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# Desensitized Optimal Guidance Using Adaptive Radau Collocation

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*Abstract*— **An optimal guidance method is developed that reduces sensitivity to parametric uncertainties in the dynamic model. The method combines a previously developed method for guidance and control using adaptive Legendre-Gauss-Radau (LGR) collocation and a previously developed approach for desensitized optimal control. Guidance updates are performed such that the desensitized optimal control problem is re-solved on the remaining horizon at the start of each guidance cycle. The effectiveness of the method is demonstrated on a simple example using Monte Carlo simulation. The application of the method results in a smaller final state error distribution when compared to desensitized optimal control without guidance as well as a previously developed method for optimal guidance and control.**

## I. Introduction

The goal of an optimal control problem is to determine the state and control of a controlled dynamical system that optimizes a specified performance index while satisfying dynamic constraints, path constraints, and boundary conditions. Due to their complexity, most optimal control problems cannot be solved analytically and, thus, must be solved numerically. Numerical methods for solving optimal control problems fall into two categories: indirect methods and direct methods. In an indirect method, the calculus of variations is employed to formulate the first-order optimality conditions, leading to a Hamiltonian boundary-value problem (HBVP). In a direct method, the control and/or the state are parameterized and the optimal control problem is transcribed into a finite-dimensional nonlinear programming problem (NLP) which is solved using well known optimization methods [1], [2].

Optimal control problems are typically formulated with a reference (nominal) dynamic model and the optimized trajectory and control are obtained in an open-loop manner along the horizon of interest using this reference model. Typically, the reference optimal control is computed based on a deterministic model. However, in real systems, uncertainties in the model exist. As a result, the utilization of the reference optimal control can lead to significant perturbations from the optimal trajectory. In the case of sufficiently large uncertainties, employing the reference optimal control for any significant duration will lead to large perturbations in the state and, in order to reduce such perturbations, some form of correction will be required. Often, course corrections in the form of guidance updates are performed where the optimal control problem may be solved periodically (that is, at specified guidance update times), thus leading to closed-loop optimal control.

In the application of optimal control to high performance vehicles, the vehicle is subject to uncertainty in model parameters. When designing a reference trajectory, it is desirable to reduce sensitivities to these parametric uncertainties to promote robustness while minimizing the error in the final state in response to perturbations in the state anywhere along the trajectory. The process of determining the control that reduces sensitivities of the state to parametric uncertainty is known as desensitized optimal control. The study of desensitized optimal control first appeared in Ref. [3] for a simple optimal control problem. An extension of the work of Ref. [3] for problems with constraints can be found in Ref. [4]. Next, Ref. [5] and [6] use desensitized optimal control to solve the Mars pinpoint landing problem. The work of Ref. [5] and [6] was then extended in Ref. [7] to study perturbations resulting from parametric uncertainties in aerodynamic characteristics and atmospheric density and was further implemented using direct collocation and nonlinear programming in Ref. [8]. Furthermore, Ref. [9] studied a relationship between covariance trajectory shaping and desensitized optimal control, while Ref. [10] studied this same relationship in conjunction with trajectory design. Next, desensitized trajectory optimization was studied in Ref. [11] where the sensitivity dynamics were explored as functions of the partial derivatives of the original dynamics with respect to the state and parameters with uncertainties. In particular, a simplified form of the sensitivity dynamics from Ref. [3] was derived. This method was then applied to hypersonic trajectory optimization of a reentry problem [12] in which parameter uncertainties existed in the parasitic drag and scaling height. The cost was augmented to include the expected deviations in a user-defined penalty term.

The objective of this work is to develop a method for optimal guidance that desensitizes the reference control to parametric uncertainties while providing guidance updates to allow for corrections in the desensitized optimal control. The method developed employs a closed-loop adaptation of the method developed in Ref. [12] while employing a

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guidance strategy that is based on the work of Ref. [13]. The adaptation of the method of Ref. [12] offers an approach for desensitized optimal control while the inclusion of the method developed in Ref. [13] provides an efficient approach for performing guidance updates using adaptive Legendre-Gauss-Radau (LGR) collocation. The method developed in this paper is demonstrated on a simple example where it is shown that combining guidance updates via solving a desensitized optimal control problem reduces the variation in the final state error compared with using either open-loop desensitized optimal control or optimal guidance alone.

