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# Improved Proportional-integral Observer-based Fault-tolerant Control for MASs Against Unbounded FDIA

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*Abstract*— **An improved proportional-integral observer (PIO) based fault-tolerant control problem is addressed for multi-agent systems (MASs) with disturbance under unbounded false data injection attack (FDIA) over an undirected graph. The FDIAs are modeled as a class of unbounded attack signals. Notably, an augmented descriptor system is formulated by letting the FDIA be an auxiliary state vector. Then, an improved PIO is constructed to achieve the estimation of process faults and FDIA simultaneously. A compensation term is incorporated into the PIO to attenuate the effects of external disturbances and thus improving the accuracy of the PIO. Besides, an improved PIO-based fault-tolerant secure control scheme against unbounded FDIA is developed to achieve consensus even in the presence of process faults. Finally, the effectiveness and advantages of the proposed control strategy is verified through a simulation result.**

## I. INTRODUCTION

Nowadays, multi-agent systems (MASs) have attracted keen interests in the control community and achieved practical applications in smart-grid [1], smart cities [2], unmanned aerial vehicles [3], etc. Nevertheless, in real applications, because of the use of open network, MAS is vulnerable to cyberattacks, including false-data-injection attacks (FDIAs), denial-of-service (DoS) attacks, and replay attacks, [4]–[6]. Different from DoS and replay attacks, FDIAs pose greater harm by tampering with transmitted information, damaging the integrity of data [4]. MASs may be subject to FDIAs in an open network environment. Hence, secure consensus issues for MASs subject to FDIAs is a very important research topic. Consensus and fault detection problems under stochastic FDIAs have already been studied in [4]. Control laws with FDIA compensation have been designed to achieve the uniformly-ultimately-bounded secure consensus in unsafe network environments [7], [8]. It should be noted that FDIA dynamics introduced in the above literatures requires that the sensor attack signals or their derivatives be bounded, which is beneficial to constructing the sensor attack compensation. Nevertheless, in real-world applications, to maximize their damage for sensor networks, instead of launching sabotaged measurement data, tricky hackers can launch unbounded FDIA to the agents [9]. In addition, the measured system states is usually required while only the output information is available in practical applications [10]–[12]. In recent years, due to its excellent robustness, proportional-integral observer (PIO) has been applied to various practical systems [13]. But to the best knowledge of the authors, the design of PIO in unsafe network environment is still in its infancy. Hence, how to construct a PIO-based consensus control strategy when MASs are subjected to unbounded sensor FDIA is an objective of this paper.

On the other hand, because the interconnection among MAS agents, control performance deterioration and even instability might appear when faults happen. The fault-tolerant control problem of MAS has been widely investigated [14]–[16]. Nonetheless, fault-tolerant control performance will degrade more when process noises and disturbances exist. Therefore, how to achieve fault-tolerant control for MASs with process noises and disturbances is another objective of this paper.

Inspired by the aforementioned analysis, the improved PIO-based fault-tolerant control problem for MASs under unbounded FDIA is studied in this paper. The main contributions of this paper are exhibited as following:

1) *An improved PIO*: In contrast to [21], an improved PIO is constructed for attenuating the influence of process noise or external disturbance. Unlike the scheme based on disturbance observer in [17], the drawback caused by the unmeasurable states is removed and the accuracy of the state estimate is improved.
2) *An improved PIO-based fault-tolerant control strategy*: In order to defend against unbounded sensor FDIAs and achieve fault-tolerant control simultaneously, an improved PIO is designed to estimate system states, process faults, and sensor FDIA signals simultaneously. Different from bounded sensor FDIAs in [7], [8], unbounded sensor FDIAs are considered in this paper.

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Based on the improved PIO, the fault-tolerant consensus is achieved with uniform-ultimate-boundedness in finite time, even in the presence of process faults, unbounded sensor FDIAs, and external disturbances.

*Notation*: Table I sums up the notations in this paper.

TABLE I
NOTATION SUMMARY

| | |
|---|---|
| $\mathbb{R}^{m\times n}$ | $m\times n$-order real matrix |
| $I_N$ | the $N$-order identity matrix |
| $\mathrm{diag}\{\cdot\}$ | a diagonal matrix |
| $\mathrm{col}\{\cdot\}$ | a column vector |
| $\otimes$ | the Kronecker product |
| $Q>0$ | A symmetric matrix $Q$ is positive definite |
| $\lambda_{\max}(Q)$ | the maximal eigenvalue of the matrix $Q>0$ |
| $Q^T$ | the transpose of matrix $Q$ |
| $1_N$ | the $N$-dimensional column vector with all elements being 1 |
| $Q^{\dagger}$ | the pseudo-inverse of matrix $Q$ |

