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Data-Driven Output Containment Control of Heterogeneous Multiagent Systems: A Hierarchical Scheme

Malika Sader, Wenyu Li, Yanhui Yin, Zhongmei Li, Dexian Huang, Zhongxin Liu, *Member, IEEE*, Xiao He, and Chao Shang, *Member, IEEE*

Abstract— **In this article, we propose a data-driven solution to the output containment control problem of multiagent systems characterized by heterogeneous and unknown dynamics. The proposed data-driven scheme is hierarchical, which includes a network layer and a physical layer, bypassing the traditional modeling exercise and eliminating the need for explicit state-space models. In the network layer, a fully distributed observer without knowing the dynamics of leaders is designed to generate a containment trajectory. A data-driven approach based on data sampled from an auxiliary system is developed to overcome the dependence on the explicit state-space models for design of control gains. This paves the way for deriving a data-driven solution to the regulator equation, which is derived by employing the data informativity condition and relevant data in the physical layer. The results demonstrate that the closed-loop system is asymptotically stable, and the regulated output containment converges to zero. Compared to generic model-based hierarchical schemes, the assumptions that the system matrices are completely known and homogeneous on system dynamics can be relaxed.**

I. Introduction

In recent years, there has been a booming interest in multiagent systems (MAS) across various disciplines, including computer science, social science, control engineering, and software engineering [1], with a variety of applications ranging from sensor networks to unmanned aerial vehicles [2]. Due to the pervasiveness of uncertainty in practical MAS, the output containment control problem has been extensively investigated to drive each follower into the convex hull spanned by the multiple leaders [3]. Recently, fruitful results have been achieved in this vein, for example, the finite-time approach [4], adaptive sliding-mode protocols [5], and observer-based protocols [6]. The same problem was investigated for nonlinear MAS such as rigid body attitude systems [7], Lagrange systems [8], and uncertain nonlinear MAS [9].

The increasing complexity of engineering systems makes the MAS more vulnerable to homogeneous cases, where all the agents have identical dynamics. In a broad class of practical applications, however, individual systems can be heterogeneous in that the dynamics and even state-space dimensions are different. Therefore, the containment control of heterogeneous systems is more worthy of study. For example, Haghshenas et al. [10] studied the containment control of heterogeneous MAS in a cooperative output regulation scheme using a state-feedback protocol. Chu et al. [11] investigated the adaptive containment control of heterogeneous MAS using state feedback and output feedback protocols. A distributed containment control policy was designed in [12] for autonomous agents under fixed and switching communication topologies with static and dynamic leaders.

However, a critical limitation of these control protocols is that they require a precise model of the system being controlled, which can be costly and time-consuming. This has stimulated the development of data-driven control methods, which aim to learn the control policy directly from data without the need for a mathematical model of the plant. Various methods have been presented to solve data-driven control problems, including model-free adaptive control [13], reinforcement learning [14], and behavioral systems theory [15]–[18], the last of which does not rely on parameter identification and has rigorous stability analysis and thereby offers a new route towards addressing widespread black-box systems. Motivated by this, data-driven solutions to leader-follower consensus and event-triggered consensus in MAS have been developed recently [19], [20]. Nonetheless, data-driven output containment control of heterogeneous MAS has not been investigated yet.

In this paper, we propose a data-driven solution to the output containment control problem of MAS, by drawing ideas from recent advances in data-driven regulatory control [21]. The problem is investigated under unknown discrete-time agent dynamics, which does not require prior knowledge of the model information, and also takes into account the heterogeneity that the leaders and followers have different dynamics. Our data-driven scheme is hierarchical, which includes a network layer and a physical layer. As compared to the generic model-based hierarchical scheme [4]–[7], our data-driven scheme exhibits the following characteristics:

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M. Sader is with Department of Automation, Tsinghua University, Beijing 100084, China (e-mail: mlksdr@tsinghua.edu.cn).

W. Li and Z. Liu are with the College of Artificial Intelligence, and also with the Tianjin Key Laboratory of Interventional Brain-Computer Interface and Intelligent Rehabilitation, Nankai University, Tianjin 300350, China (e-mail: liwenyu@nankai.edu.cn, lzhx@nankai.edu.cn)

Y. Yin is with School of Automation Engineering, North-East Electric Power University, Jilin, 132000, China (e-mail: yinyanhui2013@163.com)

Z. Li is with the Key Laboratory of Smart Manufacturing in Energy Chemical Process, Ministry of Education, East China University of Science and Technology, 200237, Shanghai, China (e-mail: lizhongmei@sina.cn)

D. Huang, X. He and C. Shang are with Department of Automation, Beijing National Research Center for Information Science and Technology, Tsinghua University, Beijing 100084, China (e-mail: huangdx@tsinghua.edu.cn, hexiao@tsinghua.edu.cn, c-shang@tsinghua.edu.cn).

- In the network layer, as compared to the model-based observers [10]–[12], a fully distributed observer without knowing the dynamics of leaders is designed to generate a containment trajectory, by only using sampled data from an auxiliary system. This overcomes the dependence on explicit state-space models for design of control gains. More importantly, the observer error system is proved to be asymptotically stable.
- In the physical layer, as compared to model-based approaches [22]–[25], the assumptions of completely known system matrices are homogeneous system dynamics are relaxed. A data-driven solution to the regulator equation is derived by employing the data informativity condition and relevant data. Based on this, we prove that the state of the resulting closed-loop system is asymptotically stable, and the regulated output converges to zero.

The rest of this article unfolds as follows. In Sec. II, we introduce the graph theory, system description, and control objective. The design of the observer and containment controller is given in Sec. III. A simulation example is given in Sec. IV, followed by final conclusions.