## II. Bolza Optimal Control Problem

Without loss of generality, consider the following Bolza optimal control problem in terms of the elapsed time, $t$, that has the domain $t \in [t_0, t_f]$. Determine the state $\mathbf{x}(t)$ and the control $\mathbf{u}(t)$ that minimizes the objective functional

$$J = \mathcal{M}(\mathbf{x}(t_0), t_0, \mathbf{x}(t_f), t_f) + \int_{t_0}^{t_f} \mathcal{L}(\mathbf{x}(t), \mathbf{u}(t), t)dt, \quad (1)$$

subject to the dynamic constraints

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t), \quad (2)$$

inequality path constraints

$$\mathbf{c}_{\min} \leq \mathbf{c}(\mathbf{x}(t), \mathbf{u}(t), t) \leq \mathbf{c}_{\max}, \quad (3)$$

and boundary conditions

$$\mathbf{b}_{\min} \leq \mathbf{b}(\mathbf{x}(t_0), t_0, \mathbf{x}(t_f), t_f) \leq \mathbf{b}_{\max}. \quad (4)$$

Suppose now that the domain, $t \in [t_0, t_f]$, is mapped to a new domain, $\tau \in [-1, +1]$ where

$$t \equiv t(\tau, t_0, t_f) = \frac{t_f + t_0}{2}\tau + \frac{t_f - t_0}{2}. \quad (5)$$

Next, let us further divide the mesh into $K$ intervals $\{\mathcal{I}_1, \ldots, \mathcal{I}_K\}$ such that $\mathcal{I} = [T_{k-1}, T_k]$, $(k = 1, \ldots, K)$ where the mesh points $(T_0, \ldots, T_K)$ are defined such that $-1 = T_0 < T_1 < \cdots < T_{K-1} < T_K = +1$. Furthermore, let $\mathbf{x}^k(\tau)$ and $\mathbf{u}^k(\tau)$ denote, respectively, the state and control in mesh interval $\mathcal{I}_k$, $(k = 1, \ldots, K)$. The Bolza optimal control problem can then be written as follows: Minimize the objective functional

$$J = M(\mathbf{x}^{(1)}(T_0), t_0, \mathbf{x}^{(K)}(T_K), t_f) + \frac{t_f - t_0}{2} \sum_{k=1}^{K} \int_{T_{k-1}}^{T_k} \mathcal{L}\left(\mathbf{x}^{(k)}(\tau), \mathbf{u}^{(k)}(\tau), t\right) d\tau, \quad (6)$$

subject to the dynamic constraints

$$\dot{\mathbf{x}}^{(k)}(\tau) = \frac{t_f - t_0}{2}\mathbf{f}(\mathbf{x}^{(k)}(\tau), \mathbf{u}^{(k)}(\tau), t), \ (k = 1, \ldots, K), \quad (7)$$

the path constraints

$$\mathbf{c}_{\min} \leq \mathbf{c}(\mathbf{x}^{(k)}(\tau), \mathbf{u}^{(k)}(\tau), t) \leq \mathbf{c}_{\max}, \ (k = 1, \ldots, K), \quad (8)$$

and the boundary conditions

$$\mathbf{b}_{\min} \leq \mathbf{b}(\mathbf{x}(T_0), t_0, \mathbf{x}(T_K), t_f) \leq \mathbf{b}_{\max}. \quad (9)$$

Finally, continuity in the state at each interior mesh point $T_k$, $(k = 1, \ldots, K-1)$ is enforced via the condition: $\mathbf{x}^k(T_k) = \mathbf{x}^{k+1}(T_k)$, $(k = 1, \ldots, K-1)$ that is enforced through utilizing the same variable in the NLP.

