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Passivity-Based Hybrid Systems Approach to Repetitive Learning Control for FES-Cycling with Control Input Saturation

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***Abstract*—Functional electrical stimulation (FES)-cycling is an effective method of rehabilitation for people with neuromuscular disorders. Muscle stimulation and electric motor inputs are designed to complement the rider's volitional pedaling, but open challenges remain in the analysis of the stability and robustness of the human-machine system under the influence of switching between muscle and motor inputs. Discontinuous switching between muscle stimulation inputs and motor input motivates the use of a hybrid systems analysis, reducing gain conditions compared to a switched systems analysis and yielding robustness to disturbances. In this paper, repetitive learning control (RLC)-based feedforward terms for each muscle group and electric motor are designed to improve cadence tracking and reduce high-gain feedback terms that can cause chattering effects. Muscle stimulation limits are systematically considered for the safety and comfort of the rider, and a cadence controller is designed integrating RLC and robust control terms to account for input saturation. A passivity-based analysis ensures the hybrid system is flow output strictly passive from the rider's volitional effort to the tracking error output. Moreover, the position and cadence tracking errors are shown to asymptotically converge based on a Lyapunov-like stability analysis.**

***Index Terms*—Functional Electrical Stimulation (FES), Repetitive Learning Control, Hybrid Systems, Passivity**

I. Introduction

Functional electrical stimulation (FES)-induced cycling is an effective rehabilitation method for individuals with spinal cord injuries, cerebral palsy, hemiparesis secondary to stroke, and a variety of other conditions [1]–[3]. Continuous high-gain stimulation during the exercise can lead to premature fatigue. Thus, motivation exists to reduce high-gain and high-frequency control for muscle activation via FES, when possible, and to replace FES input with electric motor input in regions of the crank cycle called kinematic deadzones (KDZs), where stimulation is kinematically ineffective [4]. Furthermore, because high levels of stimulation can be uncomfortable for the rider even before fatigue buildup, stimulation intensity should be capped at some rider-defined threshold, and the motor controller should be designed to provide supplementary assistance when the muscle input is saturates [5]. However, explicitly accounting for the muscle input saturation poses technical challenges in the control design stability analysis, potentially leading to uniformly ultimately bounded results that are typical for saturated systems.

Due to the periodic nature of many rehabilitative tasks such as FES-cycling, learning control techniques such as iterative learning control (ILC) or repetitive learning control (RLC) are well-suited for closed-loop FES control. Learning-based inputs can be included as feedforward terms in the control input to compensate for the periodic cycling dynamics reducing high-gain, high-frequency feedback terms. ILC-based controllers typically require the initial conditions to be reset at the start of each iteration and have been used in upper-limb exercises [6]–[8]. Although ILC can be used for cycling as in [9], RLC-based designs (that do not require state resetting conditions) are more fitting for the continuous operation of a cycle since the position of the cycle crank naturally resets after each revolution. Several works investigate the use of RLC in FES-cycling control [10]–[12]. Recent work in [12] develops separate RLC-based feedforward terms for each of the stimulated muscle groups and the electric motor to track a periodic cycle cadence trajectory using a switched systems approach, but only considers unsaturated muscle control inputs that could produce unsafe or uncomfortable stimulation levels. Moreover, the control design and analysis in [12] does not account for the rider's volitional input during cycling. The addition of stimulation limits and volitional input complicates the stability analysis of the learning-based control approach, requiring the consideration of the case when some combination of muscle groups hit their saturation limit. In addition, it is unclear if embedding learning inputs into a saturated control input can break down traditional asymptotic tracking results.

Hybrid systems are a generalization of switched systems that are able to capture complex dynamics by modeling both continuous- and discrete-time behavior [13]. Switching between different control inputs at different points in the crank cycle naturally motivates the use of hybrid systems approach that has been shown to be robust to discontinuous dynamics and reduce gain conditions in FES-cycling when

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compared to a switched systems analysis [14]. The close physical interaction of an electric motor with a human rider prompts a passivity-based analysis to ensure safe and compliant interaction between stimulation and electric motor control inputs and human rider. Several passivity-based approaches have been used to design controllers for FES rehabilitation systems and exercise machines such as [9], [11], [14], [15] and [15]. While motorized FES-cycles are modeled as a hybrid system in recent works such as [14], [16], [17], results in [14] and [16] do not develop adaptive or learning control inputs. Thus, this paper fills a significant gap for the design of learning-based controllers for hybrid systems in the context of safe human-machine interaction.

