n}$	Set of $m \times n$ real matrices
$Col(F)$	Set $\{[F]_{:,j} : j = 1, \cdots, n\}$ for $F \in \mathbb{R}^{m\times n}$
$\ltimes$	Semi-tensor product
$M^{(s)}$	$\underbrace{M \ltimes M \ltimes \cdots \ltimes M}_{s}$, $M^{(0)} = I_{m\times n}$, $M \in \mathbb{R}_{m\times n}$
$\mathbf{0}_n$ ($\mathbf{1}_n$)	$n \times 1$ vector with all entries being 0 (1)
$\xi = [\xi_1 \ \cdots \ \xi_n]^\top \succcurlyeq \mathbf{0}$	$\exists\ \xi_i > 0$ for some $i \in [1,n]_{\mathbb{N}}$
$\vee$ ($\wedge$)	Disjunction (conjunction) operator

specification to obtain an acyclic aggregation of the MAS. Such decompositions are conflict-free for collaborative tasks and can reduce the couplings between subsystems. Then, by algebraic state space representation (ASSR) method, an algorithm is presented to synthesize local controllers for subsystems, based on which criterion on controller design is proposed for the MAS with the global STL specification.

The key notations are summarized in Table I. Throughout this paper, semi-tensor product ($\ltimes$) is the basic matrix product [17], and the symbol "$\ltimes$" is omitted in most places.

II. PROBLEM FORMULATION

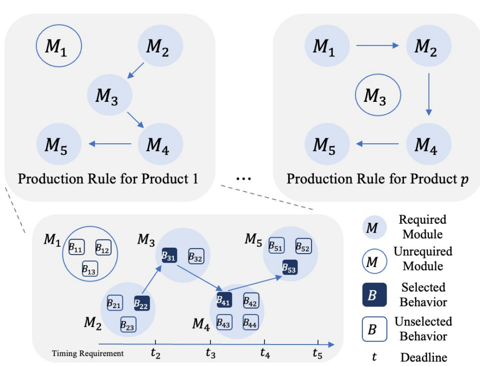

Fig. 1. Framework of a modular CPPS capable of manufacturing p kinds of products by virtue of 5 production modules. Each product corresponds to a set of required manufacturing operations, and each operation can be independently completed by a production module exhibiting one or more behaviors. Production module behaviors are collaboratively performed by agents following timing requirements such as deadline constraints.

Consider a modular CPPS capable of manufacturing several kinds of products in the workspace by virtue of a set of production modules (Fig. 1). Production module behaviors are performed by a heterogeneous interconnected MAS consisting of n agents with different capabilities such as assembling, monitoring and so on. We only control a part of agents in the MAS to complete production tasks. Specifically, the dynamics of the j-th agent is

$$x_j(t+1) = \{_\kappa \sum_{l\in\bar{\mathscr{I}}_j}\} a_{j,l} \times_\kappa x_l(t) +_\kappa b_j \times_\kappa u_j(t), \qquad (1)$$

where κ is a prime number, $\bar{\mathscr{I}}_j := \mathscr{I}_j \cup \{j\}$, $\mathscr{I}_j$ is the set of in-neighbors of j, $b_j \in [1,\kappa]_{\mathbb{N}}$, $j = 1, \cdots, m$, $b_j = 0$, $j = m+1, \cdots, n$ $m \leq n$, $a_{j,l}$, $x_j(t)$, $u_j(t) \in \mathscr{D}_\kappa$ with $x_j(t)$, $u_j(t)$ representing the state and control inputs of agent j at time t, respectively, and operations $+_\kappa$ and $\times_\kappa$ are modular addition and modular multiplication over $\mathscr{D}_\kappa$, respectively [18].

The workspace of the MAS is discretized into κ planar subregions $\{0, 1, \cdots, \kappa-1\}$ called cells. Cells $r \in \mathscr{R} \subseteq [0, \kappa-1]_{\mathbb{N}}$ related to production modules are prespecified, which are labled by a set of atomic propositions $\Pi = \{\pi_j^r : j \in \mathscr{A} := [1,n]_{\mathbb{N}}, r \in \mathscr{R}\}$, where $\pi_j^r = 1$ if and only if agent j is in cell r, that is, $x_j(\cdot) = r$. Assume that there are s tasks in the workspace over κ subregions, of which each task l can be collaboratively completed by agents in set $\mathscr{N}_l$. Assume that the task allocation has been performed according to the capabilities supported by agents. Denote the set of cells related to task l for the MAS by $\mathscr{R}_l$. Corresponding to different behaviors exhibited by a production module, each task l can be completed via w_l ways, which can be represented by $\mathscr{J}_{l,k} \subseteq S(\Pi_{\text{aug}}^l)$, $\Pi_{\text{aug}}^l := \{\pi_j^r, \neg\pi_j^r : j \in \mathscr{N}_l, r \in \mathscr{R}_l\}$, $k \in [1, w_l]_{\mathbb{N}}$. According to timing requirements of production operations, denote the earliest and the latest time to execute task l by ε_l and ι_l with $1 \leq \varepsilon_l \leq \iota_l < \infty$, respectively, and represent the duration of task l by $1 \leq \tau_l < \infty$. Then, the execution of these s tasks with timing constraints is equivalent to the satisfaction of STL_{MAS} as follows.

Definition 1: (Fragment of STL) The fragment of STL_{MAS} is defined as the class of STL specifications of the form

$$\Phi = \bigwedge_{l=1}^{s} \phi_l,$$

where $\phi_l := \mathscr{T}_{\varepsilon_l,\iota_l}^{\tau_l}\psi_l$, $\psi_l = \bigvee_{k=1}^{w_l}(\bigwedge_{\pi\in\mathscr{J}_{l,k}} \pi)$, $\mathscr{J}_{l,k} \subseteq S(\Pi_{\text{aug}}^l)$, $k \in [1, w_l]_{\mathbb{N}}$, $1 \leq \varepsilon_l \leq \iota_l < \infty$ and $1 \leq \tau_l < \infty$, $l \in [1,s]_{\mathbb{N}}$.