## III. Legendre-Gauss-Radau Collocation Method

The proposed desensitized optimal control guidance scheme employs the $hp$ form of the LGR collocation method to discretize the multiple-interval form of the trajectory optimization optimal control problem [14]–[20]. The state is approximated using a basis of Lagrange polynomials $\ell_j^{(k)}(\tau)$ [13]

$$\mathbf{x}(\tau) \approx \mathbf{X}^{(k)}(\tau) = \sum_{j=1}^{N_k+1} \mathbf{X}_j^{(k)} \ell_j^{(k)}(\tau), \quad (10)$$

the derivative of the state with respect to $\tau$ is then

$$\frac{d\mathbf{x}^{(k)}(\tau)}{d\tau} \approx \frac{d\mathbf{X}^{(k)}(\tau)}{d\tau} = \sum_{j=1}^{N_k+1} \mathbf{X}_j^{(k)} \frac{d\ell_j^{(k)}(\tau)}{d\tau}, \quad (11)$$

where

$$\ell_j^{(k)}(\tau) = \prod_{l=1, l\neq j}^{N_k+1} \frac{\tau - \tau_l^{(k)}}{\tau_j^{(k)} - \tau_l^{(k)}}. \quad (12)$$

The mesh domain is again defined on $\tau \in [-1, +1]$ where $\tau^{(k)} = (\tau_1^{(k)}, ..., \tau_{N_k}^{(k)})$ are the LGR collocation points in the $k^{th}$ mesh interval and $\tau_{N_k+1}^{(k)} = T_k$ is a noncollocated point. The problem can then be converted to a nonlinear programming problem (NLP) by writing the cost in terms of the $N_k$ collocated points as

$$J \approx \mathcal{M}(\mathbf{X}_1^{(1)}, t_0, \mathbf{X}_{N_K+1}^{(K)}, t_f) + \frac{t_f - t_0}{2} \sum_{k=1}^{K} \sum_{j=1}^{N_k} w_j^{(k)} \mathcal{L}(\mathbf{X}_j^{(k)}, \mathbf{U}_j^{(k)}, t(\tau_j^{(k)}, t_0, t_f)), \quad (13)$$

where the running cost is approximated using an $N_k$-point LGR quadrature such that $w_j^{(k)}$ are the $N_k$ LGR weights in each mesh interval. The NLP is then subject to the dynamic constraints

$$\sum_{j=1}^{N_k+1} D_{ij}^{(k)} \mathbf{X}_j^{(k)} - \frac{t_f - t_0}{2} \mathbf{f}(\mathbf{X}_i^{(k)}, \mathbf{U}_i^{(k)}, t(\tau_i^{(k)}, t_0, t_f)) = \mathbf{0}, \quad (14)$$

the boundary conditions

$$\mathbf{b}_{\min} \leq \mathbf{b}(\mathbf{X}_1^{(1)}, t_0, \mathbf{X}_{N_K+1}^{(K)}, t_f) \leq \mathbf{b}_{\max}, \quad (15)$$

and any path constraints

$$\mathbf{c}_{\min} \leq \mathbf{c}(\mathbf{X}_i^{(k)}, \mathbf{U}_i^{(k)}, t(\tau_i^{(k)}, t_0, t_f)) \leq \mathbf{c}_{\max}. \quad (16)$$

The elements of the interval LGR differentiation matrix of size $N_k \times (N_k + 1)$ are denoted $D_{ij}^{(k)}$ where $D_{ij}^{(k)} = d\ell_j^{(k)}(\tau_i^{(k)})/d\tau$ for $(i = 1, ..., N_k, j = 1, ..., N_k + 1)$.

## IV. Desensitized Optimal Control

The objective of desensitized optimal control is to determine the state and control that minimizes some performance index along with sensitivities of a user-specified function to state perturbations while satisfying dynamic constraints, boundary conditions, and any path constraints. The user-specified function acts as a penalty term in the cost that quantifies the influence of state perturbations on the final state. In the context of this research, the state perturbations are a result of parametric uncertainties in the dynamic model. In general, desensitized optimal control involves the introduction of sensitivities, $\mathbf{S}(t)$, in either matrix or function form, as states to the original problem formulation. A sensitivity matrix, as formulated by Ref. [3], allows for consideration of uncertainties with respect to time varying parameters. As a result, evaluating the objective requires propagating $(n \times m)^2 + n + m$ states where $n$ is the number of states in the original optimal control problem formulation and $m$ is the number of parameters with uncertainties. Alternatively, the sensitivity function derived in Ref. [11] assumes the parameters of interest to be constant and therefore requires only $n \times m$ states to be propagated. While different approaches have been conceived for introducing sensitivities, in this research the approach developed in Ref. [11] is employed because it reduces dimensionality compared with the approach developed in Ref. [3].

