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Multi-crop scheduling of sowings in adaptive vertical farms with model predictive control

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Abstract— **An approach based on model predictive control is presented for the scheduling of sowings in an adaptive vertical farm, that is, a pioneering vertical greenhouse where shelf spacing is adjusted automatically according to the growth of crops. We consider the case in which the greenhouse is used for the simultaneous cultivation of multiple types of crops, with plants that may deviate from the ideal growth curve due to suboptimal temperature or humidity (modeled as a system noise). Model predictive control is used to account for such possible deviations and determine the best time instants to perform sowings in the various shelves composing the greenhouse, with the aim of maximizing the production yield. Simulation results are presented in a case study involving the simultaneous cultivation of three different types of crops. They showcase the effectiveness of the proposed method in maximizing production yield while effectively using almost all the vertical space available in the greenhouse, with various control horizons and types of disturbances.**

I. Introduction

Precision agriculture is a strategy that collects, processes, and analyzes data to support management decisions to improve efficiency, productivity, quality, profitability, and sustainability of agricultural production [1]. Precision farming includes smart greenhouses, i.e., a revolution in agriculture, creating crop environments with self-regulating microclimates suitable for plant growth through sensors, actuators, monitoring and control systems that optimize and automate the growing process [2]. In this context, vertical farms concern growing and harvesting plants in high-density urban areas and selling them directly within the urban community, thus reducing transportation compared to standard rural farming models. The advantages are multiplication of agricultural land (through cultivation in vertically mounted stacks), increased crop yields (through the use of optimized production methods), protection of crops from weather issues and diseases, and reduction of water requirements (through water recycling) [3]. The main disadvantages are production costs, which make the final product more expensive than in traditional agriculture. Thus, innovation in this field is important, as it could help to make the production in vertical farms less expensive. While vertical farms increase productivity per unit of floor area occupied in plants, they still fail to fully exploit the total available volume since, during the growing phase of the crop, the volume at disposal to the plant is fixed and therefore not fully utilized. The use of artificial lighting systems and air conditioning also results in high energy consumption compared to outdoor cultivations.

In [4]–[6], a novel concept for a vertical greenhouse, referred to as *adaptive vertical farm* (AVF), was introduced for cultivating vegetables in space applications (in orbit and future lunar bases) and for industrial vertical farms on Earth. The fundamental concept behind the AVF, initially described in the patent [7], is that crops occupy a volume depending on their level of development. In more detail, crops at the beginning of their growth stage require a volume that is lower than the one needed at a later stage. The main innovation behind the AVF is the possibility of adapting to the growth of the plants being grown. In fact, in a standard vertical farms the stacked growing shelves maintain a fixed distance (defined at the design stage) throughout the crop development cycle, whereas in AVFs the height of the growing shelves is adaptable to different stages of crop development. This requires an intelligent management of the position of the growing shelves that automatically assigns plants only the conditional volume they actually need during their growth. Therefore, an AVF necessitates an optimal sowing scheduling to leverage the adaptive principle and maximize production. In fact, if seeding were done at the same time across all the shelves, plants would surpass the total greenhouse height before harvest due to the increased number of cultivating shelves. Thus, a proper planning of sowings is essential to optimize the use of available space and resources. Since during much of their early development plants are in a growth phase that requires little space, continuous "intelligent" adaptation of the position of the shelves allows plants to share the same volumes during different growth stages. Thus, shelves can move vertically in an automatic way in order to vary such a volume and optimize the occupancy of the available vertical volume, thus allowing to cultivate more shelves per unit of volume as compared to existing vertical farms with fixed shelves.

In order to fully exploit the adaptive principle and maximize the production yield, an optimal static scheduling algorithm was proposed in [4], [5] for the space cultivation of both single and multiple types of crops, respectively. In conditions without disturbances, such a static scheduling algorithm enables a careful planning of seedings and the optimization of production, while ensuring that there is enough space for crop growth until harvest. However, if one or more crops deviate from their expected growth rate due to factors like suboptimal humidity or temperature, an adjustment of the static schedule is necessary. This adjustment ensures that the maximum available volume is not

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surpassed and that crops reach their intended height before being harvested. This problem was first investigated in [8] for an AVF devoted to the cultivation of a single crop type by using model predictive control (MPC). The goal is to design robust seeding schedules despite unforeseen disturbances, even without predictions of their future occurrences.