## II. System Introduction and Preliminaries

### A. Graph Theory

The MAS communication is presumed to be an undirected communication topology $\mathcal{G}=(\mathcal{V},\mathcal{E})$ in which $\mathcal{V}=\{1,\cdots,N\}$ and $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ are the node and the edge sets, respectively. Each node corresponds to an agent. Let $\mathcal{N}_i=\{j\in\mathcal{V}|(j,i)\in\mathcal{E},j\neq i\}$ denote the set of neighbor nodes of the *i*th node, and the adjacency matrix is represented as $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$. In an undirected communication topology graph $\mathcal{G}$, if node *i* and node *j* are mutually neighbors (namely $(j,i)\in\mathcal{E}$), nodes *i* and *j* can exchange information with each other (i.e. $a_{ij}=a_{ji}=1$); otherwise, $a_{ij}=0$. Furthermore, define the degree matrix $\mathcal{D}=\mathrm{diag}\{d_1,\ldots,d_N\}\in\mathbb{R}^{N\times N}$ with $d_i=\sum_{j\in\mathcal{N}_i}a_{ij}$; and the Laplacian matrix $\mathcal{L}$ is denoted by $\mathcal{L}=\mathcal{D}-\mathcal{A}$.

### B. System Model

Consider a class of MASs consisting of agents 1-*N* and the dynamics for the agent *i* is modeled by

$$\begin{cases}\dot{x}_i(t)=Ax_i(t)+Bu_i(t)+Df_i(t)+Ed_i(t),\\ y_i(t)=Cx_i(t)+F\theta_i(t),\ i=1,\cdots,N,\end{cases}\tag{1}$$

where $x_i(t)\in\mathbb{R}^n$, $u_i(t)\in\mathbb{R}^b$, $f_i(t)\in\mathbb{R}^p$, $d_i(t)\in\mathbb{R}^r$, $y_i(t)\in\mathbb{R}^q$, and $\theta_i(t)\in\mathbb{R}^s$ $(q\geq s)$ denote the system state, controller input, process fault, process noises/disturbances, system output, and sensor FDIA, respectively. It is worth mentioning that the FDIA $\theta_i(t)$ can be unbounded. Furthermore, $A$, $B$, $C$, $D$, $E$, and $F$ are constant matrices with compatible dimensions.

*Assumption 1:* $(A,B)$ is controllable and $(A,C)$ is observable, and condition $rank\begin{bmatrix}I_n & 0\\ C & F\end{bmatrix}=n+s$ holds.

*Assumption 2 ( [15]):* The process fault signal $f_i(t)$ is a class of abrupt fault and satisfies $\dot{f}_i(t)=0$.

*Assumption 3:* The noise/disturbance $d_i(t)$ is norm bounded, i.e., $\|d_i(t)\|\leq\bar{d}_i$, and $\bar{d}_i\geq0$ is known.

*Lemma 1 ( [19]):* For matrices $\mathcal{R}_1$ and $\mathcal{R}_2$ of appropriate dimensions, under Assumption 1, the equation $[\mathcal{R}_1\quad\mathcal{R}_2]\times\begin{bmatrix}I_n & 0\\ C & F\end{bmatrix}=I_{n+s}$ can be solved and the solution is given by

$$\mathcal{R}_1=\begin{bmatrix}I_n & 0\\ C & F\end{bmatrix}^{\dagger}\begin{bmatrix}I_n\\ 0_{p\times n}\end{bmatrix},\mathcal{R}_2=\begin{bmatrix}I_n & 0\\ C & F\end{bmatrix}^{\dagger}\begin{bmatrix}0_{n\times p}\\ I_p\end{bmatrix}.$$

*Lemma 2 ( [20]):* For a continuous function $V(t)>0$ with $V(0)$ being of boundedness, if it satisfies

$$\dot{V}(t)\leq-\gamma_1V(t)+\gamma_2,$$

where $\gamma_1$ and $\gamma_2$ are positive constants, then, $V(t)$ is of boundedness in finite time.

## III. Main Results

### A. Improved PIO-based fault-tolerant control strategy

To estimate system state $x_i(t)$ and sensor attack $\theta_i(t)$ in unison for MAS (1), an augmented descriptor system is formulated as

$$\begin{cases}S\dot{\breve{x}}_i(t)=\breve{A}\breve{x}_i(t)+Bu_i(t)+Df_i(t)+Ed_i(t),\\ y_i(t)=\breve{C}\breve{x}_i(t),\ i=1,\cdots,N,\end{cases}\tag{2}$$

where $\breve{x}_i(t)=[x_i^T(t)\quad\theta_i{}^T(t)]^T\in\mathbb{R}^{n+s}$, $S=[I_n\quad 0_{n\times s}]$, $\breve{A}=[A\quad 0_{n\times s}]$, and $\breve{C}=[C\quad F]$.