Consider a desensitized optimal control problem with dynamics of the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{p}, \mathbf{u}, t), \tag{17}$$

which are assumed to be continuous in $(\mathbf{x}, \mathbf{p}, \mathbf{u}, t)$ and continuously differentiable with respect to the state, $\mathbf{x}$, and the nominal parameter values, $\mathbf{p}$. Now suppose the solution to those dynamics is given as

$$\mathbf{x}(\mathbf{p}, t) = \mathbf{x}_0 + \int_{t_0}^{t_f} \mathbf{f}(\mathbf{x}(\mathbf{p}, \tau), \mathbf{p}, \mathbf{u}(\tau), \tau) d\tau, \tag{18}$$

where the initial condition on the original state vector is known. Let the partial derivative of the state with respect to the parameters now be taken as a function of the elapsed time, $t$, and parameters

$$\frac{\partial \mathbf{x}}{\partial \mathbf{p}}(\mathbf{p}, t) = \int_{t_0}^{t_f} \left[ \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{x}(\mathbf{p}, \tau), \mathbf{p}, \mathbf{u}(\tau), \tau) \frac{\partial \mathbf{x}}{\partial \mathbf{p}}(\mathbf{p}, \tau) + \frac{\partial \mathbf{f}}{\partial \mathbf{p}}(\mathbf{x}(\mathbf{p}, \tau), \mathbf{p}, \mathbf{u}(\tau), \tau) \right] d\tau. \tag{19}$$

Taking the derivative with respect to the elapsed time yields the following expression for the sensitivity dynamics:

$$\frac{d}{dt}\left[ \frac{\partial \mathbf{x}}{\partial \mathbf{p}}(\mathbf{p}, t) \right] = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{x}(t), \mathbf{p}, \mathbf{u}(t), t) \frac{\partial \mathbf{x}}{\partial \mathbf{p}}(\mathbf{p}, t) + \frac{\partial \mathbf{f}}{\partial \mathbf{p}}(\mathbf{x}(t), \mathbf{p}, \mathbf{u}(t), t), \tag{20}$$

where the sensitivity is now defined as the change in the state with respect to nominal parameter values

$$\mathbf{S}(t) = \frac{\partial \mathbf{x}}{\partial \mathbf{p}}(\mathbf{p}, t), \tag{21}$$

with $n \times m$ elements. The sensitivity dynamics are then

$$\dot{\mathbf{S}}(t) = \frac{d}{dt}\left[ \frac{\partial \mathbf{x}}{\partial \mathbf{p}}(\mathbf{p}, t) \right], \tag{22}$$

where the initial condition on the sensitivity matrix is the zero matrix due to a fixed initial state. The augmented cost is now a function of the original cost and the sensitivity function at the final time subject to some user-defined weighting term, $Q(t)$, also evaluated at the final time

$$J_A = J + \int_{t_0}^{t_f} ||\mathbf{S}(t_f)||^2_{Q(t_f)}, \tag{23}$$

where, again, the sensitivities are a function of the propagated state under nominal parameter conditions and the nominal parameters are assumed to be constant to improve computational efficiency.

