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Improved Quasi-Min-Max MPC for Constrained LPV Systems via Nonlinearly Parameterized State Feedback Control

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***Abstract*— We consider the regulation problem of linear parameter varying systems with input and state constraints. It is assumed that the time-varying parameters are available at the current time, but their future behavior is unknown and contained in a polytopic set. The aim is to design a new stabilizing quasi-min-max MPC algorithm via a nonlinearly parameterized state feedback control law. It is shown that the use of such a control law leads to less conservative results compared to those derived from linearly parameterized state feedback control laws. At each time instant, a convex semi-definite optimization problem is required to solved. Two numerical examples, including a non-quadratically stabilizable system, are given with comparison to earlier solutions from the literature to illustrate the effectiveness of the proposed approaches.**

I. Introduction

Model Predictive Control (MPC) is widely used in industry as an effective method for dealing with constrained multivariable control problems [1], [2], [3]. In essence, this control scheme consists of the on-line optimization technique: at each time instant, an open-loop optimization problem is solved online using the current available measurements. The first element of the resulting optimal control sequence is applied to the system. At the next time instant, the entire procedure is repeated: the optimization problem is refreshed with the new measurements and re-solved. The issues of feasibility of the on-line optimization problem, and of stability of the closed-loop system are largely understood for linear discrete-time systems with input and state constraints.

In the last three decades, various research efforts have been made to synthesize an MPC controller for nonlinear systems, see, e.g., [4], [5]. In this context, linear parameter varying (LPV) system modeling framework offers a powerful tool. LPV systems are dynamical models capable of capturing the behavior of nonlinear plants in terms of a linear structure [6]. Signal relations between the inputs and outputs in an LPV representation are linear. Nonetheless, this linearity is parameter-dependent. The parameters affecting the system dynamics are often referred to as scheduling variables, which are assumed to be measurable online and taking values from a so-called scheduling region, often restricted to be a polytopic set.

Frequently used MPC approaches for the control of LPV systems with input and state constraints are based on the online design of closed-loop linear feedback strategies, e.g., [7], [8], [9]. The optimization is done over a feedback law. In [10], a quasi-min-max MPC algorithm was proposed. The MPC algorithm is called quasi since the first stage cost can be calculated without any uncertainty. At each time instant, a solution of a convex semi-definite program (SDP) is required. It was shown that the quasi-min-max MPC control law can improve the performance and reduce the conservativeness with respect to other min-max MPC techniques, e.g., [11]. However, the feedback control law for $u(k+t), \forall t \geq 1$ in [10] is a linear function of the scheduling parameters.

It is worth noticing that the application of the Linear Matrix Inequality (LMI) technique in the context of fuzzy control has opened up new ideas for the design of MPC strategies for LPV systems. In [12], a nonlinearly parameterized Lyapunov function and an associated nonlinearly parameterized state feedback control law were proposed for the stabilization problem of a discrete-time Takagi-Sugeno fuzzy system. It was shown that the results are less conservative than what can be obtained with a quadratic Lyapunov function. Using this class of Lyapunov function and of the control law, an LMI based MPC control law was considered in [13]. The proposed algorithm can be classified as a closed-loop linear feedback strategy, as the optimization is done over a feedback law.

The objective of this paper is to provide new stabilizing quasi-min-max MPC algorithms for LPV systems. Both input and state constraints are considered. The main contributions of the paper are

- We consider the class of nonlinearly parameterized state feedback control laws for $u(k+t), t \geq 1$ in our quasi-min-max MPC algorithms.
- Our nonlinearly parameterized Lyapunov function is simpler than the one in [12], [13].
- We provide a new LMI condition for state constraints.

The paper is organized as follows. Section II covers the problem formulation and preliminaries. Section III is dedicated to the main results of the paper. Two simulated examples with comparison to earlier solutions from the literature are evaluated in Section IV before drawing the conclusions in Section V.

Notation: We denote by $\mathbb{R}$ the set of real number, by $\mathbb{R}^{n\times m}$ the set of $n\times m$ real matrices. Given an integer $L > 0$, we use $\overline{1, L}$ to denote the set $\{1, 2, \ldots, L\}$. We denote a positive definite matrix P by $P \succ 0$. We use $\mathbb{S}^n$ to denote the set of positive definite matrices in $\mathbb{R}^{n\times n}$. For symmetric matrices, the symbol $(*)$ denotes each of its symmetric block. For a given matrix $P \in \mathbb{S}^n$, $\mathcal{E}(P)$ is used to denote the

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ellipsoid

$$\mathcal{E}(P) = \{x \in \mathbb{R}^n : x^T P^{-1} x \leq 1\}$$

We use $0, I$ to denote the zero and the identity matrices of appropriate dimension.

