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A Robust Interval MPC for Uncertain LPV Systems via Integral Sliding–Mode Control

Susana V. Gutiérrez–Martínez[†◇], Héctor Ríos[†⋆], Manuel Mera[‡], Tarek Raïssi[§] and Denis Efimov[∗⋆]

Abstract—**This paper presents the design of a robust control strategy for linear parameter–varying (LPV) systems. The proposed strategy involves the design of a robust control law based on an integral sliding–mode control (ISMC) approach with an interval predictor–based state feedback controller and a Model Predictive Control (MPC) scheme. The proposed controller is robust to some external disturbances and parameter uncertainties, and deals with state and input constraints. The integral sliding–mode compensates for matched perturbations starting from the initial moment, *i.e.*, ensuring the sliding–mode from the initial time instance. Then, the interval predictor–based state feedback controller and the MPC deal with the state and input constraints. The proposed strategy guarantees the exponential stability of the system. Furthermore, the simulation results show high performance of the proposed controller.**

Index Terms—**LPV Systems, Sliding–Mode Control, Constrained System.**

I. Introduction

In recent decades, the control of nonlinear systems has become an important area of research. This is driven by a wide range of critical engineering applications, such as flight control systems, robotic manipulators, automated highway systems, aircraft wing structures, high–performance fuel injection systems, etc. However, all of these systems possess parameter uncertainties, and state and input constraints; moreover, they might be affected by external disturbances.

With regards to the control design of constrained dynamical systems, it is known that this is a very complex task to be tackled using classic feedback solutions [1]. Since this scenario is recurrent, the MPC emerges as a way of handling constrained optimal control problems for uncertain dynamical systems [2]. Moreover, the MPC has grown to be very popular and successful in many industrial applications (see, *e.g.*, [3], [4], and [5]), and in the academic research [6]. The MPC solves an optimal control problem at each sampling instant, where a sequence of inputs is optimized and only the first input is injected into the system. Then, an MPC scheme is said to be robust if it achieves the control task, with guaranteed satisfaction of the state and input constraints, for all possible realizations of a certain range of uncertainties [7].

However, there still exist some challenges in the treatment of uncertain systems based on MPC: the prediction is based on models that are often discrepant with respect to the real system, by cause of uncertainties, noise, and disturbances [8]; normally, the MPC requires full state measurement, which is not always available and leads to the need for estimation tools that add more complexity to the problem due to estimation errors [9]. In this sense, the linear parameter varying (LPV) systems are typically utilized to represent systems with uncertainties, and/or nonlinearities and they can be used for the design of MPC approaches. For instance, in [10], the authors propose an MPC algorithm, based on an interval state predictor, in order to deal with the stabilization problem for a class of continuous–time LPV systems, under state and control constraints, and subject to bounded disturbances and parameter uncertainties. In [11], the authors provide a robust output–feedback predictive control for discrete–time LPV under the effect of system constraints, state, and measurement disturbances. However, in both [10] and [11], only Input–to–State Stability (ISS) is guaranteed in the presence of external disturbances.

Regarding robust control techniques, the sliding–mode control (SMC) is a popular methodology frequently adopted due to its insensibility, against a certain class of external perturbations, and its finite–time convergence (see, *e.g.*, [12] and [13]). In the literature, there have been several proposals for the combination of MPC and SMC. In [14], a hierarchical MPC, at a higher level, and SMC, at a lower level, is proposed for a class of nonlinear constrained uncertain systems. A model–based event–triggered control scheme is proposed by [15] for nonlinear constrained continuous–time uncertain systems in a networked configuration. The proposed strategy is based on the use of MPC and ISMC, it complies with state and input constraints, and deals with matched uncertainties.

[†]Tecnológico Nacional de México/I.T. La Laguna, C.P. 27000, Torreón, Coahuila, México. Emails: svgutierrezmr@gmail.com, hriosb@lalaguna.tecnm.mx
[⋆]CONAHCYT IxM, C.P. 03940, CDMX, México.
[◇]Facultad de Ingeniería Mecánica y Eléctrica, Universidad Autónoma de Nuevo León, C.P. 66451, San Nicolás de Los Garza, Nuevo León, México.
[‡]Instituto Politécnico Nacional, Section of Graduate Studies and Research, ESIME–UPT, C.P. 07340, CDMX, México. Email: mlmera@ipn.mx
[§]Conservatoire National des Arts et Métiers (CNAM), Cedric 292, Rue St–Martin, 75141 Paris, France. Email: tarek.raissi@cnam.fr
[∗]Inria, Univ. Lille, CNRS, UMR 9189–CRISTAL, F–59000, Lille, France. Email: denis.efimov@inria.fr
[⋆]Department of Control Systems and Informatics, Information Technologies Mechanics and Optics University, Saint Petersburg 197101, Russia.