## V. Desensitized Optimal Guidance

The following framework for desensitized optimal guidance combines the desensitized trajectory optimization method from Ref. [12] and mesh remapping guidance method from Ref. [13]. Consider the aforementioned Bolza optimal control problem. Suppose now that the state, $\mathbf{x}(\mathbf{p}, t)$, is subject to parametric uncertainties where the nominal values of the parameters are denoted by $\mathbf{p}$. As a result, it is desirable to reduce sensitivities to these uncertainties so that the resultant error in the final state is minimized. The sensitivity, $\mathbf{S}(t)$, of the state with respect to perturbations at any point along the trajectory is given as the solution to the ordinary differential equation [11]

$$\dot{\mathbf{S}}(t) = \mathbf{A}(t)\mathbf{S}(t) + \mathbf{B}(t), \tag{24}$$

where $\mathbf{S}(t_0) = \mathbf{0}$. Here, $\mathbf{A}(t)$ and $\mathbf{B}(t)$ are defined as

$$\mathbf{A}(t) = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{x}(\mathbf{p}, t), \mathbf{u}(t), \mathbf{p}, t), \tag{25}$$

$$\mathbf{B}(t) = \frac{\partial \mathbf{f}}{\partial \mathbf{p}}(\mathbf{x}(\mathbf{p}, t), \mathbf{u}(t), \mathbf{p}, t). \tag{26}$$

To minimize perturbations in the final state, the objective functional must be augmented, $J_A$, to include variations in a user-defined penalty term, $\mathbf{h} = \mathbf{g}(x)$.

$$J_A = J + \mathbb{E}\left( ||\delta \mathbf{h}(t_f)||^2_{\mathbf{Q}_f} + \int_{t_0}^{t_f} ||\delta \mathbf{h}(t)||^2_{\mathbf{Q}(t)} dt \right) \tag{27}$$

The penalty error term is then defined as [12]

$$\mathbb{E}(||\delta \mathbf{h}||^2_{\mathbf{Q}}) = \text{tr}\mathbf{Q}\mathbb{E}(\delta \mathbf{h} \delta \mathbf{h}^\top) \approx \text{tr}\mathbf{Q}\mathbf{G}\mathbf{S}\mathbf{P}\mathbf{S}^\top\mathbf{G}^\top, \tag{28}$$

where $\mathbf{Q}$ is a user-defined weighting matrix that is positive semi-definite, $\mathbf{G}$ is the jacobian of the penalty term, $\mathbf{h}$, and $\mathbf{S}$ is the sensitivity function with $n \times m$ elements. $\mathbf{P}$ is a user-defined, positive semi-definite covariance matrix which is a function of the nominal values of the parameters. This formulation allows for the user to define the uncertainty in the parameter through $\mathbf{P}$ while adjusting how much it is desired to desensitize the control through $\mathbf{Q}$. If $\mathbf{Q}$ is set to zero at all points in time, then the objective returns to that of the original optimal control problem.

The aforementioned problem formulation determines a desensitized reference trajectory and therefore a control which is less sensitive to perturbations in the state due to parametric uncertainties. Suppose now that it is desired to perform guidance updates such that the desensitized optimal trajectory is recalculated on the remaining horizon to allow for corrections in the desensitized optimal control. Let $s$ denote the current guidance cycle where $s \in [1, 2, 3, ..., S]$ and $D$ denote the duration of the guidance cycle. The current guidance cycle iteration then occurs on the time interval $t \in [t_0^{(s)}, t_e^{(s)}]$ where

$$t_0^{(s)} = t_0 + sD, \tag{29}$$

$$t_e^{(s)} = t_0 + (s+1)D. \tag{30}$$

At the end of each guidance cycle, the expired horizon is removed and the initial conditions are updated before the problem can be resolved on the remaining horizon. The terminal time of the previous cycle occurs between two mesh points as seen in Fig. 1. To delete the expired horizon, the desensitized optimal control, state, and sensitivity are interpolated to $t_e^{(s-1)}$ and the initial conditions become

$$\mathbf{x}_0 = \mathbf{x}(t_0^{(s)}) = \tilde{\mathbf{x}}(t_e^{(s-1)}), \tag{31}$$

$$\mathbf{S}_0 = \mathbf{0}, \tag{32}$$

while the final boundary conditions remain the same. The mesh is then remapped such that the first mesh point corresponds to $t_e^{(s-1)}$ and $t \in [t_0^{(s)}, t_f]$ maps to $\tau \in [-1, +1]$. Note that $t_e^{(s)}$ is not the same as $t_f$. The former is the final time for the simulated dynamics of the relevant guidance cycle while the latter is the terminal boundary condition on the time for the desensitized optimal control problem.