Based on Assumption 1 and Lemma 1, MAS (1) can be transformed into the following form

$$\begin{cases}\dot{\breve{x}}_i(t)=\mathcal{R}_1\breve{A}\breve{x}_i(t)+\mathcal{R}_1Bu_i(t)+\mathcal{R}_1Df_i(t)\\ \qquad\quad+\mathcal{R}_1Ed_i(t)+\mathcal{R}_2\dot{y}_i(t),\\ y_i(t)=\breve{C}\breve{x}_i(t),\ i=1,\cdots,N,\end{cases}\tag{3}$$

noting that $\mathcal{R}_1S+\mathcal{R}_2\breve{C}=I_{n+s}$ from Lemma 1. Inspired by [21], an improved PIO for (3) can be constructed as

$$\begin{cases}\dot{\hbar}_i(t)=\mathcal{R}_1\breve{A}\hat{\breve{x}}_i(t)+\mathcal{R}_1Bu_i(t)+L(y_i(t)-\hat{y}_i(t))\\ \qquad\quad+\mathcal{R}_1D\hat{f}_i(t)+\mathcal{R}_1EJ_i(t),\\ \hat{\breve{x}}_i(t)=\hbar_i(t)+\mathcal{R}_2y_i(t),\\ \dot{\hat{f}}_i(t)=H(y_i(t)-\hat{y}_i(t)),\\ \hat{y}_i(t)=\breve{C}\hat{\breve{x}}_i(t),\ i=1,\cdots,N.\end{cases}\tag{4}$$

where $\hat{\breve{x}}_i(t)$, $\hat{f}_i(t)$, and $\hat{y}_i(t)$ represent the estimation of $\breve{x}_i(t)$, $f_i(t)$, and $y_i(t)$, respectively; $H$ and $L$ are the gain matrices to be designed; the specific form of the compensation term $J_i(t)$ is as follows

$$J_i(t)=\bar{d}_i\frac{M_1e_{y_i}(t)}{\|M_1e_{y_i}(t)\|+r_1},\tag{5}$$

where $M_1$ is a matrix that enables the improved estimate of the PIO observer under process fault and sensor unbounded FDIA and the output estimation error is defined as $e_{y_i}(t) = y_i(t) - \hat{y}_i(t)$.

*Remark 1:* Different from bounded sensor FDIAs in [7], [8], unbounded sensor FDIAs are considered in this article. Notably, the construction in (4) avoids the appearance of $\dot{y}_i(t)$ which is difficult to obtain in practice.

*Remark 2:* Unlike the PIO observer in [21] where noise/disturbance is not considered, the compensation term $J_i(t)$ introduced into the above PIO compensates the adverse effects of the noise/disturbance, hence increases the accuracy of the PIO observer (4). Besides, different from the scheme based on disturbance observer in [17], output (rather than state) information is used for feedback control.

For convenience of subsequent description, define

$$\xi_i(t) = \sum_{j\in\mathcal{N}_i}(x_i(t) - x_j(t)),$$

$$\bar{x}(t) = \frac{1}{N}\sum_{i=1}^{N} x_i(t),$$

and consensus error

$$\delta_i(t) = x_i(t) - \bar{x}(t).$$

It is not difficult to see that

$$\delta(t) \triangleq \mathrm{col}\{\delta_1(t), \cdots, \delta_N(t)\} = (U \otimes I_n)x(t)$$

where

$$U{=}I_N - \frac{1}{N}1_N 1_N^T.$$

In addition, we have $\mathcal{L}U = U\mathcal{L} = \mathcal{L}$. Then, we have that

$$\xi(t){=}(\mathcal{L} \otimes I_n)\delta(t)$$

where

$$\xi(t) \triangleq \mathrm{col}\{\xi_1(t), \cdots, \xi_N(t)\}.$$

Next, based on improved PIO (4), the control law for the agent $i$ is designed as

$$u_i(t) = \mu K_x\hat{\xi}_i(t) - K_a\hat{f}_i(t), \tag{6}$$

where $\mu$ is a positive constant, $K_x$ and $K_a$ are the control input gains, and $\hat{\xi}_i(t) \triangleq \sum_{j\in\mathcal{N}_i}(\hat{x}_i(t) - \hat{x}_j(t))$.

### *B. Consensus Analysis*

First of all, estimation errors of system state and sensor attack signals are defined by

$$\breve{x}_{e,i}(t) = \breve{x}_i(t) - \hat{\breve{x}}_i(t), \tag{7}$$

and

$$f_{e,i}(t) = f_i(t) - \hat{f}_i(t). \tag{8}$$

Note that

$$\begin{aligned}&\mathcal{R}_1 S\breve{x}_i(t) - \hbar_i(t)\\ =&\mathcal{R}_1 S\breve{x}_i(t) + \mathcal{R}_2 y_i(t) - \hat{\breve{x}}_i(t)\\ =&\breve{x}_i(t) - \hat{\breve{x}}_i(t) = \breve{x}_{e,i}(t).\end{aligned} \tag{9}$$

Using (3) and (4), the error dynamic of $\breve{x}_{e,i}(t)$ is given by

$$\begin{aligned}\dot{\breve{x}}_{e,i}(t) =&\dot{\breve{x}}_i(t) - \dot{\hat{\breve{x}}}_i(t)\\ =&(\mathcal{R}_1\breve{A} - L\breve{C})\breve{x}_{e,i}(t) + \mathcal{R}_1 Df_{e,i}(t) - \mathcal{R}_1 J_i(t).\end{aligned} \tag{10}$$