Inspired by the control scheme in [12], an RLC-based control law is developed and analyzed in this paper using a hybrid passivity-based approach. Motivated to ensure rider comfort and safety, stimulation limits are enforced in the developed saturated FES controller. The electric motor input is designed to provide assistance both in the KDZs and when the FES input saturates. Despite discrete changes to the torque input to the system due to discontinuous switching between muscle and motor control inputs, the error trajectories evolve continuously prompting a hybrid systems analysis. A passivity-based analysis shows the developed adaptive control scheme is flow output strictly passive from the rider's volitional input to the tracking error output, and in the absence of the rider's volition, a Lyapunov-like stability analysis guarantees asymptotic stability of the position and cadence tracking errors and robustness to unmodeled disturbances despite potential FES input saturation with reduced gain conditions than would be achieved with a switched systems approach.

II. Dynamic Model

The single degree-of-freedom cycle-rider system can be modeled as [4]:

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) + P(q,\dot{q}) + b_c\dot{q} + \tau_{vol}(t) = B_M u_m(t) + B_E u_e(t), \tag{1}$$

where $q : \mathbb{R}_{\geq 0} \to \mathcal{Q}$ denotes the measurable angle of the cycle's crank, $\dot{q} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ denotes the calculable angular velocity, $\ddot{q} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ denotes the unmeasurable angular acceleration of the crank, and $\mathcal{Q} \subseteq \mathbb{R}$ denotes the set of all possible crank angles. The combined inertial effects of both the cycle and rider is denoted as $M : \mathcal{Q} \to \mathbb{R}_{>0}$. The centripetal-Coriolis forces, gravitational forces, passive viscoelastic tissue forces of the rider, viscous damping coefficient of the cycle, and torque due to the volitional efforts of the rider are denoted as $V : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$, $G : \mathcal{Q} \to \mathbb{R}$, $P : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$, $b_c \in \mathbb{R}_{>0}$, and $\tau_{vol} : \mathbb{R}_{\geq 0} \to \mathbb{R}$, respectively. The terms on the right-hand side of (1) represent the torque applied about the crank axis by the FES-induced lower-limb muscle contractions and the electric motor, respectively, where $B_M,\ B_E \in \mathbb{R}_{\geq 0}$ are lumped, switched control effectiveness terms for the muscle stimulation and electric motor, respectively, $u_m : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is the stimulation input applied to each of the six muscles $m \in \mathcal{M}$, where $\mathcal{M} \triangleq \{Quad_L, Quad_R, Gl_L, Gl_R, Ham_L, Ham_R\}$, representing the left (L) and right (R) quadriceps (Quad), gluteal (Gl), and hamstring (Ham) muscle groups, and $u_e : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is the motor current control input. The terms B_M and B_E are defined as [4]

$$B_M \triangleq \sum_{m\in\mathcal{M}} B_m(q,\dot{q})\, k_m \sigma_m(q), \tag{2}$$

$$B_E \triangleq B_e k_e \sigma_e(q), \tag{3}$$

respectively, where the uncertain and nonlinear muscle control effectiveness for each stimulated muscle $B_m : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}_{\geq 0}$ relates the stimulation input to output torque, the unknown motor control torque constant $B_e \in \mathbb{R}_{\geq 0}$ relates the motor's input current to its output torque, $\sigma_m \in \{0,1\}$ and $\sigma_e \in [0,1]$ denote subsequently defined switching signals, and $k_m \in \mathbb{R}_{>0}$ and $k_e \in \mathbb{R}_{>0}$ are constant control gains developed for each muscle group and the electric motor, respectively.

The system in (1) has the following properties [4]:

Property 1. The unknown terms in (1) can be bounded as $c_m \leq M \leq c_M$, $|V| \leq c_V |\dot{q}|$, $|G| \leq c_G$, and $|P| \leq c_{P1} + c_{P2}|\dot{q}|$, for all $q \in \mathcal{Q}$ and $\dot{q} \in \mathbb{R}$, where c_m, c_M, c_V, c_G, c_{P1}, $c_{P2} \in \mathbb{R}_{>0}$ are known constants.

Property 2. The first time derivative of the inertial and centripetal-Coriolis terms satisfy $\frac{1}{2}\dot{M}(q) - V(q,\dot{q}) = 0$, for all $q \in \mathcal{Q}$ and $\dot{q} \in \mathbb{R}$.

Property 3. The muscle control effectiveness B_m and the motor control effectiveness B_e can be bounded as $c_{b_m} \leq B_m \leq c_{B_m}$ for each $m \in \mathcal{M}$, $q \in \mathcal{Q}$, $\dot{q} \in \mathbb{R}$, and $c_{b_e} \leq B_e \leq c_{B_e}$, respectively, where c_{b_m}, c_{B_m}, c_{b_e}, $c_{B_e} \in \mathbb{R}_{>0}$ are known constants.