Notation: The Euclidean norm and Kronecker product are denoted by $\|\cdot\|$ and $\otimes$, respectively. The positive-definiteness (negative-definiteness) of a matrix P is indicated by $P \succ 0$ ($P \prec 0$). A block diagonal matrix with $Q_1, Q_2, \cdots, Q_n$ on its principal diagonal is denoted by $\text{diag}\{Q_1, Q_2, \cdots, Q_n\}$. $\lambda_{\min}(Q)$ and $\lambda_{\max}(Q)$ denote the minimum and maximum eigenvalue of Q, respectively. The identity matrix and the column vector with all ones are denoted by I and $\mathbf{1}$, respectively. The superscript "$\top$" denotes matrix transpose. The convex hull of a set S is denoted as $\text{Co}(S)$. The distance from x to a set S is defined as $\text{dist}(x, S) = \inf_{y\in S}\|x - y\|$.

II. Preliminaries and Problem Statement

A. Graph Theory

This paper considers a digraph $\mathscr{G}$, which includes m leaders and n followers. $\mathscr{K} = \{n+1, n+2, \cdots, n+m\}$ and $\mathscr{F} = \{1, 2, \cdots, n\}$ denote the sets of leaders and followers, respectively. $\mathscr{G}_f = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ indicates the interactions among the followers, where $\mathscr{V} = \{v_1, v_2, \cdots, v_n\}$, $\mathscr{E} \subset \mathscr{V} \times \mathscr{V}$, and $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$. (v_j, v_i) indicates the information flow from node j to node i. $a_{ij} \neq 0$ if $(v_j, v_i) \in \mathscr{E}$, otherwise $a_{ij} = 0$. $N_i = \{j \mid (v_j, v_i) \in \mathscr{E}\}$ represents the neighbours of agent i. The directed path from agent i to agent j is described by $\{(v_i, v_k), (v_k, v_l), \cdots, (v_m, v_j)\}$. The diagonal matrix $\mathscr{G}_k = \text{diag}\{g_i^k\} \in \mathbb{R}^{n\times n}, i \in \mathscr{F}, k \in \mathscr{K}$ indicates the pinning gains. $g_i^k \neq 0$ if there is a link from the k-th leader to the i-th follower, otherwise $g_i^k = 0$. The degree matrix is denoted by $\mathscr{D} = \text{diag}\left\{\sum_{j\in N_i} a_{ij}\right\}$ and the Laplacian matrix is $\mathscr{L} = \mathscr{D} - \mathscr{A}$.

Assumption 1 (Network topology): The communication network topology composed of all leaders and followers is undirected and each leader has a path to all the followers.

Assumption 1 is a necessary condition for undirected graph's networks connectivity. Otherwise, there exists at least one follower that has no path to each leader. Then the containment control is not obtainable. Assumption 1 is also used in the existing containment control results.

Lemma 1 ([26]): Let $\Phi_k = \frac{1}{m}\mathscr{L} + \mathscr{G}_k$. Under Assumption 1, Φ_k and $\sum_{k=n+1}^{n+m} \Phi_k$ are positive-definite.

B. System Description

Consider the following heterogeneous MAS in which the dynamics of the follower and the leader are described by

$$\begin{cases} x_i(t+1) = A_i x_i(t) + B_i u_i(t), \\ \quad\; y_i(t) = C_i x_i(t), \end{cases} \tag{1}$$

and

$$\begin{cases} x_{0k}(t+1) = A_0 x_{0k}(t), \\ \quad\; y_{0k}(t) = C_0 x_{0k}(t), \end{cases} \tag{2}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ and $x_{0k}(t) \in \mathbb{R}^q$ are the states. $u_i(t) \in \mathbb{R}^{m_i}$ is the input. $y_i(t) \in \mathbb{R}^p$ and $y_{0k}(t) \in \mathbb{R}^p$ are the outputs. All matrices, i.e. $A_i \in \mathbb{R}^{n_i\times n_i}, B_i \in \mathbb{R}^{n_i\times m_i}$, $C_i \in \mathbb{R}^{p_i\times n_i}$, $A_0 \in \mathbb{R}^{q\times q}$, and $C_0 \in \mathbb{R}^{p\times q}$ are assumed to be unknown. Besides, (A_i, B_i) is stabilizable, (A_i, C_i) is detectable, and the following assumptions are made throughout.

Assumption 2: The leader dynamics A_0 has all its poles within the unit circle and non-repeated.

Assumption 3: The output regulation equation

$$\begin{cases} \Pi_i A_0 = A_i \Pi_i + B_i \Gamma_i, \\ C_i \Pi_i - C_0 = 0 \end{cases} \tag{3}$$

admits a solution pair (Π_i, Γ_i).

C. Control Objective

This work is oriented towards a data-driven hierarchical scheme of the MAS described by (1) and (2) with heterogeneous and unknown dynamics, such that:

- The state of the resultant closed-loop system is asymptotically stable.
- The outputs of each follower move asymptotically into the dynamic convex hull spanned by the outputs of leaders, i.e.

$$\lim_{t\to\infty} \text{dist}\Big(y_i(t), \text{Co}\left(Y_{\mathscr{K}}(t)\right)\Big) = 0. \tag{4}$$

Such a containment control problem has been extensively studied in a model-based scheme [4]–[7], which requires a complete knowledge of state-space representations for control design. However, in cases of MAS or large-scale systems where model parameters are entirely unknown, generic model-based solutions may become inapplicable. Consequently, a completely data-driven control design is practically appealing. To this end, define the local neighborhood output containment error of the i-th follower as:

$$e_i(t) = \sum_{j=1}^{n} a_{ij}\big(y_j(t) - y_i(t)\big) + \sum_{k=n+1}^{n+m} g_i^k\big(y_{0k}(t) - y_i(t)\big).$$

Letting $e(t) = \left[e_1^\top(t), \cdots, e_n^\top(t)\right]^\top$, the global output containment error can be written as:

$$e(t) = \sum_{k=n+1}^{n+m} \left(\Phi_k \otimes I_q\right)\left(\bar{y}_{0k}(t) - y(t)\right), \tag{5}$$

where $y(t) = \left[y_1^\top(t), \cdots, y_n^\top(t)\right]^\top$, $\bar{y}_{0k}(t) = \mathbf{1}_n \otimes y_{0k}(t)$. Based on (5) and [27], the output containment control objective is said to be achieved if $\lim\limits_{t\to\infty} e(t) = 0$.