To reiterate, the desensitized optimal control problem is first solved using the nominal parameter values to obtain a desensitized optimal control for the entirety of the trajectory. The dynamics are then simulated for the first guidance cycle using the perturbed parameter values and the reference desensitized optimal control. At the end of the guidance cycle, the final value of the simulated dynamics is used as the initial state at the start of the next guidance cycle. The expired horizon is removed and the mesh is remapped after interpolating the desensitized reference state, control, and sensitivity to the current time. The problem is then resolved on the remaining horizon to obtain a new desensitized optimal control used to simulate the dynamics for the next cycle. This process repeats until the end of the horizon with the goal of minimizing sensitivities to parametric uncertainties while allowing for corrections in the control to reduce perturbations in the final state. A simple numerical example is introduced in the next section to demonstrate this guidance method.

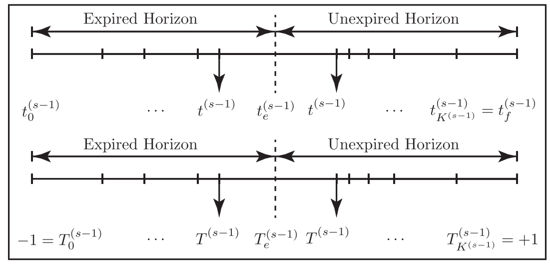


Fig. 1: Mesh remapping for guidance cycle $(s-1)$ featuring the expired and unexpired horizon [13].

## VI. Example

The approach described in Section V is now applied to the following free-flying robot optimal control problem taken from [1]. Determine the state, $\mathbf{x}(t)$, and control, $\mathbf{u}(t)$, that minimize the objective functional

$$J = \int_{t_0}^{t_f} \left(u_1^2(t) + u_2^2(t) + u_3^2(t) + u_4^2(t)\right) dt, \tag{33}$$

subject to the dynamic constraints

$$\begin{array}{rcl} (\dot{x}, \dot{y}) & = & (v_x, v_y), \\ (\dot{v}_x, \dot{v}_y) & = & (\xi \cos\theta, \xi \sin\theta), \\ (\dot{\theta}, \dot{\omega}) & = & (\omega, \alpha T_1 - \beta T_2), \end{array} \tag{34}$$

where $\xi = T_1 + T_2$, and the boundary conditions

$$\begin{array}{rcl} (x_0, x_f) & = & (-10, 0), \\ (y_0, y_f) & = & (-10, 0), \\ (v_{x0}, v_{xf}) & = & (0, 0), \\ (v_{y0}, v_{yf}) & = & (0, 0), \\ (\theta_0, \theta_f) & = & (\pi/2, 0), \\ (\omega_0, \omega_f) & = & (0, 0), \\ (t_0, t_f) & = & (0, 12), \end{array} \tag{35}$$

and the control inequality path constraints

$$\begin{array}{ll} 0 \leq u_i(t) \leq 1000, & (i = 1, 2, 3, 4), \\ |T_i(t)| \leq 1, & (i = 1, 2), \end{array} \tag{36}$$

where

$$\begin{array}{rcl} T_1 & = & u_1 - u_2, \\ T_2 & = & u_3 - u_4, \\ \alpha & = & 0.2, \\ \beta & = & 0.2. \end{array} \tag{37}$$

Suppose now that it is desired to design both a reference solution and a guidance solution that reduces sensitivities in the terminal state with respect to perturbations in the parameters $\mathbf{p} = (\alpha, \beta)$. A desensitized optimal control problem that meets this aforementioned objective is then given as follows. Minimize

$$J_A = J + \mathbb{E}\left(||\delta \mathbf{h}(t_f)||_{Q_f}^2\right), \tag{38}$$

subject to (34), (35), (36), and (24) where $\mathbf{S}(t_0) = \mathbf{0}$. The objective in desensitizing the control is to minimize the final state error; therefore, let the penalty term $\mathbf{h}$ be a function of the state such that $\mathbf{h} = (x, y)$. The parameter covariance, $\mathbf{P}$, is set such that three standard deviations is equal to two percent of the nominal value of the parameters. The desensitization weight, $Q$, is altered depending on the magnitude of desensitization desired. When $Q$ is assigned a value of zero, the problem returns to a standard optimal control problem. As the value of $Q$ increases, more emphasis is placed on minimizing the deviation of the terminal state from its desired target.