III. Control Formulation

A. Control Objective

The control objective is for the cycle crank to track a smooth desired trajectory $q_d : \mathbb{R}_{\geq 0} \to \mathbb{R}$. It is assumed that the desired trajectory is selected such that the crank position and its first two time derivatives are bounded as $|q_d(t)| \leq \bar{q}_d$, $|\dot{q}_d(t)| \leq \bar{\dot{q}}_d$, and $|\ddot{q}_d(t)| \leq \bar{\ddot{q}}_d$ for all $t \in \mathbb{R}_{\geq 0}$, where $\bar{q}_d$, $\bar{\dot{q}}_d$, $\bar{\ddot{q}}_d \in \mathbb{R}_{>0}$ are known positive constants. The desired trajectory is designed to be periodic in the sense that $q_d(t) = q_d(t-T)$, $\dot{q}_d(t) = \dot{q}_d(t-T)$, and $\ddot{q}_d(t) = \ddot{q}_d(t-T)$, where $T \in \mathbb{R}_{>0}$ is the known constant time period. To quantify the tracking performance, an integral position error term $e_0 : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is defined as

$$e_0 \triangleq \int_0^t (q_d(\varphi) - q(\varphi))\, d\varphi. \tag{4}$$

The measurable auxiliary error signals $e_1 : \mathbb{R}_{\geq 0} \to \mathbb{R}$ and $r : \mathbb{R}_{\geq 0} \to \mathbb{R}$ are defined as

$$e_1 \triangleq \dot{e}_0 + \alpha_0 e_0, \tag{5}$$

$$r \triangleq \dot{e}_1 + \alpha_1 e_1, \tag{6}$$

respectively, where $\alpha_0,\ \alpha_1 \in \mathbb{R}_{\geq 0}$ are user-defined constants.

To aid in the subsequent development, auxiliary signals $W_d : \mathbb{R}_{\geq 0} \to \mathbb{R}$ and $N_d : \mathbb{R}_{\geq 0} \to \mathbb{R}$ are defined as

$$W_d\left(t\right) \triangleq \sum_{i \in \mathcal{I}} \left(M_i\left(q_d\right)\ddot{q}_d + V_i\left(q_d, \dot{q}_d\right)\dot{q}_d + G_i\left(q_d\right)\right), \quad (7)$$

$$N_d \triangleq c_{P1} + \left(c_{P2} + b_c\right)\dot{q}_d, \quad (8)$$

respectively, where $\mathcal{I} \triangleq \{\mathcal{M}, \mathrm{e}\}$. Because of the boundedness of the desired trajectory, (7) can be bounded as $\|W_d\left(t\right)\| \leq \beta_r$ where $\beta_r \in \mathbb{R}_{>0}$ is a known positive constant. Furthermore, (7) is periodic in the sense that $W_d\left(t\right) = \sum_{i\in\mathcal{I}} \mathrm{sat}_{\ \beta_i}\left(W_{d,i}\left(t-T\right)\right)$, where $\beta_i \in \mathbb{R}_{>0}, \forall i \in \mathcal{I}$, are selectable bounding constants satisfying $\beta_r < \beta_i$. The auxiliary signal in (8) can be upper-bounded by $|N_d| \leq \overline{N}_d$, where $\overline{N}_d \in \mathbb{R}_{>0}$ is a known positive constant.

Taking the time derivative of (6), pre-multiplying by M, substituting (1), and performing algebraic manipulation yields the open-loop error system

$$\begin{aligned} M\dot{r} = & -Vr + \chi + \tau_{vol}\left(t\right) - B_M u_m\left(t\right) \\ & - B_E u_e\left(t\right) + W_d + N_d - e_1, \end{aligned} \quad (9)$$

where the auxiliary signal $\chi : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is defined as

$$\begin{aligned} \chi \triangleq\ & M\left(q\right)\left(\ddot{q}_d + \left(\alpha_0 + \alpha_1\right)\dot{e}_1\right) - M\alpha_0^2\dot{e}_0 \\ & + V\left(q,\dot{q}\right)\left(\dot{q}_d - \alpha_0^2 e_0 + \left(\alpha_0 + \alpha_1\right)e_1\right) + G\left(q\right) \\ & + P\left(q,\dot{q}\right) + b_c\dot{q} - W_d - N_d + e_1. \end{aligned} \quad (10)$$

Using Property 1 and the Mean Value Theorem, χ can be upper-bounded as $\chi \leq \rho\left(\|z\|\right)\|z\|$, where $\rho\left(\cdot\right) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a known positive, strictly increasing, radially unbounded function, and the error signal vector $z \in \mathbb{R}^3$ is defined as $z \triangleq \left[e_0, e_1, r\right]^{\top}$ [18, Appendix A].