This paper builds upon the successful results of [8] and presents a scheduling approach based again on the formalism of MPC to compute the optimal seedings within AVFs for cultivating multiple crop types at the same time. The basic idea consists of solving a series of optimal control problems at different discrete time instants over a rolling time horizon, by using the actual growth level of crops at the various steps as a starting point. Each problem is formulated based on a dynamic model that predicts how shelf occupancy and distance between shelves evolve over time. When unexpected disturbances affect plant growth, the repeated solution of different optimization problems with updated information on the growth level of crops implement a feedback control mechanism, which enables taking resilient decisions to enhance production yield while ensuring that the total height of the greenhouse is not exceeded. To evaluate the effectiveness of the proposed approach, results with various disturbance intensities in an AVF growing lettuce, wheat, and basil simultaneously are presented and discussed.

MPC has been effectively used in various agricultural applications, such as managing flow and water levels in irrigation systems, guiding autonomous tractors, regulating environmental parameters (temperature, humidity, CO_2 levels, etc.), and controlling energy consumption [9], [10]. However, to our knowledge, its implementation in vertical farming, particularly for the optimal scheduling of sowings in response to disturbances, remains limited, establishing the novelty and challenge of our research.

The rest of this paper is organized as follows. Section II formulates the problem statement. Section III presents the predictive control approach to generate an optimal schedule. Section IV reports the results of simulations in a case study. Section V draws conclusions and outlines future works.

II. Dynamic Model of Shelf Occupancy and Optimization Problem

In this section, we describe the discrete-time dynamic system devised to model the growth of crops within an AVF. We focus on an AVF consisting of N shelves and a total height H_{tot}, designed with a modular structure. Each shelf can reach a maximum height equal to H_m when plants are fully grown. The maximum height H_m for each module consists of a variable, adaptive component H_c representing the crop height, and a static portion $H_{\text{fixed}} := H_h + H_s + H_l$, where H_h accounts for the technical space including ventilation structures and lighting systems, H_s represents the height of the cultivation substrate (such as soil for traditional farming or water for hydroponic systems), and H_l denotes the space above the crop canopy required for adequate air flow and lighting. The adaptive feature H_c ensures that the height occupied by a shelf equals H_m only at the time of harvest, while at other times the actual occupied vertical space is less since plants have not yet reached their full size.

We consider cultivation in the continuous time interval $[0, \mathcal{T}]$, where $\mathcal{T}$ denotes a specified duration, which is divided into T periods of equal length Δt. Hence, we focus on the discrete time instants $t = 0, 1\Delta t, \dots, T\Delta t$, or simply $t = 0, 1, \dots T$ with a little abuse of notation. We focus on the case study where the AVF is used to grow K kinds of crops simultaneously. The nominal growth cycle from seeding to harvest for plants of type k, $k = 1, \dots, K$, is assumed to have a duration of C_K sampling intervals. Plants are harvested when they reach the maximum height, which is equal to H_k for type-k crops. Likewise in [5], [8], we assume a nominal linear growth curve over time, that is, plants experience an increase in height equal to H_k/C_k from a given time step to the following one. This linear approximation serves as an initial, practical approach to modeling crop growth, due to the absence of detailed, empirical growth-height relationships in existing literature.

However, in real-world scenarios, actual plant growth may deviate from the nominal growth curve due to various influences, such as suboptimal humidity or temperature conditions. We take such potential deviations into account by introducing a disturbance variable, as it will be detailed in the following. More specifically, we account for the dynamics of the cultivation in the various shelves of the AVF by means of a discrete-time dynamical system, which describes, at each time instant, the type of crop cultivated in each shelf along with the distance with respect to the above shelf (or to the top of the greenhouse if the shelf is the last one).

Let us define the following state variables to track the evolution of crop growth over time for $t = 0, 1, \dots, T$:

- $x_{i,k,t} \in \{0, 1\}$, $i = 1, \dots, N$, $k = 1, \dots, K$: it is equal to 1 if shelf i is cultivated with crop type k at time t, otherwise it is equal to zero;
- $h_{i,k,t} \geq 0$, $i = 1, \dots, N$, $k = 1, \dots, K$: it is the height occupied in shelf i by the crop type k at time t; if no crops of type k are cultivated in shelf i, then $h_{i,k,t} = 0$.

For the sake of compactness, let us collect all the state variables in the vector $\underline{x}_t := \text{col}\,[x_{i,k,t}, h_{i,k,t}, i = 1, \dots, N, k = 1, \dots, K] \in \mathbb{R}^{2NK}$ for $t = 0, 1, \dots, T$.