II. Problem Formulation and Preliminaries

A. Problem Formulation

Consider the following discrete-time linear parameter varying (LPV) system

$$x(k+1) = A(k)x(k) + B(k)u(k) \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ is the measured state, $u \in \mathbb{R}^{n_u}$ is the control input. The matrices $[A(k)\ B(k)] \in \Omega$ with

$$\Omega = \text{Co}\left([A_1\ B_1], [A_2\ B_2], \ldots, [A_L\ B_L]\right) \tag{2}$$

where $A_j \in \mathbb{R}^{n_x \times n_x}$ and $B_j \in \mathbb{R}^{n_x \times n_u}$ are constant matrices, $\forall j = \overline{1, L}$, L is the number of subsystems, and $\text{Co}(\cdot)$ denotes the convex hull operator. It is well known [14] that the condition $[A(k)\ B(k)] \in \Omega$ holds if and only if $\exists p_j(k), \forall j = \overline{1, L}$ such that

$$A(k) = \sum_{j=1}^{L} p_j(k) A_j, B(k) = \sum_{j=1}^{L} p_j(k) B_j \tag{3}$$

where $p(k) = [p_1(k)\ p_2(k)\ \ldots p_L(k)]^T$ satisfies

$$p(k) \in \Sigma_L := \left\{ p \in \mathbb{R}^L \mid \sum_{j=1}^{L} p_j = 1,\ p_j \geq 0 \right\} \tag{4}$$

LPV hypothesis: $p(k)$ is available in each time instant k.

The state $x(k)$ and the input $u(k)$ are subject to the constraints, $\forall k \geq 0$

$$\begin{cases} x(k) \in \mathcal{X} := \left\{x \in \mathbb{R}^{n_x} \mid |x_l| \leq x_{l,max}, l = \overline{1, n_x}\right\} \\ u(k) \in \mathcal{U} := \left\{u \in \mathbb{R}^{n_u} \mid |u_l| \leq u_{l,max}, l = \overline{1, n_u}\right\} \end{cases} \tag{5}$$

We denote $x(k+t), u(k+t), t = 0, 1, \ldots$, respectively, as the predicted states and the predicted control inputs at time $k+t$ from time k. Consider the following infinite horizon objective function

$$J_0^\infty(k) = \sum_{t=0}^{\infty} x(k+t)^T Q x(k+t) + u(k+t)^T R u(k+t) \tag{6}$$

where $Q \in \mathbb{S}^{n_x}, R \in \mathbb{S}^{n_u}$ are weighting matrices.

At a generic time k, the objective of the paper is to design a control law

$$u(k+t) = g(x(k+t), p(k+t)), t = 0, 1, \ldots \tag{7}$$

that asymptotically stabilizes the system (1). Furthermore, $u(k+t) = g(x(k+t), p(k+t))$ should solve the following min-max problem

$$\begin{aligned} &\min_{U_0^\infty(k)} \max_{[A(p(k+t)), B(p(k+t))] \in \Omega} J_0^\infty(k) \\ &\text{s.t. } (1), (5) \end{aligned} \tag{8}$$

where

$$U_0^\infty(k) = [u(k)^T\ u(k+1)^T\ \ldots]^T \tag{9}$$

One way to solve the problem (8) is to apply the quasi-min-max MPC control strategy [10]. Using this method, we split the cost function $J_0^\infty(k)$ into two parts

$$J_0^\infty(k) = x(k)^T Q x(k) + u(k)^T R u(k) + J_1^\infty(k) \tag{10}$$

with

$$J_1^\infty(k) = \sum_{t=1}^{\infty} x(k+t)^T Q x(k+t) + u(k+t)^T R u(k+t) \tag{11}$$

Using (10), (11), and the principle of optimality [15], problem (8) is equivalent to

$$\begin{aligned} &\min_{u(k)} \left(u(k)^T R u(k) + \min_{U_1^\infty(k)} \max_{[A(p(k+t)), B(p(k+t))] \in \Omega} J_1^\infty(k) \right) \\ &\text{s.t. } (1), (5) \end{aligned} \tag{12}$$

with $U_1^\infty(k) = [u(k+1)^T\ u(k+2)^T\ \ldots]^T$. Note that the term $x(k)^T Q x(k)$ is removed from the cost (12) as it does not influence the optimal argument.