B. Control Design

To delay the onset of muscle fatigue, switching signals allow for each muscle to only be stimulated in the region of the crank cycle where it is able to effectively produce forward motion of the crank [4], and high-gain high-frequency terms are reduced in favor of RLC-based feedforward terms that compensate for the system's periodic dynamics. The motor is activated in the regions where no muscle group is able to meaningfully contribute to the tracking objective, called the kinematic deadzones (KDZs). Limits are placed on the stimulation level for the comfort and safety of the rider. Because the intensity of FES to each muscle may be limited in the stimulation region by the rider's stimulation comfort threshold, the motor switching signal is designed to supplement the muscle input when necessary.

To limit the stimulation level preventing rider discomfort, a maximum stimulation threshold $\beta_l \in \mathbb{R}_{>0}$ for each muscle group is selected by the rider. The stimulation intensity applied to each muscle group $u_m : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is defined as[1]

$$u_m\left(t\right) \triangleq \mathrm{sat}_{\beta_l}\left(u_{\mathrm{FES}}\right), \forall m \in \mathcal{M}, \quad (11)$$

[1]The saturation function $\mathrm{sat}_{\beta}\left(\cdot\right)$ is defined such that $\mathrm{sat}_{\beta}\left(\Omega\right) \triangleq \Omega$ for $|\Omega| \leq \beta$ and $\mathrm{sat}_{\beta}\left(\Omega\right) \triangleq \mathrm{sgn}\left(\Omega\right)\beta$ for $|\Omega| > \beta$.

where $u_{FES} \in \mathbb{R}_{\geq 0} \to \mathbb{R}$ is the FES control input.

Based on the subsequent analysis, the input to each muscle group and the electric motor is defined as

$$\begin{aligned} u_{\mathrm{FES}}\left(t\right) = u_e\left(t\right) \triangleq\ & k_1 r + \left(k_2 + k_3\rho\left(\|z\|\right)\|z\|\right)\mathrm{sgn}\left(r\right) \\ & + \widehat{W}_d + k_4\left\|\widehat{W}_d\right\|\mathrm{sgn}\left(r\right), \end{aligned} \quad (12)$$

where $k_1,\ k_2,\ k_3,\ k_4 \in \mathbb{R}_{>0}$ are user-defined constants and $\mathrm{sgn}\left(\cdot\right) : \mathbb{R} \to [-1, 1]$ is the signum function. The distributed RLC law $\widehat{W}_d : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is defined as

$$\widehat{W}_d\left(t\right) = \sum_{i\in\mathcal{I}} \widehat{W}_{d,i}\left(t\right) \triangleq \sum_{m\in\mathcal{M}} \widehat{W}_{d,m} + \widehat{W}_{d,e}, \quad (13)$$

which represents the sum of the RLC laws for each muscle group and the electric motor. The RLC laws denoted for each actuator $\widehat{W}_{d,m}, \widehat{W}_{d,e} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ are defined as [12]

$$\widehat{W}_{d,m}\left(t\right) \triangleq \sigma_m\left(\mathrm{sat}_{\beta_m}\left(\widehat{W}_{d,m}\left(t-T\right)\right) + k_{L,m}r\right), \quad (14)$$

$$\widehat{W}_{d,e}\left(t\right) \triangleq \sigma_e\left(\mathrm{sat}_{\beta_e}\left(\widehat{W}_{d,e}\left(t-T\right)\right) + k_{L,e}r\right), \quad (15)$$

where $k_{L,i} \in \mathbb{R}_{>0}, \forall i \in \mathcal{I}$ are learning control gains. Leveraging the cyclic nature of the system, the distributed learning control law $\widehat{W}_d$ exploits data from time $t - T$, to continuously adjust the muscle and motor control inputs through the feedforward learning terms in (12).