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In [16], the authors propose a combination of MPC with output ISMC techniques for output tracking in uncertain linear systems. The exponential convergence to zero for the output tracking error is guaranteed. An optimal control, based on MPC and ISMC, is proposed in [17] for linear systems represented by a multi–model approach and affected by parameter uncertainties, and control–matched external disturbances. Then, by solving a min–max multi–model MPC problem, the authors provide the asymptotic stability of the multi–model system. However, for most of the previous results, the system model and parameters are required to achieve regulation accuracy, and most of them only provide ISS properties.

The main contribution of this paper focuses on the design of a robust controller, based on an ISMC approach together with an interval predictor–based state feedback controller and an MPC scheme, for a class of uncertain LPV systems. The proposed control strategy is robust against some external disturbances and parameter uncertainties, and deals with state and input constraints. The ISMC compensates for matched perturbations ensuring the sliding–mode from the initial time instance. On the other hand, the interval predictor–based state feedback controller and the MPC, which establish a switching controller, deal with the state and input constraints. The interval predictor-based state feedback controller possesses a constructive synthesis in terms of Linear Matrix Inequalities (LMIs), which do not depend on the upper bound of the external disturbances due to the robustness properties of the ISMC. It is shown that the proposed control approach guarantees exponential stability and the computational burden is relaxed due to the switching structure of the controller. It is worth mentioning that this proposal extends the results given by [10] compensating completely the effect of the external disturbance.

This paper is organized as follows: Section II gives the problem statement. The preliminaries are presented in Section III, and the proposed controller is presented in Section IV. Section V illustrates the proposed approach with a numerical example. Finally, Section VI concludes this study.

Notation: The absolute value is represented as $|\cdot|$ while the euclidean norm of a vector $x \in \mathbb{R}$ is denoted by $||x||$, and for a matrix $A \in \mathbb{R}^{m\times n}$, the induced norm is the spectral norm, *i.e.,* $||A|| = \sqrt{\lambda_{\max}(A^T A)}$. The identity matrix of dimension n is defined by I_n. Denote a sequence of integers $1,\ldots,n$ as $\overline{1,n}$, for any $n \in \mathbb{N}$. The set of all inputs $u : \mathbb{R}_{\geq 0} \to \mathbb{R}^p$ such that its L_∞ norm on $[0,\infty]$ is less than infinity, *i.e.,* $||u||_\infty := ||u||_{[0,\infty]} = ess\sup_{t\geq 0}||u(t)|| < \infty$, is denoted as $\mathcal{L}_\infty$. For a couple of vectors $x_1, x_2 \in \mathbb{R}^n$, and a couple of matrices A_1, $A_2 \in \mathbb{R}^{n\times n}$, the relation $x_1 \leq x_2$ and $A_1 \leq A_2$ are understood in the component–wise sense. For a matrix $A \in \mathbb{R}^{n\times n}$, define $A^+ = \max\{0, A\}$, $A^- = A^+ - A$, and $|A| = A^+ + A^-$, similarly for a vector. The term $\mathrm{He}(A)$ denotes $A + A^\top$, for a matrix $A \in \mathbb{R}^{n\times n}$. In addition, for a symmetric matrix $A \in \mathbb{R}^{n\times n}$ the relation $A \prec 0$ ($A \succeq 0$) means that $A \in \mathbb{R}^{n\times n}$ is negative (positive) definite. A matrix $A \in \mathbb{R}^{n\times n}$ is called Metzler when all its non–diagonal elements are non–negative.

II. Problem Statement

Consider the following class of LPV system:

$$\dot{x} = A(\theta)x + B(u + w(t)), \tag{1}$$

where $x \in \mathbb{R}^n$ is the vector state, $u \in \mathbb{R}^m$ is the control input, the term $w : \mathbb{R}_{\geq 0} \to \mathbb{R}^m$ represents some bounded external disturbances, $\theta \in \mathbb{R}^p$ is an unknown scheduling parameter vector in the matrix $A : \mathbb{R}^p \to \mathbb{R}^{n\times n}$, which belongs to a compact set $\Theta \in \mathbb{R}^p$, and the matrix $B \in \mathbb{R}^{n\times m}$ is known and such that $\mathrm{rank}(B) = m$.