To evaluate the effectiveness of the desensitized optimal guidance method, 100 Monte Carlo simulations were performed with the desensitization weight $Q = 5$. Furthermore, for this study, parameters $\alpha$ and $\beta$ were sampled from the Gaussian distribution $\tilde{\mathbf{p}} \sim N(\mathbf{p}, \mathbf{P})$. Figures 2−4 display the nominal desensitized optimal state and control. Figure 5 shows the distribution of perturbed parameter values used. For the full set of Monte Carlo simulations, the deviation in the final state relative to the reference, denoted $\epsilon$, is then shown in Figs. 6−7. For each error distribution, a $3\sigma$ confidence ellipse is computed using the sample mean and sample covariance of the 100 Monte Carlo simulations. The probability that a sample will lie in the $3\sigma$ confidence ellipse of each method is then 99.97%. Results are compared for standard optimal control with guidance (OG), desensitized optimal control with guidance (DOG), and desensitized optimal control without guidance (DOC). For methods with guidance updates, the guidance cycle duration was set to three seconds with a total of three cycles performed. The mesh error and NLP tolerance were set to $1 \times 10^{-5}$ and $1 \times 10^{-7}$, respectively.

The results shown in Fig. 6 demonstrate that DOG provides a tighter distribution compared with DOC. Figure 7 shows that when compared with the distribution obtained using OG, the variation in the final state error for DOG is smaller. Thus, the results of this example show that a guidance method that combines the desensitized optimal control approach of Ref. [12] with the optimal guidance method of Ref. [13] leads to a guidance method that reduces the errors in the terminal state error when compared with using only the method of Ref. [12] without guidance updates or using only the optimal guidance method of Ref. [13] without including desensitization.

## VII. CONCLUSIONS

A method has been developed for employing desensitized optimal control with guidance updates. The optimal control has been desensitized to state perturbations resulting from parametric uncertainties to promote robustness. For each guidance update, the portion of the mesh corresponding to the expired horizon was deleted and the remaining mesh was remapped to the remaining horizon. The desensitized optimal control problem was then re-solved at the start of the next guidance cycle. This approach was applied to an example, and numerical results demonstrated that combining desensitized optimal control with guidance updates produces results where the distribution of errors in the final state is tighter when compared with using either desensitized optimal control without guidance updates or using a previously developed optimal guidance method without desensitization.

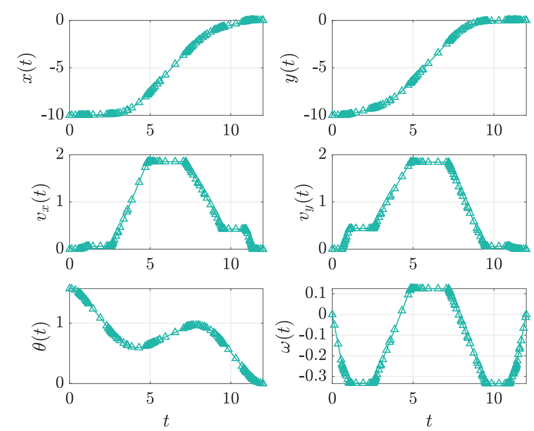


Fig. 2: Reference desensitized optimal state solutions for free-flying robot problem.

