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On $\mathcal{L}_p$-gains in Noisy Event-triggered Control

K. J. A. Scheres, R. Postoyan, W. P. M. H. Heemels

***Abstract*— This paper investigates the design of event-triggered control strategies that guarantee $\mathcal{L}_p$-stability properties when measurement noise is present. It is well known that in many event-based transmission schemes, the inclusion of measurement noise can lead to the so-called Zeno phenomenon, where an infinite number of transmissions occur in a finite amount of time, even when a minimum inter-event time is guaranteed in absence of noise. In this paper, we present a solution to the open problem of designing triggering rules, which ensure bounded $\mathcal{L}_p$-gains from the exogenous inputs to a desired performance output in the closed-loop system, in the presence of measurement noise. We guarantee a global minimum inter-event time by design. Additionally, we show that suitable choices of the tuning parameters allow us to affect the "steady-state" inter-event times (when close to the attractor) by exploiting the design freedom in the parameter selection, which may result in improved behavior when close to the attractor. We showcase our results through a consensus example.**

I. Introduction

In recent years, event-triggered control (ETC), see, e.g., [1] and references therein, has been suggested as an alternative transmission paradigm in (packet-based) networked control systems, to time-triggered control (TTC). Whereas in TTC, the transmission times are determined by a local clock, in ETC, the transmissions are generated using a threshold function which incorporates (local) state or output information of the system. The intuition behind this strategy is that transmissions only occur when "needed" to guarantee certain stability or performance criteria, contrary to TTC, where they occur based on a timer. In TTC, the performance of the system is strongly coupled to the inter-transmission times, which effectively means that the sampling times have to be chosen for the worst case scenario. However, when (network) resources are scarce, this may result in redundant transmissions when not operating in the worst-case scenario. In ETC, however, this is not the case in principle. As the transmissions occur only when necessary, the network utilization can be reduced while still meeting the desired stability and/or performance criteria.

A notoriously hard problem in ETC is dealing with measurement noise. As was shown in [2], when a popular *relative* trigger is used, it is impossible to guarantee Zeno freeness in presence of additive measurement noise. Some works in the literature can ensure Zeno freeness by design, see, e.g., [3], [4], however, they require that the noise is differentiable and that its derivative is bounded. A notable exception is [5], wherein *space-regularization* is used to ensure the absence of Zeno behavior, while requiring only boundedness of the measurement noise (and no other conditions on, e.g., the existence or boundedness of its derivative). In [5], a practical input-to-state stability property is ensured, which means that, even for vanishing noises, the solutions to the system are not guaranteed to converge to the attractor, but rather to remain in a bounded neighborhood of it. A notable exception is when *time-regularization* is used. In that case, no space-regularization is needed to ensure Zeno freeness, thereby retaining the asymptotic behavior in presence of vanishing noises, see [5, Remark 5], although, as also illustrated in [5], in some cases, it may still be beneficial to also use space-regularization.

Due to the fact that the stability property is practical in [5], i.e., because of the application of *space-regularization*, it is not possible to extend the results in a straightforward manner to the case where other stability properties are required, such as, e.g., $\mathcal{L}_p$-stability, which is often essential in, e.g., platooning.

In this paper, we showcase some of the difficulties that arise when dealing with measurement noise in an $\mathcal{L}_p$-stability setting. Due to the zero-order-hold present as a sampling mechanism, additional conditions are required to ensure that, when the measurement signal itself is bounded in an $\mathcal{L}_p$ sense, the *sampled* version of the noise signal is also bounded in an $\mathcal{L}_p$ sense. Under these conditions, we design triggers that ensure bounded $\mathcal{L}_p$-gains of the closed-loop system. We also investigate (using simulations) what the effect of certain choices are on the qualitative behavior of the closed-loop system. Using these observations, we put forth some suggestions on how to select the tuning parameters based on the specific scenario in mind, and discuss some open problems related to the notion of $\mathcal{L}_p$ stability in settings where only noisy measurements are available.

Hence, the contributions of this work are three-fold:

1) the proposed setup ensures an $\mathcal{L}_p$-stability property of the closed-loop system in presence of measurement noise;
2) the triggering rules are thus robust to measurement noises and process disturbances and have a guaranteed global minimum inter-event time by design;
3) we showcase how the selection of tuning parameters may affect the qualitative behavior of the closed-loop system.

All proofs are omitted for space reasons.

Koen Scheres and Maurice Heemels are with the Department of Mechanical Engineering, Eindhoven University of Technology, The Netherlands. k.j.a.scheres@tue.nl, m.heemels@tue.nl

Romain Postoyan is with the Université de Lorraine, CNRS, CRAN, F-54000 Nancy, France. romain.postoyan@univ-lorraine.fr

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II. Preliminaries

A. Notation

The sets of all non-negative and positive integers are denoted $\mathbb{N}$ and $\mathbb{N}_{>0}$, respectively. The field of all reals and all non-negative reals are indicated by $\mathbb{R}$ and $\mathbb{R}_{\geqslant 0}$, respectively. The identity matrix of size $N \times N$ is denoted by I_N, and the vectors in $\mathbb{R}^N$ whose elements are all ones or zeros are denoted by $\mathbf{1}_N$ and $\mathbf{0}_N$, respectively. For any vector $u \in \mathbb{R}^m$, $v \in \mathbb{R}^n$, the stacked vector $\begin{bmatrix} u^\top & v^\top \end{bmatrix}^\top$ is denoted by (u, v). By $\langle \cdot, \cdot \rangle$ and $|\cdot|$ we denote the usual inner product of real vectors and the Euclidean norm, respectively. For any $x \in \mathbb{R}^N$, the distance to a closed non-empty set $\mathcal{A}$ is denoted by $|x|_\mathcal{A} := \min_{y \in \mathcal{A}} |x - y|$. We use $U^\circ(x; v)$ to denote the generalized directional derivative of Clarke of a locally Lipschitz function U at x in the direction v, i.e., $U^\circ(x; v) := \limsup_{h \to 0^+,\, y \to x} (U(y + hv) - U(y))/h$, which reduces to the standard directional derivative $\langle \nabla U(x), v \rangle$ when U is continuously differentiable; see [6] for more details. We denote the function space of Lebesgue measurable signals from $\mathbb{R}_{\geqslant 0}$ to $\mathbb{R}^n$ by $\mathcal{M}^n$ with $n \in \mathbb{N}_{>0}$. Given $x \in \mathcal{M}$, if $\operatorname{range} x \subset \mathcal{A} \subset \mathbb{R}^n$, *i.e.*, if $x(t) \in \mathcal{A}$ for all $t \in \mathbb{R}_{\geqslant 0}$, we write $x \in \mathcal{M}_\mathcal{A}$. By $\wedge$ and $\vee$ we denote the logical *and* and *or* operators respectively. We use the usual definitions for comparison functions $\mathcal{K}$, $\mathcal{K}_\infty$ and $\mathcal{KL}$, see [7].