Remark 1: It is easy to see

$$\mathscr{T}_{\varepsilon,\iota}^{\tau}\psi = \begin{cases} G_{[a,b]}\psi, & \varepsilon = \iota = a, \tau = b-a+1, \\ F_{[a,b]}\psi, & \varepsilon = a, \iota = b, \tau = 1, \\ F_{[a,b]}G_{[\bar{a},\bar{b}]}\psi, & \varepsilon = a+\bar{a}, \iota = b+\bar{a}, \tau = \bar{b}-\bar{a}+1, \end{cases}$$

where $G_{[a,b]}$ and $F_{[a,b]}$ are the always and eventually operators, respectively. Then, STL specifications in Definition 1 can characterize ubiquitous fixed-time reachability and finite-time persistence requirements in CPPSs and contains commonly used STL formulae [6], [12], [14] as special cases. Compared with [12], [14], Definition 1 contains disjunction operator. Such feature can capture more complex production rules such as different ways to complete a certain task.

Given a state trajectory of MAS (1) as $X = \{x(t) = (x_1(t), \cdots, x_n(t)) : t = 0, 1, \cdots\} \subseteq \mathscr{D}_\kappa^n$, for the STL specification in Definition 1, the semantics of the satisfaction relation, denoted by $\models$, are recursively defined as follows:

(i) $x(t) \models \pi_j^r$, if and only if $\sigma_{\mathscr{A},\{j\}}(x(t)) = r$; $x(t) \models \neg\pi_j^r$, if and only if $\sigma_{\mathscr{A},\{j\}}(x(t)) \neq r$, where $\sigma_{\mathscr{A},\{j\}}(\cdot) : \mathscr{D}_\kappa^n \to \mathscr{D}_\kappa$ denotes the natural projection from the state of agents in set $\mathscr{A}$ to that of agent j;

(ii) For $\pi_1,\ \pi_2 \in \Pi_{\text{aug}} := \{\pi_j^r, \neg\pi_j^r : j \in \mathscr{A}, r \in \mathscr{R}\}$, $x(t) \models \pi_1 \wedge \pi_2$ (or $\pi_1 \vee \pi_2$), if and only if $x(t) \models \pi_1$ and (or) $x(t) \models \pi_2$;

(iii) $X \models \mathscr{T}_{\varepsilon_l, \iota_l}^{\tau_l} \psi_l$, if and only if there exists $t' \in [\varepsilon_l, \iota_l]_{\mathbb{N}}$ such that $x(t) \models \psi_l$ holds for all $t \in [t', t'+\tau_l-1]_{\mathbb{N}}$.

(iv) $X \models \phi_i \wedge \phi_j,\ i,j \in [1,s]_{\mathbb{N}}$, if and only if $X \models \phi_i$ and $X \models \phi_j$.

Due to the disjunction operator, one can not directly obtain the local task for each agent by decomposing the global STL specification in Definition 1. In addition, due to the interactions between agents, the satisfaction of each local task may not imply that of the collaborative tasks and so as that of the global one. A formal statement of the problem considered in the paper is stated as follows.

Problem 1 Consider MAS (1) with initial state $x(0) = x_0$ and global STL specification Φ in Definition 1.

(i) Find a decomposition $\Phi = \bigwedge_{i=1}^{\rho} \Phi_i$ of Φ which is conflict-free for collaborative tasks and can reduce the couplings between subsystems;

(ii) Synthesize local controllers for subsystems such that the state trajectory of MAS (1) from state x_0 satisfies the global STL specification.

III. ACYCLIC DECOMPOSITION OF GLOBAL SPECIFICATION

In this section, we study the decomposition of global specification to address the conflicts in collaborative tasks.

Partition agents by finding strongly connected components of graph $G_1 = (A(G_1), E(G_1))$, where each vertex $a_l \in A(G_1)$ represents a set $\mathscr{N}_l$, $l \in [1,s]_{\mathbb{N}}$, and $(a_i, a_j) \in E(G_1)$ if and only if $\mathscr{N}_i \cap \mathscr{N}_j \neq \emptyset$. Using existing algorithms such as Tarjan's algorithm [19], we partition agents into ω blocks $\mathscr{A} = \bigcup_{i=1}^{\omega} \mathscr{C}_i$ with $\mathscr{C}_i = \bigcup_{a_j \in A(G_1^i)} \mathscr{N}_j$, where ω is the number of strongly connected components for G_1, and G_1^i is the i-th strongly connected component of G_1. Correspondingly, a decomposition of global STL specification in Definition 1 is $\Phi = \bigwedge_{i=1}^{\omega} \Phi_1^i$, $\Phi_1^i := \bigwedge_{a_j \in A(G_1^i)} \phi_j$, where $\mathscr{C}_i$ is the set of agents assigned to tasks captured by Φ_1^i. Since $\mathscr{C}_i \cap \mathscr{C}_{i'} = \emptyset$, $i \neq i'$, this composition is conflict-free for collaborative tasks.

We further seek acyclic aggregations with unidirectional influence to reduce the couplings between subsystems.

Construct an acyclic aggregation from graph $G_2 = (A(G_2), E(G_2))$, where each vertex $b_i \in A(G_2)$ represents a set $\mathscr{C}_i$, $i \in [1,\omega]_{\mathbb{N}}$, and $(b_i, b_j) \in E(G_2)$ if and only if $(\cup_{k \in \mathscr{C}_j} \mathscr{I}_k) \cap \mathscr{C}_i \neq \emptyset$. Assume that there are ρ components in the graph of strongly connected components for G_2, denoted by $G_2^1, \cdots, G_2^{\rho}$. Since the graph of strongly connected components is acyclic, we have the following result.

Lemma 1: Aggregation $\mathscr{A} = \bigcup_{i=1}^{\rho} \mathscr{A}_i$ with $\mathscr{A}_i := \bigcup_{b_j \in A(G_2^i)} \mathscr{C}_j$ and $\mathscr{C}_j := \bigcup_{a_k \in A(G_1^j)} \mathscr{N}_k$ is an acyclic aggregation of MAS (1).

Corresponding to the acyclic aggregation in Lemma 1, we obtain a decomposition of global STL specification in Definition 1, called acyclic decomposition as

$$\Phi = \bigwedge_{i=1}^{\rho} \Phi_i,$$

where $\Phi_i := \bigwedge_{b_j \in A(G_2^i)} \bigwedge_{a_k \in A(G_1^j)} \phi_k$ and $\mathscr{A}_i$ is the set of agents assigned to tasks captured by Φ_i. Since $\mathscr{A}_i$ is constructed by aggregating $\mathscr{C}_i$, it is clear that they are disjoint and thus the obtained decomposition is also conflict-free for collaborative tasks. Thus, (i) in Problem 1 is solved.