The control inputs are given by the seedings performed at each time steps for $t = 0, 1, \dots, T - 1$:

- $s_{i,k,t} \in \{0, 1\}$, $i = 1, \dots, N$: it is equal to 1 if a seeding of crop type k occurs in shelf i at time t, otherwise it is equal to zero;

We collect again all the control inputs in the vector $\underline{u}_t := \text{col}\,[s_{i,k,t}, i = 1, \dots, N, k = 1, \dots, K] \in \mathbb{R}^{NK}$ for $t = 0, 1, \dots, T - 1$.

As said, we consider the presence of disturbances acting on the system that may delay or speedup the growth of the plants. The following disturbance is defined for $t = 0, 1, \dots, T$:

- $\xi_{i,t} \in \mathbb{R}$, $i = 1, \dots, N$: it is a disturbance affecting the growth of the plants cultivated in shelf i at time t. It does not affect the dynamics if the shelf is empty at time t (see the equations (1)–(2) defined later on). A positive value of $\xi_{i,t}$ denotes a speedup ins the growth rate of plants, whereas a negative one indicates a slowdown.

We collect all the disturbances in the vector $\underline{\xi}_t := \mathrm{col}\,[\xi_{i,t}, i = 1,\ldots,N] \in \mathbb{R}^N$ for $t = 0,1,\ldots,T$.

The evolution of the state variables over time is governed by the discrete-time dynamic system

$$\underline{x}_{t+1} = f\left(\underline{x}_t, \underline{u}_t, \underline{\xi}_t\right), \quad t = 0,1,\ldots,T-1,$$

where the mapping $\mathbb{R}^{2NK} \times \mathbb{R}^{NK} \times \mathbb{R}^N \mapsto \mathbb{R}^{2NK}$ defined by the function f is the following:

$$x_{i,k,t+1} = \begin{cases} 0 & \text{if } s_{i,k,t} = 0 \text{ and } h_{i,k,t} = 0, \\ 0 & \text{if } s_{i,k,t} = 0 \text{ and } h_{i,k,t} \geq H_k, \\ x_{i,k,t} & \text{if } x_{i,k,t} = 1 \text{ and } h_{i,t} < H_k, \\ 1 & \text{if } s_{i,k,t} = 1, \end{cases}$$

$$i = 1,\ldots,N, \quad k = 1,\ldots,K, \quad t = 0,1,\ldots,T-1, \tag{1}$$

$$h_{i,k,t+1} = \begin{cases} 0 & \text{if } s_{i,k,t} = 0 \text{ and } h_{i,t} = 0, \\ 0 & \text{if } s_{i,k,t} = 0 \text{ and } h_{i,t} \geq H_k, \\ h_{i,k,t} + \frac{H_k}{C_k} + \xi_{i,t} & \text{if } x_{i,k,t} = 1 \text{ and } h_{i,k,t} < H_k, \\ \frac{H_k}{C_k} + \xi_{i,t} & \text{if } s_{i,k,t} = 1, \end{cases}$$

$$i = 1,\ldots,N, \quad k = 1,\ldots,K, \quad t = 0,1,\ldots,T-1. \tag{2}$$

If no sowing occurs at time t and shelf i is empty, then the shelf remains empty also at the next time instant $t+1$; in this case, the height of shelf i at time $t+1$ remains equal to 0. If shelf i is cultivated with crops of type k at time t, crops have reached the height for the harvest, and no new seedings are performed, then harvest occurs and the shelf becomes free at time $t+1$; as a consequence, the height of shelf i fro crop k is set equal to 0. On the contrary, if shelf i is cultivated with type-k plants at time t, but crops have not yet reached the height for harvest, then the shelf remains cultivated also at time $t+1$; the corresponding height increases of the nominal growth equal to H_k/C_k, plus the effect of the disturbance. Lastly, if a new sowing is performed in shelf i at time t, then the shelf will be cultivated at time $t+1$; its height will increase of the nominal growth rate H_k/C_k plus a disturbance.