B. Hybrid systems

Based on an extension [8] of the formalism of [7], we model hybrid systems $\mathcal{H}(F, \mathcal{C}, G, \mathcal{D}, \mathbb{X}, \mathbb{V})$ as

$$\begin{cases} \dot{\xi} \in F(\xi, \nu) & (\xi, \nu) \in \mathcal{C}, \\ \xi^+ \in G(\xi, \nu) & (\xi, \nu) \in \mathcal{D}, \end{cases} \tag{1}$$

where $\xi \in \mathbb{X} \subseteq \mathbb{R}^{n_\xi}$ denotes the state, ν an external input taking values in $\mathbb{V} \subseteq \mathbb{R}^{n_\nu}$, $\mathcal{C} \subseteq \mathbb{X} \times \mathbb{V}$ the flow set, $\mathcal{D} \subseteq \mathbb{X} \times \mathbb{V}$ the jump set, $F : \mathbb{X} \times \mathbb{V} \rightrightarrows \mathbb{R}^{n_\xi}$ the (set-valued) flow map and $G : \mathbb{X} \times \mathbb{V} \rightrightarrows \mathbb{R}^{n_\xi}$ the (set-valued) jump map. Sets $\mathcal{C}$ and $\mathcal{D}$ are assumed to be closed. We refer to [7] for notions related to (1) such as hybrid time domains or hybrid arcs. For a hybrid time domain E, $\sup_t E := \sup \{t \in \mathbb{R}_{\geqslant 0} : \exists j \in \mathbb{N} \text{ such that } (t, j) \in E\}$, $\sup_j E := \sup \{j \in \mathbb{N} : \exists t \in \mathbb{R}_{\geqslant 0} \text{ such that } (t, j) \in E\}$ and $\sup E := (\sup_t E, \sup_j E)$. We consider the notion of solutions proposed in [8]. The set of maximal solutions for the hybrid system $\mathcal{H}$ with initial condition $\phi(0, 0) = x$ and input v are denoted $\mathcal{S}_\mathcal{H}(v, x)$.

For $1 < p < \infty$, we introduce the $\mathcal{L}_p$-norm of a signal z defined on a hybrid time domain $\operatorname{dom} z = \cup_{j=0}^{J-1} [t_j, t_{j+1}] \times \{j\}$ with J possibly ∞ and/or $t_J = \infty$ by

$$\|z\|_{\mathcal{L}_p} = \left(\sum_{j=0}^{J-1} \int_{t_j}^{t_{j+1}} |z(t, j)|^p dt \right)^{\frac{1}{p}}. \tag{2}$$

Definition 1. *Hybrid system $\mathcal{H}$ is said to be $\mathcal{L}_p$-*stable*, $1 < p < \infty$, from input v to output $z = h(\xi, \nu)$ with respect to a closed non-empty set $\mathcal{A}$ with an $\mathcal{L}_p$-gain less than or equal to $\vartheta \in \mathbb{R}_{>0}$, if there exists $\beta \in \mathcal{K}_\infty$ such that for any initial condition $\xi_0 \in \mathbb{X}$ and all $\phi \in \mathcal{S}_\mathcal{H}(v, \xi_0)$ with $v \in \mathcal{M}$,*

$$\|z\|_{\mathcal{L}_p} \leqslant \beta(|\phi(0, 0)|_\mathcal{A}) + \vartheta \|v\|_{\mathcal{L}_p}, \tag{3}$$

Proposition 1. *Consider the hybrid system $\mathcal{H}$ with a set of inputs $\mathcal{V} \subseteq \mathcal{L}_p$ and let $\mathcal{A} \subset \mathbb{R}^{n_x}$ be a non-empty closed set. If there exist a locally Lipschitz $V : \mathbb{R}^{n_x} \to \mathbb{R}_{\geqslant 0}$, $\underline{\alpha}, \overline{\alpha} \in \mathcal{K}_\infty$ and $\gamma, \vartheta \in \mathbb{R}_{>0}$ such that*

i) for any $(\xi, v) \in \mathcal{C} \cup \mathcal{D}$,

$$\underline{\alpha}(|\xi|_\mathcal{A}) \leqslant V(\xi) \leqslant \overline{\alpha}(|\xi|_\mathcal{A}),$$

ii) for all $(\xi, v) \in \mathcal{C}$ and $f \in F(\xi, v)$,

$$V^\circ(\xi; f) \leqslant \gamma(\vartheta^p |v|^p - |z|^p),$$

iii) for all $(\xi, v) \in \mathcal{D}$ and any $g \in G(\xi, v)$,

$$V(g) - V(\xi) \leqslant 0,$$

then $\mathcal{H}$ is $\mathcal{L}_p$-stable from input v to output z with $\mathcal{L}_p$ gain less than or equal to ϑ with respect to $\mathcal{A}$.