The piecewise right-continuous muscle stimulation switching signal $\sigma_m \in \{0, 1\}$ is defined such that

$$\sigma_m\left(q\right) \triangleq \begin{cases} 1 & q \in \mathcal{Q}_m, \\ 0 & q \notin \mathcal{Q}_m, \end{cases}$$

$\forall m \in \mathcal{M}$, where the region of the crank cycle where a particular muscle is stimulated is denoted $\mathcal{Q}_m \subset \mathcal{Q}, \forall m \in \mathcal{M}$ [4]. The switching signal for the activation of the electric motor is denoted by $\sigma_e \in [0, 1]$ and is defined as

$$\sigma_e\left(q\right) \triangleq \begin{cases} 1 & \text{if } q \notin \mathcal{Q}_{\mathcal{M}}, \\ \Gamma & \text{if } q \in \mathcal{Q}_{\mathcal{M}} \text{ and } u_m = \beta_l, \\ 0 & \text{if } q \in \mathcal{Q}_{\mathcal{M}} \text{ and } u_m < \beta_l, \end{cases}$$

$\forall m \in \mathcal{M}$, where $\mathcal{Q}_{\mathcal{M}} \triangleq \underset{m\in\mathcal{M}}{\cup} \mathcal{Q}_m, \forall m \in \mathcal{M}$ and $\Gamma \in (0, 1)$ is defined as $\Gamma \triangleq \min\left(1, \gamma\right)$. The function $\gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is the ratio between the control effort of the electric motor and the sum of effort from each muscle group [5] and is defined as

$$\gamma \triangleq \sum_{m\in\mathcal{M}} \left(\frac{k_s\sigma_m\left(q\right)u_{\mathbf{FES}}}{\beta_l} - 1\right),$$

where $k_s \in \mathbb{R}_{>0}$ is a control gain that should be selected such that $k_s > \frac{\beta_l}{u_{\mathrm{FES}}}, \forall m \in \mathcal{M}$. The design of γ and the motor switching signal allows for the motor to provide selectable supplementary assistance when the muscle saturates.

Substituting (11) and (12) into the open-loop error system in (9) and performing algebraic manipulation yields the closed-loop error system

$$\begin{aligned}\dot{r} = M^{-1}\Big(&-Vr+\chi+\tau_{vol}(t)-B_M\mathrm{sat}_{\beta_l}\Big(k_1r\\&+(k_2+k_3\rho(\|z\|)\|z\|)\,\mathrm{sgn}(r)\\&+\widehat{W}_{d,m}(t)+k_4\left\|\widehat{W}_{d,m}\right\|\mathrm{sgn}(r)\Big)\\&-B_E\Big(k_1r+(k_2+k_3\rho(\|z\|)\|z\|)\,\mathrm{sgn}(r)\\&+\widehat{W}_{d,e}(t)+k_4\left\|\widehat{W}_{d,e}\right\|\mathrm{sgn}(r)\Big)\\&+\widetilde{W}_d+\widehat{W}_d+N_d-e_1\Big),\end{aligned}\tag{16}$$

where the repetitive learning estimation error $\widetilde{W}_d \in \mathbb{R}$ is defined as $\widetilde{W}_d \triangleq W_d - \widehat{W}_d$. Because of the periodicity and boundedness of W_d, $W_d(t) = \sum_{i\in\mathcal{I}} \mathrm{sat}_{\beta_i}(W_{d,i}(t)) = \sum_{i\in\mathcal{I}} \mathrm{sat}_{\beta_i}(W_{d,i}(t-T))$. Thus, by using (13) and the RLC laws in (14) and (15),

$$\begin{aligned}\widetilde{W}_d = &\sum_{i\in\mathcal{I}} \mathrm{sat}_{\beta_i}(W_{d,i}(t-T))\\&-\sum_{m\in\mathcal{M}}\sigma_m\left(\mathrm{sat}_{\beta_m}\left(\widehat{W}_{d,m}(t-T)\right)+k_{L,m}r\right)\\&-\sigma_e\left(\mathrm{sat}_{\beta_e}\left(\widehat{W}_{d,e}(t-T)\right)+k_{L,e}r\right).\end{aligned}\tag{17}$$

IV. Hybrid System

With different combinations of muscle and motor inputs at different points in the crank cycle, the torque input to the system discretely changes. Despite these discrete changes, the error trajectories evolve continuously. This combination of discrete- and continuous-time dynamics motivates the use of a hybrid systems approach to model and analyze the system.

A. Hybrid System Preliminaries

A hybrid plant $\mathcal{H}_P \triangleq (C_P, F_P, D_P, G_P)$ with state $\xi \in \mathcal{X} \subset \mathbb{R}^n$, flow input $\nu_c \in \mathbb{R}^m$, and jump input $\nu_d \in \mathbb{R}^m$ is structured as

$$\mathcal{H}_P \quad : \quad \begin{cases}\dot{\xi} \in F_P(\xi,\nu_c) & (\xi,\nu_c)\in C_P\\ \xi^+ \in G_P(\xi,\nu_d) & (\xi,\nu_d)\in D_P,\end{cases}\tag{18}$$