In the quasi-min-max MPC, the first control move $u(k)$ is a decision variable in the problem (12). The rest of the future control moves is parameterized as [10]

$$u(k+t) = Fx(k+t), \forall t = 1, 2, \ldots \tag{13}$$

where $F \in \mathbb{R}^{n_u \times n_x}$ is a decision variable in (12). In [10], [8], [9], less conservative results can be achieved where linearly scheduled state feedback control laws of the form, $\forall t = 1, 2, \ldots$

$$u(k+t) = \left(\sum_{j=1}^{L} p_j(k+t) F_j \right) x(k+t) \tag{14}$$

are employed, where $F_j \in \mathbb{R}^{n_u \times n_x}, \forall j = \overline{1, L}$ are decision variables in (8).

With the aim of obtaining even less conservative results, in this paper we will adopt the following class of nonlinearly parameterized state feedback control laws, $\forall t = 1, 2, \ldots$

$$u(k+t) = F(k+t) G(k+t)^{-1} x(k+t) \tag{15}$$

where $F(k+t) = \sum_{j=1}^{L} p_j(k+t) F_j, G(k+t) = \sum_{j=1}^{L} p_j(k+t) G_j$ with $F_j \in \mathbb{R}^{n_u \times n_x}, G_j \in \mathbb{R}^{n_x \times n_x}, \forall j = \overline{1, L}$, are decision variables in (8). Clearly if $G_j = I, \forall j = \overline{1, L}$ then (15) coincides with (14).

It should be stressed that the use of (15) is not new. In [12], the authors employed (15) for the stabilization problem of unconstrained discrete-time Takagi-Sugeno fuzzy systems. The control law (15) was also used in the context of MPC [13] for constrained LPV systems.

It is worth noticing that both in [12], [13], the following nonlinearly parameterized Lyapunov function

$$V(x, p) = x^T \left(\sum_{j=1}^{L} p_j G_j \right)^{-T} \left(\sum_{j=1}^{L} p_j P_j \right) \left(\sum_{j=1}^{L} p_j G_j \right)^{-1} x \tag{16}$$

was employed to associate with the control law (15). In our companion paper, we will show that the use of (16) does not bring any advantages compared to our simpler "nonlinearly" paramerized Lyapunov function in Section III.

B. Preliminaries

We will deal several times with the following double sum positivity problem of the form

$$x^T \left(\sum_{i,j=1}^{L} p_i p_j M_{ij} \right) x \geq 0 \tag{17}$$

where $x \in \mathbb{R}^n$, $M_{ij} \in \mathbb{R}^{n\times n}$. The parameters p_i satisfy

$$\sum_{i=1}^{L} p_i = 1, p_i \geq 0, \forall i = \overline{1, L} \tag{18}$$

Lemma 1: [16] Inequality (17) holds $\forall x \in \mathbb{R}^n$, $\forall p_i$ satisfying (18) if

$$\begin{cases} M_{ii} \succeq 0, \forall i = \overline{1, L} \\ \frac{2}{L-1} M_{ii} + M_{ij} + M_{ji} \succeq 0, 1 \leq i \neq j \leq L \end{cases} \tag{19}$$

We will use the following lemma to construct a parameter dependent Lyapunov function.

Lemma 2: [17] Given $Q \in \mathbb{S}^{n\times n}$, $G \in \mathbb{R}^{n\times n}$. The following relation holds

$$G^T Q^{-1} G \succeq G + G^T - Q \tag{20}$$

III. Main Results

In this section, we will first provide a way to calculate the upper bound of $J_1^\infty(k)$ using the control law (15). Substituting (15) into (1), one obtains, $\forall t \geq 1$

$$x(k+t+1) = A_c(k+t)x(k+t) \tag{21}$$

where $A_c(k+t) = A(k+t) + B(k+t)F(k+t)G(k+t)^{-1}$. We will provide a way to design $F_j, G_j, \forall j = \overline{1, L}$ such that the closed-loop system (21) is asymptotically stable. For the moment, it is assumed that $\lim_{t\to\infty} x(k+t) = 0$ for any admissible state $x(k)$.