Then, combining Assumption 2 and (4), the error dynamic of $f_{e,i}(t)$ is given by

$$\begin{aligned}\dot{f}_{e,i}(t) =& - H(y_i(t) - \hat{y}_i(t))\\ =& - H\breve{C}\breve{x}_{e,i}(t).\end{aligned} \tag{11}$$

Combining (1) and (6), the dynamics of the state can be described as follows

$$\begin{aligned}\dot{x}_i(t) =&Ax_i(t) + \mu BK_x\hat{\xi}_i(t) + Df_i(t) - D\hat{f}_i(t)\\ =&Ax_i(t) + \mu BK_x\hat{\xi}_i(t) + Df_{e,i}(t).\end{aligned} \tag{12}$$

Hence, the consensus error $\delta(t)$ is given by

$$\begin{aligned}\dot{\delta}(t) =&[I_N \otimes A]\delta(t) + [U \otimes \mu BK_x]\hat{\xi}(t) + [U \otimes D]f_e(t)\\ &+ [U \otimes E]d(t),\end{aligned} \tag{13}$$

where $\hat{\xi}(t) = \mathrm{col}\{\hat{\xi}_1(t), \cdots, \hat{\xi}_N(t)\}$, $d(t) = \mathrm{col}\{d_1(t), \cdots, d_N(t)\}$, and $f_e(t) = \mathrm{col}\{f_{e,1}(t), \cdots, f_{e,N}(t)\}$. Next, the main results are given as follows.

*Theorem 1:* Under Assumptions 1-3, the proposed controller (6) with the observer (4) can ensure that the consensus error $\delta(t)$, the estimation errors $\breve{x}_{e,i}(t)$ and $f_{e,i}(t)$ of the MAS (1) with process faults, unbounded sensor FDIA, and process disturbances are of uniform-ultimately-boundedness in finite time, if there exist symmetric positive definite matrices $Q_1$, $Q_2$, $Q_3$ and matrices $K_x$, $K_a$, $L$, $H$, and $M_1$ satisfying

$$\begin{bmatrix}\pi_{11} & 0 & \pi_{13}\\ * & \pi_{22} & \pi_{23}\\ * & * & \pi_{33}\end{bmatrix} \leq 0, \tag{14}$$

$$D = BK_a, \tag{15}$$

$$Q_2\mathcal{R}_1 E = \breve{C}^T M_1^T, \tag{16}$$

where

$$\begin{aligned}\pi_{11} =&I_N \otimes [Q_1A + A^TQ_1] + I_N \otimes (\mu^2 Q_1Q_1)\\ &+ [U \otimes (Q_1Q_1^T)] + \mathcal{L}^2 \otimes (2K_x^TB^TBK_x),\\ \pi_{13} =&I_N \otimes (Q_1D),\\ \pi_{22} =&I_N \otimes (Q_2Y + Y^TQ_2) + \mathcal{L}^2 \otimes 2\begin{bmatrix}K_x^TB^TBK_x & 0\\ 0 & 0\end{bmatrix},\\ \pi_{23} =&I_N \otimes (Q_2\mathcal{R}_1 D - Q_3H\breve{C}).\end{aligned}$$

*Proof:* Let

$$\begin{aligned}Y =&\mathcal{R}_1\breve{A} - L\breve{C},\\ \bar{d} =&\mathrm{col}\{\bar{d}_1, \cdots, \bar{d}_N\},\\ \tilde{Q} =&\mathrm{diag}\{Q_1, Q_2, Q_3\},\\ \aleph_i(t) =&\mathrm{col}\{\delta_i(t)\ \ \breve{x}_{e,i}(t)\ \ f_{e,i}(t)\},\end{aligned}$$

$$\aleph(t) = \text{col}\{\aleph_1(t), \cdots, \aleph_N(t)\},$$
$$\breve{x}_e(t) = \text{col}\{\breve{x}_{e,1}(t), \cdots, \breve{x}_{e,N}(t)\},$$

and define the Lyapunov function $V(t)$ as

$$V(t) = \sum_{i=1}^{N} \aleph_i^T(t)\tilde{Q}\aleph_i(t) = \aleph^T(t)[I_N \otimes \tilde{Q}]\aleph(t). \tag{17}$$

From (10), (11), and (13), it can be derived that

$$\begin{aligned}\dot{V}(t) = &\sum_{i=1}^{N} \dot{\delta}_i^T(t)Q_1\delta_i(t) + \sum_{i=1}^{N} \dot{\breve{x}}_{e,i}^T(t)Q_2\breve{x}_{e,i}(t)\\
&+ \sum_{i=1}^{N} \delta_i^T(t)Q_1\dot{\delta}_i(t) + \sum_{i=1}^{N} \breve{x}_{e,i}^T(t)Q_2\dot{\breve{x}}_{e,i}(t)\\
&+ \sum_{i=1}^{N} \dot{f}_{e,i}^T(t)Q_3 f_{e,i}(t) + \sum_{i=1}^{N} \dot{f}_{e,i}^T Q_3 f_{e,i}(t)\\
=&\delta^T[I_N \otimes (Q_1A + A^TQ_1)]\delta(t)\\
&+ 2\delta^T(t)[U \otimes \mu Q_1BK_x]\hat{\xi}(t)\\
&+ 2\delta^T(t)[U \otimes Q_1D]f_e(t) + 2\delta^T(t)[U \otimes E]d(t)\\
&+ \breve{x}_e^T(t)[I_N \otimes (Q_2Y + Y^TQ_2)]\breve{x}_e(t)\\
&+ 2\breve{x}_e^T(t)[I_N \otimes Q_2\mathcal{R}_1E]\left[d(t) - \bar{d}\frac{V_1e_y(t)}{\|V_1e_y(t)\| + r_1}\right]\\
&+ 2\breve{x}_e^T(t)[I_N \otimes Q_2\mathcal{R}_1D]f_e(t)\\
&- 2f_e^T(t)[I_N \otimes \breve{C}^TH^TQ_3]\breve{x}_e(t).\end{aligned} \tag{18}$$