III. Main Results

As a major contribution, a data-driven scheme is put forward to address the output containment control problem. The proposed scheme is hierarchical involving a network layer and a physical layer.

A. Network Layer: Design of Distributed Observer

The following fully distributed observer is considered, which is motivated by [28]:

$$\hat{x}_i(t+1) = A_0\hat{x}_i(t) + v_i(t), \tag{6}$$

where $\hat{x}_i(t) \in \mathbb{R}^{n_i}$ is used to estimate $x_{0k}(t)$ in agent i. v_i denotes the fully distributed virtual cooperative control protocol, which is designed as:

$$v_i(t) = \alpha z_i(t), \tag{7}$$

where $\alpha > 0$ is a constant respresenting the coupling gain among neighboring agents. $z_i(t)$ is the local form of the observer state containment error, which is denoted as:

$$z_i(t) = \sum_{j=1}^{n} a_{ij}\left(\hat{x}_j(t) - \hat{x}_i(t)\right) + \sum_{k=n+1}^{n+m} g_i^k\left(x_{0k}(t) - \hat{x}_i(t)\right).$$

Letting $z(t) = \left[z_1^\top(t), z_2^\top(t), \cdots, z_n^\top(t)\right]^\top$ and according to the property of Kronecker product, the dynamics of the observer state containment error can be written as:

$$z(t) = \hat{x}(t) - \left(\sum_{r=n+1}^{n+m} \left(\Phi_r \otimes I_q\right)\right)^{-1} \sum_{k=n+1}^{n+m} \left(\mathscr{G}_k \otimes I_q\right)\bar{x}_{0k}(t), \tag{8}$$

where $\hat{x}(t) = \left[\hat{x}_1^\top(t), \hat{x}_2^\top(t), \cdots, \hat{x}_n^\top(t)\right]^\top$ and $\bar{x}_{0k}(t) = \mathbf{1}_n \otimes x_{0k}(t)$. According to (8), the observer state containment error dynamics equation is obtained as:

$$z(t+1) = \left(I_n \otimes A_0 - \alpha \sum_{k=n+1}^{n+m} \left(\Phi_k \otimes I_q\right)\right) z(t). \tag{9}$$

When A_0 is unknown, it is challenging to ensure the asymptotic stability of the observer state containment error system (9) in a data-driven fashion through the design of α. To bypass the dependence of exactly knowing A_0, it is assumed that the following auxiliary dynamics enables to encode information of A_0 within sampled data:

$$\tilde{x}_i(t+1) = A_0\tilde{x}_i(t) + \gamma_i\tilde{u}_i(t), \tag{10}$$

where $\tilde{x}_i(t) \in \mathbb{R}^q$ and $\tilde{u}_i(t) \in \mathbb{R}^q$ are the state and input of auxiliary dynamics, respectively. The data matrices generated by (10) is used to encode useful information about A_0. γ_i is the input gain of auxiliary dynamics to be designed. We define the following data matrices:

$$\begin{aligned} \tilde{X}_i &= \left[\tilde{x}_i(0) \quad \tilde{x}_i(1) \quad \cdots \quad \tilde{x}_i(L)\right], \\ \tilde{X}_{i+} &= \left[\tilde{x}_i(1) \quad \tilde{x}_i(2) \quad \cdots \quad \tilde{x}_i(L)\right], \\ \tilde{X}_{i-} &= \left[\tilde{x}_i(0) \quad \tilde{x}_i(1) \quad \cdots \quad \tilde{x}_i(L-1)\right], \\ \tilde{U}_i &= \left[\tilde{u}_i(0) \quad \tilde{u}_i(1) \quad \cdots \quad \tilde{u}_i(L-1)\right], \end{aligned} \tag{11}$$

where L is the length of data sequence. In the following, we give the rank condition on the data informativity.

Definition 1: The dataset $(\tilde{U}_i, \tilde{X}_i)$ *is said to be informative if the stacked matrix* $\begin{bmatrix}\tilde{U}_i \\ \tilde{X}_i\end{bmatrix}$ *has full row rank.*

Remark 1: Considering the unknown matrix A_0*, it is crucial to fulfill the rank condition so that data matrices* (11) *maintain linear independence, which is vital for deriving an exact data-driven representation. Therefore, it is meaningful to design auxiliary dynamics* (10).