Consider the following parameterized quadratic function:

$$V(k+t) = x(k+t)^T \left(\sum_{j=1}^{L} p_j(k+t) S_j \right)^{-1} x(k+t) \tag{22}$$

where $S_j \in \mathbb{S}^{n_x \times n_x}, \forall j = \overline{1, L}$ are chosen to satisfy, $\forall t \geq 1$

$$\begin{aligned} x^T(k+t)Qx(k+t) + u(k+t)^T Ru(k+t) \leq \\ V(k+t) - V(k+t+1) \end{aligned} \tag{23}$$

As $\lim_{t\to\infty} x(k+t) = 0$, one has $\lim_{t\to\infty} V(k+t) = 0$. Summing (23) from $t = 1$ to $t = \infty$, one obtains

$$\sum_{t=1}^{\infty} x^T(k+t)Qx(k+t) + u(k+t)^T Ru(k+t) \leq V(k+1)$$

or equivalently, $\forall k \geq 0$

$$J_1^\infty(k) \leq V(k+1) \tag{24}$$

Hence $V(k+1)$ is an upper bound of the cost $J_1^\infty(k)$.

Define $R_c \in \mathbb{R}^{n_u \times n_u}, Q_c \in \mathbb{R}^{n_x \times n_x}$, respectively, as a squared root of R, Q, i.e., $R_c^T R_c = R, Q_c^T Q_c = Q$. For a given $x(k)$, the following result provides a way to calculate $u(k), F_j, G_j, S_j$ that satisfies condition (23), and that solves the problem (12).

Proposition 1: For a given $x(k)$ at a generic time instant k, a solution to the problem (12) may be found by solving the following SDP program

$$\min_{u(k), F_j, G_j, \tilde{S}_j, \alpha} \{\alpha\} \tag{25}$$

subject to

$$\begin{bmatrix} 1 & * & * \\ A(k)x(k) + B(k)u(k) & \widetilde{S}_s & * \\ R_c u(k) & 0 & \alpha I \end{bmatrix} \succeq 0, s = \overline{1, L} \tag{26}$$

$$M_{iis} \succeq 0, \; i = \overline{1, L}, s = \overline{1, L} \tag{27}$$

$$\frac{2}{L-1} M_{iis} + M_{ijs} + M_{jis} \succeq 0, 1 \leq i \neq j \leq L, s = \overline{1, L} \tag{28}$$

$$|u_l(k)| \leq u_{l,max}, \forall l = \overline{1, n_u} \tag{29}$$

$$\begin{bmatrix} u_{l,max}^2 & * \\ F_i^T f_{l,u} & G_i^T + G_i - \widetilde{S}_i \end{bmatrix} \succeq 0, \; i = \overline{1, L}, l = \overline{1, n_u} \tag{30}$$

$$\begin{bmatrix} x_{l,max}^2 & * \\ \widetilde{S}_i f_{l,x} & \widetilde{S}_i \end{bmatrix} \succeq 0 \; i = \overline{1, L}, l = \overline{1, n_x} \tag{31}$$

where $\tilde{S}_s = \alpha S_s, \forall s = \overline{1, L}$, and, $\forall i, \forall j, \forall s = \overline{1, L}$

$$M_{ijs} = \begin{bmatrix} G_i^T + G_i - \widetilde{S}_i & * & * & * \\ A_i G_j + B_i F_j & \widetilde{S}_s & * & * \\ Q_c G_i & 0 & \alpha I & * \\ R_c F_i & 0 & 0 & \alpha I \end{bmatrix} \tag{32}$$

and $f_{l,u}, f_{l,x}$ are, respectively, the lth element of the standard basis in $\mathbb{R}^{n_u}$ and $\mathbb{R}^{n_x}$, i.e.,

$$\begin{cases} f_{l,u} = [0 \; \dots \; 0 \; 1 \; 0 \; \dots \; 0]^T, \\ f_{l,x} = [0 \; \dots \; 0 \; 1 \; 0 \; \dots \; 0]^T \end{cases} \tag{33}$$

Proof: See Appendix. □

For further use, denote the solution of (25), (26), (27), (28), (29), (30), (31) as $u^*(k), F_j^*, G_j^*, S_j^*, \forall j = \overline{1, L}$.