III. Problem formulation

As in [5], we consider a collection of $N \in \mathbb{N}_{>0}$ continuous-time plants $\mathcal{P}_i$, $i \in \mathcal{N} := \{1, 2, \ldots, N\}$, of the form

$$\mathcal{P}_i : \begin{cases} \dot{x}_{p,i} = f_{p,i}(x_p, u_i, v_i), \\ y_i = g_{p,i}(x_{p,i}, u_i), \\ \widetilde{y}_i = y_i + w_i, \end{cases} \tag{4}$$

where $x_{p,i} \in \mathbb{R}^{n_{x_{p,i}}}$ is the plant state, $x_p := (x_{p,1}, x_{p,2}, \ldots, x_{p,N}) \in \mathbb{R}^{n_{x_p}}$ with $n_{x_p} = \sum_{i \in \mathcal{N}} n_{x_{p,i}}$ is the collection of the plant states, $u_i \in \mathbb{R}^{n_{u,i}}$ is the control input, $v_i \in \mathcal{M}^{n_{v,i}}$ a process disturbance taking values in $\mathbb{R}^{n_{v,i}}$, $y_i \in \mathbb{R}^{n_{y,i}}$ the output of the system unaffected by measurement noise, $\widetilde{y}_i \in \mathbb{R}^{n_{y,i}}$ the output of the system affected by additive measurement noise and $w_i \in \mathcal{M}^{n_{y,i}}$ the measurement noise taking values in $\mathbb{R}^{n_{y,i}}$, $f_{p,i} : \mathbb{R}^{n_{x_p}} \times \mathbb{R}^{n_{u,i}} \times \mathbb{R}^{n_{v,i}} \to \mathbb{R}^{n_{x_{p,i}}}$ is continuous and $g_{p,i} : \mathbb{R}^{n_{x_{p,i}}} \times \mathbb{R}^{n_{u,i}} \to \mathbb{R}^{n_{x_{y,i}}}$ is continuously differentiable. Each system $\mathcal{P}_i$, $i \in \mathcal{N}$ is controlled by a local dynamic feedback law $\mathcal{C}_i$ with dynamics

$$\mathcal{C}_i : \begin{cases} \dot{x}_{c,i} = f_{c,i}(x_{c,i}, \widetilde{y}_i, \widehat{\widetilde{y}}), \\ u_i = g_{c,i}(x_{c,i}, \widetilde{y}_i, \widehat{\widetilde{y}}), \end{cases} \tag{5}$$

where $x_{c,i} \in \mathbb{R}^{n_{x_{c,i}}}$ is the controller state, $\widehat{\widetilde{y}} := (\widehat{\widetilde{y}}_1, \widehat{\widetilde{y}}_2, \ldots, \widehat{\widetilde{y}}_N) \in \mathbb{R}^{n_y}$ denotes the "networked" version of the outputs y_i of the systems $\mathcal{P}_i$, $i \in \mathcal{N}$ with $f_{c,i} : \mathbb{R}^{n_{c,i}} \times \mathbb{R}^{n_{y,i}} \times \mathbb{R}^{n_y} \to \mathbb{R}^{n_{c,i}}$ and $g_{c,i} : \mathbb{R}^{n_{x_{c,i}}} \times \mathbb{R}^{n_y} \to \mathbb{R}^{n_{u,i}}$ continuous and $n_y = \sum_{i \in \mathcal{N}} n_{y,i}$.

We impose the following assumption on the measurement noises w_i and process disturbances v_i.

Assumption 1. *For each $i \in \mathcal{N}$, $v_i, w_i \in \mathcal{L}_p$ and $w_i(t) \in \mathcal{W}_i$ for all $t \in \mathbb{R}_{\geqslant 0}$, where $\mathcal{W}_i := \left\{ w_i \in \mathbb{R}^{n_{y,i}} \,\middle|\, |w_i| \leqslant \overline{w}_i \right\}$ for some known $\overline{w}_i \in \mathbb{R}_{\geqslant 0}$. Moreover, there exists $\Upsilon_i : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, which is monotonically decreasing and satisfies $|w_i(t)|^p \leqslant \Upsilon_i(t)$ and $\int_0^\infty \Upsilon(s) ds < \infty$ for all $p \in [1, \infty)$.*

Assumption 1 imposes boundedness conditions on the measurement noise and it does not impose restrictions on the existence or boundedness of their derivatives. The function Υ_i in Assumption 1 is required in order to ensure that the "sampled" version of w_i, which we will formalize below, also

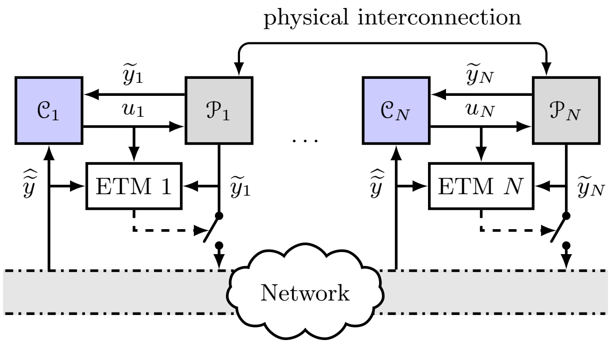

Fig. 1. Networked control setup where the controller and plant are (physically) separated and communicate via a packet-based network.

has bounded $\mathcal{L}_p$ norm, see Remark 3 below. This assumption is satisfied if, *e.g.*, w_i is bounded by an exponential function, *i.e.*, if for all $t \in \mathbb{R}_{\geqslant 0}$ we have $|w_i(t)| \leqslant ce^{-\alpha t}$ for some $c, \alpha \in \mathbb{R}_{\geqslant 0}$. Essentially, it is required that w_i vanishes *sufficiently fast* when $t \to \infty$.

Remark 1. *Without Assumption 1, we still obtain a finite $\mathcal{L}_p$-gain from $(v, w, \widehat{w})$ to z, hence, $\|z\|_{\mathcal{L}_p}$ is finite if $\|w\|_{\mathcal{L}_p}$, $\|\widehat{w}\|_{\mathcal{L}_p}$ and $\|v\|_{\mathcal{L}_p}$ are finite.*