Definition 2: The dataset $(\tilde{U}_i, \tilde{X}_i)$ *is said to be informative for stabilization by state feedback if there exists a controller* $\tilde{u}_i = \tilde{K}_i\tilde{x}_i$ *for all* $(A_0, \gamma_i I) \in \Sigma_{\tilde{D}}$*, where*

$$\Sigma_{\tilde{D}} := \left\{(A_0, \gamma_i I) \middle| X_{i+} = \left[\gamma_i I \quad A_0\right]\begin{bmatrix}\tilde{U}_i \\ \tilde{X}_{i-}\end{bmatrix}\right\}.$$

Lemma 2: The dataset $(\tilde{U}_i, \tilde{X}_i)$ *is informative for stabilization of the auxiliary system* (10) *by a controller* $\tilde{u}_i = \tilde{K}_i\tilde{x}_i$ *with state feedback* $\tilde{K}_i = \tilde{U}_i\tilde{Q}_i\tilde{P}_i^{-1}$ *if there exist* $\tilde{P}_i \succ 0$ *and* $\tilde{Q}_i$ *such that*

$$\begin{bmatrix} \tilde{P}_i & \tilde{X}_{i+}\tilde{Q}_i \\ \tilde{Q}_i^\top\tilde{X}_{i+}^\top & \tilde{P}_i \end{bmatrix} \succ 0, \quad \tilde{X}_{i-}\tilde{Q}_i = \tilde{P}_i. \tag{12}$$

Moreover, there exists $\tilde{W}_i \succ 0$ *such that*

$$\begin{bmatrix} -\tilde{P}_i & A_0 + \gamma_i\tilde{K}_i \\ (A_0 + \gamma_i\tilde{K}_i)^\top & -\tilde{P}_i^{-1} \end{bmatrix} \prec -\tilde{W}_i. \tag{13}$$

Proof: For any given $\tilde{K}_i$, $A_0 + \gamma_i\tilde{K}_i$ is Schur stable if there exists a matrix $\tilde{P}_i \succ 0$ such that

$$(A_0 + \gamma_i\tilde{K}_i)\tilde{P}_i(A_0 + \gamma_i\tilde{K}_i)^\top - \tilde{P}_i \prec 0. \tag{14}$$

By the Rouché–Capelli theorem, there exists $\tilde{G}_i$ such that

$$\begin{bmatrix}\tilde{K}_i \\ I\end{bmatrix} = \begin{bmatrix}\tilde{U}_i \\ \tilde{X}_{i-}\end{bmatrix}\tilde{G}_i \tag{15}$$

holds. It then follows from (15) that

$$A_0 + \gamma_i\tilde{K}_i = \left[\gamma_i I \quad A_0\right]\begin{bmatrix}\tilde{U}_i \\ \tilde{X}_{i-}\end{bmatrix}\tilde{G}_i = \tilde{X}_{i+}\tilde{G}_i. \tag{16}$$

Substituting (16) into (14) yields

$$(\tilde{X}_{i+}\tilde{G}_i)\tilde{P}_i(\tilde{X}_{i+}\tilde{G}_i)^\top - \tilde{P}_i \prec 0. \tag{17}$$

Letting $\tilde{Q}_i = \tilde{G}_i\tilde{P}_i$, (17) is equivalent to $\tilde{P}_i - \tilde{X}_{i+}\tilde{Q}_i\tilde{P}_i^{-1}(\tilde{X}_{i+}\tilde{Q}_i)^\top \succ 0$. Using Schur complement argument, we can obtain the first part of (12). Note that in (15), one can regard $\tilde{G}_i$ as a decision variable, which satisfies $\tilde{X}_{i-}\tilde{G}_i = I$ and $\tilde{K}_i = \tilde{U}_i\tilde{G}_i$. By exploiting $\tilde{P}_i = \tilde{X}_{i-}\tilde{Q}_i$, seeking stability is equivalent to finding a matrix $\tilde{Q}_i$ such that

$$\tilde{X}_{i-}\tilde{Q}_i - \tilde{X}_{i+}\tilde{Q}_i(\tilde{X}_{i-}\tilde{Q}_i)^{-1}(\tilde{X}_{i+}\tilde{Q}_i)^\top \succ 0, \tag{18}$$

with the choice $\tilde{K}_i = \tilde{U}_i\tilde{Q}_i\tilde{P}_i^{-1} = \tilde{U}_i\tilde{Q}_i(\tilde{X}_{i-}\tilde{Q}_i)^{-1}$. Then, it can be concluded that $A_0+\gamma_i\tilde{K}_i$ is Schur stable as $\tilde{P}_i \succ 0$. Multiplying (18) by the reversible matrix $\begin{bmatrix} I & 0 \\ 0 & \tilde{P}_i^{-1}\end{bmatrix}$ on its left and right sides, respectively, yields

$$\begin{bmatrix} I & 0 \\ 0 & \tilde{P}_i^{-1}\end{bmatrix}\begin{bmatrix} \tilde{P}_i & \tilde{X}_{i+}\tilde{Q}_i \\ \tilde{Q}_i^\top \tilde{X}_{i+}^\top & \tilde{P}_i\end{bmatrix}\begin{bmatrix} I & 0 \\ 0 & \tilde{P}_i^{-1}\end{bmatrix} \succ 0. \tag{19}$$

Substituting $A_0 + \gamma_i\tilde{K}_i = \tilde{X}_{i+}\tilde{G}_i$ and $\tilde{Q}_i = \tilde{G}_i\tilde{P}_i$ into (19) eventually yields (13). ∎

In view of Lemma 2, we arrive at the following theorem in the network layer.