where $F_P : \mathbb{R}^n \times \mathbb{R}^m \rightrightarrows \mathbb{R}^n$ represents the flow map, $C_P \subset \mathbb{R}^n \times \mathbb{R}^m$ denotes the flow set, $G_P : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ denotes the jump map, and $D_P \subset \mathbb{R}^n$ denotes the jump set. A solution to $\mathcal{H}_P$ is a function $(t, j) \to \phi(t, j)$ defined on a hybrid time domain $\mathrm{dom}\phi \subset \mathbb{R}_{\geq 0} \times \mathbb{N}$, where continuous time is denoted $t \in \mathbb{R}_{\geq 0}$ and $j \in \mathbb{N}$ represents the discrete jump variable where j indicates the j^{th} instance that ϕ jumps. A solution ϕ to $\mathcal{H}_P$ is considered maximal if there is no other solution ψ to $\mathcal{H}_P$ such that $\mathrm{dom}\phi \subset \mathrm{dom}\psi$ and $\phi(t,j) = \psi(t,j), \forall (t,j) \in \mathrm{dom}\phi$.

B. Cycle-Rider Hybrid Plant

The closed-loop error system in (16) can be modeled by a hybrid plant $\mathcal{H}_P = (C_P, F_P, D_P, G_P)$ with state $\xi = [z, \sigma_{\mathcal{I}}, Q_L, \tau]^\top \in \mathcal{X}$, where $\sigma_{\mathcal{I}} \in \{0,1\}^6 \times [0,1]$ denotes the vector of switching signals σ_i, for all $i \in \mathcal{I}$, the state $\tau \in \mathbb{R}_{\geq 0}$ denotes a timer variable, and the state space $\mathcal{X}$ is defined as $\mathcal{X} \triangleq \mathbb{R}^4 \times \{0,1\}^6 \times [0,1] \times \mathbb{R}_{\geq 0}$. An auxiliary function $Q_L : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is defined as

$$Q_L \triangleq \sum_{i\in\mathcal{I}} \int_{t-T}^{t} \left(\mathrm{sat}_{\beta_i}\left(W_{d,i}(\varphi)\right) - \mathrm{sat}_{\beta_i}\left(\widehat{W}_{d,i}(\varphi)\right)\right)^2 d\varphi,$$

and is included to aid in the following passivity analysis by helping to incorporate the repetitive learning error into the hybrid system.

The flow set $C_P \subset \mathcal{X}$ is defined as $C_P(\xi,\nu_c) \triangleq \{(\xi,\nu_c) \in \mathcal{X}\times\mathbb{R} : \tau \in [0,\tau_d]\}$, where τ_d represents the dwell-time the system must reach before switching. The timer variable τ evolves continuously with time, and resets to zero when the dwell-time τ_d is reached. In practice, τ_d corresponds to the sampling frequency. The function $F_P : \mathcal{X} \rightrightarrows \mathcal{X}$ represents the continuous-time dynamics of the system and is defined as

$$F_P(\xi,\nu_c) \triangleq \begin{bmatrix}\dot{e}_0\\ \dot{e}_1\\ \dot{r}\\ \dot{\sigma}_{\mathcal{I}}\\ \dot{Q}_L\\ \dot{\tau}\end{bmatrix} \in \begin{bmatrix} e_1-\alpha_0e_0\\ r-\alpha_1e_1\\ F_{CL}(\xi,\nu)\\ 0\\ F_Q(\xi,\nu)\\ 1\end{bmatrix},\tag{19}$$

where $F_{CL} : \mathcal{X} \rightrightarrows \mathbb{R}$ represents the closed-loop error system in (16) and $F_Q : \mathcal{X} \rightrightarrows \mathbb{R}$ is the time derivative of the auxiliary function Q_L which can be written as

$$\begin{aligned}F_Q(\xi,\nu_c) = \sum_{i\in\mathcal{I}}\Bigg(&\left(\mathrm{sat}_{\beta_i}(W_{d,i}(t)) - \mathrm{sat}_{\beta_i}\left(\widehat{W}_{d,i}(t)\right)\right)^2\\&-\left(\mathrm{sat}_{\beta_i}(W_{d,i}(t-T)) - \mathrm{sat}_{\beta_i}\left(\widehat{W}_{d,i}(t-T)\right)\right)^2\Bigg).\end{aligned}$$