Define the sets of external input nodes and output nodes, if not empty, respectively for each block $\mathscr{A}_i$ as $\mathscr{E}_i := (\bigcup_{j \in \mathscr{A}_i} \mathscr{I}_j) \setminus \mathscr{A}_i := \{\check{i}_1, \cdots, \check{i}_{q_i}\}$, $\mathscr{O}_i := (\bigcup_{j \in \mathscr{A} \setminus \mathscr{A}_i} \mathscr{I}_j) \bigcap \mathscr{A}_i := \{\hat{i}_1, \cdots, \hat{i}_{p_i}\}$. Set $\mathscr{A}_i := \{i_1, \cdots, i_{n_i}\}$, $i \in [1,\rho]_{\mathbb{N}}$. Then, each block $\mathscr{A}_i$ forms a subsystem Ξ_i, the dynamics of which can be given as

$$x_{i_j}(t+1) = \{_{\kappa}\sum_{l=1}^{n_i}\} a_{i_j, i_l} \times_{\kappa} x_{i_l}(t) +_{\kappa} \{_{\kappa}\sum_{l=1}^{q_i}\} a_{i_j, \check{i}_l} \times_{\kappa} x_{\check{i}_l}(t) +_{\kappa} b_{i_j} \times_{\kappa} u_{i_j}(t),\ j = 1, \cdots, n_i,\ a_{i_j,k} = 0, k \notin \mathscr{I}_{i_j}. \quad (2)$$

In the following, we study the STL synthesis of the overall MAS (1) by synthesizing local controllers for subsystems Ξ_i, $i = 1, \cdots, \rho$ with local specification Φ_i.

IV. CONTROLLER DESIGN BASED ON LOCAL CONTROLLER SYNTHESIS

In this section, we firstly study the satisfaction of local specifications, and then synthesize controllers for MAS (1).

A. ASSR Reformulation of the MAS

By ASSR method, we convert MAS (2) into a bilinear equivalent algebraic form, which facilitates the studies.

Represent $k-1 \in \mathscr{D}_{\kappa}$ by a vector δ_{κ}^k. Collect all i_j, $j \in [1,n_i]_{\mathbb{N}}$ satisfying $i_j \in [1,m]_{\mathbb{N}}$ to form a set as $\mathscr{U}_i = \{i_{j_1}, \cdots, i_{j_{m_i}}\}$. Let $\lambda_i(t) = \ltimes_{j=1}^{n_i} x_{i_j}(t)$, $\gamma_i(t) = \ltimes_{j=1}^{q_i} x_{\check{i}_j}(t)$ and $v_i(t) = \ltimes_{k=1}^{m_i} u_{i_{j_k}}(t)$. By similar calculation to [20], for the dynamics of agent $i_{j_k} \in \mathscr{U}_i$ and $i_j \in \mathscr{A}_i \setminus \mathscr{U}_i$, we can respectively obtain

$$x_{i_{j_k}}(t+1) = L_{i_{j_k}} \gamma_i(t) v_i(t) \lambda_i(t),$$

and

$$x_{i_j}(t+1) = L_{i_j} \gamma_i(t) v_i(t) \lambda_i(t),$$

where $L_{i_{j_k}} = L_{i_{j_k},1}(I_{\kappa^{q_i}} \otimes L_{i_{j_k},2})(I_{\kappa^{q_i+n_i}} \otimes L_{i_{j_k},3})$, $L_{i_j} = L_{i_j,1}(I_{\kappa^{q_i}} \otimes (S_{\kappa}^{(n_i-1)} L_{i_j,3}))(I_{\kappa^{q_i}} \otimes \mathbf{1}_{\kappa^{m_i}}^{\top})$, $L_{i_j,1} = S_{\kappa}^{(q_i)} M_{\kappa} a_{i_j,\check{i}_1} \ltimes_{l=1}^{q_i-1} [I_{\kappa^l} \otimes (M_{\kappa} a_{i_j,\check{i}_{l+1}})]$, $L_{i_{j_k},2} = S_{\kappa}^{(n_i)} M_{\kappa} b_{i_{j_k}} (\mathbf{1}_{\kappa^{k-1}}^{\top} \otimes I_{\kappa} \otimes \mathbf{1}_{\kappa^{m_i-k}}^{\top})$, $L_{i_j,3} = M_{\kappa} a_{i_j,i_1} \ltimes_{l=1}^{n_i-1} [I_{\kappa^l} \otimes (M_{\kappa} a_{i_j,i_{l+1}})]$, $\otimes$ denotes the Kronecker product, S_{κ}, M_{κ} respectively denote the structural matrices for $+_{\kappa}$, $\times_{\kappa}$. The equivalent ASSR of (2) is

$$\lambda_i(t+1) = \ltimes_{j=1}^{n_i} x_{i_j}(t+1) = L_i \gamma_i(t) v_i(t) \lambda_i(t), \quad (3)$$

where $Col_k(L_i) = \ltimes_{j=1}^{n_i} Col_k(L_{i_j})$.

B. Satisfaction of Local Specifications

In this part, we consider the satisfaction of local specifications for subsystems Ξ_i, $i = 1, \cdots, \rho$.

For the acyclic decomposition $\Phi = \bigwedge_{i=1}^{\rho} \Phi_i$ with $\Phi_i = \bigwedge_{k=1}^{s_i} \phi_{i_k}$, there must be some root blocks $\mathscr{A}_i$ called level-0 blocks which have no external input, that is, $\mathscr{E}_i = \emptyset$. For each other block, it is level-h if the length of its longest path from the root blocks is h. Denote $\mathscr{L}^h$ as the union of all the blocks of level-0 to the level-h. We can see that $\mathscr{L}^h$, as a block, has no input. We firstly investigate the case for $i \in \mathscr{L}^0$, and then generalize the results to subsystems Ξ_i, $i \in \mathscr{L}^h$, $h \geq 1$.