The dynamic model (1)–(2) is completed by the following constraints. First of all, at each time t, a shelf cannot be cultivated with more than one type of crop, that is, the following holds:

$$\textstyle\sum_{k=1}^{K} x_{i,k,t} \leq 1, \quad i = 1,\ldots,N, \quad t = 0,1,\ldots,T. \tag{3}$$

Similarly, new seedings can be only of one crop type at a time, i.e., we impose:

$$\textstyle\sum_{k=1}^{K} s_{i,k,t} \leq 1, \quad i = 1,\ldots,N, \quad t = 0,1,\ldots,T-1. \tag{4}$$

Furthermore, if shelf i is cultivated at time t and crops have not yet reached the height for harvest, then a new seeding cannot be performed, i.e., we can write

$$s_{i,k,t} = 0 \text{ if } x_{i,k,t} = 1 \text{ and } h_{i,t} < H_k, \quad i = 1,\ldots,N, \quad k = 1,\ldots,K, \quad t = 0,1,\ldots,T-1. \tag{5}$$

Lastly, the sum of the heights of the shelves at a given time t cannot exceed the total height of the greenhouse, i.e.,

$$\textstyle\sum_{i=1}^{N} H_{\text{fixed}} + \sum_{k=1}^{K} h_{i,k,t} \leq H_{\text{tot}}, \quad t = 0,1,\ldots,T. \tag{6}$$

We re-write the constraints (3)–(6) at each time t by defining a function $g : \mathbb{R}^{2NK} \times \mathbb{R}^{NK} \times \mathbb{R}^N \to \mathbb{R}^{2N+NK+1}$ such that $g(\underline{x}_t, \underline{u}_t, \underline{\xi}_t) \leq \underline{0}$, where $\underline{0}$ is the zero vector with $2N + NK + 1$ components.

We now formulate an optimal control problem with the goal of determining the best time instant when to perform seedings in the various shelves, in order to maximize the number of sowings (and therefore of harvests) from time $t = 0$ up to time T. The problem is the following:

$$\begin{aligned} &\max_{\underline{u}_0,\ldots,\underline{u}_{T-1}} E_{\underline{\xi}_0,\ldots,\underline{\xi}_T} \left\{ \textstyle\sum_{t=0}^{T} \sum_{i=1}^{N} \sum_{k=1}^{K} c_k\, s_{i,k,t} \right\}, \\ &\text{subject to} \\ &\underline{x}_{t+1} = f(\underline{x}_t, \underline{u}_t, \underline{\xi}_t), \quad t = 0,1,\ldots,T-1, \\ &g(\underline{x}_t, \underline{u}_t, \underline{\xi}_t) \leq \underline{0}, \quad t = 0,1,\ldots,T, \\ &\underline{x}_0 = \hat{\underline{x}}, \end{aligned} \tag{7}$$

where $\hat{\underline{x}} \in \mathbb{R}^{2NK}$ is a given initial condition for the dynamic system (1)–(2), representing the occupancy and height of the shelves of the AVF at $t = 0$, $E\{\cdot\}$ is the expected value performed with respect to the sequence of disturbances, and $c_k > 0$, $k = 1,\ldots,K$ are weight coefficients. The value of such coefficients can be tuned to regulate the number of seedings of one crop type over the other.

III. Scheduling of Sowings with Model Predictive Control

As also discussed in Section II, in practical scenarios, the growth trajectory of plants may differ from the expected one due to a variety of reasons, such as suboptimal humidity or temperature conditions. This potential deviation is incorporated into our model through disturbance variables $\xi_{i,t}$ for $i = 1,\ldots,N$ and $t = 0,1,\ldots,T$, within the dynamics system (1)–(2). The inclusion of disturbances introduces the need for an expectation operator in the cost function of (7), thus complicating the search for an optimal solution. In fact, the knowledge of the probability density function of the disturbances or the availability of accurate forecasts for the disturbance values over the time interval $[0, T]$ are required.

To mitigate the aforesaid limitations, we employ MPC to determine the optimal timing for sowings across the various shelves, ensuring resilience against disturbances. Specifically, we set a prediction horizon $\overline{T}$, and for every time step $t = 0,1,\ldots,T$, we construct an optimal control problem spanning the discrete interval $[t, t+\overline{T}-1]$. The goal is to maximize the number of sowings (and consequently, the number of harvests) within this timeframe. In each of these optimal control scenarios, it is assumed that crops progress at their standard growth rate, meaning that there is a height increase at every time step equal to H_k/C_k for shelves cultivated with crops of type k. The control variables to be determined are the components of the vector $\underline{u}_\tau$ for

$\tau = t, t+1, \ldots, t+\overline{T}-1$. In more detail, we have to solve the following problem at each time instant $t = 0, 1, \ldots, T$:

$$\begin{aligned}
&\max_{\underline{u}_t,\ldots,\underline{u}_{t+\overline{T}-1}} \sum_{\tau=t}^{t+\overline{T}} \sum_{i=1}^{N} \sum_{k=1}^{K} c_k \, s_{i,k,\tau}, \\
&\text{subject to} \\
&\underline{x}_{\tau+1} = f(\underline{x}_\tau, \underline{u}_\tau, \underline{0}), \quad \tau = t, t+1, \ldots, t+\overline{T}-1, \\
&g(\underline{x}_\tau, \underline{u}_\tau, \underline{0}) \leq \underline{0}, \quad \tau = t, t+1, \ldots, t+\overline{T}, \\
&\underline{x}_t = \underline{x}_t^\star,
\end{aligned} \tag{8}$$

where $\underline{x}_t^\star \in \mathbb{R}^{NK+N}$ is the state of the system at time t, while f and g are the same functions defined in (7).