The goal of this work is to design a robust controller such that the trajectories of the system (1) converge to zero. Such a controller must cope with some external disturbances and parameter uncertainties, and it has to take into account some state and input constraints, *i.e.*, $x(t) \in \mathbb{X} \subset \mathbb{R}^n$, and $u(t) \in \mathbb{U} \subset \mathbb{R}^m$, for all $t \geq 0$ with some bounded constraint sets $\mathbb{X}$ and $\mathbb{U}$.

In order to deal with the control design we consider that the state x is measurable and the following assumptions are imposed on the system (1).

Assumption 1. *There exists a Metzter matrix $A_0 \in \mathbb{R}^{n\times n}$, and some matrices $A_i \in \mathbb{R}^{n\times n}$, for $i = \overline{1,\eta}$, for some $\eta \in \mathbb{N}_+$ such that the following constraints*

$$A(\theta) = A_0 + \sum_{i=1}^{\eta} \lambda_i(\theta)A_i, \ \sum_{i=1}^{\eta} \lambda_i(\theta) = 1,$$

with $\lambda_i(\theta) \in [0,1]$, hold for the system (1).

Assumption 2. *The external disturbances are bounded, i.e., $w \in \mathbb{W} := \{w \in \mathcal{L}_\infty : ||w||_\infty \leq w^+\}$, with w^+ a positive known constant; and there exists two vectors $\underline{x}_0, \overline{x}_0 \in \mathbb{X}$ such $\underline{x}_0 \leq x(0) \leq \overline{x}_0$.*

III. Preliminaries

Consider the system

$$\dot{x} = f(t,x), \ t \in \mathbb{R}_{\geq 0}, \ x(0) = x_0, \tag{2}$$

where $x \in \mathbb{R}^n$ is the state vector; $f : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}^n$ is assumed to be locally bounded uniformly in t. For f locally measurable but discontinuous with respect to x, the solutions are understood in the sense of Filippov [18]. Then, a solution of the system (2), for an initial condition $x_0 \in \mathbb{R}^n$ at time instant $t_0 \in \mathbb{R}$ is denoted as $x(t, t_0, x_0)$, and defined on some finite time interval $[t_0, t_0 + T)$ such that $0 \leq T < \infty$. Let Ω be open neighborhood of the origin in $\mathbb{R}^n$, $0 \in \Omega$.

Definition 1. *[19], [20]. At the steady state $x = 0$, the system (2) is said to be*

1) Uniformly Stable (US) *if for any $\epsilon > 0$ there is $\delta(\epsilon)$ such that for any $x_0 \in \Omega$, if $||x_0|| \leq \delta(\epsilon)$; then, $||x(t, t_0, x_0)|| \leq \epsilon$, for all $t \geq t_0$, and any to $t_0 \in \mathbb{R}$;*

2) Uniformly Exponentially Stable (UES) *if it is US and exponentially converging from* Ω, i.e., *for any* $x_0 \in \Omega$, *there exist* k, $\gamma > 0$ *such that* $||x(t,t_0,x_0)|| \leq k||x_0||e^{-\gamma(t-t_0)}$, *for all* $t \leq t_0$ *and any* $t_0 \in \mathbb{R}$;

3) Uniformly Finite–Time Stable (UFTS) *if it is US and finite–time converging from* Ω, i.e., *for any* $x_0 \in \Omega$, *there exists* $0 \leq T_{x_0} < +\infty$ *such that* $x(t,t_0,x_0) = 0$, *for all* $t \geq t_0 + T_{x_0}$, *and any* $t_0 \in \mathbb{R}$. *The function* $T_0(x_0) = \inf\{T_{x_0} \geq 0 : x(t,t_0,x_0) = 0 \; \forall t \geq t_0 + T_{x_0}\}$ *is called the settling–time of the system* (2).

The following lemma is also used.