Denoting by ϑ_{i_k} the latest time to complete task i_k, it holds $\vartheta_{i_k} = \iota_{i_k} + \tau_{i_k} - 1$. For simplicity, we assume that the execution time intervals corresponding to propositions in Φ_i do not overlap, and Φ_i is arranged in chronological order, that is, $\vartheta_{i_{k_1}} < \varepsilon_{i_{k_2}}$ holds for any $k_1, k_2 \in [1, s_i]_{\mathbb{N}}$, $k_1 < k_2$.

1) Equivalence Between the Propostion and the State Set of the MAS : Recursively define the state set $\mathscr{S}(\psi_{i_k})$ corresponding to proposition ψ as follows:

(i) $\mathscr{S}(\pi_l^r) = \{\ltimes_{j=1}^{n_i} x_{i_j} : x_l = \delta_\kappa^{r+1}, x_{l'} \in Col(I_\kappa), l' \in \mathscr{A}_i \setminus \{l\}\}$, $\mathscr{S}(\neg\pi_l^r) = Col(I_{\kappa^{n_i}}) \setminus \mathscr{S}(\pi_l^r)$;

(ii) $\mathscr{S}(\psi_{i_k}) = \bigcup_{j=1}^{w_{i_k}} \big(\bigcap_{\pi \in \mathscr{I}_{i_k,j}} \mathscr{S}(\pi) \big)$.

Denote the state trajectory of Ξ_i starting from the initial state $\lambda_i(0) = \sigma_{\mathscr{A},\mathscr{A}_i}(x_0) := \lambda_i^0 \in \mathscr{D}_\kappa^{n_i}$ under control input sequence $V_i = \{v_i(t) : t = 0, 1, \cdots\}$ and external input sequence $\bar{\Gamma}_i := \{\bar{\gamma}_i(t) : t = 0, 1, \cdots\}$ by $\Lambda_i(\lambda_i^0, \bar{\Gamma}_i, V_i) := \{\lambda_i(t; \lambda_i^0, \bar{\Gamma}_i, V_i) : t = 0, 1, \cdots\} \subseteq \mathscr{D}_\kappa^{n_i}$. We say $\Lambda_i(\cdot; \lambda_i^0, \bar{\Gamma}_i) \models \phi_{i_k}$, $k \in [1, s_i]_{\mathbb{N}}$, if there exists $\bar{V}_i$ such that $\Lambda_i(\lambda_i^0, \bar{\Gamma}_i, \bar{V}_i) \models \phi_{i_k}$. By the definition of set $\mathscr{S}(\psi_{i_k})$, we have the following result.

Lemma 2: Consider subsystem Ξ_i, $i \in [1, \rho]_{\mathbb{N}}$. Give initial state $x(0) = x_0 \in \mathscr{D}_\kappa^n$ and external input sequence $\bar{\Gamma}_i$. $\Lambda_i(\cdot; \lambda_i^0, \bar{\Gamma}_i) \models \Phi_i$, if there exist $t_{i_k} \in [\varepsilon_{i_k}, \iota_{i_k}]_{\mathbb{N}}$, $k = 1, \cdots, s_i$, and a control input sequence $\bar{V}_i$ such that $\lambda_i(t; \lambda_i^0, \bar{\Gamma}_i, \bar{V}_i) \in \mathscr{S}(\psi_{i_k})$ holds for any $t \in [t_{i_k}, t_{i_k} + \tau_{i_k} - 1]_{\mathbb{N}}$, $k \in [1, s_i]_{\mathbb{N}}$.

2) Satisfaction of Local STL Specification for Ξ_i, $i \in [1, \rho]_{\mathbb{N}}$: Since $\mathscr{E}_i = \emptyset$, $i \in \mathscr{L}^0$, we denote $\Lambda_i(\cdot; \lambda_i^0, \bar{\Gamma}_i)$ and $\Lambda_i(\lambda_i^0, \bar{\Gamma}_i, \bar{V}_i)$ by $\Lambda_i(\cdot; \lambda_i^0)$ and $\Lambda_i(\lambda_i^0, \bar{V}_i)$, respectively.

Denote the one-step reachability matrix for Ξ_i, $i \in \mathscr{L}^0$ by $M_i = \sum_{\mu=1}^{\kappa^{m_i}} L_i \delta_{\kappa^{m_i}}^{\mu}$, and Boolean matrix $S_0 \in \mathbb{R}_{n \times n}$ for a given set $\mathscr{S} \subseteq Col(I_n)$ by

$$Col_j(S_0) = \begin{cases} \delta_n^j, & \delta_n^j \in \mathscr{S}, \\ \mathbf{0}, & \text{otherwise}. \end{cases}$$

Then, we have the following relationship between S_0 and $\mathscr{S}$.

Lemma 3: Given a set $\mathscr{S} \subseteq Col(I_n)$ and $\xi = [\xi_1 \ \cdots \ \xi_n]^\top$ with $\xi_i \geq 0$, $S_0\xi \succcurlyeq \mathbf{0}$ if and only if there exists $j \in [1, n]_{\mathbb{N}}$ satisfying $\delta_n^j \in \mathscr{S}$ and $\xi_j > 0$.

Proof: Since $S_0\xi = \sum_{j=1}^n \xi_j S_0 \delta_n^j = \sum_{j=1}^n \xi_j Col_j(S_0)$, we have $Row_j(S_0\xi) = \begin{cases} \xi_j, & \delta_n^j \in \mathscr{S}, \\ 0, & \text{otherwise}. \end{cases}$ □

We have the following criteria on the satisfaction of fixed-time reachability, finite-time persistence tasks and single task with reachability and persistence requirements for subsystem Ξ_i, $i \in \mathscr{L}^0$. The proof is omitted due to page limitation.

Lemma 4: Consider subsystem Ξ_i, $i \in \mathscr{L}^0$ with ASSR (3) and initial state $\lambda_i^0 := \delta_{\kappa^{n_i}}^{\varsigma_i}$. Let $\mathscr{S} := \mathscr{S}(\psi)$, $S_j := S_{j-1} M_i S_0 = (S_0 M_i)^{(j)} S_0$, $j \geq 1$.