The i-th system, $i \in \mathcal{N}$, broadcasts its output $\widetilde{y}_i$ to the controllers $\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_N$ over the digital network. The corresponding transmissions occur at time instants t_k^i, $k \in \mathbb{N}$, which are generated by a local Event-Triggering Mechanism (ETM), which is to be designed. Because of the packet-based communication over the network, the i-th controller, which depends on the outputs of $\mathcal{P}_m$, $m \in \mathcal{N}$, does not have continuous access to $\widetilde{y} := (\widetilde{y}_1, \widetilde{y}_2, \dots, \widetilde{y}_N)$, but only to its estimate $\widehat{\widetilde{y}} := (\widehat{\widetilde{y}}_1, \widehat{\widetilde{y}}_2, \dots, \widehat{\widetilde{y}}_N)$ and to its local output $\widetilde{y}_i$. When ETM $i \in \mathcal{N}$, transmits the measured output of plant i over the network, $\widehat{\widetilde{y}}_i$ is updated according to

$$\widehat{\widetilde{y}}_i((t_k^i)^+) = \widetilde{y}_i(t_k^i). \tag{6}$$

In between transmissions, a zero-order-holding device is used to produce a continuous-time estimate, *i.e.*,

$$\dot{\widehat{\widetilde{y}}}_i = 0. \tag{7}$$

For modeling purposes only, we define $\widehat{y}_i$ and $\widehat{w}_i$, where

$$\begin{aligned} \widehat{y}_i((t_k^i)^+) &= y_i(t_k^i), & \dot{\widehat{y}}_i &= 0, \\ \widehat{w}_i((t_k^i)^+) &= w_i(t_k^i), & \dot{\widehat{w}}_i &= 0. \end{aligned} \tag{8}$$

Hence, $\widehat{w}_i$ is the value of w_i at the last transmission instant of ETM i. Due to the aforementioned definitions, we obtain that $\widehat{\widetilde{y}}_i = \widehat{y}_i + \widehat{w}_i$.

We define the *ideal* network-induced error e_i as the difference between the sampled output $\widehat{y}_i$ *without* measurement noise and the current output y_i *without* measurement noise:

$$e_i := \widehat{y}_i - y_i. \tag{9}$$

Note that e_i is *not* known by the ETM, and therefore, cannot be used by the corresponding local triggering condition for determining t_k^i, $k \in \mathbb{N}$. Hence, we also define the *measured* network-induced error $\widetilde{e}_i$ as the difference between the estimated output $\widehat{\widetilde{y}}_i$ and the current measured output $\widetilde{y}_i$, which are both affected by noise, i.e.,

$$\widetilde{e}_i := \widehat{\widetilde{y}}_i - \widetilde{y}_i = e_i + \widehat{w}_i - w_i. \tag{10}$$

The local ETM at plant i does have access to $\widetilde{e}_i$. We denote the concatenated variables corresponding to (9) and (10) as $e := (e_1, e_2, \dots, e_N)$ and $\widetilde{e} := (\widetilde{e}_1, \widetilde{e}_2, \dots, \widetilde{e}_N)$, respectively.

To determine the triggering times, we define the local auxiliary variables $\eta_i \in \mathbb{R}$, $i \in \mathcal{N}$, whose dynamics are given by

$$\eta_i((t_k^i)^+) = \varrho_i(o_i), \qquad \dot{\eta}_i = \Psi_i(o_i), \tag{11}$$

where $o_i := (\widetilde{y}_i, \widehat{\widetilde{y}}, \widetilde{e}_i, u_i, \eta_i) \in \mathbb{R}^{n_{o,i}}$ with $n_{o,i} := 2n_{y,i} + n_y + n_{u,i} + 1$ is the locally available information at ETM i, and the functions Ψ_i and ϱ_i are to be designed. The triggering times are then given by

$$t_{k+1}^i := \min\left\{ \inf\left\{ t > t_k^i + \tau_{\text{miet}}^i \mid \eta_i(t) \leqslant 0 \right\}; T_i \right\} \tag{12}$$

with $T_i \in \mathbb{R}_{>0}$ an arbitrarily large constant. The constant T_i forces transmissions to occur indefinitely, which is needed to ensure boundedness of the "sampled" noise signal $\widehat{w}$, see also Remark 3 below.

Remark 2. *The upper-bound $\overline{w}_i$ of the noise is used only in the reset maps ϱ_i presented in Section V. Consequently, if we design $\varrho_i \equiv 0$, we can forego Assumption 1 and instead just assume that $w_i \in \mathcal{L}_p$ and that it is bounded from above by the function Υ_i. However, the inclusion of a nontrivial reset improves the inter-event behavior of the closed-loop significantly. Therefore, we choose to impose Assumption 1.*

We are now interested in designing robust event-triggering conditions which ensure bounded $\mathcal{L}_p$-gains with respect to some output variable

$$z = q(x, v, w, \widehat{w}), \tag{13}$$

where $x := (x_1, x_2, \dots, x_N) \in \mathbb{R}^{n_x}$, $x_i := (x_{p,i}, x_{c,i}) \in \mathbb{R}^{n_{x_{p,i}} + n_{x_{c,i}}}$, $v := (v_1, v_2, \dots, v_N) \in \mathbb{R}^{n_v}$, $w := (w_1, w_2, \dots, w_N) \in \mathcal{W}$ and $\widehat{w} := (\widehat{w}_1, \widehat{w}_2, \dots, \widehat{w}_N)$ with $n_v := \sum_{i \in \mathcal{N}} n_{v,i}$, $n_x := n_{x_p} + n_{x_c}$, $\mathcal{W} := \mathcal{W}_1 \times \mathcal{W}_2 \times \ldots \times \mathcal{W}_N$ and $\mathcal{W}_i$, $i \in \mathcal{N}$ as defined in Assumption 1.

IV. Hybrid model

We model the overall system as a hybrid system $\mathcal{H}$ as in Section II-B, for which a jump corresponds to the broadcasting of the noisy output $\widetilde{y}_i$ over the network for some $i \in \mathcal{N}$. We allow the local triggering (transmission) conditions to depend on two local auxiliary variables denoted $\eta_i, \varrho_i \in \mathbb{R}$, as is the case in dynamic triggering [9], [10]. The dynamics of η_i and ϱ_i are designed in the following.