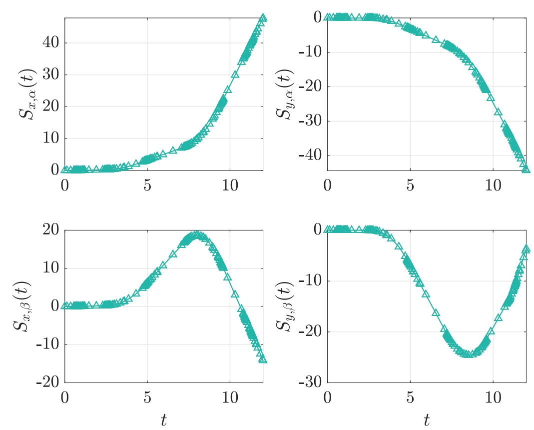


Fig. 3: Sensitivities of states $x$ and $y$ with respect to uncertainties in parameters $\alpha$ and $\beta$ for free-flying robot problem.

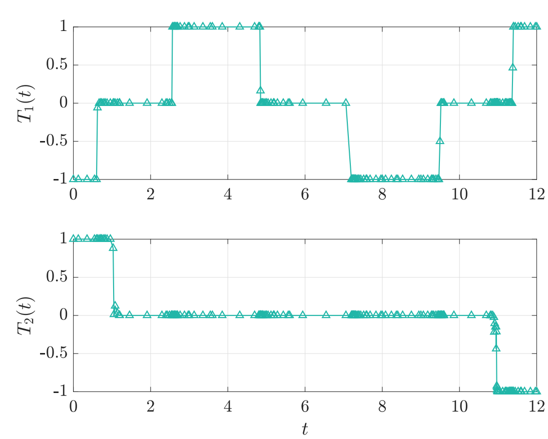


Fig. 4: Desensitized optimal control solution for free-flying robot problem.

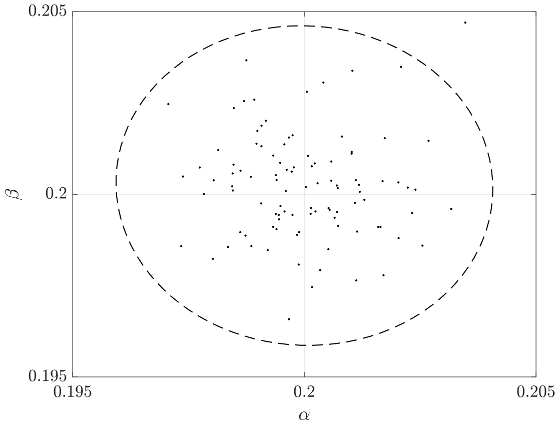


Fig. 5: Perturbed parameter values for $\alpha$ and $\beta$ with $3\sigma$ confidence bounds.

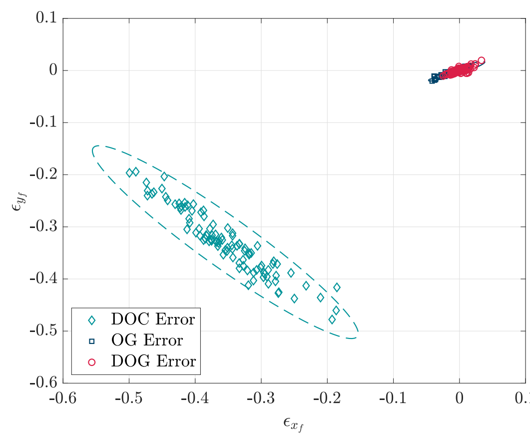


Fig. 6: Final position error for all three methods with $3\sigma$ confidence bounds.

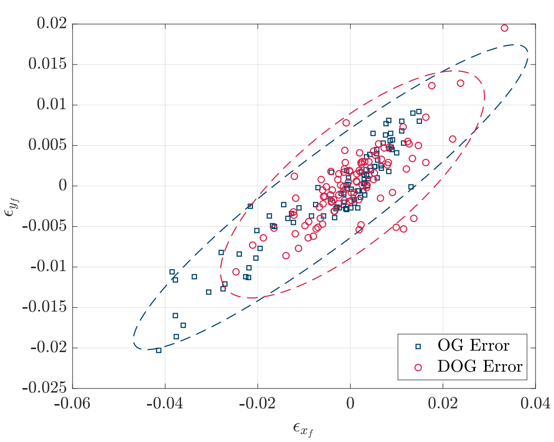


Fig. 7: Final position error for DOG and OG algorithms with $3\sigma$ confidence bounds.

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