From Young's inequality [4], it can be derived that

$$\begin{aligned}&2\delta^T(t)[U \otimes \mu Q_1BK_x]\hat{\xi}(t)\\
\leq&\delta^T(t)[U \otimes (\mu^2Q_1Q_1^T) + \mathcal{L}^2 \otimes (2K_x^TB^TBK_x)]\delta(t)\\
&+ \breve{x}_e^T(t)\left[\mathcal{L}^2 \otimes 2\begin{bmatrix}K_x^TB^TBK_x & 0\\ 0 & 0\end{bmatrix}\right]\breve{x}_e(t),\end{aligned} \tag{19}$$

and

$$\begin{aligned}&2\delta^T(t) \times [U \otimes Q_1E]d(t)\\
\leq&\delta^T(t)[U \otimes (Q_1Q_1^T)]\delta(t) + d^T(t)[U \otimes (EE^T)]d(t)\\
\leq&\delta^T(t)[U \otimes (Q_1Q_1^T)]\delta(t) + \bar{d}^T[U \otimes (EE^T)]\bar{d}(t).\end{aligned} \tag{20}$$

Further,

$$\begin{aligned}&2\breve{x}_e^T(t)[I_N \otimes Q_2\mathcal{R}_1E]\left[d(t) - \bar{d}\frac{M_1e_y(t)}{\|M_1e_y(t)\| + r_1}\right]\\
=&-2\breve{x}_e^T(t)\left[I_N \otimes \frac{\bar{d}Q_2\mathcal{R}_1EM_1\breve{C}}{\|M_1e_y(t)\| + r_1}\right]\breve{x}_e(t)\\
&+ 2\breve{x}_e^T(t)[I_N \otimes Q_2\mathcal{R}_1E]d(t) \leq 0,\end{aligned} \tag{21}$$

where (16) has been used. Then, combining (19)−(21) into (18), $\dot{V}(t)$ can be collated as

$$\begin{aligned}\dot{V}(t) \leq &\begin{bmatrix}\delta(t)\\ \breve{x}_e(t)\\ f_e(t)\end{bmatrix}^T \begin{bmatrix}\pi_{11} & 0 & \pi_{13}\\ * & \pi_{22} & \pi_{23}\\ * & * & 0\end{bmatrix} \begin{bmatrix}\delta(t)\\ \breve{x}_e(t)\\ f_e(t)\end{bmatrix}\\
&+ \bar{d}^T[U \otimes (EE^T)]\bar{d}(t).\end{aligned} \tag{22}$$

Then, condition (14) and Lemma 2 ensure the uniform-ultimate-boundedness of $\aleph(t)$. The proof is complete. ■

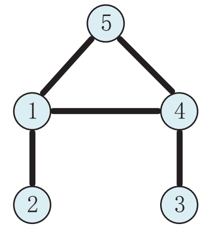



Fig. 1. Communication topology.

### *C. Improved PIO and Controller Synthesis*

Because it is difficult to solve the nonlinear matrix inequality constraint (14), linear matrices equation constraints (15) and (16) by applying the LMI toolbox, Theorem 2 is proposed to convert them into LMIs.

*Theorem 2:* Under Assumptions 1-3, for given positive parameters $\mu$ and the small constant $\ell$, the matrix parameters $K_x$, $K_a$, $H$ and $L$ can be obtained, if there exist symmetric positive definite matrices $Q_1$, $Q_2$, $Q_3$, $Q_4$, and $M_1$ satisfying the LMI conditions:

$$\begin{bmatrix}\ell I & D - BK_a\\ * & \ell I\end{bmatrix} < 0, \tag{23}$$

$$\begin{bmatrix}\ell I & Q_2\mathcal{R}_1E - C^TM_1^T\\ * & \ell I\end{bmatrix} < 0, \tag{24}$$

$$\begin{bmatrix}\Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} & 0\\ * & -I & 0 & 0 & 0 & 0\\ * & * & -I & 0 & 0 & 0\\ * & * & * & \Pi_{44} & \Pi_{45} & \Pi_{46}\\ * & * & * & * & -I & 0\\ * & * & * & * & * & 0\end{bmatrix} < 0, \tag{25}$$

where

$$\begin{aligned}\Pi_{11} =&Q_1A + A^TQ_1,\\
\Pi_{12} =&(\lambda_{max}(U) + \mu^2)Q_1,\\
\Pi_{13} =&\sqrt{2}\lambda_{max}(\mathcal{L})K_x^TB^T,\\
\Pi_{14} =&Q_1D,\\
\Pi_{44} =&Q_2\mathcal{R}_1\breve{A} + \breve{A}^T\mathcal{R}_1^TQ_2 - Q_4C - C^TQ_4,\\
\Pi_{45} =&\sqrt{2}\lambda_{max}(\mathcal{L})\begin{bmatrix}K_x^TB^T\\ 0\end{bmatrix},\\
\Pi_{46} =&Q_3H\breve{C} - Q_2\mathcal{R}_1D.\end{aligned}$$