Theorem 1: Consider the MAS described by (1) *and* (6). *Suppose that Assumptions 1-3 hold. The dataset* $(\tilde{U}_i, \tilde{X}_i)$ *is informative for the asymptotical stability of both the auxiliary system* (10) *and the observer state containment error system* (9), *if there exist* α, $\tilde{P}_i \succ 0$, $\tilde{Q}_i$, $\tilde{K}_i$, $\tilde{W}_i \succ 0$ *satisfying conditions in Lemma 2 and*

$$\begin{bmatrix} \alpha\lambda_{\max}^2(\tilde{\Phi})I & -\gamma_i\tilde{K}_i \\ -\gamma_i\tilde{K}_i^\top & \alpha I\end{bmatrix} \prec \tilde{W}_i. \tag{20}$$

Proof: From Lemma 2, $\tilde{u}_i = \tilde{K}_i\tilde{x}_i$ stabilizes the auxiliary system (10) if (12) holds. Defining $\tilde{A} = I \otimes A_0$ and $\tilde{\Phi} = \sum_{k=n+1}^{n+m}(\Phi_k \otimes I_q)$, then $A_s = \tilde{A} - \alpha\tilde{\Phi}$ is Schur stable if there exists $\tilde{P} = I \otimes \tilde{P}_i$ such that $A_s^\top \tilde{P} A_s - \tilde{P} \prec 0$. Invoking the Schur complement lemma, we have

$$\begin{bmatrix} -\tilde{P} & (\tilde{A} - \alpha\tilde{\Phi})\tilde{P} \\ \tilde{P}(\tilde{A} - \alpha\tilde{\Phi})^\top & -\tilde{P}^{-1}\end{bmatrix} \prec 0,$$

which is equivalent to

$$\begin{bmatrix} -\tilde{P} & \tilde{A} - \alpha\tilde{\Phi} \\ (\tilde{A} - \alpha\tilde{\Phi})^\top & -\tilde{P}^{-1}\end{bmatrix} \prec 0,$$

if and only if

$$\begin{bmatrix} -\tilde{P} & \tilde{A} \\ \tilde{A}^\top & -\tilde{P}^{-1}\end{bmatrix} \prec \begin{bmatrix} 0 & \alpha\tilde{\Phi} \\ \alpha\tilde{\Phi}^\top & 0\end{bmatrix}. \tag{21}$$

Note that the following relation always holds:

$$\begin{bmatrix} -\alpha\lambda_{\max}^2(\tilde{\Phi})I & 0 \\ 0 & -\alpha I\end{bmatrix} \prec \begin{bmatrix} 0 & \alpha\tilde{\Phi} \\ \alpha\tilde{\Phi}^\top & 0\end{bmatrix}.$$

Then (21) holds if

$$\begin{bmatrix} -\tilde{P} + \alpha\lambda_{\max}^2(\tilde{\Phi})I & \tilde{A} \\ \tilde{A}^\top & -\tilde{P}^{-1} + \alpha I\end{bmatrix} \prec 0,$$

which is equivalent to

$$\begin{bmatrix} -\tilde{P}_i + \alpha\lambda_{\max}^2(\tilde{\Phi})I & A_0 \\ A_0^\top & -\tilde{P}_i^{-1} + \alpha I\end{bmatrix} \prec 0, \quad \forall i \in \mathscr{V}. \tag{22}$$

Combining (13) and (22) yields (20), which completes the proof. ∎

B. Physical Layer: Distributed Data-Driven Feedback Control Protocol Design

1) Solution to Regulator Equations: In the physical layer, defining $s_i(t) = x_i(t) - \Pi_i\hat{x}_i(t)$, one obtains

$$\begin{aligned} s_i(t+1) =& x_i(t+1) - \Pi_i\hat{x}_i(t+1) \\ =& A_i s_i(t) - B_i\Gamma_i\hat{x}_i(t) + B_i u_i - \Pi_i\alpha z_i(t). \end{aligned} \tag{23}$$

According to the distributed observer (6), we can design the following distributed feedback control protocol:

$$\begin{aligned} u_i(t) =& K_{1i}x_i(t) + K_{2i}\hat{x}_i(t) \\ =& K_{1i}\big(x_i(t) - \Pi_i\hat{x}_i(t)\big) + (K_{1i}\Pi_i + K_{2i})\hat{x}_i(t), \end{aligned} \tag{24}$$

where K_{1i} and K_{2i} are feedback gain matrices to be designed. Π_i and Γ_i are the solutions to output regulator equation (3). Letting $K_{1i}\Pi_i + K_{2i} = \Gamma_i$, $\hat{u}_i(t) = u_i(t) - \Gamma_i\hat{x}_i(t)$, and substituting (24) into (23), one obtains:

$$s_i(t+1) = (A_i + B_iK_{1i})s_i(t) - \Pi_i\alpha z_i(t), \tag{25}$$

where the coupling gain α and $z_i(t)$ are defined in (7). Theorem 1 indicates that $\lim_{t\to\infty}\|z(t)\| = 0$. With this result at hand, we proceed to the problem of identifying a gain matrix K_{1i} for follower i that ensures the Schur stability of $A_i + B_iK_{1i}$.

The objective is to design the distributed feedback control protocol (24) in a model-agnostic way. Specifically, akin to (11), we construct data matrices $\{X_i, X_{i+}, X_{i-}, S_i, S_{i+}, S_{i-}\}$ using $x_i(t)$ and $s_i(t)$, and $\{U_i, \hat{U}_i\}$ using $u_i(t)$. Before proceeding, the first step is to solve the regulator equations (3), and the following informativity condition is given.

Definition 3: The dataset (U_i, X_i) *is said to be informative for output regulation if there exists a controller* (24) *for all* $(A_i, B_i, C_i) \in \Sigma_D$, *where* $\Sigma_{\mathcal{D}} := \big\{(A, B, C)\big| X_{i+} = AX_{i-} + BU_i, Y_i = CX_{i-}\big\}$.

Lemma 3 ([21]): The dataset (U_i, X_i) is informative for output regulation if the following conditions hold.

- X_{i-} has full row rank and there exists a right inverse $X_{i-}^\dagger$ of X_{i-} such that $X_{i+}X_{i-}^\dagger$ is stable.
- The following linear equations have a solution W_i:

$$\begin{cases} X_{i-}W_i\tilde{X}_{i+}V_1 - X_{i+}W_i = 0, \\ \Pi_i = X_{i-}W_i, \ \Gamma_i = U_iW_i. \end{cases} \tag{26}$$

2) Solvability of the Problem: Now, we apply the techniques developed above to solve the output containment control problem by a distributed dynamic state feedback control law having the form (24). More precisely, K_{1i} is the controller gain if and only if $A_i + B_iK_{1i}$ is Schur stable. The following set is given based on matrix K_{1i}: $\Sigma := \{(A_i, B_i)| A_i + B_iK_{1i} \text{ is Schur stable}\}$.