The state changes according to the jump map when it is in the jump set $D_P \subset \mathcal{X}$ defined as $D_P \triangleq \{(\xi,\nu_d) \in \mathcal{X} : \tau = \tau_d\}$. The states e_0, e_1, r, and Q_L evolve in continuous time and do not change when jumps occur. The jump map and jump set model the discrete changes in torque that the system may experience due to the crank angle transitioning between muscle control effectiveness regions and the activation of the electric motor. Thus, the jump map $G_P : \mathcal{X} \rightrightarrows \mathcal{X}$ is defined as

$$G_P(\xi,\nu_d) \triangleq \begin{bmatrix} e_0^+\\ e_1^+\\ r^+\\ \sigma_{\mathcal{I}}^+\\ Q_L^+\\ \tau^+\end{bmatrix} \in \begin{bmatrix} e_0\\ e_1\\ r\\ G_{\mathcal{I}}\\ Q_L\\ 0\end{bmatrix},\tag{20}$$

where the outer semicontinuous Krasovskii regularization of the switching rules σ_m and σ_e is denoted by $G_{\mathcal{I}} : \mathcal{X} \rightrightarrows$

$\{0,1\}^6 \times [0,1]$. The components of $G_{\mathcal{I}}, \forall i \in \mathcal{I}$ are defined as

$$G_m \triangleq \begin{cases} 1 & q \in \mathcal{Q}_m \\ \{0,1\} & q \in \partial\mathcal{Q}_m \\ 0 & \text{otherwise,} \end{cases}$$

$\forall m \in \mathcal{M}$, and by

$$G_e \triangleq \begin{cases} \sigma_e & q \in \mathcal{Q}_e \\ \{0,\sigma_e\} & q \in \partial\mathcal{Q}_e \\ 0 & \text{otherwise,} \end{cases}$$

for the electric motor, where the electric motor region $\mathcal{Q}_e \subseteq \mathcal{Q}$ is defined as $\mathcal{Q}_e \triangleq \{q \in \mathcal{Q} : \sigma_e > 0\}$. These definitions represent the discrete changes in torque that the system may experience due to the crank angle transitioning between control effectiveness regions.

To facilitate the subsequent analysis, the Krasovskii regularization of the flow map in (19) is evaluated as

$$F_P^r(\xi,\nu_c) \in \begin{bmatrix} e_1 - \alpha_0 e_0 \\ r - \alpha_1 e_1 \\ F_{CL}^r(\xi,\nu) \\ 0 \\ F_Q(\xi,\nu) \\ 1 \end{bmatrix}, \tag{21}$$

where $F_{CL}^r : \mathcal{X} \rightrightarrows \mathbb{R}$ is defined as

$$\begin{aligned} F_{CL}^r(\xi,\nu_c) \triangleq M^{-1}\Big(&-Vr + \chi + \tau_{vol}(t) - B_M \mathrm{sat}_{\beta_l}\Big(k_1 r \\ &+ (k_2 + k_3\rho(\|z\|)\|z\|)\,\mathrm{SGN}(r) + \widehat{W}_{d,m}(t) \\ &+ k_4\left\|\widehat{W}_{d,m}\right\|\mathrm{SGN}(r)\Big) - B_E\Big(k_1 r \\ &+ (k_2 + k_3\rho(\|z\|)\|z\|)\,\mathrm{SGN}(r) + \widehat{W}_{d,e}(t) \\ &+ k_4\left\|\widehat{W}_{d,e}\right\|\mathrm{SGN}(r)\Big) + \widetilde{W}_d + \widehat{W}_d \\ &+ N_d - e_1\Big), \end{aligned} \tag{22}$$

where $\mathrm{SGN}(r) \triangleq \overline{\mathrm{co}}\{\mathrm{sgn}(r)\}$ denotes the closed convex hull of $\mathrm{sgn}(r)$.

V. Passivity and Stability Analysis

This section outlines the passivity and stability properties of the hybrid cycle-rider system and the developed RLC-based control input. Theorem 1 shows that the hybrid plant in (18) is flow-output strictly passive from the rider's volitional input to the system's output. Theorem 2 demonstrates that the designed input in (12) with the RLC law in (13) ensure asymptotic convergence of the cadence tracking errors. Moreover, the stability analysis of the Krasovskii regularization of the closed-loop system yields robustness to perturbations allowing for less conservative gain conditions when compared to a switched systems analysis.