(i) $\Lambda_i(\cdot; \lambda_i^0) \models \mathscr{T}_{\varepsilon,\varepsilon}^1 \psi$, if and only if $Col_{\varsigma_i}(S_0 M_i^{(\varepsilon)}) \succcurlyeq \mathbf{0}$;

(ii) $\Lambda_i(\cdot; \lambda_i^0) \models \mathscr{T}_{1,1}^\tau \psi$, if and only if $Col_{\varsigma_i}\Big((S_0 M_i)^{(\tau)}\Big) \succcurlyeq \mathbf{0}$;

(iii) $\Lambda_i(\cdot; \lambda_i^0) \models \mathscr{T}_{\varepsilon,\iota}^\tau \psi$, if and only if

$$Col_{\varsigma_i}\Big(\sum_{t=\varepsilon}^{\iota} S_{\tau-1} M_i^{(t)}\Big) \succcurlyeq \mathbf{0}.$$

Fixing external input $\gamma_i = \delta_{\kappa^{q_i}}^{\xi}$, define the one-step reachability matrix for subsystem Ξ_i, $i \in \mathscr{L}^h$, $h \geq 1$ as $M_i(\xi) = \sum_{\mu=1}^{\kappa^{m_i}} L_{i,\xi} \delta_{\kappa^{m_i}}^{\mu}$, $L_{i,\xi} := L_i \delta_{\kappa^{q_i}}^{\xi}$. By Lemma 4, the satisfaction of local specifications for Ξ_i, $i \in [1, \rho]_{\mathbb{N}}$ can be verified.

C. Controller Synthesis for the Overall MAS

If all the subsystems Ξ_i, $i \in \mathscr{L}^h$ satisfy their local specifications, by projecting outputs, called valid outputs, from $\mathscr{L}^h$ onto blocks in $\bar{\mathscr{L}}^{h+1}$, $\bar{\mathscr{L}}^{h+1} = \mathscr{L}^{h+1} \setminus \mathscr{L}^h$, $h \geq 0$, subsystems corresponding to these blocks are turned into switched MAS. In this way, for each switched subsystem Ξ_i, $i \in \bar{\mathscr{L}}^{h+1}$, one just need to find feasible control input sequences enforcing local STL specifications. Repeat these procedures until all blocks can be driven by the outputs from their parent blocks, we solve the controller synthesis for the overall MAS (1).

For subsystem Ξ_i with $\Lambda_i(\cdot; \lambda_i^0, \bar{\Gamma}_i) \models \Phi_i$ under $\bar{\Gamma}_i = \{\delta_{\kappa^{q_i}}^{\xi_i^t} : t = 0, 1, \cdots, \vartheta\}$, $\vartheta := \max\{\vartheta_{i_{s_i}} : i = 1, 2, \cdots, \rho\}$, $i \in \mathscr{L}^h$, Algorithm 1 is established to find feasible $\bar{V}_i$ satisfying $\Lambda_i(\lambda_i^0, \bar{\Gamma}_i, \bar{V}_i) \models \Phi_i$. For the convenience of statement, assume $s_i > 1$, and denote $\bar{t}_k := t_k + \bar{\iota}_{i_{k-1}} - t_{k-1}$, $k = 1, \cdots, s_i$ for given t_k, $k = 0, \cdots, s_i$, $\bar{\iota}_{i_k} := \iota_{i_k} - \vartheta_{i_{k-1}}$, $\vartheta_{i_0} := 0$, $\bar{\iota}_{i_0} := 0$.

In Algorithm 1, the construction of the execution trajectory for subsystem Ξ_i, $i \in \mathscr{L}^0$ is divided into two steps. Firstly, find each ending state that Ξ_i reaches when completing the subtask (Line 3). For each end state $\delta_{\kappa^{n_i}}^{\varsigma_k}$ for the k-th subtask ϕ_{i_k}, it is reachable from that for the $k-1$-th subtask, that is, $[S_{\tau_{i_k}-1}^{i_k} M_i^{(\bar{t}_k)}]_{\zeta_k, \zeta_{k-1}} > 0$, $\mathscr{S}^{i_k} := \mathscr{S}(\psi_{i_k})$. In addition, Ξ_i can complete the rest subtasks starting from $\delta_{\kappa^{n_i}}^{\varsigma_k}$, that is, $Col_{\zeta_k}(M_{i,s_i} \prod_{j=s_i-1}^{k+1} M_{i,j} M_i^{(\bar{\iota}_{i_k} - t_k)}) \succcurlyeq \mathbf{0}$, where $\bar{\varepsilon}_{i_k} := \varepsilon_{i_k} - \vartheta_{i_{k-1}}$,

$$M_{i,k} = \begin{cases} \sum_{t=\bar{\varepsilon}_{i_k}}^{\bar{\iota}_{i_k}} M_i^{(\bar{\iota}_{i_k} - t)} S_{\tau_{i_k}-1}^{i_k} M_i^{(t)}, & k = 1, \cdots, s_i - 1, s_i > 1, \\ \sum_{t=\bar{\varepsilon}_{i_k}}^{\bar{\iota}_{i_k}} S_{\tau_{i_k}-1}^{i_k} M_i^{(t)}, & k = s_i. \end{cases}$$

Then, for each two adjacent end states, find the intermediate state trajectory as $\{\delta_{\kappa^{n_i}}^{\zeta_{k-1}^t} : t = 1, \cdots, \bar{t}_{k-1} + \tau_{i_{k-1}} - 2\}$ (Line 8). According to the one-step reachability between two adjacent intermediate states, it holds $[M_i]_{\zeta_{k-1}^j, \zeta_{k-1}^{j-1}} > 0$. The intermediate states are divided into two groups: ones are that generated before executing subtask ϕ_{i_k}, and the others are generated by the task execution. Ξ_i can reach the end state $\delta_{\kappa^{n_i}}^{\varsigma_k}$ starting from each intermediate states between $\delta_{\kappa^{n_i}}^{\varsigma_{k-1}}$ and $\delta_{\kappa^{n_i}}^{\varsigma_k}$, which respectively leads to $[S_{\tau_{i_k}-1}^{i_k} M_i^{(\bar{t}_k - j)}]_{\zeta_k, \zeta_{k-1}^j} > 0$,

Algorithm 1 : Construction of feasible control input sequence $\bar{V}_i$ satisfying $\Lambda_i(\lambda_i^0,\bar{\Gamma}_i,\bar{V}_i)\models\Phi_i$, $i\in[1,\rho]_{\mathbb{N}}$