Remark 1: It is worth noting that, unlike [10], we omit the term $x(k)^T Qx(k)$ in (12). Consequently, the computational effort is reduced. □

Based on Proposition 1, we propose the following quasi-min-max MPC algorithm

Algorithm 1 Improved quasi-min-max MPC

1: Set $k \leftarrow 0$
2: Measure and/or estimate $x(k)$ and $p(k)$.
3: Solve the optimization problem (25), (26), (27), (28), (29), (30), (31).
4: Feed the plant with $u^*(k)$.
5: Set $k \leftarrow k + 1$ and go to step 2.

Theorem 1: Assuming feasibility at the initial condition $x(0)$. Then, Algorithm 1 guarantees recursive feasibility, and the closed-loop system with the MPC control law yields is asymptotically stable.

Feasibility Proof: The basic idea of the feasibility proof is to show that the optimal solution at time k is a feasible solution at time $k+1$.

As condition (26) is for the performance, one needs only to check conditions (27), (28), (29), (30), (31).

Using the proof of Proposition 1, one has that conditions (27), (28) do not contain the parameters $p(k)$, and are for the upper bound of the cost $J_1^{\infty}(k)$. Using (23), it is clear that if $J_1^{\infty}(k) \leq V(k+1)$, then $J_1^{\infty}(k+1) \leq V(k+2)$. Hence conditions (27), (28) are satisfied at time $k+1$.

Using (30), one has $|u_l(k+1)| \leq u_{l,max}, \forall l = \overline{1, L}$ with

$$u(k+1) = F^*(k+1)G^*(k+1)^{-1}x(k+1)$$

Hence, condition (29) is satisfied. It remains to show (30), (31), which are for the constraint admissibility. As (30), (31) do not contain the parameters $p(k)$, hence if it holds at time k, then it holds at time $k+1$.

Stability Proof: Consider the following parameter-dependent Lyapunov candidate function

$$\begin{aligned}\phi(k) &= x(k)^T Qx(k) + u^*(k)^T Ru^*(k) \\ &\quad + x(k+1)^T S^*(k+1)^{-1}x(k+1)\end{aligned} \tag{34}$$

where $u^*(k), S_j^*, \forall j = \overline{1, L}$ are the outputs of Algorithm 1 at time k. Note that $\phi(k) > 0, \forall x(k) \neq 0$. Note also that in Algorithm 1, if $x(k) = 0$, then $u(k) = 0$ is the solution. In this case $\phi(k) = 0$.

As $u(k+1) = F^*(k+1)G^*(k+1)^T x(k+1)$ and $S^*(k+1)$ is a feasible solution at time $k+1$, one obtains

$$\begin{aligned}&x(k+1)^T Qx(k+1) + u(k+1)^T Ru(k+1) \\ &+ x(k+2)^T S(k+2)^{-1}x(k+2) \\ &\leq x(k+1)S^*(k+1)^{-1}x(k+1)\end{aligned} \tag{35}$$

Using (34), (35), one gets

$$\begin{aligned}\phi(k) \geq\ & x(k)^T Qx(k) + u^*(k)^T Ru^*(k) \\ & + x(k+1)^T Qx(k+1) + u(k+1)^T Ru(k+1) \\ & + x(k+2)^T S(k+2)^{-1}x(k+2)\end{aligned} \tag{36}$$

At time instant $k+1$, one has

$$\begin{aligned}\phi(k+1) &= x(k+1)^T Qx(k+1) + u^*(k+1)^T Ru^*(k+1) \\ &\quad + x(k+2)^T S^*(k+2)^{-1}x(k+2) \\ &\leq x(k+1)^T Qx(k+1) + u(k+1)^T Ru(k+1) \\ &\quad + x(k+2)^T S(k+2)^{-1}x(k+2)\end{aligned} \tag{37}$$

Combining (36), (37), one obtains

$$\phi(k) \geq \phi(k+1) + x(k)^T Qx(k) + u^*(k)^T Ru^*(k)$$

Hence, $\phi(k)$ is a Lyapunov function for the closed-loop system with the control law $u(k) = u^*(k)$. In the other words, asymptotic stability is guaranteed. This complete the proof. □

IV. Numerical Examples

Two example systems are shown in this section. The CVX toolbox [14] was used to solve the SDP optimization problems.