Lemma 1. [21]. *Let* $\xi \in \mathbb{R}^n$ *be a varying vector such that* $\underline{\xi} \leq \xi \leq \overline{\xi}$, *for some* $\underline{\xi}, \overline{\xi} \in \mathbb{R}^n$. *If* $A \in \mathbb{R}^{m\times n}$ *is a constant matrix; then*

$$A^+\underline{\xi} - A^-\overline{\xi} \leq A\xi \leq A^+\overline{\xi} - A^-\underline{\xi}.$$

If $A \in \mathbb{R}^{m\times n}$ *is a variable matrix such that* $\underline{A} \leq A \leq \overline{A}$, *for some* $\underline{A}, \overline{A} \in \mathbb{R}^{m\times n}$; *then*

$$A_m \leq A\xi \leq A_M,$$

with $A_m = \underline{A}^+\underline{\xi}^+ - \overline{A}^+\underline{\xi}^- - \underline{A}^-\overline{\xi}^+ + \overline{A}^-\overline{\xi}^-$ *and* $A_M = \overline{A}^+\overline{\xi}^+ - \underline{A}^+\overline{\xi}^- - \overline{A}^-\underline{\xi}^+ + \underline{A}^-\underline{\xi}^-$.

IV. Control Design

The proposed controller has two components

$$u(t) = u_0(t) + u_1(t). \quad (3)$$

The nonlinear element u_1 is based on an integral sliding–mode approach that compensates for the effect of the matched disturbances; whereas u_0 is the nominal control part, which is composed of an interval predictor–based state–feedback controller and an MPC scheme that will deal with the state and input constraints. The proposed approach will guarantee exponential stability for the dynamics system.

Considering that $u(t) \in \mathbb{U}$, this implies that $||u|| \leq u_{\max}$, for a given $u_{\max} > 0$; then, a specific control effort can be assigned to each part of controller (3), *i.e*, $||u_0|| \leq u_{0\max}$ and $||u_1|| \leq u_{1\max}$, with $u_{0\max}, u_{1\max} > 0$, such that $u_{0\max} + u_{1\max} \leq u_{\max}$.

A. Integral Sliding–Mode Control

Consider the following sliding variable

$$s(x) = G[x(t) - x(0)] - G\int_0^t [A_0x(\tau) + Bu_0(\tau)]d\tau, \quad (4)$$

where $G \in \mathbb{R}^{m\times n}$ is such that $\det(GB) \neq 0$. Such a matrix can be selected as $G = (B^TB)^{-1}B^T$, for more details see [22] and [23]. Thus, the dynamics of the sliding variable holds

$$\dot{s} = G\sum_{i=1}^{\eta} \lambda_i(\theta)A_ix + GB(u_1 + w(t)). \quad (5)$$

Then, the following sliding–mode control is proposed

$$u_1 = -\gamma(x)\frac{(GB)^Ts}{||(GB)^Ts||}, \quad (6)$$

where $\gamma(x) > 0$ for all $x \in \mathbb{X}$. The following lemma provides the conditions to ensure the finite–time convergence of the sliding variable to zero fulfilling the input constraint.

Lemma 2. *Let the ISMC* (6) *be applied to the system* (5), *for a given* $u_{1\max} > 0$. *If the gain* $\gamma(x)$ *is selected as*

$$\gamma(x) = \rho + w^+ + ||G||A_{\max}||x||, \quad (7)$$

with $A_{\max} = \sum_{i=1}^{\eta} ||A_i||$, *and some* $\rho > 0$ *such that*

$$0 < \rho \leq u_{1\max} - w^+ - ||G||A_{\max}x_{\max}, \quad (8)$$

is satisfied for a given $u_{1\max} > 0$; *then,* $s = 0$ *is UFTS.*

Remark 1. *Since* u_1 *is selected as in* (6), *a discontinuous control signal is obtained. However, it is possible to approximate the control signal as*

$$u_1 = -\gamma(x)\frac{(GB)^Ts}{||(GB)^Ts||} \approx -\gamma(x)\frac{(GB)^Ts}{||(GB)^Ts|| + \tau},$$

where τ *is a small positive constant.*

Therefore, the robust controller u_1 will deal with the perturbations w and part of the parametric uncertainty $\sum_{i=1}^{\eta} \lambda_i(\theta)A_i$, satisfying the input constraint $||u_1|| \leq u_{1\max}$. However, the parametric uncertainty $\sum_{i=1}^{\eta} \lambda_i(\theta)Ax$ is not matched with the control u_1, and only its projection into the matched space of B could be compensated by u_1.

Additionally, it is possible to fix $u_{1\max}$ according to the upper bound of the disturbances; then, to assign the rest of the control effort to u_0, *i.e.*, $u_{0\max} \leq u_{\max} - u_1$.