Require: L_i, $\bar{\Gamma}_i$, ς_i, s_i, $\mathscr{S}^{i_k}$, ε_{i_k}, ι_{i_k}, τ_{i_k}, $i=1,2,\cdots,\rho$, $k=1,\cdots,s_i$

Ensure: $\bar{V}_i=\{\delta_{\kappa^{n_i}}^{\mu_1^0},\cdots,\delta_{\kappa^{n_i}}^{\mu_1^{\bar{t}_1+\tau_{i_1}-2}},\cdots,\delta_{\kappa^{n_i}}^{\mu_{s_i}^0},\cdots,\delta_{\kappa^{n_i}}^{\mu_{s_i}^{\bar{t}_{s_i}+\tau_{i_{s_i}}-2}}\}$

1: $\zeta_0\leftarrow\varsigma_i$, $\iota_{i_0}\leftarrow 0$, $t_0\leftarrow 0$
2: **for** $k=1,\cdots,s_i$ **do**
3: Find $t_k\in[\bar{\varepsilon}_{i_k},\bar{\iota}_{i_k}]_{\mathbb{N}}$, ζ_k with $\delta_{\kappa^{n_i}}^{\zeta_k}\in\mathscr{S}^{i_k}$ satisfying (4a), (4b), $k\in[1,s_i-1]_{\mathbb{N}}$; or only (4a), $k=s_i$, where

$$[A_i^1(k,t_k,\bar{\Gamma}_i)]_{\zeta_k,\zeta_{k-1}}>0, \tag{4a}$$

$$Col_{\zeta_k}(A_i^2(k,\bar{\Gamma}_i))\succcurlyeq\mathbf{0} \tag{4b}$$

4: **end for**
5: **for** $k=1,\cdots,s_i$ **do**
6: Calculate $\bar{t}_k$, $\zeta_{k-1}^0\leftarrow\zeta_{k-1}$, $\zeta_{k-1}^{\bar{t}_k+\tau_{i_k}-1}\leftarrow\zeta_k$
7: **for** $j=1,\cdots,\bar{t}_k+\tau_{i_k}-2$ **do**
8: Find ζ_{k-1}^j satisfying (5a) and (5b), $j\in[1,\bar{t}_k-1]_{\mathbb{N}}$; or (5a) and (5c), $j\in[\bar{t}_k,\bar{t}_k+\tau_{i_k}-2]_{\mathbb{N}}$, where

$$[A_i^3(j,k,\bar{\Gamma}_i)]_{\zeta_{k-1}^j,\zeta_{k-1}^{j-1}}>0 \tag{5a}$$

$$[A_i^4(j,k,\bar{\Gamma}_i)]_{\zeta_k,\zeta_{k-1}^j}>0 \tag{5b}$$

$$[A_i^5(j,k,\bar{\Gamma}_i)]_{\zeta_k,\zeta_{k-1}^j}>0 \tag{5c}$$

9: Find μ_k^{j-1} with $\delta_{\kappa^{n_i}}^{\mu_k^{j-1}}\in Col(I_{\kappa^{n_i}})$ satisfying

$$[A_i^6(j,k,\bar{\Gamma}_i)\delta_{\kappa^{n_i}}^{\mu_k^{j-1}}]_{\zeta_{k-1}^j,\zeta_{k-1}^{j-1}}>0$$

10: **end for**
11: **end for**

and $[S^{i_k}_{\tau_{i_k}-1-(j-\bar{t}_k)}]_{\zeta_k,\zeta_{k-1}^j}>0$. Finally, control input $\delta_{\kappa^{n_i}}^{\mu_k^{j-1}}$ is designed to connect adjacent states (Line 9), that is, $[L_i\delta_{\kappa^{n_i}}^{\mu_k^{j-1}}]_{\zeta_{k-1}^j,\zeta_{k-1}^{j-1}}>0$.

Let

$$M_{i,k}(\bar{\Gamma}_i)=\begin{cases}\sum\limits_{t=\bar{\varepsilon}_{i_k}}^{\bar{\iota}_{i_k}}M_{i,k}^2(\bar{\Gamma}_i,t)S^{i_k}_{\tau_{i_k}-1}(\bar{\Gamma}_i,t)M_{i,k}^1(\bar{\Gamma}_i,t), & k=1,\cdots,s_i-1,s_i>1,\\ \sum\limits_{t=\bar{\varepsilon}_{i_k}}^{\bar{\iota}_{i_k}}S^{i_k}_{\tau_{i_k}-1}(\bar{\Gamma}_i,t)M_{i,k}^1(\bar{\Gamma}_i,t), & k=s_i.\end{cases}$$