A. Example 1

This example is taken from [18]. Consider system (1) with

$$A_1 = \begin{bmatrix} 1.0 & 0.1 \\ 0 & 0.99 \end{bmatrix}, \ A_2 = \begin{bmatrix} 1.0 & 0.1 \\ 0 & 0 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0 & 0.0787 \end{bmatrix}^T, B_2 = B_1$$

Note that the open-loop system is unstable. The input and state constraints are $|u(k)| \leq 1$. The weighting matrices are $Q = I, R = 1$.

For the initial condition $x(0) = [2 \ 1]^T$, Fig. 1 and Fig. 2 present the state and the input trajectories as functions of time for Algorithm 1 (solid blue), for [10] (dashed red). Using Fig. 1, Fig. 2, the superior performance of Algorithm 1 compared to [10] is clearly observed. Finally, Fig. 3 presents

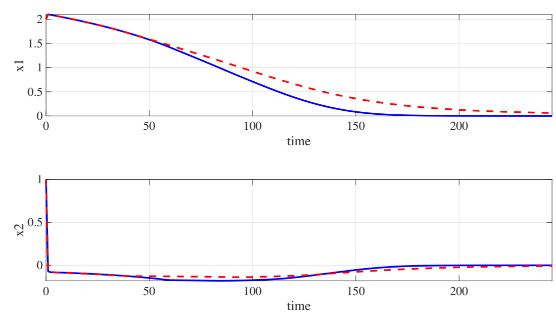

Fig. 1. State trajectories for Algorithm 1 (solid blue), for [10] (dashed red) for example 1.

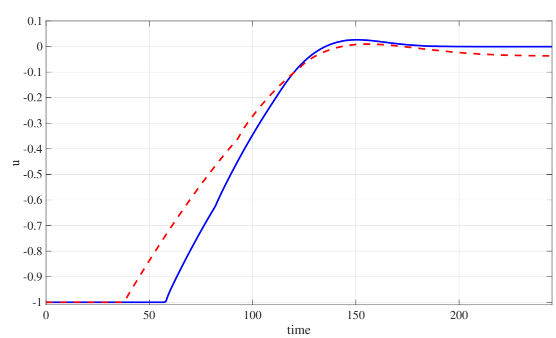

Fig. 2. Input trajectories for Algorithm 1 (solid blue), for [10] (dashed red) for example 1.

the accumulated cost $\mathcal{V}(k)$, which is computed as, $\mathcal{V}(-1) = 0$

$$\mathcal{V}(k) = \mathcal{V}(k-1) + x(k)^T Qx(k) + u(k)^T Ru(k)$$

for Algorithm 1 (solid blue), for [10] (dashed red). Fig. 3 also presents the p_1 realization as a function of time.

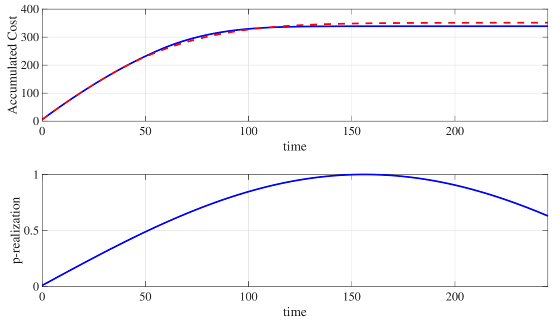

Fig. 3. Accumulated cost for Algorithm 1 (solid blue), for [10] (dashed red), and p_1-realization for example 1.

B. Example 2

Consider system (1) with

$$A_1 = \begin{bmatrix} 1.0 & -1.4 \\ -1.0 & -0.8 \end{bmatrix}, A_2 = \begin{bmatrix} 1.0 & 1.4 \\ -1.0 & -0.8 \end{bmatrix} \quad (38)$$
$$B_1 = \begin{bmatrix} 5.9 & 2.8 \end{bmatrix}, B_2 = \begin{bmatrix} 3.1 & -2.8 \end{bmatrix}$$

Note that system (1), (38) is unstable. It can also be verified that it is not quadratically stabilizable. Since $B_1 \neq B_2$, the linear parameterized control law in [10] is not applicable.

The input constraints are $|u(k)| \leq 1$. There are no state constraints. The weighting matrices are $Q = I, R = 0.01$.