Remark 2. *It is also possible to design the sliding–mode control* (6) *by means of continuous control laws, e.g., by means of multi–variable Super–Twisting algorithms (see [24], [25], or [26]). However, one needs to deal with the saturation constraints.*

B. Interval Predictor

Since the sliding–mode takes place, from (5), it follows that the equivalent control is

$$u_{1eq} = -(GB)^{-1}G\sum_{i=1}^{\eta} \lambda_i(\theta)A_ix - w, \quad (9)$$

so the trajectories of the system at the sliding surface are given by

$$\dot{x} = \left[A_0 + \sum_{i=1}^{\eta} \lambda_i(\theta)\tilde{A}_i\right]x + Bu_0, \quad (10)$$

where $\tilde{A}_i = (I - B(GB)^{-1}G)A_i$. Then, according to Lemma 1 and [27], it follows that

$$-\overline{A}\underline{x}^- - \underline{A}\overline{x}^+ \leq \sum_{i=1}^{\eta} \lambda_i(\theta)\tilde{A}_ix \leq \overline{A}\overline{x}^+ + \underline{A}\underline{x}^-, \quad (11)$$

where $\overline{A} = \sum_{i=1}^{n} \tilde{A}_i^+$, $\underline{A} = \sum_{i=1}^{n} \tilde{A}_i^-$. Thus, it is possible to design the following interval predictor [10] for the system (10)

$$\dot{\xi} = \mathcal{A}_0\xi + \mathcal{A}_1\xi^+ + \mathcal{A}_2\xi^- + \mathcal{B}u_0, \quad (12)$$

where $\xi = \left(\underline{x}^T,\ \overline{x}^T\right)^T \in \mathbb{R}^{2n}$, and the system matrices

$$\mathcal{A}_0 = \begin{pmatrix} A_0 & 0 \\ 0 & A_0 \end{pmatrix}, \quad \mathcal{A}_1 = \begin{pmatrix} 0 & -\underline{A}^- \\ 0 & \overline{A}^+ \end{pmatrix},$$

$$\mathcal{A}_2 = \begin{pmatrix} -\overline{A} & 0 \\ \underline{A} & 0 \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} B \\ B \end{pmatrix}.$$

Note that, the boundedness of ξ implies the same property of x. Thus, in order to stabilize the dynamics of the system (1), it is required to design a state–feedback u_0 to take the trajectories of the system (12) to zero [21].

C. State–Feedback Control Design

The control signal u_0 is proposed as

$$u_0(t) = \begin{cases} \mathcal{U}_0(t), & \xi(t_i) \notin \mathbb{X}_f, \\ \overline{u}_0(t), & \xi(t_i) \in \mathbb{X}_f, \end{cases} \quad (13)$$

where $\mathcal{U}_0$ is the control signal provided by the MPC scheme, for all $t \in [t_i, t_i+h)$, with $h \in (0, T)$ and T as the application time and the prediction interval for the MPC, respectively; and $\overline{u}_0$ is the state–feedback controller. The switching set $\mathbb{X}_f$ is defined further on.

The nonlinear state–feedback controller $\overline{u}_0$ is given as follows

$$\overline{u}_0 = K_0\xi + K_1\xi^+ + K_2\xi^-, \quad (14)$$

with K_0, K_1, $K_2 \in \mathbb{R}^{m\times 2n}$ are the gains to be designed. The following lemma provides a constructive way to design state feedback gains to ensure convergence of the trajectories of the system (12) to zero.

Lemma 3. *Let Assumptions 1 and 2 be satisfied and the state–feedback control law* (14) *be applied to the system* (12)*,* i.e., $u_0(t) = \overline{u}_0(t)$*. Suppose that there also exist diagonal matrices* $0 < \Gamma_l \in \mathbb{R}^{2n\times 2n}$*,* $0 \le R_1, R_2 \in \mathbb{R}^{2n\times 2n}$*, some diagonal matrices* Ψ_l*,* $R_0 \in \mathbb{R}^{2n\times 2n}$ *and some matrices* $Y_l \in \mathbb{R}^{m\times 2n}$*, for* $l = \overline{0,2}$*, such that the following LMIs*

$$\Omega = \begin{pmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ \star & \Omega_{22} & \Omega_{23} \\ \star & \star & \Omega_{33} \end{pmatrix} \preceq 0, \quad (15)$$