The full state for $\mathcal{H}$ is $\xi := (x, e, \widehat{w}, \tau, \eta) \in \mathbb{X}$, where $\mathbb{X} := \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \times \mathcal{W} \times \mathbb{R}_{\geqslant 0}^N \times \mathbb{R}_{\geqslant 0}$. We define the concatenated exogenous inputs $\nu := (v, w) \in \mathbb{V}$, where $\mathbb{V} := \mathbb{R}^{n_v} \times \mathcal{W}$. The flow map $F : \mathbb{X} \times \mathbb{V} \rightrightarrows \mathbb{X}$ can then be written as

$$F(\xi,\nu) := \big(f(x,e,\widehat{w},v,w), g(x,e,\widehat{w},v,w), \mathbf{0}_{n_y}, \mathbf{1}_N, \Psi(o)-\sigma(\eta)\big).$$

Based on (4), (5) and (10), we obtain $f(x,e,\widehat{w},v,w) := (f_1(x,e,\widehat{w},v_1,w_1), f_2(x,e,\widehat{w},v_2,w_2), \ldots, f_N(x,e,\widehat{w},v_N,w_N))$, where $f_i : \mathbb{R}^{n_x}\times\mathbb{R}^{n_y}\times\mathcal{W}\times\mathbb{R}^{n_{v,i}}\times\mathbb{R}^{n_{y,i}} \to \mathbb{R}^{n_{x,i}}$ is given by $f_i(x,e,\widehat{w},v_i,w_i) := \Big(f_{p,i}\big(x_p, g_{c,i}(x_{c,i}, g_{p,i}(x_{p,i})+w_i, g_p(x_p)+e+\widehat{w}), v_i\big), f_{c,i}\big(x_{c,i}, g_{p,i}(x_{p,i})+w_i, g_p(x_p)+e+\widehat{w}\big)\Big)$ with $g_p(x_p) := (g_{p,1}(x_{p,1}), g_{p,2}(x_{p,2}), \ldots, g_{p,N}(x_{p,N}))$. Based on (4), (7) and (10), we obtain $g(x,e,\widehat{w},v,w) := (g_1(x,e,\widehat{w},v_1,w_1), g_2(x,e,\widehat{w},v_2,w_2), \ldots, g_N(x,e,\widehat{w},v_N,w_N))$, where $g_i(x,e,\widehat{w},v_i,w_i) := f_{h,i}(g_{p,i}(x_{p,i})+e_i+\widehat{w}_i) - f_{y,i}(x,e,\widehat{w},v_i,w_i)$ with $f_{y,i}(x,e,\widehat{w},v_i,w_i) := \dfrac{\partial g_{p,i}}{\partial x_{p,i}} f_{p,i}\left(x_p, g_{c,i}(x_{c,i}, g_{p,i}(x_{p,i})+w_i, g_p(x_p)+e+\widehat{w}), v_i\right)$.
The function Ψ defines the dynamics of the local triggering variables η and are defined as

$$\Psi(\widetilde{y},\widehat{\widetilde{y}},\widetilde{e},u,\varrho) := (\Psi_1(o_1), \Psi_2(o_2), \ldots, \Psi_N(o_N)), \quad (14)$$

where Ψ_i will be constructed in the following.

The flow set $\mathcal{C}\subseteq\mathbb{X}\times\mathbb{V}$ is given by $\mathcal{C} := \cap_{i\in\mathcal{N}}\mathcal{C}_i$ with $\mathcal{C}_i := \{(\xi,\nu)\in\mathbb{X}\times\mathbb{V} \mid \tau_i \leqslant T_i \wedge \eta_i \geqslant 0\}$, where τ^i_{miet} is to be designed.

The jump set is given by $\mathcal{D} := \cup_{i\in\mathcal{N}}\mathcal{D}_i$ with $\mathcal{D}_i := \big\{(\xi,\nu)\in\mathbb{X}\times\mathbb{V} \mid \tau_i \geqslant \tau^i_{\text{miet}} \wedge \eta_i \leqslant 0\big\}$. The jump map $G : \mathbb{X}\times\mathbb{V} \rightrightarrows \mathbb{X}$ is, for any $(\xi,\nu)\in\mathbb{X}\times\mathbb{V}$, given by

$$G(\xi,\nu) := \bigcup_{i\in\mathcal{N}} \begin{cases} G_i(\xi,\nu), & \text{when } (\xi,\nu)\in\mathcal{D}_i, \\ \emptyset, & \text{otherwise} \end{cases} \quad (15)$$

with

$$G_i(\xi,\nu) := (x, \overline{\Gamma}_i e, \overline{\Gamma}_i\widehat{w} + \Gamma_i w, \overline{\Gamma}_i\eta + \Gamma_i\eta^0(\xi,\nu)) \quad (16)$$

where $\overline{\Gamma}_i := I_{n_y} - \Gamma_i$ and $\Gamma_i := \operatorname{diag}(\Delta_{i,1}, \Delta_{i,2}, \ldots, \Delta_{i,N})$ with

$$\Delta_{i,j} := \begin{cases} \mathbf{0}_{n_{y,j},n_{y,j}}, & \text{if } i\neq j, \\ I_{n_{y,j}}, & \text{if } i=j. \end{cases} \quad (17)$$

Observe that, by construction of the sets $\mathcal{C}_i$ and $\mathcal{D}_i$, a trigger is enforced when $\tau_i \geqslant T_i$, which is consistent with (12). Similarly, a trigger cannot occur when $\tau_i < \tau^i_{\text{miet}}$. Although the setup is similar to the setup in [5], the essential differences here are the inclusion of the timers τ_i which enforce a global minimum inter-event time by design. This difference is instrumental in showing Zeno-freeness.

V. Main results

With the model in place, we can now present the conditions that are required to ensure $\mathcal{L}_p$-stability.

Condition 1. *For all $i\in\mathcal{N}$, there exist a constant $L_i \geqslant 0$ and a continuous function $H_i : \mathbb{R}^{n_x}\times\mathbb{R}^{n_y}\times\mathcal{W}\times\mathbb{R}^{n_v} \to \mathbb{R}_{\geqslant 0}$ such that for all $x\in\mathbb{R}^n$, $e\in\mathbb{R}^{n_y}$, $\widehat{w}\in\mathbb{R}^m$ and $v\in\mathbb{R}^{n_v}$,*

$$|g_i(x,e,\widehat{w},v,w)| \leqslant L_i|e_i| + H_i(x,e,\widehat{w},v,w). \quad (18)$$

The inequality in (18) is loosely speaking an upper bound on the growth of e_i between successive transmission instants. This condition is always satisfied, e.g., for linear systems or when the map g_i satisfies a linear growth condition. In the following, we will omit the arguments of H_i, as specified in Condition 1, for brevity.