Definition 4: The dataset $(\hat{U}_i, S_i)$ *is said to be informative for stabilization of the compensate error system* (25) *by the distributed dynamic state feedback if there exists a control gain* K_{1i} *such that* $A_i + B_iK_{1i}$ *is Schur stable for all* $(A_i, B_i) \in \Sigma$.

Lemma 4: The dataset $(\hat{U}_i, S_i)$ *is informative for stabilization of the compensate error system* (25) *by the* *distributed dynamic state feedback, if there exist* $K_{1i} = \hat{U}_i Q_i P_i^{-1}$ *and* Q_i *with* (27)

$$\begin{bmatrix} P_i & S_{i+}Q_i \\ Q_i^\top S_{i+}^\top & P_i \end{bmatrix} \succ 0, \quad S_{i-}Q_i = P_i, \quad P_i \succ 0, \tag{27}$$

such that $A_i + B_i K_{1i}$ *is Schur stable.*

Proof: For any given K_{1i}, the $A_i + B_i K_{1i}$ is Schur stable, if there exists a matrix $P_i \succ 0$ such that

$$(A_i + B_i K_{1i}) P_i (A_i + B_i K_{1i})^\top - P_i \prec 0. \tag{28}$$

By the Rouché–Capelli theorem, there exists G_i satisfying

$$\begin{bmatrix} K_{1i} \\ I \end{bmatrix} = \begin{bmatrix} \hat{U}_i \\ S_{i-} \end{bmatrix} G_i, \tag{29}$$

which implies that

$$A_i + B_i K_{1i} = \begin{bmatrix} B_i & A_i \end{bmatrix} \begin{bmatrix} \hat{U}_i \\ S_{i-} \end{bmatrix} G_i = S_{i+} G_i. \tag{30}$$

Substituting (30) into (28) yields

$$(S_{i+}G_i) P_i (S_{i+}G_i)^\top - P_i \prec 0. \tag{31}$$

Defining $Q_i = G_i P_i$, (31) amounts to $P_i - S_{i+}Q_i P_i^{-1} (S_{i+}Q_i)^\top \succ 0$. Using Schur complement arguments, we obtain the first part of (27). Note that in (29), one can regard G_i as a decision variable, which satisfies $S_{i-}G_i = I$ and $K_{1i} = \hat{U}_i G_i$. Using $P_i = S_{i-}Q_i$, seeking stability is equivalent to finding a matrix Q_i such that $S_{i-}Q_i - S_{i+}Q_i (S_{i-}Q_i)^{-1} (S_{i+}Q_i)^\top \succ 0$ with the choice $K_{1i} = \hat{U}_i Q_i P_i^{-1} = \hat{U}_i Q_i (S_{i-}Q_i)^{-1}$. Then, $A_i + B_i K_{1i}$ is Schur stable. ■

Building upon above results, we present our distributed data-driven output containment solution for unknown heterogeneous MAS (1)-(2) in Algorithm 1, together with stability guarantees.

Theorem 2: Consider the heterogeneous MAS described by (1) *and* (2)*. Suppose that Assumptions 1-3 hold. Denoting* Π_i *and* Γ_i *are the solutions to the regulator equation* (3)*. The output containment control problem is addressed under the distributed data-driven feedback protocol* (24) *and* (6) *for any initial state and all* $i \in \{1, 2, \cdots, n\}$ *such that:*

- *The error of state of the resultant closed-loop system is asymptotically stable.*
- *The outputs of each follower move into the dynamic convex hull spanned by the outputs of leaders, that is*

$$\lim_{t\to\infty} \operatorname{dist}\Big(y_i(t), \operatorname{Co}\big(Y_{\mathscr{K}}(t)\big)\Big) = 0.$$

Proof: Letting $\tilde{y}(t) = \bar{y}_{0k}(t) - y(t)$, $\bar{C} = \operatorname{diag}\{C_1, C_2, \cdots, C_n\}$, $\bar{\Pi} = \operatorname{diag}\{\Pi_1, \Pi_2, \cdots, \Pi_n\}$, then we have

$$\begin{aligned} \tilde{y}(t) &= \bar{C}x(t) - C_0 \bar{x}_{0k}(t) \\ &= -\Big[\bar{C}\left(x(t) - \bar{\Pi}\hat{x}(t)\right) - C_0\bar{x}_{0k}(t) + \bar{C}\bar{\Pi}\hat{x}(t)\Big] \\ &= -\bar{C}s(t) - \bar{C}\bar{\Pi}\big(\hat{x}(t) - \bar{x}_{0k}(t)\big), \end{aligned} \tag{32}$$

where $s(t) = \left[s_1^\top(t), s_2^\top(t), \cdots, s_n^\top(t)\right]^\top$. According to $\Phi_k = \frac{1}{m}\mathscr{L} + \mathscr{G}_k$, we have

$$\begin{aligned} &\hat{x}(t) - \bar{x}_{0k}(t) \\ =&\hat{x}(t) - \left(\sum_{r=n+1}^{n+m} (\Phi_r \otimes I_q)\right)^{-1} \sum_{k=n+1}^{n+m} (\Phi_k \otimes I_q)\, \bar{x}_{0k}(t) \\ =&\hat{x}(t) - \left(\sum_{r=n+1}^{n+m} (\Phi_r \otimes I_q)\right)^{-1} \sum_{k=n+1}^{n+m} (\mathscr{G}_k \otimes I_q)\, \bar{x}_{0k}(t) \\ =&z(t). \end{aligned} \tag{33}$$