$$\Psi_0 + \min\{\Psi_1, \Psi_2\} + 2\min\{R_1, R_2\} > 0, \quad (16)$$

$$\Omega_{11} = \mathrm{He}(\mathcal{A}_0\Gamma_0 + \mathcal{B}Y_0) + \Psi_0,$$
$$\Omega_{12} = \mathcal{A}_1\Gamma_1 + \mathcal{B}Y_1 + \Gamma_0\mathcal{A}_0^T + Y_0^T\mathcal{B}^T + R_1,$$
$$\Omega_{13} = \mathcal{A}_2\Gamma_2 + \mathcal{B}Y_2 - \Gamma_0\mathcal{A}_0^T - Y_0^T\mathcal{B}^T - R_2,$$
$$\Omega_{22} = \mathrm{He}(\mathcal{A}_1\Gamma_1 + \mathcal{B}Y_1) + \Psi_1,$$
$$\Omega_{23} = \mathcal{A}_2\Gamma_2 + \mathcal{B}Y_2 - \Gamma_1\mathcal{A}_1^T - Y_1^T\mathcal{B}^T + R_0,$$
$$\Omega_{33} = \Psi_2 - \mathrm{He}(\mathcal{A}_2\Gamma_2 + \mathcal{B}Y_2),$$

are feasible. If the state–feedback gains are designed as $K_j = Y_j\Gamma_j^{-1}$*, for* $j = \overline{0,2}$*; then, the trajectories of the system* (12) *exponentially converge to zero.*

The diagonal structure required for Γ_0 is natural since the existence of a diagonal matrix Γ_0, as a solution of the Lyapunov equation $\mathrm{He}(\overline{\mathcal{A}}_0\Gamma_0) \preceq 0$, is equivalent to the stability of a Metzler matrix $\overline{\mathcal{A}}_0 = \mathcal{A}_0 + \mathcal{B}K_0$.

Thus, the control (14) provides exponential convergence to zero for the interval predictor (12). Furthermore, considering the fact that $\mathcal{A}_0$ is Metzler and $\underline{x}_0 \le x(0) \le \overline{x}_0$, for two vectors $\underline{x}_0$, $\overline{x}_0 \in \mathbb{R}^n$, the inclusion property $\underline{x}(t) \le x(t) \le \overline{x}(t)$ is satisfied (for further details, see [21] and [28]). Therefore, the trajectories of system (1) will converge to zero and the considered problem will be properly solved provided that $x(t) \in \mathbb{X}$, and $u(t) \in \mathbb{U}$, for all $t \ge 0$, hold.

Now, it is possible to define the set $\mathbb{X}_f$ as follows

$$\mathbb{X}_f = \left\{\xi \in \mathbb{R}^{2n} : V_\xi(\xi) \le \kappa^{-1}\epsilon\right\}, \quad (17)$$

where $V_\xi = \xi^\top\Gamma_0^{-1}\xi + \xi^\top\Gamma_1^{-1}\xi^+ - \xi^\top\Gamma_2^{-1}\xi^-$, ε is a positive constant, and $\kappa = \min_{\forall i=\overline{1,2p}} \lambda_i\left[\Upsilon(\Gamma_0 + \Gamma_1^+ + \Gamma_2^+)\right]$, with $\Upsilon = \bar{\Psi}_0 + \min\{\bar{\Psi}_1, \bar{\Psi}_2\} + 2\min\{\bar{R}_1, \bar{R}_2\} > 0$, $\bar{\Psi}_j = \Gamma_j^{-1}\Psi_j\Gamma_j^{-1}$, and $\bar{R}_j = \Gamma_j^{-1}R_j\Gamma_j^{-1}$, for $l = \overline{0,2}$. Note that $\mathbb{X}_f$ is an invariant set for the system (12); in addition, all the trajectories, outside of $\mathbb{X}_f$, are attracted inside it and converge to zero. Therefore, it is possible to select ϵ such that $\mathbb{X}_f \subset \mathbb{X} \times \mathbb{X}$, *i.e.,* such that $x(t) \in \mathbb{X}$ holds, for all $t \ge t_f \ge 0$.

D. MPC Design

Before proceeding with the description of the MPC scheme, it needs to introduce the following assumption.

Assumption 3. *There exist* $\gamma(x)$*,* K_0*,* K_1*, and* K_2 *satisfying the conditions of Lemma 2 and 3, such that*

$$-\gamma(x(t))\frac{(GB)^Ts}{||(GB)^Ts||} + K_0\xi(t) + K_1\xi_1^+(t) + K_2\xi^-(t) \in \mathbb{U},$$

for any $\xi \in \mathbb{X}_f$ *and all* $t \ge t_f \ge 0$*.*

The previous assumption implies that there always exists a controller (3), given by (6) and (14), such that system (1) is stabilized, and inside $\mathbb{X}_f$, the state and input constraints hold.