*Proof:* Firstly, with the support of Schur complement lemma, (15) and (16) are transformed into (23) and (24). Secondly, let $Q_4 = Q_2L$. Then, we can derive (25) by applying the Schur complement lemma. This completes the proof. ■

## IV. Numerical Examples

Consider a MAS consisting of 5 agents with the topology graph shown in Fig. 1, and dynamics as in (1) with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.24 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -22.85 & 0 \end{bmatrix}, B = D = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ 0.2 \\ 0 \\ 0 \end{bmatrix}.$$

In addition, the parameters for (5), (6) and $\ell$ in Theorem 2 are $r_1 = 0.00001$, $\mu = 0.02$, and $\ell = 0.000001$, respectively. Then, from Fig. 1, it can be obtained that $\lambda_{max}(\mathcal{L}) = 4.3028$.

By solving the LMI conditions (23), (24) and (25), the gain matrices in improved PIO observer (4) and control law (6) are obtained as

$$K_a = 1,$$
$$K_x = \begin{bmatrix} 0.0005 & 0.0011 & -0.0005 & 0.0002 \end{bmatrix},$$

and

$$H = \begin{bmatrix} 1.4228 & -1.6360 & 0.8620 \end{bmatrix},$$
$$M_1 = 10^6 * \begin{bmatrix} -0.1354 & 0.4334 & -1.2932 \end{bmatrix},$$
$$L = \begin{bmatrix} 1.3533 & -1.1982 & 0.5110 \\ -20.8133 & 11.9494 & 41.7307 \\ 1.7102 & -2.1056 & 3.3671 \\ 20.2445 & -21.3587 & 2.9737 \\ 19.8011 & -10.2544 & -43.7662 \end{bmatrix},$$

respectively.

To test the performance of the proposed control scheme in this paper, the actuator faults and the unbounded sensor FDIAs are given by

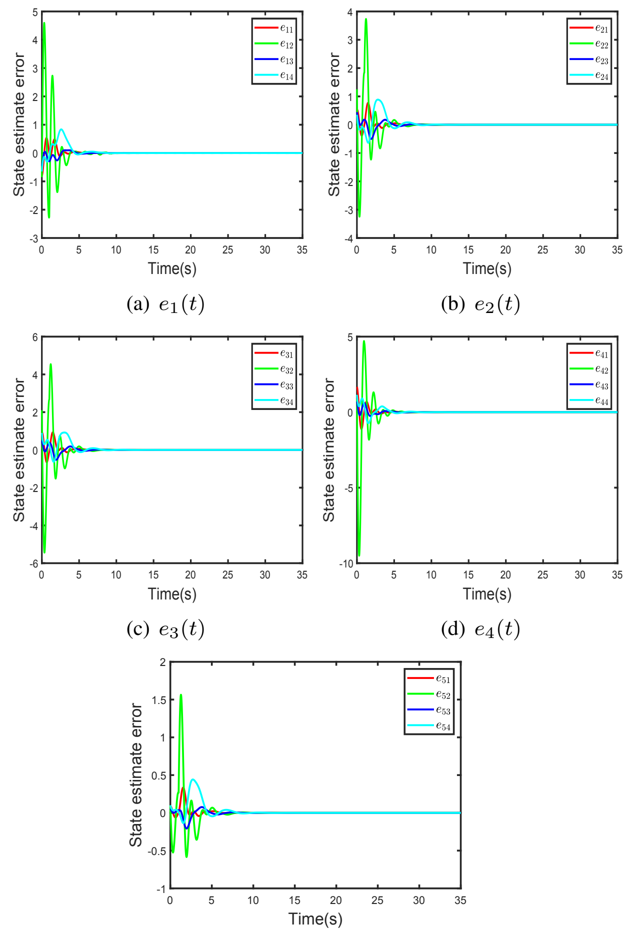
$$f_i(t) = \begin{cases} 0.7, & t > 1s, \\ 0, & 0 < t < 1, \end{cases}$$

and

$$\theta_i(t) = \begin{cases} 1.5e^{0.5t}, & t > 12s, \\ 0, & 0 < t < 12. \end{cases}$$

Consider that the disturbance $d_1(t) = 0.7\sin(t)$ and $d_2(t) = 0.5\cos(3t + 2)$ acting on agent 1 and agent 2, and there exist no disturbances on agents 3-5. Fig. 2 displays the state estimation errors of the five agents. Fig. 3 shows the process fault signals, sensor unbounded FDIAs, and their estimates, from which it is obvious that the improved PIO (4) can achieve accurate estimation even in the presence of disturbances. Compared to Fig. 3, the poorer estimation performance of the observer without $J_i(t)$ is shown in Fig. 4, which indicates that the accuracy of the PIO observer can be improved by applying the disturbance compensation term $J_i(t)$.