For the initial condition $x(0) = [2.78 \ \ 2]^T$, Fig. 4 and Fig. 5 show the state and the input trajectories as function of time for Algorithm 1 (solid blue), and for [13] (dashed red). Note that for this example both Algorithm 1 and Algorithm 2 have the identical performance. Finally, Fig. 6 shows the accumulated cost for Algorithm 1 (solid blue), for [13] (dashed red). Fig.6 also shows the p_1-realization as a function of time.

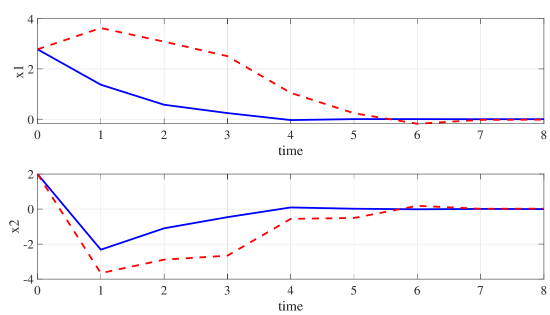

Fig. 4. State trajectories for Algorithm 1 (solid blue), for [13] (dashed red) for example 2.

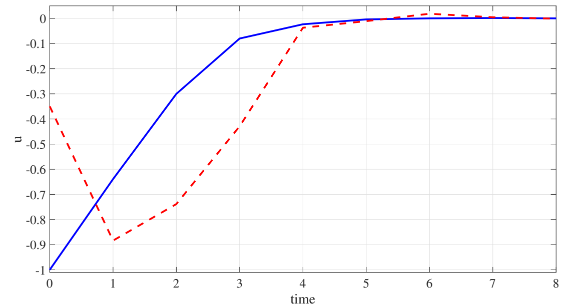

Fig. 5. Input trajectories for Algorithm 1 (solid blue), for [13] (dashed red) for example 2.

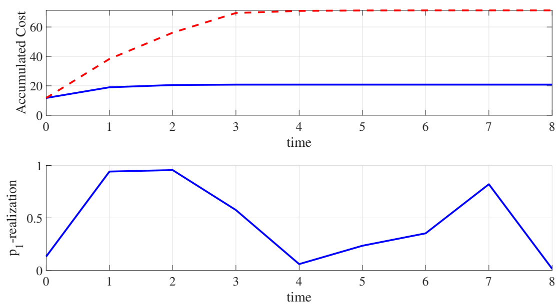

Fig. 6. Accumulated cost for Algorithm 1 (solid blue), for [13] (dashed red), and p_1-realization for example 2.

V. Conclusion

In this paper we proposed a new quasi-min-max MPC algorithm for discrete-time LPV systems with input and state constraints. The main idea is to use a class of nonlinearly parameterized state feedback control laws. Recursive feasibility of the online optimization problem and asymptotic stability of the closed system are guaranteed. Two numerical examples with comparison to earlier solutions in the literature demonstrate the effectiveness of the new methods.

The requirement of solving a convex semi-definite program at each time instant for the proposed algorithms imposes a computational burden, especially for systems with high complexity or fast sampling rates. In the future, we aim to reduce the online computational burden of the proposed techniques.

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VI. Appendix - Proof of Proposition 1

A. Cost Function

Using (12), (24), the problem of minimizing J_0^∞ can be rewritten as

$$[A(k)x(k)+B(k)u(k)]^T S(k+1)^{-1}[A(k)x(k)+B(k)u(k)] + u(k)^T Ru(k) \le \alpha \tag{39}$$

Thus, using Schur complements, as $\tilde{S}_j = \alpha S_j$, $\forall p(k+1) \in \Sigma_L$, (26) can be proved.