Let us now describe the application of the MPC, which will deal with the state and input constraints for all $\xi(t_i) \notin \mathbb{X}_f$. Define T and $h \in (0, T)$ as the prediction interval and the application time for the MPC, respectively. Hence, the optimal control problem for the MPC algorithm is given as follows (see [1] and [29]):

Problem 1. *For given matrices* $0 \preceq Q_j \in \mathbb{R}^{2n\times 2n}$*,* $j = \overline{0,1}$*,* $0 \preceq Q_2 \in \mathbb{R}^{m\times m}$*, and* $t_i = ih$*, with* $i \in \mathbb{N}_+$*, to find the control signals*

$$\mathcal{U}_0 = \underset{u:[t_i, t_i+T]\to\mathbb{R}^m}{\arg\min}\ \xi^T(t_i+T)Q_0\xi(t_i+T) + \int_{t_i}^{t_i+T} [\xi^T(\sigma)Q_1\xi(\sigma) + u_0^T(\sigma)Q_2u_0(\sigma)]d\sigma, \quad (18)$$

such that the following constraints hold:

a) $\xi : [t_i, t_i + T] \to \mathbb{R}^{2n}$ *is a solution of* (12).
b) $\xi(\sigma) \in \mathbb{X} \times \mathbb{X}$ *and* $u_0(\sigma) \in \mathbb{U}$ *for* $\sigma \in [t_i, t_i + T]$.
c) $\xi(t_i + T) \in \mathbb{X}_f$.

Thus, if the above–mentioned optimal control is feasible, the trajectories of the system (14) will converge to the terminal set $\mathbb{X}_f$, when $u_0(t) = \mathcal{U}_0(t)$; then, inside $\mathbb{X}_f$, when $u_0(t) = \overline{u}_0(t)$, the trajectories will converge to zero satisfying the state and input constraints.

Finally, the statements of Lemma 1 and the solution to problem 1 will provide the main result of this paper, which is described by the following Theorem.

Theorem 1. *Let Assumption 1, the conditions of Lemmas 2 and 3 be satisfied and Problem 1 be feasible. If the control law* (3)*, given by* (6) *and* (13)*, is applied to the system* (1) *and designed according to Lemmas 2 and 3, and the solution of Problem 1; then, the origin of the system* (1) *is Uniformly Exponentially Stable, and the state and input constraints are satisfied.*

Due to space limitations, the proofs of all the results are omitted.

V. Simulation Results

In this section a numerical example is presented in order to illustrate the usefulness of the proposed methodology. Consider the LPV system (1) with:

$$A(\theta) = \begin{pmatrix} -1 & 6 + \theta(t) \\ \theta(t) & -1 \end{pmatrix}, \; B = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

with $\theta(t) = 2\sin(t)$ and $w(t) = 2\sin(3t) + 1$. The constraint sets are defined as $\mathbb{X} = [-3, 3] \times [-2, 2]$ and $\mathbb{U} = [-10, 10]$, while the initial conditions for the system are $x(0) = (-2, 1)^T$. It is possible to show that this system satisfies Assumption 1 and the matrices A_0 and A_i, with $i = 1, 2$, can be obtained by a convex polytopic method, and the external disturbance holds Assumption 2, with $w^+ = 3$. On the other hand, the initial conditions for the predictor are $\underline{x}_0 = (-2.5, -1.5)^T$ and $\overline{x}_0 = (0.5, 1.5)^T$. The gains of the state–feedback controller are assessed with the LMIs of Lemma 2, *i.e.,*

$$K_0 = (-0.0226, -1.6266, -0.0226, -1.6266),$$
$$K_1 = (-0.0535, -0.0997, -0.0535, -9.7960),$$
$$K_2 = (0.0535, 9.7970, 0.0535, 0.0997).$$

The prediction horizon is selected as $N = 10$ and we compute the corresponding switching set $\mathbb{X}_f$ according to (17). All the simulations have been done in MATLAB with the Euler discretization method, sampling–time equal to 0.001, and the solutions for the corresponding LMIs have been found by means of SDPT3 solver among YALMIP in MATLAB while the MPC has been implemented using the nlmpc toolbox in MATLAB.