Condition 2. *There exist a locally Lipschitz function $V : \mathbb{R}^n \to \mathbb{R}_{\geqslant 0}$, a supply rate $s : \mathbb{R}^{n_x}\times\mathbb{R}^{n_y}\times\mathcal{W}\times\mathbb{V} \to \mathbb{R}$, a non-empty closed set $\mathcal{X}\subseteq\mathbb{R}^{n_x}$, $\mathcal{K}_\infty$-functions $\underline{\alpha}, \overline{\alpha}$, and, for all $i\in\mathcal{N}$, positive semi-definite functions $\delta_i : \mathbb{R}^{n_{o,i}} \to \mathbb{R}_{\geqslant 0}$ and constants $\zeta_i, \gamma_i, \mu_i > 0$ such that for all $x\in\mathbb{R}^{n_x}$*

$$\underline{\alpha}(|x|_{\mathcal{X}}) \leqslant V(x) \leqslant \overline{\alpha}(|x|_{\mathcal{X}}), \quad (19)$$

and for all $\nu\in\mathbb{V}$, $e\in\mathbb{R}^{n_y}$, $\widehat{w}\in\mathcal{W}$, and almost all $x\in\mathbb{R}^{n_x}$,

$$\begin{aligned} &\langle \nabla V(x), f(x,e,\widehat{w},v,w)\rangle \\ &\quad \leqslant \alpha(\vartheta^p|(\widehat{w},\nu)|^p - |q(x,v,w,\widehat{w})|^p) \\ &\quad + \textstyle\sum_{i\in\mathcal{N}}\big(-\delta_i(o_i) - \zeta_i H_i^2 + (\gamma_i^2-\mu_i)|e_i|^2\big). \end{aligned} \quad (20)$$

Condition 2 is akin to an $\mathcal{L}_2$-gain condition from $|e_i|$ to H_i like in [11], which translates the impact of the network-induced error on the output H_i. When $e\equiv 0$, i.e., in absence of the network, the condition in (20) ensures the desired $\mathcal{L}_p$-gain property.

Condition 3. *There exist continuous functions $\underline{H}_i : \mathbb{R}^{n_{o,i}} \to \mathbb{R}_{\geqslant 0}$, $i\in\mathcal{V}$, that satisfy for all $x\in\mathbb{R}^{n_x}$, $e\in\mathbb{R}^{n_y}$, $\widehat{w}\in\mathcal{W}$, $v\in\mathbb{R}^{n_v}$, $w\in\mathcal{W}$,*

$$\underline{H}_i(o_i) \leqslant H_i(x,e,\widehat{w},v,w) \quad (21)$$

for functions H_i as in Condition 1.

Condition 3 is trivially satisfied by taking $\underline{H}_i(o_i) = 0$ for all $i\in\mathcal{V}$, $o_i\in\mathbb{R}^{n_{o,i}}$ as the function H_i is non-negative. However, the introduction of the function $\underline{H}_i$ is an important feature that allows to capture relevant systems such as consensus systems with single-integrator dynamics, as we will show in the case study in Section VI.

We are now ready to state the main result of this paper.

Theorem 1. *Suppose Assumption 1 and Conditions 1-3 hold. For all $i\in\mathcal{N}$, select $\overline{\lambda}_i > \underline{\lambda}_i > 0$ and $\epsilon_i\in(0,1]$. We define for all $\xi\in\mathbb{X}$ and $\nu\in\mathbb{V}$,*

$$\begin{aligned} \Psi(o_i) &:= \delta_i(o_i) + (1-\epsilon_i)\zeta_i\underline{H}_i^2(o_i) - \omega_i(\tau_i)\overline{\gamma}_i|\widetilde{e}_i|^2, \\ \varrho_i(o_i) &:= \underline{\lambda}_i\gamma_i(\max\{|\widetilde{e}_i| - 2\overline{w}_i, 0\})^2, \\ \overline{\gamma}_i &:= \gamma_i(2L_i\underline{\lambda}_i + \gamma_i(1+\underline{\lambda}_i^2)) \end{aligned} \quad (22)$$

with $\omega_i : \mathbb{R}_{\geqslant 0} \rightrightarrows [0,1]$ defined as

$$\omega(\tau) := \begin{cases} \{0\}, & \text{if } \tau_i\in[0,\tau^i_{\text{miet}}), \\ [0,1], & \text{if } \tau_i = \tau^i_{\text{miet}}, \\ \{1\}, & \text{if } \tau_i > \tau^i_{\text{miet}}. \end{cases} \quad (23)$$

and τ^i_{miet} given by

$$\tau^i_{\text{miet}} := \begin{cases} \frac{1}{L_i r_i}\arctan(\theta_i), & \sqrt{\epsilon_i\zeta_i}\gamma > L_i, \\ \frac{1}{L_i}\frac{\overline{\lambda}_i - \underline{\lambda}_i}{(\overline{\lambda}_i+1)(\underline{\lambda}_i+1)}, & \sqrt{\epsilon_i\zeta_i}\gamma_i = L_i, \\ \frac{1}{L_i r_i}\operatorname{arctanh}(\theta_i), & \sqrt{\epsilon_i\zeta_i}\gamma_i < L_i, \end{cases} \quad (24)$$

where

$$\theta_i := \frac{r_i(\overline{\lambda}_i - \underline{\lambda}_i)}{\frac{\gamma_i}{L_i}(1+\overline{\lambda}_i\underline{\lambda}_i) + \overline{\lambda}_i + \underline{\lambda}_i}, \quad r := \sqrt{\left|\epsilon_i\zeta_i\left(\frac{\gamma_i}{L_i}\right)^2 - 1\right|},$$

and L_i*, and* γ_i *come from Conditions 1 and 2. System* $\mathcal{H}(F,\mathcal{C},G,\mathcal{D})$ *with triggering dynamics (22) renders the closed-loop system* $\mathcal{L}_p$*-stable with finite* $\mathcal{L}_p$*-gain from input* $(\nu, \widehat{w})$ *to output* z *with respect to* $\mathcal{A} := \{\xi \in \mathbb{X} \mid x \in \mathcal{X}, e = \mathbf{0}, \eta = \mathbf{0}\}$ *and has a guaranteed minimum inter-event time given by* $\tau^i_{\text{miet}} > 0$.

The function ω_i, $i \in \mathcal{N}$, in (23) is defined such that the flow map F is outer semi-continuous to ensure that the hybrid system $\mathcal{H}$ satisfies the hybrid basic conditions as presented in [7, Assumption 6.5].