By means of (32) and (33), the output containment error $e(t)$ in (5) can be expressed as:

$$e(t) = -\sum_{k=n+1}^{n+m} (\Phi_k \otimes I_q) \left[\bar{C}s(t) + \bar{C}\bar{\Pi}z(t)\right]. \tag{34}$$

Theorem 1 implies that $\lim_{t\to\infty} \|z(t)\| = 0$ when the coupling weight α satisfies the condition (20). Furthermore, according to Lemma 4, $\lim_{t\to\infty} \|s(t)\| = 0$ when the controller gain matrix $K_{1i} = \hat{U}_i Q_i P_i^{-1}$, and Q_i satisfies the condition (27). Recall that $\sum_{k=n+1}^{n+m} \Phi_k$ is a positive-definite in Lemma 1. Therefore, $\lim_{t\to\infty} e(t) = 0$ is guaranteed by (34). This ensures the outputs of all followers to move into the convex hull formed by leaders under the proposed distributed data-driven feedback protocol (24) and (6) for any initial state and all $i \in \{1, 2, \cdots, n\}$. ■

Algorithm 1 Data-Driven Algorithm for Solving Output Containment Control Problem

1: Given initial conditions $x_i(0)$, $x_{0k}(0)$.
2: Select a coupling weight α according to (20) such that A_s is Schur stable.
3: Collect data matrix U_i.
4: Find a right inverse $X_{i-}^\dagger$ of X_{i-} such that the matrix $X_{i+}X_{i-}^\dagger$ is stable.
5: Find a solution W_i that satisfies the linear equation (26).
6: Define $\Pi_i = X_{i-}W_i$, $\Gamma_i = U_i W_i$, $K_{1i} = \hat{U}_i Q_i P_i^{-1}$, and $K_{2i} = \Gamma_i - K_{1i}\Pi_i$.
7: Design the control input $u_i(t) = K_{1i}s_i(t) + K_{2i}\hat{x}_i(t)$ based on the observer (6).

Remark 2: A salient feature of the proposed hierarchical scheme lies in the ability to partially decouple the closed-loop dynamics of heterogeneous agents through the observer layer, thus obviating the need for a unified model-based solution to the output regulation equation (3)*. A data-driven solution is developed to design control gains of the agents that are heterogeneous. This not only enhances design flexibility but also is useful for improving network robustness.*

IV. Numerical Example

Consider a heterogeneous MAS with six followers indexed as 1-6 and four leaders indexed as 7-10. The undirected communication network topology is illustrated in Fig 1. The system matrices are selected as:

$$A_0 = \begin{bmatrix} 1 & -3 \\ 1 & -1 \end{bmatrix}, \quad C_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$(A_i, B_i, C_i)$$

$$= \begin{cases} \left(\begin{bmatrix} 0.3 & -2 \\ 0.1 & -0.2 \end{bmatrix}, \begin{bmatrix} 1.8 & -0.8 \\ 0.9 & 1.6 \end{bmatrix}, \begin{bmatrix} -0.1 & 1.2 \\ 0.4 & 1.4 \end{bmatrix} \right), i = 1, 2, 3, \\ \left(\begin{bmatrix} 4 & -1 \\ 3 & -5 \end{bmatrix}, \begin{bmatrix} 2 & -6 \\ 1 & 7 \end{bmatrix}, \begin{bmatrix} -5 & 3 \\ 2 & 4 \end{bmatrix} \right), i = 4, 5, 6. \end{cases}$$

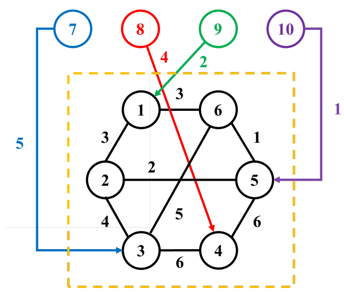


Fig. 1. Communication topology of MAS.

The control design procedure is implemented using MATLAB. We generate the data with random initial conditions and by applying to each input channel a random input sequence of length $L = 10$, which satisfies the persistence excitation condition according to [17]. From (3), we obtain the following solutions of the output regular equation:

$$\Pi_i = \begin{bmatrix} -2.25 & 1.93 \\ 0.64 & 0.16 \end{bmatrix}, \; \Gamma_i = \begin{bmatrix} 0.98 & 1.53 \\ 0.16 & -2.27 \end{bmatrix}, \; i = 1, 2, 3,$$

$$\Pi_i = \begin{bmatrix} -0.15 & 0.11 \\ 0.07 & 0.19 \end{bmatrix}, \; \Gamma_i = \begin{bmatrix} 0.56 & 0.08 \\ 0.07 & 0.01 \end{bmatrix}, \; i = 4, 5, 6.$$

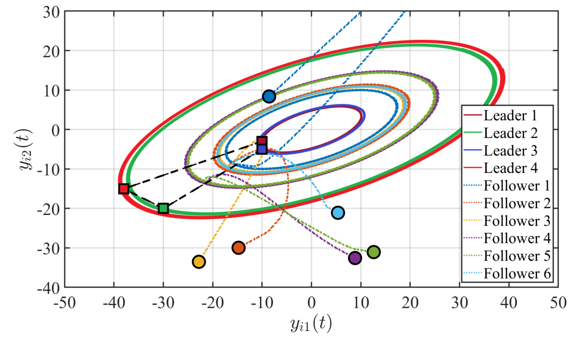
Then, by solving the linear inequalities (12) in Theorem 2, we obtain the stabilizing controller gains $\{K_{1i}\}$ by the toolbox CVX in MATLAB:

$$K_{11} = \begin{bmatrix} 79.99 & -156.38 \\ 214.27 & -184.93 \end{bmatrix}, K_{12} = \begin{bmatrix} -137.20 & 345.71 \\ 116.13 & -460.78 \end{bmatrix},$$