Define the consensus error as $W(t) = \frac{1}{N}\sqrt{\sum_{i=1}^{N} \|x_i(t) - \bar{x}(t)\|^2}$. To demonstrate the superiority and effectiveness of our proposed scheme, comparisons with the fault-tolerant strategy [15] are given in Fig. 5. From Fig. 5, we observe that the consensus error $W(t)$ for the control strategy in [15] cannot converge due to the existence of the unbounded FDIA. However, the scheme of this paper can achieve better consensus performance even in the presence of the FDIAs and external disturbances. Therefore, better consensus performance is obtained by using the scheme of this paper.

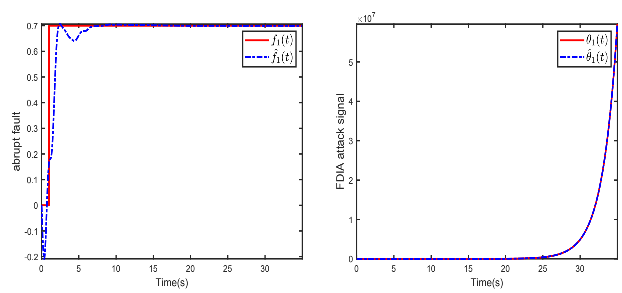



(a) $e_1(t)$ (b) $e_2(t)$

(c) $e_3(t)$ (d) $e_4(t)$

(e) $e_5(t)$

Fig. 2. State estimation errors of five agents.

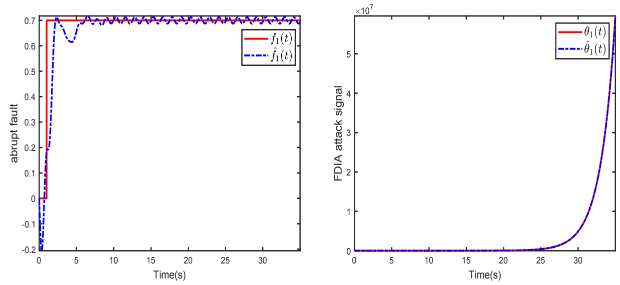



(a) $f_1(t)$ (b) $\theta_1(t)$

Fig. 3. The estimation performance of the improved PIO of Agent 1

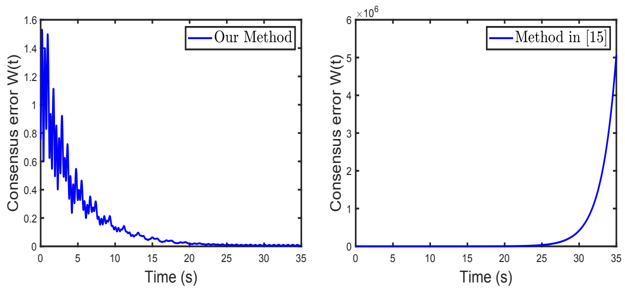
## V. Conclusion

In this article, the improved PIO based fault-tolerant control problem for MASs subject to unbounded sensor FDIAs is studied. The improved PIO is designed to estimate the system state and process actuator fault simultaneously




(a) $f_1(t)$ (b) $\theta_1(t)$

Fig. 4. The estimation performance of the improved PIO without $J_1(t)$ of Agent 1




(a) Our method (b) Method in [15]

Fig. 5. Comparison between our method and method in [15]

without being affected by the sensor unbounded FDIA. Moreover, a compensation term is introduced to compensate the adverse effects of the noise/disturbance, increasing the accuracy of the PIO observer. An improved PIO-based fault-tolerant control strategy is constructed to achieve secure consensus under disturbances and unbounded FDIAs. The controller and PIO gains are solved by MATLAB LMI toolbox. In the end, a numerical example shows the superiority and effectiveness of our proposed scheme. Noted that only undirected communication topology is considered currently. Generalization of the results to directed topology and time-varying topology is the focus of our future research. Moreover, fast consensus problems with finite-time or predefined-time will also be an important future research direction.

## REFERENCES


[1] S. R. Islam, D. Sutanto, and K. M. Muttaqi, "A distributed multi-agent based emergency control approach following catastrophic disturbances in interconnected power systems," *IEEE Trans. Power Syst.*, vol. 31, no. 4, pp. 2764-2775, Jul. 2016.

[2] A. Colombo, G. R. de Campos, and F. D. Rossa, "Control of a city road network: distributed exact verification of traffic safety," *IEEE Trans. Autom. Control*, vol. 62, no. 10, pp. 4933-4948, Oct. 2017.