$S^{i_k}_{\tau_{i_k}-1}(\bar{\Gamma}_i,t)=(\prod_{j=t+\vartheta_{i_{k-1}}+\tau_{i_k}-2}^{t+\vartheta_{i_{k-1}}}S_0^{i_k}M_i(\xi_i^j))S_0^{i_k}$, $M_{i,k}^1(\bar{\Gamma}_i,t)=\prod_{j=t+\vartheta_{i_{k-1}}-1}^{\vartheta_{i_{k-1}}}M_i(\xi_i^j)$ and $M_{i,k}^2(\bar{\Gamma}_i,t)=\prod_{j=\vartheta_{i_k}-1}^{t+\vartheta_{i_{k-1}}+\tau_{i_k}-1}M_i(\xi_i^j)$. For $i\in[1,\rho]_{\mathbb{N}}\setminus\mathscr{L}^0$, same procedure can be achieved by $A_i^1(k,t_k,\bar{\Gamma}_i)=S^{i_k}_{\tau_{i_k}-1}(\bar{\Gamma}_i,t_k)M_{i,k}^1(\bar{\Gamma}_i,t_k)M_{i,k-1}^2(\bar{\Gamma}_i,t_{k-1})$, $A_i^2(k,\bar{\Gamma}_i)=M_{i,s_i}(\bar{\Gamma}_i)\prod_{j=s_i-1}^{k+1}M_{i,j}(\bar{\Gamma}_i)M_{i,k}^2(\bar{\Gamma}_i,t_k)$, $A_i^3(j,k,\bar{\Gamma}_i)=M_i(\xi_i^{\vartheta_{i_{k-1}}+j-1})$, $A_i^5(j,k,\bar{\Gamma}_i)=(\prod_{l=t_k+\vartheta_{i_{k-1}}+\tau_{i_k}-2}^{t_k+\vartheta_{i_{k-1}}+j-\bar{t}_k}S_0^{i_k}M_i(\xi_i^l))S_0^{i_k}$, $A_i^6(j,k,\bar{\Gamma}_i)=L_{i,\xi_i^{\vartheta_{i_{k-1}}+j-1}}$, and $A_i^4(j,k,\bar{\Gamma}_i)=S^{i_k}_{\tau_{i_k}-1}(\bar{\Gamma}_i,t_k)M_{i,k}^1(\bar{\Gamma}_i,t_k)M_{i,k-1}^2(\bar{\Gamma}_i,t_{k-1}+j)$, $j\in[1,\bar{\vartheta}_{i_k}]_{\mathbb{N}}$, $A_i^4(j,k,\bar{\Gamma}_i)=S^{i_k}_{\tau_{i_k}-1}(\bar{\Gamma}_i,t_k)\prod_{l=t_k+\vartheta_{i_{k-1}}-1}^{\vartheta_{i_{k-1}}+j-\bar{\vartheta}_{i_k}+1}M_i(\xi_i^l)$, $j\in[\bar{\vartheta}_{i_k}+1,\bar{t}_k-1]_{\mathbb{N}}$ with $\bar{\vartheta}_{i_k}:=\vartheta_{i_k}-(t_k+\vartheta_{i_{k-1}}+\tau_{i_k})$.

Theorem 1: Consider MAS (1) with initial state $x(0)=x_0\in Col(I_{\kappa^n})$ and global STL specification Φ in Definition 1. For the decomposition of global specification $\Phi=\wedge_{i=1}^{\rho}\Phi_i$ with acyclic aggregation $\mathscr{A}=\cup_{i=1}^{\rho}\mathscr{A}_i$, we have $X(\cdot;x_0)\models\Phi$, if and only if the following two conditions hold:

(i) For all $i\in\mathscr{L}^0$, $\Lambda_i(\cdot;\lambda_i^0)\models\Phi_i$;

(ii) For each $h\geq 1$, there exists a valid output $\bar{Y}^{h-1}$ of $\mathscr{L}^{h-1}$, such that $\Lambda_i(\cdot;\lambda_i^0,\bar{\Gamma}_i)\models\Phi_i$ holds for any $i\in\mathscr{L}^h$, $\bar{\Gamma}_i:=\sigma_{\cup_{i\in\mathscr{L}^h}\mathscr{E}_i,\mathscr{E}_i}(\bar{Y}^{h-1})$.

Proof: The necessity is obvious. If conditions (i) and (ii) hold, then there exist $\bar{V}_i=\{\bar{v}_i(0),\bar{v}_i(1),\cdots\}\subseteq Col(I_{\kappa^{m_i}})$, $i=1,\cdots,\rho$ such that $\Lambda_i(\lambda_i^0,\bar{V}_i)\models\Phi_i$, $i\in\mathscr{L}^0$ and $\Lambda_i(\lambda_i^0,\bar{\Gamma}_i,\bar{V}_i)\models\Phi_i$, $i\in\mathscr{L}^h$, $h\geq 1$, where $\bar{\Gamma}_i=\sigma_{\cup_{i\in\mathscr{L}^h}\mathscr{E}_i,\mathscr{E}_i}(\bar{Y}^{h-1})$ and $\bar{Y}^h=\{\ltimes_{j\in\cup_{i\in\mathscr{L}^h}\mathscr{O}_i}x_j(t):t=0,1,\cdots\}$. Then, under the control of $\bar{U}=\{\bar{u}(t)=\ltimes_{j=1}^n\bar{u}_j(t):t=0,1,\cdots\}$, it holds that $X(\bar{x},\bar{U})\models\wedge_{i=1}^{\rho}\Phi_i$, that is, $X(\bar{x},\bar{U})\models\Phi$, where $\bar{x}=\ltimes_{j=1}^n\sigma_{\mathscr{A}_i,\{j\}}(\lambda_i^0)$. Since $\lambda_i^0:=\sigma_{\mathscr{A},\mathscr{A}_i}(x_0)$, $i=1,2,\cdots,\rho$, it is obvious that $\bar{x}=x_0$. $\square$

By local controllers $\bar{V}_i=\{\bar{v}_i(0),\bar{v}_i(1),\cdots\}$ obtained by Algorithm 1, control sequences $\bar{U}=\{\bar{u}(t)=\ltimes_{j=1}^m\bar{u}_j(t):t=0,1,\cdots\}$ enforcing $X(x_0,\bar{U})\models\Phi$ are $\bar{u}_j(t)=\sigma_{\mathscr{U}_i,\{j\}}(\bar{v}_i(t))$, $j\in\mathscr{A}_i$. Moreover, one can prove that the existence of controllers do not depend on the choice of acyclic aggregation. Thus, the synthesis of MAS (1) with global specification Φ in Definition 1 is solved, which solves (ii) in Problem 1.

V. ILLUSTRATIVE EXAMPLE

Consider a modular CPPS with 10 agents performing production module behaviors. Discretize the the workspace into 5 planar subregions. There are 6 tasks with timing constraints in the workspace of which each task l can be collaboratively completed by agents in set $\mathscr{N}_l$ in w_l ways. The constraints on each task is shown in Table II. Then, the execution of these s tasks is equivalent to the satisfaction of the global STL specification