Problem (8) differs from problem (7) since the former is defined on the discrete interval $[t, t+\overline{T}]$ rather than in the entire horizon $[0, T]$, and with the disturbance vector $\underline{\xi}_\tau = \underline{0}$ for all $\tau = t, t+1, \ldots, t+\overline{T}$. This allows avoiding the expectation operator in the cost function of (8). Moreover, the initial condition $\underline{x}_t^\star$ at the beginning of the interval $[t, t+\overline{T}]$ is the state of the system at time t. Let $\underline{u}_t^\star, \ldots, \underline{u}_{t+\overline{T}-1}^\star$ be the optimal control inputs obtained by solving (8). According to the receding horizon principle of MPC, we retain and apply to the system only the first control inputs $\underline{u}_t^\star$.

In more detail, starting from time $t = 0$ and an initial condition $\hat{\underline{x}}_0$ for the occupancy and height of the various shelves of the greenhouse, we solve the noise-free problem (8) defined over the discrete interval $[0, \overline{T}]$ and obtain the optimal control input $\underline{u}_0^\star$, after having discarded all the subsequent control inputs within the interval. This control input is applied to the system, which evolves according to the state equation (1)–(2), with the possible effect of the disturbance $\underline{\xi}_0$, thus obtaining the new state vector $\underline{x}_1$. At this point, the new control input $\underline{u}_1^\star$ is obtained by solving a new optimal control problem (8) with a one-step-forward shift of the control horizon, i.e., optimization is performed in the interval $[1, 1+\overline{T}]$. The procedure is iterated up to time T. The use of the updated state $\underline{x}_t$ at each time step accounts for the presence of disturbances acting on the system and implements the typical feedback mechanism of MPC.

The optimal control problem (8) can be re-written as an mixed-integer linear programming (MILP) problem with decision variables $x_{i,k,t}$, $h_{i,k,t}$, $s_{i,k,t}$, and $\delta_{i,k,t}$, where $\delta_{i,k,t}$ is an auxiliary binary variable that is equal to 1 if $h_{i,k,t} < H_k$, otherwise it is equal to 0, i.e., it is equal to 0 if the plants in shelf i have reached the height for harvest, otherwise it is equal to 1. All the constraints in the function g, i.e., (1)–(6) can be written in terms of linear constraints by introducing a very large, positive constant M, which makes each constraint active or trivially satisfied. The overall MILP problem to be solved is the following:

$$\max \textstyle\sum_{\tau=t}^{\overline{T}} \sum_{i=1}^{N} \sum_{k=1}^{K} c_k \, s_{i,k,\tau} \tag{9}$$

subject to

$$x_{i,k,\tau+1} \leq M(s_{i,k,\tau} + h_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{10}$$

$$x_{i,k,\tau+1} \geq -M(s_{i,k,\tau} + h_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{11}$$

$$x_{i,k,\tau+1} \leq M(s_{i,k,\tau} + \delta_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{12}$$

$$x_{i,k,\tau+1} \geq -M(s_{i,k,\tau} + \delta_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{13}$$

$$x_{i,k,\tau+1} \leq x_{i,k,\tau} + M(1 - x_{i,k,\tau} + 1 - \delta_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{14}$$

$$x_{i,k,\tau+1} \geq x_{i,k,\tau} - M(1 - x_{i,k,\tau} + 1 - \delta_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{15}$$

$$x_{i,k,\tau+1} \leq 1 + M(1 - s_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{16}$$

$$x_{i,k,\tau+1} \geq 1 - M(1 - s_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{17}$$

$$h_{i,k,\tau+1} \leq M(s_{i,k,\tau} + h_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{18}$$

$$h_{i,k,\tau+1} \geq -M(s_{i,k,\tau} + h_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{19}$$

$$h_{i,\tau+1} \leq M(s_{i,k,\tau} + \delta_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{20}$$