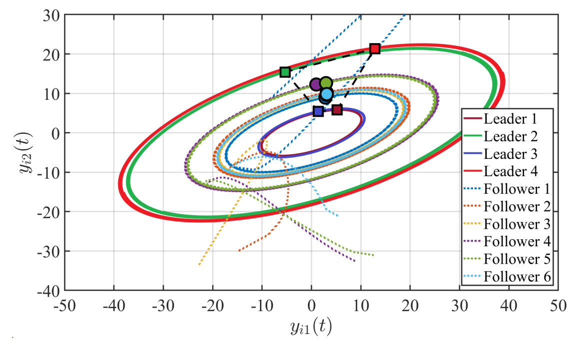
$$K_{13} = \begin{bmatrix} -376.25 & 230.83 \\ 78.21 & -76.75 \end{bmatrix}, K_{14} = \begin{bmatrix} -54.54 & -13.70 \\ 57.12 & -61.52 \end{bmatrix},$$

$$K_{15} = \begin{bmatrix} -36.63 & 0.23 \\ 57.62 & -60.03 \end{bmatrix}, K_{16} = \begin{bmatrix} -18.92 & -44.34 \\ 81.13 & -70.53 \end{bmatrix}.$$

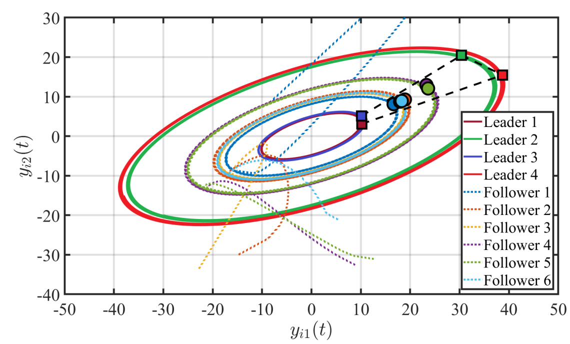
The outputs containment trajectories of all agents are given in Fig. 2. The evolution of the output containment errors and the observer errors are shown in Fig. 3 and Fig. 4, respectively. At time instant 0s, all agent states are initialized randomly. At time instant 0.2s, the system has achieved containment control. After time instant 0.2s, the followers returned to the convex hull under the proposed method (see time instant 3s and 20s). The simulation results test and verify the feasibility of our approaches, where the followers move into the convex hull formed by leaders as illustrated in the above figures.



(a) $t = 0$s



(b) $t = 3$s



(c) $t = 20$s

Fig. 2. The outputs containment trajectories of all agents. (a) $t = 0$s, (b) $t = 3$s, (c) $t = 20$s.

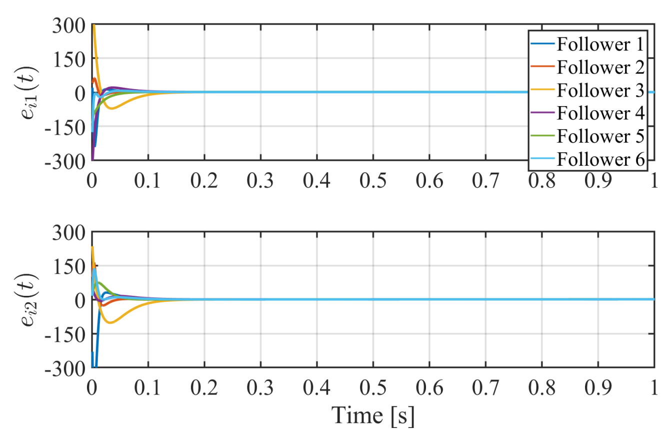


Fig. 3. Evolution of the output containment errors e_{i1} and e_{i2}, $i = 1, 2, \cdots, 6$.

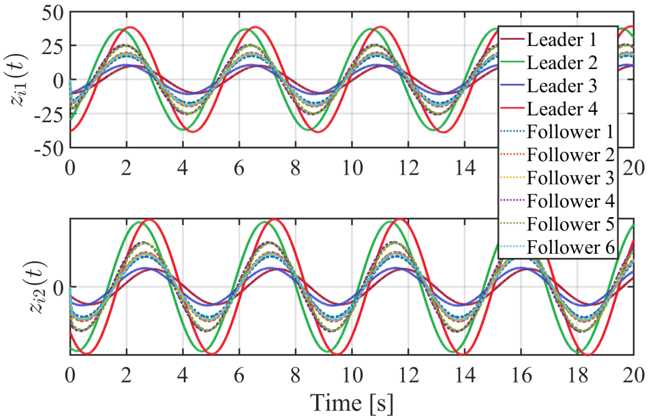


Fig. 4. Evolution of the observer containment errors z_{i1} and z_{i2}, $i = 1, 2, \cdots, 6$.

V. Conclusion

A data-driven solution to the output containment control problem of MAS with heterogeneous and unknown dynamics was derived. Our data-driven scheme was hierarchical, which includes a network layer and a physical layer, and bypasses the traditional modeling exercise and eliminates the need for explicit state-space models. To achieve the output containment control objective, in the network layer, we first designed control gains for an auxiliary system using the input and state data and then established an observer to generate a containment trajectory. In the physical layer, we developed a distributed feedback control protocol based on data-driven principles. The resulting closed-loop system is proved to be asymptotically stable, and the regulated output containment converges to zero. A numerical example demonstrated the significantly improved performance of the hierarchical approach.

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