$$\Phi=\bigwedge_{l=1}^{6}\phi_l$$

with $\phi_1=\mathscr{T}^2_{2,2}\pi_2^0$, $\phi_2=\mathscr{T}^1_{3,4}(\wedge_{j\in\{1,5,8\}}\pi_j^1)$, $\phi_3=\mathscr{T}^3_{5,6}\{(\vee_{j=0}^4(\pi_4^4\wedge\pi_9^j))\vee(\vee_{j=0}^3(\pi_4^j\wedge\pi_9^4))\}$, $\phi_4=\mathscr{T}^2_{9,9}\{(\pi_3^3\wedge\pi_7^0\wedge\pi_{10}^0)\vee(\pi_3^0\wedge\pi_7^3\wedge\pi_{10}^0)\vee(\pi_3^0\wedge\pi_7^0\wedge\pi_{10}^3)\}$, $\phi_5=\mathscr{T}^3_{11,13}(\wedge_{j\in\{2,6\}}\pi_j^2)$, $\phi_6=\mathscr{T}^1_{14,15}(\wedge_{j\in\{1,5,8\}}\pi_j^1)$. For the evolution of the MAS, let $A=[a_{j,k}]=[3\ 0\ 0\ 0\ 0\ 0\ 0\ 2\ 0\ 0;\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0;\ 0\ 0\ 0\ 0\ 0\ 0\ 2\ 0\ 0\ 0;\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 2;\ 4\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0;\ 3\ 4\ 0\ 0\ 0\ 2\ 0\ 0\ 4\ 0;\ 0\ 0\ 4\ 0\ 1\ 0\ 1\ 0\ 0\ 0;\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 2\ 0\ 0;\ 0\ 0\ 0\ 3\ 0\ 0\ 0\ 0\ 2\ 0;\ 0\ 0\ 2\ 0\ 0\ 0\ 3\ 0\ 0\ 0]$, $B=[b_j]=[1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^{\top}$. In addition, set $x_7(0)=x_{10}(0)=0$, $x_1(0)=x_8(0)=1$, $x_4(0)=x_6(0)=2$, $x_2(0)=x_3(0)=x_5(0)=x_9(0)=3$.

By Lemma 1, an acyclic decomposition of the global STL specification can be obtained as $\Phi=\wedge_{i=1}^4\Phi_i$ with $\Phi_1=\phi_4$,

TABLE II

THE CONSTRAINTS ON EACH TASK

C \ T	1	2	3	4	5	6
ε_l	2	3	5	4	11	14
ι_l	2	4	6	9	13	15
τ_l	2	1	3	2	13	15
$\mathcal{N}_l$	{2}	{1,5,8}	{4,9}	{3,7,10}	{2,6}	{1,5,8}
w_l	1	1	7	3	1	1

C: Constraints T: Task

$\Phi_2 = \phi_3$, $\Phi_3 = \phi_1 \wedge \phi_5$ and $\Phi_4 = \phi_2 \wedge \phi_6$. The corresponding acyclic aggregation of the MAS containing 4 blocks $\mathscr{A}_1 = \{3,7,10\}$, $\mathscr{A}_2 = \{4,9\}$, $\mathscr{A}_3 = \{2,6\}$, $\mathscr{A}_4 = \{1,5,8\}$. Then, all agents assigned to the same collaborative task belong to the same block. In addition, the couplings between any two blocks is unidirectional with $\mathscr{L}^0 = \{4\}$, $\mathscr{L}^1 = \{1,4\}$, $\mathscr{L}^2 = \{1,2,4\}$ and $\mathscr{L}^3 = \{1,2,3,4\}$. Thus, the decomposition is conflict-free for collaborative tasks and can effectively reduce the couplings between subsystems.

We can obtain $\varsigma_1 = 76$, $\varsigma_2 = 14$, $\varsigma_3 = 18$ and $\varsigma_4 = 42$. By Algorithm 1, local controllers can be designed as

$$\bar{V}_1 = \{\bar{u}_3(t) : t = 0,1,\cdots,14\} = \{\delta_5^5, \delta_5^5, \delta_5^5, \delta_5^1, \delta_5^5, \delta_5^2, \delta_5^1, \delta_5^3, \delta_5^2, \delta_5^1, \delta_5^4, \delta_5^2, \delta_5^4, \delta_5^1, \delta_5^2\},$$

$$\bar{V}_3 = \{\bar{u}_2(t) : t = 0,1,\cdots,14\} = \{\delta_5^1, \cdots, \delta_5^1, \delta_5^3, \delta_5^3, \delta_5^3\},$$

$$\bar{V}_4 = \{\bar{u}_1(t) : t = 0,1,\cdots,14\} = \{\delta_5^1, \cdots, \delta_5^1\}.$$

Fig. 2 shows the state trajectory for subsystems under $\bar{V}_i$ and $\bar{\Gamma}_i$. Since $\Lambda_i(\lambda_i^0, \bar{\Gamma}_i, \bar{V}_i) \models \Phi_i$, $i = 1,2,3,4$, under the local-based controller $\bar{U} = \{\bar{u}(t) = \ltimes_{j=1}^{3} \bar{u}_j(t) : t = 0,1,\cdots,14\}$, it holds $X(x_0, \bar{U}) \models \Phi$. This corroborates Theorem 1.

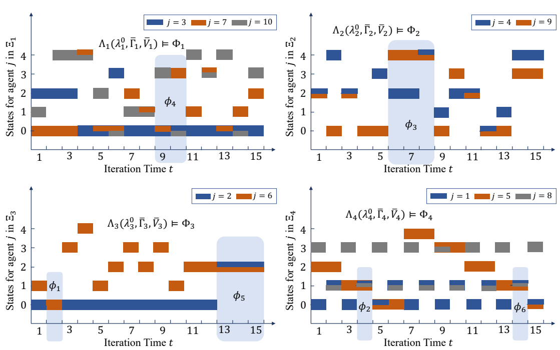

Fig. 2. The state trajectory for subsystem Ξ_i under $\bar{V}_i$ and $\bar{\Gamma}_i$, $i = 1,2,3,4$

VI. CONCLUSIONS

In this paper, using acyclic decomposition, we have synthesized controllers for interconnected MASs with STL tasks capturing production rules of CPPSs. Firstly, we define a class of STL specifications characterizing tasks as a combination of fixed-time reachability and finite-time persistence tasks. Secondly, by finding strongly connected components, we have constructed an acyclic decomposition for the global STL formula, which aggregates agents assigned to the same collaborative task to a subsystem. Criteria on the satisfaction of reachability and persistence tasks are established via establishing the equivalence between the proposition and state set of the MAS, based on which we guarantee the satisfaction of local specifications for subsystems. By utilizing local controllers of subsystems, we have synthesized controllers for the MAS with the global STL specification. Future works will devote to presenting selection methods for the acyclic decomposition to reduce the computational complexity.

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