$$h_{i,\tau+1} \geq -M(s_{i,k,\tau} + \delta_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{21}$$

$$h_{i,k,\tau+1} \leq h_{i,\tau} + H_k/C_k + M(1 - x_{i,k,\tau} + 1 - \delta_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{22}$$

$$h_{i,k,\tau+1} \geq h_{i,\tau} + H_k/C_k - M(1 - x_{i,k,\tau} + 1 - \delta_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{23}$$

$$h_{i,k,\tau+1} \leq H_k/C_k + M(1 - s_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{24}$$

$$h_{i,k,\tau+1} \geq H_k/C_k - M(1 - s_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{25}$$

$$h_{i,k,\tau} < H_k + M(1 - \delta_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{26}$$

$$h_{i,\tau} \geq H_k - M\delta_{i,k,\tau}, \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{27}$$

$$\textstyle\sum_{k=1}^{K} x_{i,k,\tau} \leq 1, \quad i = 1, \ldots, N, \quad \tau = t, \ldots, \overline{T}, \tag{28}$$

$$\textstyle\sum_{k=1}^{K} s_{i,k,\tau} \leq 1, \quad i = 1, \ldots, N, \quad \tau = t, \ldots, \overline{T}, \tag{29}$$

$$s_{i,k,\tau} \leq M(1 - x_{i,k,\tau} + 1 - \delta_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{30}$$

$$s_{i,k,\tau} \geq -M(1 - x_{i,k,\tau} + 1 - \delta_{i,k,\tau}), \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{31}$$

$$\textstyle\sum_{i=1}^{N} H_{\text{fixed}} + \sum_{k=1}^{K} h_{i,k,\tau} \leq H_{\text{tot}}, \quad \tau = t, \ldots, \overline{T}, \tag{32}$$

$$x_{i,k,0} = \hat{x}_{i,k}, \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \tag{33}$$

$$h_{i,k,0} = \hat{h}_{i,k}, \quad i = 1, \ldots, N, \tag{34}$$

$$x_{i,k,\tau} \in \{0, 1\}, \quad i = 1, \ldots, N, \quad k = 1, \ldots, K, \quad \tau = t, \ldots, \overline{T}, \tag{35}$$

$$h_{i,k,\tau} \geq 0, \quad i=1,\ldots,N, \quad k=1,\ldots,K, \quad \tau=t,\ldots,\overline{T}, \tag{36}$$

$$s_{i,k,\tau} \in \{0,1\}, \quad i=1,\ldots,N, \quad k=1,\ldots,K, \quad \tau=t,\ldots,\overline{T}, \tag{37}$$

$$\delta_{i,k,\tau} \in \{0,1\}, \quad i=1,\ldots,N, \quad k=1,\ldots,K, \quad \tau=t,\ldots,\overline{T}. \tag{38}$$

The cost function in (9) aims to maximize seedings. Constraints (10)–(11), (12)–(13), (14)–(15), and (16)–(17) accounts for the first, second, third, and fourth condition in state equation (1). Constraints (18)–(19), (20)–(21), (22)–(23), and (24)–(25) accounts for the first, second, third, and fourth condition in state equation (2). Equations (26)–(27) implement the relationship between $\delta_{i,k,t}$ and $h_{i,k,t}$. Constraints (28), (29), (30)–(31), and (32) are the equivalent version of (3), (4), (5), and (6), respectively, with a span from $\tau = t$ to $\tau = t + \overline{T}$. Constraints (33)–(34) impose initial conditions $\hat{x}_{i,k}$ and $\hat{h}_{i,k}$ for the occupancy and height in all the shelves as in (1)–(2), Lastly, (35)–(38) define the decision variables.

IV. Numerical Results

We present the results of the simulations conducted to validate the effectiveness of the proposed scheduling method. We adopted a time discretization with a sampling interval Δt equal to 1 day, spanning a year-long horizon, i.e., T was set to 365 days. The solution of each MPC optimal control problem (8) was performed over a computer equipped with a 3.6 GHz Intel i9 CPU and 64 GB or RAM.