This simulation scenario is shown in Figs. 1, 2 and 3. In Fig. 1 it is possible to see that the time evolution of the system trajectories satisfy the constraints and converge to zero.

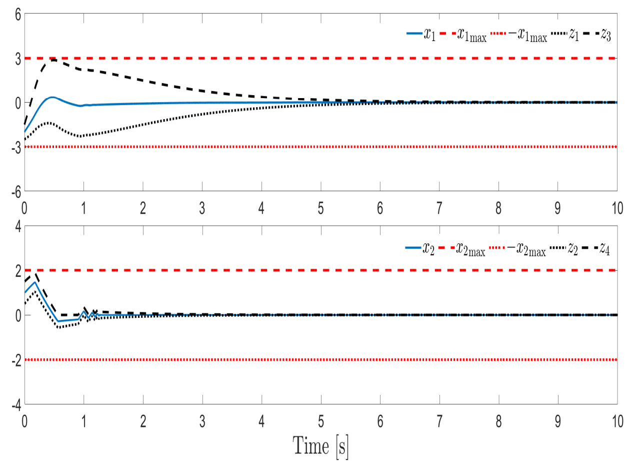

Figure 1. Evolution of the state variables

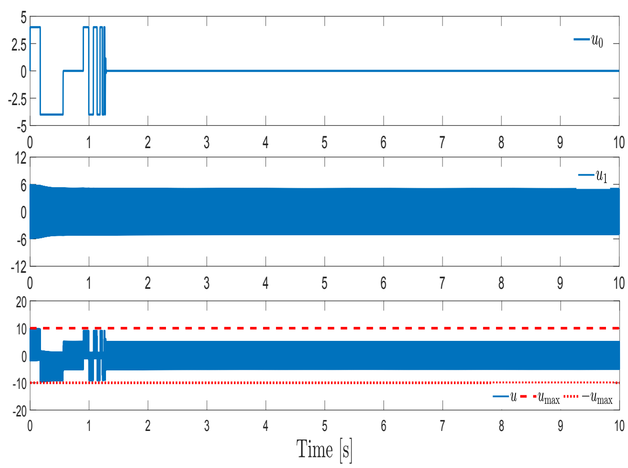

Figure 2. Time evolution of the control signals

The input u applied to the system is displayed in Fig. 2. Furthermore, it is shown the control signals u_0 and u_1.

In order to illustrate the influence of the ISMC, Fig. 3 shows a comparison between the input controller working with and without the ISMC, *i.e.*, $u_1 = 0$. As a result, the control signal u_0 is not able to stabilize the system, due to the external perturbations, and the trajectories start growing leaving the set of constraints.

VI. Conclusion

This paper presents the design of a robust controller, based on an ISMC approach together with an interval predictor–based state feedback controller and an MPC scheme, for a class of uncertain LPV systems. The contribution of this work can be summarized in the following points: first, an ISMC control strategy is designated to reject the matched perturbations ensuring the sliding–mode from the initial time instance. Second, the control is also robust against

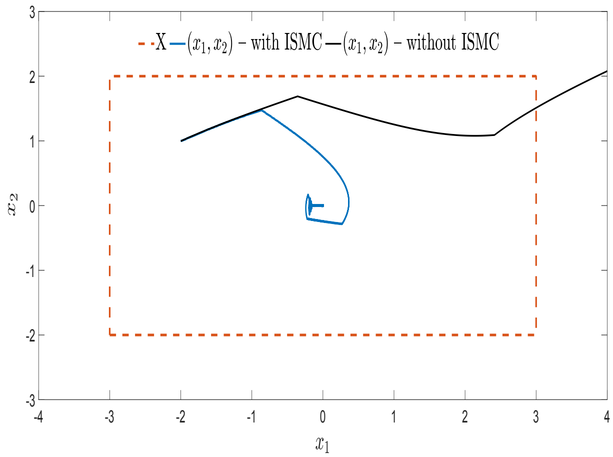

Figure 3. Comparison of the system with and without ISMC

some parameter uncertainties, and the interval predictor–based state feedback controller and the MPC deal with the state and input constraints. Third, the use of LMIs allows a constructive design of the interval predictor–based controller gains, which does not depend on the upper bound of the external disturbances due to the robustness properties of the ISMC. Fourth, the proposed control approach guarantees exponential stability and the computational burden is relaxed due to the switching structure of the controller. Finally, the proficiency of the method is demonstrated in a numerical example.

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