[3] J. J. Cui, Y. W. Liu, and A. Nallanathan, "Multi-agent reinforcement learning-based resource allocation for UAV networks," *IEEE Trans. Wireless Commun.*, vol. 19, no. 2, pp. 729-743, Feb. 2020.

[4] X. -G. Guo, D. -Y. Zhang, J. -L. Wang, J. H. Park, and L. Guo, "Observer-based event-triggered composite anti-disturbance control for multi-agent systems under multiple disturbances and stochastic FDIAs," *IEEE Trans. Autom. Sci. Eng.*, vol. 20, no. 1, pp. 528-540, Jan. 2023.

[5] G. Franz, F. Tedesco, and D. Famularo, "Resilience against replay attacks: a distributed model predictive control scheme for networked multi-agent systems," *IEEE/CAA J. Autom. Sin.*, vol. 8, no. 3, pp. 628-640, Mar. 2021.

[6] D. Zhang, C. Deng, and G. Feng, "Resilient cooperative output regulation for nonlinear multi-agent systems under DoS attacks," *IEEE Trans. Autom. Control*, vol. 68, no. 4, pp. 2521-2528, Apr. 2023.

[7] X. Huang and J. X. Dong, "Reliable leader-to-follower formation control of multiagent systems under communication quantization and attacks," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 50, no. 1, pp. 89-99, Jan. 2020.

[8] M. Meng, G. X. Xiao, and B. B. Li, "Adaptive consensus for heterogeneous multi-agent systems under sensor and actuator attacks," *Automatica*, vol. 122, p. 109242, Dec. 2020.

[9] S. Zuo and D. Yue, "Resilient containment of multigroup systems against unknown unbounded FDI attacks," *IEEE Trans. Ind. Electron.*, vol. 69, no. 3, pp. 2864-2873, Mar. 2022.

[10] X. G. Guo, D. Y. Zhang, J. L. Wang, and C. K. Ahn, "Adaptive memory event-triggered observer-based control for nonlinear multi-agent systems under dos attacks," *IEEE/CAA J. Autom. Sinica*, vol. 8, no. 10, pp. 1644-1656, May 2021.

[11] Y. Yang, D. Yue, and C. X. Dou, "Output-based event-triggered schemes on leader-following consensus of a class of multi-agent systems with Lipschitz-type dynamics," *Inf. Sci.*, vol. 459, pp. 327-340, Feb. 2018.

[12] Y. Xu and Z. G. Wu, "Distributed adaptive event-triggered fault-tolerant synchronization for multiagent systems," *IEEE Trans. Ind. Electron.*, vol. 68, no. 2, pp. 1537-1547, Feb. 2021.

[13] D. Zhao, Z. Wang, Y. Chen, and G. Wei. "Proportional-integral observer design for multidelayed sensor-saturated recurrent neural networks: a dynamic event-triggered protocol," *IEEE Trans. Cybern.*, vol. 50, no. 11, pp. 4619-4632, Nov. 2020.

[14] X. Wang and G. -H. Yang. "Fault-tolerant consensus tracking control for linear multiagent systems under switching directed network," *IEEE Trans. Cybern.*, vol. 50, no. 5, pp. 1921-1930, May 2020.

[15] C. Liu, B. Jiang, K. Zhang, and R. J. Patton. "Distributed fault-tolerant consensus tracking control of multi-agent systems under fixed and switching topologies," *IEEE Trans. Circuits Syst. I Regul. Pap.*, vol. 68, no. 4, pp. 1646-1658, Apr. 2021.

[16] Y. Wu, J. Liu, Z. Wang, and Z. Ju. "Distributed resilient tracking of multiagent systems under actuator and sensor faults," *IEEE Trans. Cybern.*, vol. 53, no. 7, pp. 4653-4664, Jul. 2023.

[17] C. L. Liu, L. Shan, Y. Y. Chen, and Y. Zhang, Average-consensus filter of first-order multi-agent systems with disturbances, IEEE Trans. Circuits Syst., II, Exp. Briefs, vol. 65, no. 11, pp. 1763-1767, Nov. 2018.

[18] X. Zhang, L. Liu, and G. Feng, "Leader-follower consensus of time-varying nonlinear multi-agent systems," *Automatica*, vol. 52, pp. 8-14, Feb. 2015.

[19] F. P. Komachali, M. Shafiee, and M. Darouach. "Design of unknown input fractional order proportional-integral observer for fractional order singular systems with application to actuator fault diagnosis," *IET Control Theory Appl.*, vol. 13, no. 14, pp. 2163-2172, 2019.

[20] X. G. Guo, D. C. Tan, C. K. Ahn, J. L. Wang. "Fully distributed adaptive fault-tolerant sliding mode control for nonlinear leader-following multiagent systems with ANASs and IQCs," *IEEE Trans. Cybern.*, vol. 52, no. 5, pp. 2763-2774, May 2022.

[21] Q. Jia, W. Chen, Y. Zhang, and H. Li. "Fault reconstruction and fault-tolerant control via learning observers in TakagiCSugeno fuzzy descriptor systems with time delays," *IEEE Trans. Ind. Electron.*, vol. 62, no. 6, pp. 3885-3895, Jun. 2015.