Similar to prior studies [4], [5], [11], our investigation is based on the cultivation of lettuce, wheat, and basil, i.e., we chose $K = 3$. Such types of crops were chosen for their disparate growth characteristics in terms of height and duration, thus enabling a comprehensive assessment of the effectiveness of the proposed approach across diverse crop types. Specifically, we assume a cultivation cycle of $C_1 = 25$ days and a harvest height of $H_1 = 30$ cm for the lettuce, a cultivation cycle of $C_2 = 70$ days and a harvest height of $H_2 = 50$ cm for the wheat, as well as a cultivation cycle of $C_3 = 40$ days and a harvest height of $H_3 = 20$ cm for the basil. The coefficients c_k in the cost function of (7) and (8) were fixed to 0.1, 1, and 0.5, respectively. The goal is to penalize more the cultivation of the crop with a shorter growth cycle, which tends to be privileged as regards the maximization of sowings. Likewise in [6], we considered an industrial AVF with a total height $H_{\text{tot}} = 600$ cm and a number of shelves equal to $N = 15$. The heights of each module composing the AVF were fixed to $H_s = 10$ cm, $H_h = 5$ cm, and $H_f = 10$ cm, summing up to a fixed module height of $H_{\text{fixed}} = 25$ cm when no plants are cultivated.

Following [8], we assessed the performances of the proposed approach across various configurations for what concerns control horizon $\overline{T}$ and disturbances. As regards the horizon $\overline{T}$, we focused on lenghts of 30 and 50 days. Concerning disturbances, we considered the combinations of random disturbances drawn from Gaussian probability density functions characterized by mean $\mu_{\underline{\xi}}$ equal to -0.5, -0.1, 0, 0.1, 0.5 and standard deviation $\sigma_{\underline{\xi}}$ equal to 0, 0.1, 0.5. These values allow to evaluate performance with a diverse range of conditions, starting from the case in which we have a slowdown with respect to the nominal growth of crops (negative values of the mean $\mu_{\underline{\xi}}$, as the height increase from a time step to the following one is reduced on the average), to the one with a speed up in the growth of crops (positive values of the mean $\mu_{\underline{\xi}}$, as the height increase from a time step to the following one is increased on the average). In all the considered cases, the greenhouse was assumed to be empty at $t = 0$, i.e., $\hat{x}_{i,k} = 0$ and $\hat{h}_{i,k} = 0$ for all $i = 1,\ldots,N$ and $k = 1,\ldots,K$.

TABLE I
Summary of the simulation results.

$\overline{T}$	$\mu_{\underline{\xi}}$	$\sigma_{\underline{\xi}}$	n_{tot}	n_{lettuce}	n_{wheat}	n_{basil}	CPU time (s)
		0	29	16	2	11	1042.72
	-0.5	0.1	28	15	4	9	1009.77
		0.5	33	18	3	12	1176.02
		0	64	19	10	35	798.06
	-0.1	0.1	67	23	11	33	841.00
		0.5	64	19	8	37	998.87
		0	94	22	13	59	1322.61
30	0	0.1	100	20	11	69	1227.48
		0.5	44	15	10	19	915.47
		0	62	17	10	35	830.64
	0.1	0.1	84	21	14	49	928.07
		0.5	87	25	11	51	864.70
		0	86	34	8	44	812.24
	0.5	0.1	84	33	9	42	833.87
		0.5	121	39	12	70	801.97
		0	65	58	0	7	2015.83
	-0.5	0.1	53	44	0	9	1966.37
		0.5	21	5	0	16	2422.61
		0	96	8	0	88	2504.56
	-0.1	0.1	20	4	1	15	1252.10
		0.5	93	13	1	79	2471.28
		0	116	8	0	108	2534.49
50	0	0.1	117	10	1	106	2867.45
		0.5	84	9	1	74	2037.66
		0	18	2	0	16	1206.73
	0.1	0.1	134	10	1	123	2869.22
		0.5	117	11	1	105	2297.61
		0	128	4	0	124	1764.64
	0.5	0.1	140	4	0	136	1774.91
		0.5	159	4	0	155	2244.65

Performances were evaluated by considering:

- the total number of sowings within the discrete interval $[0, T]$, denoted by n_{tot};
- the number of sowings for lettuce, wheat, and basil, denoted by n_{lettuce}, n_{wheat}, and n_{basil}, respectively;
- the total height occupied by crops at each time step, denoted by $\overline{h}_t = \sum_{i=1}^{N} h_{i,t}$, $t = 0,1,\ldots,T$;
- the percentage of occupied height with respect to the total height, denoted by $\overline{p}_t = \sum_{i=1}^{N} h_{i,t}/H_{\text{tot}} \cdot 100$, $t = 0,1,\ldots,T$;
- the average CPU time required to solve each MPC problem (8) at a given time instant.