B. Upper Bound of $J_1^\infty(k)$

Concerning condition (23), it is clear that it suffices to verify (23) with $t = 1$, i.e

$$\begin{aligned} x^T(k+1)Qx(k+1) + u(k+1)^T Ru(k+1) \le \\ V(k+1) - V(k+2) \end{aligned} \tag{40}$$

Using (15), (21), it follows that (40) holds if and only if

$$\begin{aligned} &S(k+1)^{-1} - A_c(k+1)^T S(k+2)^{-1} A_c(k+1) \\ &- Q - [F(k+1)G(k+1)^{-1}]^T RF(k+1)G(k+1)^{-1} \succeq 0 \end{aligned} \tag{41}$$

By pre- and post-multiplying with $\sqrt{\frac{1}{\alpha}}G(k+1)^T$ and $\sqrt{\frac{1}{\alpha}}G(k+1)$, one obtains

$$\begin{aligned} &S_g(k+1) - A_g(k+1)^T \times \tilde{S}(k+2)^{-1} A_g(k+1) \\ &- G(k+1)^T \frac{Q}{\alpha} G(k+1) - F(k+1)^T \frac{R}{\alpha} F(k+1) \succeq 0 \end{aligned}$$

where

$$\begin{aligned} S_g(k+1) &= G(k+1)^T \tilde{S}(k+1)^{-1} G(k+1), \\ A_g(k+1) &= A(k+1)G(k+1) + B(k+1)F(k+1) \end{aligned}$$

thus, by using Schur complements Recall that $\tilde{S}(k+2) = \sum_{s=1}^{L} p_s(k+2)\tilde{S}_s$. Hence (41) is satisfied, $\forall p(k+2) \in \Sigma_L$ if and only if

$$\begin{bmatrix} S_g(k+1) & * & * & * \\ A_g(k+1) & \tilde{S}_s & * & * \\ Q_c G(k+1) & 0 & \alpha I & 0 \\ R_c F(k+1) & 0 & 0 & \alpha I \end{bmatrix} \succeq 0, \forall s = \overline{1, L} \tag{42}$$

Using Lemma 2, Therefore, if the following condition

$$\begin{bmatrix} G(k+1)^T + G(k+1) - \tilde{S}(k+1) & * & * & * \\ A_g(k+1) & \tilde{S}_s & * & * \\ Q_c G(k+1) & 0 & \alpha I & 0 \\ R_c F(k+1) & 0 & 0 & \alpha I \end{bmatrix} \succeq 0 \tag{43}$$

holds $\forall s = \overline{1, L}$, then (42) holds.

Note that (43) can be written in the following double sum form, $\forall s = \overline{1, L}$

$$\sum_{i=1}^{L}\sum_{j=1}^{L} p_i(k+1)p_j(k+1)M_{ijs} \succeq 0 \tag{44}$$

with M_{ijs} being defined in (32). Using Lemma 1, if (27), (28) hold then (44) holds.

C. Input Constraints

Consider first the case where $u(k)$ is a scalar. In quasi-min-max MPC, $u(k)$ is a decision variable. Hence (29) imposes directly the constraint on $u(k)$. For the remaining inputs U_1^∞, one has, $\forall t \ge 1$

$$|F(k+t)G(k+t)^{-1}x(k+t)| \le u_{max} \tag{45}$$

Using (39), one obtains

$$x(k+1)^T \tilde{S}(k+1)^{-1} x(k+1) \le 1$$

Using the feasibility proof of Theorem 1, it follows that, $\forall t \ge 1$

$$x(k+t) \in \mathcal{E}(\sum_{j=1}^{L} p_j(k+t)\tilde{S}_j) \tag{46}$$

Define, $\forall t \ge 1$

$$\begin{aligned} v_u(t) &= \max_{x(k+t)} \left\{ F(k+t)G(k+t)^{-1}x(k+t) \right\}, \\ &\text{s.t. } x(k+t)^T \tilde{S}(k+t)^{-1} x(k+t) \le 1 \end{aligned} \tag{47}$$

Using the method of Lagrange multipliers, it can be shown that

$$v_u(t) = \sqrt{F(k+t)G(k+t)^{-1}\tilde{S}(k+t)G(k+t)^{-T}F(k+t)^T}$$

As the set $\mathcal{E}(\sum_{j=1}^{L} p_j(k+t)\tilde{S}_j)$ is symmetric. Hence (45) hold if and only if

$$u_{max}^2 - F(k+t)G(k+t)^{-1}\tilde{S}(k+t)G(k+t)^{-T}F(k+t)^T \ge 0$$

Thus, using Schur complements

$$\begin{bmatrix} u_{max}^2 & * \\ F(k+t)^T & G(k+t)^T \tilde{S}(k+t)^{-1} G(k+t) \end{bmatrix} \succeq 0 \tag{48}$$

Using Lemma 2, it follows that if (30) holds, then (48) is satisfied $\forall t \ge 1$.