Remark 3. *The requirement that the inter-transmission times are bounded from above by* T_i *together with Assumption 1 are essential requirements to ensure that* $\widehat{w}_i \in \mathcal{L}_p$*. Indeed, as* $w_i(t) \to 0$ *for* $t \to \infty$*, its sampled version* $\widehat{w}_i \to 0$ *as well, given that transmissions occur at least every* T_i *time-units. This is instrumental due to the term* $\widehat{w}$ *entering the system through the controller. As a consequence, if, at some point, transmissions stop and* $\widehat{w} \neq 0$ *for all* $t > \tilde{T} \in \mathbb{R}_{\geqslant 0}$*,* $\|\widehat{w}\|_{\mathcal{L}_p} = \infty$*, even though* $\|w\|_{\mathcal{L}_p} < \infty$*. Similarly, the requirement that* $\widehat{w} \in \mathcal{L}_p$ *essentially also requires the upper bound on* $|w_i(t)|^p$ *given by* Υ_i *in Assumption 1. These assumptions together allow us to ascertain that* $\|\widehat{w}\|_{\mathcal{L}_p} < \infty$ *if* $\|w\|_{\mathcal{L}_p} < \infty$*. Of course, when it can be a priori inferred that* $\|\widehat{w}\|_{\mathcal{L}_p} < \infty$*, this requirement is not necessary and* $T_i = \infty$ *can be chosen in that case.*

Remark 4. *To illustrate the necessity of Assumption 1, consider the signal given by* $z(t) = 1$ *if* $t \in \mathcal{T}$*, and* $z(t) = 0$ *if* $t \notin \mathcal{T}$ *with* $\mathcal{T} := \bigcup_{k\in\mathbb{N}} \left[k, k+\frac{1}{2^k}\right]$*. The signal* z *is depicted in Figure 2. For any* $p \in \mathbb{R}_{\geqslant 1}$*, we have that*

$$\|z\|_{\mathcal{L}_p} = \left(\int_0^\infty |z(t)|^p dt\right)^{\frac{1}{p}} = \left(\sum_{k=0}^{\infty} \frac{1}{2^k}\right)^{\frac{1}{p}} = 2^{\frac{1}{p}}.$$

However, if we sample this signal at every $t \in \mathbb{N}$*, we find that its sampled version* $\widehat{z}(t) = 1$ *for all* $t \in \mathbb{R}_{\geqslant 0}$*. Hence,* $\widehat{z} \notin \mathcal{L}_p$*.*

Remark 5. *In the literature, the design parameters* $\overline{\lambda}_i$ *and* $\underline{\lambda}_i$ *are typically chosen to satisfy* $\underline{\lambda}_i \in (0,1)$ *and* $\overline{\lambda}_i = 1/\underline{\lambda}_i$*, see,* e.g.*, [12]. As we illustrate using simulations below, in some cases, a significant contribution to the favorable inter-event times in dynamic event-triggered control originate from the reset maps* ϱ_i *in (22). Thus, often, the resulting* average *inter-event times improve when* $\lambda_i \in (0,1)$ *is selected larger, even though, in these cases, the MIET is smaller due to the larger* λ_i*. This has also been reported in other literature, mainly in the context of self-triggered control, see [13]. The (re)selection or shifting of* $\underline{\lambda}_i$ *and* $\overline{\lambda}_i$ *may help in improving the (average) inter-event behavior, however, the* $\mathcal{L}_p$*-gain depends on* $\underline{\lambda}_i$ *in the sense that larger* $\underline{\lambda}_i$ *will result in a larger* $\mathcal{L}_p$*-gain.*

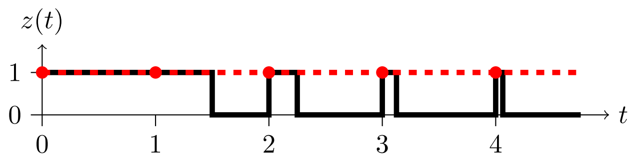

Fig. 2. Example of a signal $z \in \mathcal{L}_p$ whose sampled version $\widehat{z} \notin \mathcal{L}_p$.

TABLE I

MEAN AND STANDARD DEVIATION OF THE NUMBER OF EVENTS IN 100 SIMULATIONS WITH THE INITIAL CONDITIONS $x_i(0,0)$ CHOSEN RANDOMLY IN $[-10, 10]$.

$\overline{\lambda}_i^{-1} = \underline{\lambda}_i = 0.01$	$\overline{\lambda}_i^{-1} = \underline{\lambda}_i = 0.4$	$\overline{\lambda}_i = 100$, $\underline{\lambda}_i = 0.4$
350.02 ± 50.39	315.66 ± 10.14	268.30 ± 30.23

VI. CASE STUDY

To illustrate our results, we focus on a consensus problem. Specifically, we are interested in the consensus of single integrator systems, where each plant P_i, which we call agent in this section, has dynamics $\dot{x}_i = u_i$, with $x_i, u_i \in \mathbb{R}$, and the output $y_i = x_i$.

For a network topology described by a connected weight-balanced digraph $\mathcal{G}$ with Laplacian L, it is known that agents achieve consensus when the *ideal* (static) control law $\bar{u}_i = \sum_{m\in\mathcal{V}_i^{\text{in}}}(x_i - x_m)$, with $\mathcal{V}_i^{\text{in}}$ the in-neighbors of agent i, is applied, see [14]. In vector notation, this is written as $\bar{u} = -Lx$, where $\bar{u} := (u_1, u_2, \ldots, u_N)$ and L is the Laplacian matrix of the graph. We use the noisy sampled states for each agent instead of the actual states, resulting in the *actual* control law

$$u_i = \textstyle\sum_{m\in\mathcal{V}_i^{\text{in}}}(x_i + e_i + \widehat{w}_i - x_j - e_m - \widehat{w}_m),$$

written in vector notation as $u = -L(x + e + \widehat{w})$. Hence, the closed-loop system dynamics are $\dot{x} = -Lx - Le - L\widehat{w}$. We are interested in the $\mathcal{L}_p$-stability properties with $p = 2$ from inputs (v, w) to the output $q(x) := Lx$ with respect to the consensus set

$$\mathcal{X} := \left\{x \in \mathbb{R}^N \mid x_1 = x_2 = \ldots = x_N\right\}. \tag{25}$$

For this particular system, we have the following result.