The obtained results are summarized in Table I. Figure 1 depicts the scheduling of the three crop types, the occupied height of the greenhouse divided in the components of the modules, and the percentage of occupied height with respect to the total height for $N = 15$, $\overline{T} = 30$, $\mu_{\underline{\xi}} = 0.1$, and $\sigma_{\underline{\xi}} = 0.1$. It turns out that the proposed MPC approach guarantees the

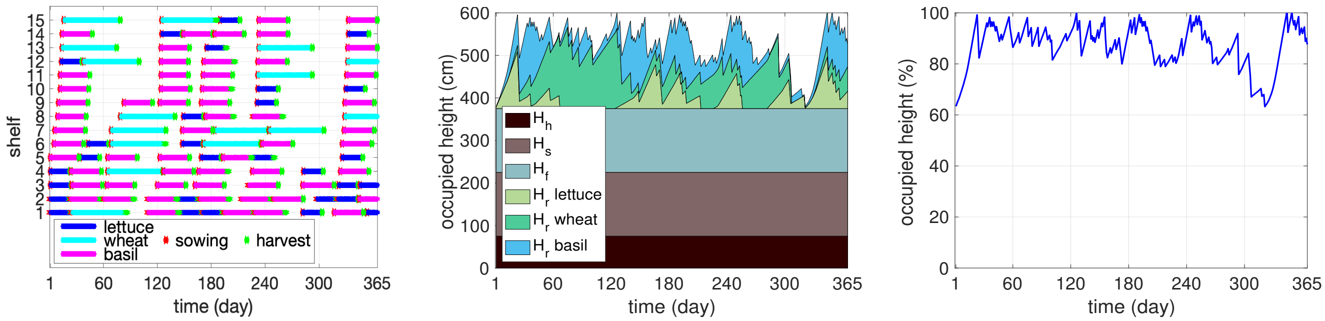

Fig. 1. Scheduling of sowings, growing, and harvests (left), occupied height of the greenhouse divided in the components of the various modules (center), and percentage of occupied height (right). The plots correspond to $\overline{T} = 30$, $\mu_{\underline{\xi}} = 0.1$, and $\sigma_{\underline{\xi}} = 0.1$.

cultivation of the crops even with large deviations from the nominal growth. A slowdown in the growth results into a reduced amount of sowings as compared to the nominal case with $\mu_{\underline{\xi}} = 0$ and $\sigma_{\underline{\xi}} = 0$. In fact, plants need more cultivation days before being ready for harvest. On the contrary, a speedup in the growth allows an increase of the production with respect to the nominal growth curve, as the length of the cultivation cycle is reduced. The basil is the preferred crop type for cultivation, as it is characterized by a reduced height at harvest, which allow a reduced occupation of the overall height of the greenhouse. Instead, the wheat is the crop time with the minimum amount of sowings due to the large height at harvest and long cultivation cycle. As a consequence, the cultivation of wheat requires a large amount of resources, i.e., both time and vertical space, and therefore too many sowings of such a type of crop result into a reduced number of overall harvests. The vertical space available in the greenhouse is well exploited in almost all configurations. For instance, in the configuration reported in Figure 1, the mean over time of the percentage of occupied height is 87.13%.

As regards the control horizon $\overline{T}$ used within the MPC problem (8), the larger the control horizon, the larger the number of sowings. However, concerning CPU times, the longer the control horizon $\overline{T}$, the higher the required computational burden, as optimal control problems with a larger number of unknowns have to be solved. However, the maximum required CPU time is always less than 3000 s, which is quite a reduced value as compared to the slow dynamics of crop growth, thus guaranteeing the possibility of applying the proposed approach on line during crop cultivation. The increased total number of sowings is obtained by reducing the number of sowings of lettuce and wheat, which require more resources as compared to the basil. This result may suggest that a better tuning of the coefficients c_k, $k = 1, \dots, K$, appearing in the cost function of (8), or the formulation of the optimization problem as a multi-objective one, with the computation of the full Pareto front to allow a better distribution of sowings among the various crop types.

V. Conclusions and Future Works

We have presented an approach based on MPC to schedule the simultaneous sowing of several types of crops within an AVF. An MPC problem has been formulated as a linear mixed-integer programming problem over a certain time horizon to account for the disturbances affecting the crop growth. Preliminary simulation results have validated the effectiveness of the proposed approach in optimizing production while maintaining the total height used by crops within prescribed limits. Future efforts will be devoted to combine the proposed scheduling approach with real crop growth models, to study the effectiveness of the approach with a different number of cultivation shelves, and to investigate the existence of theoretical guarantees on the MPC formulation.

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