To facilitate the subsequent analysis, let the set $\mathcal{M}_s \subseteq \mathcal{M}$ represent the set of muscles that are saturated at a particular time, let $\mathcal{M}_u \subseteq \mathcal{M}$ represent the remaining unsaturated muscles defined as $\mathcal{M}_u \triangleq \mathcal{M} \backslash \mathcal{M}_s$. The complete set of unsaturated actuators can be defined as $\mathcal{U} \triangleq \{\mathcal{M}_u, e\}$, and the RLC law $\widehat{W}_{d,u} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is defined as

$$\widehat{W}_{d,u}(t) \triangleq \sum_{i \in \mathcal{U}} \widehat{W}_{d,i}(t). \tag{23}$$

The combined control effectiveness term for the unsaturated actuators is $B_u \triangleq \sum_{m \in \mathcal{M}_u} B_m k_m \sigma_m + B_e k_e \Gamma$ and can be bounded as $c_{b_u} \leq B_u \leq c_{B_u}$, where $c_{b_u}, c_{B_u} \in \mathbb{R}_{>0}$ are known constants. In the case when none of the muscle groups are saturated, the lumped, switched control effectiveness term $B_\sigma \in \mathbb{R}_{\geq 0}$ is defined as

$$B_\sigma(q,\dot{q}) \triangleq B_M + B_E, \tag{24}$$

where B_M and B_E are defined in (2) and (3). By Property 3, (24) can be bounded such that $c_{b_\sigma} \leq B_\sigma \leq c_{B_\sigma}, \forall \sigma \in (1,2,...,N) \subset \mathbb{N}$, where $c_{b_\sigma}, c_{B_\sigma} \in \mathbb{R}_{>0}$ are known constants and $N \in \mathbb{R}_{>0}$ is the total number of activated muscle and electric motor input combinations. Constants $c_{b_u}, c_{B_u} \in \mathbb{R}_{>0}$ are defined as $c_{b_u} \triangleq \min\{c_{b_\sigma}, \Gamma k_e c_{b_e}\}$ and $c_{B_u} \triangleq \max\{c_{B_\sigma}, \Gamma k_e c_{B_e}\}$, respectively.

Theorem 1. *Consider the hybrid plant $\mathcal{H}_P$ in (18)-(22) with flow input $\nu_c \triangleq \tau_{vol}$ and jump input $\nu_d \triangleq 0$. The system is flow output strictly passive from input $\nu \triangleq (\nu_c,\nu_d)$ to output r on the set $\mathcal{A}^* \triangleq \{((\xi,\nu) \in C_P \cup D_P : e_0 = e_1 = r = 0,\ \nu = 0)\}$ provided the following sufficient gain conditions are satisfied:*

$$\alpha_0, \alpha_1 > \frac{1}{2}, \quad k_1 > \frac{\sum_{m \in \mathcal{M}_s} k_{L,m}}{c_{b_u}},$$

$$k_2 > \frac{\bar{N}_d + \sum_{m \in \mathcal{M}_s} \beta_m}{c_{b_u}}, \quad k_3 > \frac{1}{c_{b_u}}, \quad k_4 > \frac{1 + c_{B_u}}{c_{b_u}}. \tag{25}$$

Proof available upon request.

Passivity from the volitional effort of the rider to the output r is a metric of safe interaction between human and the machine [19]. The passivity result shows that the energy stored in the system will be no greater than the energy supplied by the rider, making the controlled system robust to the perturbation from the rider. The rider's volitional input is accomodated because the FES and electric motor controllers comply to the rider's input instead of treating it as a disturbance that would need to be overcome with high-gain, high-frequency robust control terms.

Theorem 2. *Provided the gain conditions in (25) are satisfied and in the absence of volitional effort from the rider, every maximal solution ϕ to the hybrid plant $\mathcal{H}_P$ asymptotically converges to the set $\mathcal{A}^* \triangleq \{((\xi,\nu) \in C_P \cup D_P : e_0 = e_1 = r = 0),\ \nu = 0\}$.*

Proof available upon request.

Remark 1. Because of the well-posedness of $\mathcal{H}_P^r$ and the global asymptotic stability of $\mathcal{A}^*$, the set $\mathcal{A}^*$ has some robustness properties [20, Lemma 7.20], making $\mathcal{A}^*$ robust to vanishing disturbances such as muscle spasms.

VI. Conclusion

Passivity and stability properties of a developed cycle-rider control system are proven using a hybrid systems approach. The system is modeled as a hybrid system because it experiences both flows (since the states continuously evolve) and jumps (as the control effort changes discretely between different muscle stimulation and motor activation regions). Due to the cyclic dynamics of the system, an RLC-based feedforward term is used in the cadence tracking controller to improve tracking performance and reduce high-gain, high-frequency effects. Unlike in previous works, stimulation limits are considered for rider comfort. Using a hybrid systems-based passivity analysis, the hybrid plant is shown to be flow output strictly passive from the rider's volitional input to the measured output. Additionally, the RLC law guarantees the asymptotic convergence of the error signals, despite control input saturation. Future work will focus on testing the developed control scheme, including in experiments with participants with neuromuscular disorders.

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