Proposition 2 ([5], [15])**.** *Consider the above system. Conditions 1, 2 and 3 hold with* $V(x) = \frac{1}{2}x^\top Lx$*,* $\delta_i(o_i) := (1-\alpha_i)d_i(1-2aN_i)$*,* $\zeta_i = (1-d_i)(1-2aN_i)$*,* $\gamma_i = \sqrt{\frac{1}{a}N_i + \mu_i}$*,* $H_i = |u_i|$*,* $L_i = 0$ *and* $\underline{H}_i = |u_i|$*, where* $a \in (0, \frac{1}{2N_i})$*,* $\mu_i > 0$*,* $\alpha_i \in (0,1)$*,* $\epsilon_i \in (0,1)$ *and* $d_i \in (0,1)$ *are tuning parameters.*

We now run three different simulations, in the first one we select $\overline{\lambda}_i^{-1} = \underline{\lambda}_i = 0.01$. The second one, we will select $\overline{\lambda}_i^{-1} = \underline{\lambda}_i = 0.4$. For the last one, we choose $\overline{\lambda}_i = 100$ and $\underline{\lambda}_i = 0.2$. The resulting inter-event times are depicted in Fig. 3 and the total number of events are stated in Table I.

We can draw several conclusions from Fig. 3. Firstly, when close to the attractor, the triggering times are indeed close to the MIET. Thus, in case the parameters $\overline{\lambda}_i$, $\underline{\lambda}_i$ are

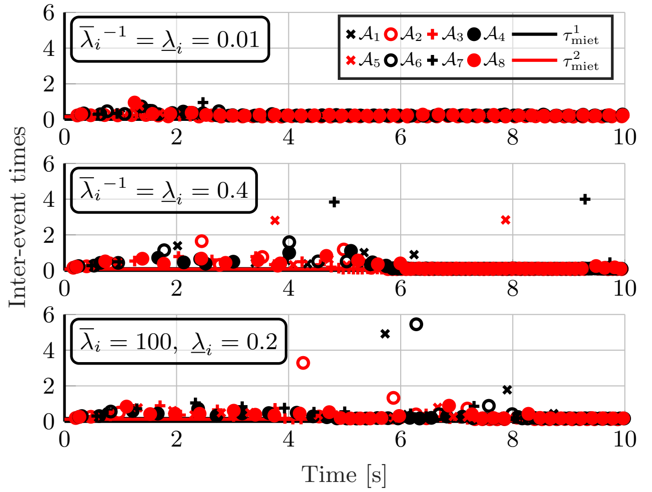

Fig. 3. Comparison of inter-event times for the different choices of $\overline{\lambda}_i$, $\underline{\lambda}_i$. From top to bottom, the minimum inter event-times are given by $(\tau^1_{\text{miet}}, \tau^2_{\text{miet}}) = (0.2132, 0.1628)$, $(\tau^1_{\text{miet}}, \tau^2_{\text{miet}}) = (0.1562, 0.1180)$ and $(\tau^1_{\text{miet}}, \tau^2_{\text{miet}}) = (0.1722, 0.1295)$, respectively.

selected "only" for the best transient response, eventually, a periodic triggering rule or the triggering rule with larger $\overline{\lambda}_i$ and smaller $\underline{\lambda}_i$ will always outperform the triggering rule designed for the best transient response. Moreover, when $\overline{\lambda}_i = 100$ and $\underline{\lambda}_i = 0.2$, the transient response is somewhere between the other two responses, i.e., it does not perform as well as taking $\overline{\lambda}_i = 1/0.4$ *during the transient*, however, it does better *vis-à-vis* the case where $\underline{\lambda}_i = 0.01$. Similarly, during the "steady-state" phase, i.e., when the noise is dominant, this reverses: due to the MIET being the dominant factor in the inter-event times, the simulation with $\overline{\lambda}_i^{-1} = \underline{\lambda}_i = 0.01$ "outperforms" the simulation with $\overline{\lambda}_i^{-1} = \underline{\lambda}_i = 0.4$ in terms of average inter-event times. Again, in this setting, the solution with $\overline{\lambda}_i = 100$ and $\underline{\lambda}_i = 0.2$ performs somewhere in between in terms of inter-event times, which is to be expected based on the MIET. Thus, freely selecting the parameters $\overline{\lambda}_i$ and $\underline{\lambda}_i$, allows us to shape the behavior both during the "transient response," i.e., when the initial condition is dominant in the behavior, as well as during "steady-state," i.e., when the noise is dominant in the behavior.

VII. Discussion & future work

Although we can ensure bounded a $\mathcal{L}_p$-gain through this particular design, it requires some additional conditions on the (measurement) noise signal on top of assuming that it is bounded in an $\mathcal{L}_p$ sense due to the zero-order-hold, which is applied to allow packet-based communication. There are several interesting directions to pursue in future work. The first issue we want to address is that, although we prove $\mathcal{L}_p$-stability, it is difficult to obtain the $\mathcal{L}_p$-gain explicitly due to the conservatism in the analysis. Secondly, we are interested in relaxing the conditions that are imposed on the noise signals. As a last potentially interesting direction, it seems that additional refinement of the reset functions ϱ_i may improve the average inter-event behavior further.

VIII. Conclusions

In this paper, we have designed time-regularized triggering rules for the event-based control of distributed systems. Our approach allows us to ascertain $\mathcal{L}_p$-stability properties of the closed-loop system. This property is ensured only under mild conditions on the noise signals and process disturbances. We ensure a minimum inter-event time by design. Furthermore, we show that by cleaverly selecting the tuning parameters, we can affect the average inter-event times both during the "transient response," i.e., when the initial condition in dominant in the behavior, as well as during "steady-state," when the noises determine the dominant behavior. This allows us to, loosely speaking, "tune" the behavior to prioritize the behavior during the "transient" or during "steady-state". We envision that this is beneficial in applications where the attractor is not (necessarily) constant, such as, e.g., tracking (motion) control. We have demonstrated the benefits of the proper selection of these parameters on a consensus example.

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