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Value of Information in Remote Estimation Subject to Delay and Packet Dropouts

Siyi Wang and Sandra Hirche

Abstract— **Emerging cyber-physical systems impel the development of advanced network scheduling schemes to utilize communication and computation resources efficiently. This paper investigates the event-based schedule for remote state estimation in networked control systems (NCSs) subject to delay and packet dropouts. The scheduler decides whether or not to send out a local estimate according to the Value of Information (VoI) metric, which measures the relative importance of an information update. In addition, we model the triggering intervals as a Markov chain and analyze the tradeoff between the estimation performance and communication cost under the proposed VoI-based scheduling for the first-order system.**

I. Introduction

In networked control systems (NCSs), the concept of remote state estimation over a communication network has been widely utilized in fields such as robotics [1], bio-inspired Vision [2] and autonomous driving [3]. With the increasing number of wireless sensors and estimators in cyber-physical systems, communication resource usage has become a significant issue. Meanwhile, due to the limited communication bandwidth, the data transmission collision leads to network-induced delay and packet dropouts. Thus, reducing unnecessary transmission or prioritizing more valuable packets in NCSs subject to network effects becomes relevant in remote estimation.

It is widely known that event trigger achieves better performance than periodic sampling while consuming the same resources [4]. A classic problem of event-based estimation is to find the optimal scheduling approach to maximize the estimation performance with limited communication resources [5]. For example, [6] restricts the signal to be a Wiener process or an Ornstein-Uhlenbeck process and demonstrates that the level-triggered sampling is almost optimal with the stable signal. Moreover, compared with the widely used emulation-based approach, which designs the event trigger and the remote estimator separately, [7] formulates the design of event-triggered estimation as a team-optimal decision problem by regarding the event trigger and estimator as two individual distributed decision makers. Generally, the dual effect exists in event-triggered control or estimation problems due to the potential non-classical information pattern between the event trigger and the remote estimator [8]. To address this issue, [9] identifies a dominating policy pair in which the scheduling policy depends only on primitive random variables. In this case, the design of the event trigger and estimator has no dual effect, and therefore the original optimal co-design becomes separable.

The concept of VoI is first proposed in [10], which denotes a metric encoding our knowledge within the considered problem context. On the basis of separation techniques, [11] decomposes the optimal co-design of control and communication in a feedback control system into two subproblems, which are optimal control and optimal scheduling, respectively. When designing the optimal scheduling, the VoI metric is utilized to quantify the importance of transmitting certain information. More spcifically, the information will be transmitted to the controller only if the benefit of this information update surpasses its transmission cost. Furthermore, the optimal scheduling policy based on the VoI is further proved to be globally optimal in [12].

In addition, network-induced delay and packet dropouts widely exist in wireless NCSs due to network coupling and data collisions [13]. The existing papers on event-triggered estimation subject to networked effects, such as [14], mainly focus on finding the maximum sampling interval guaranteeing the system stability within the emulation-based approach framework. When speaking of maximizing remote estimation performance with limited information updates, it turns out to be an optimization problem, where the scheduler is required to determine which packet is more valuable for remote estimation. This is generally difficult, especially in the presence of network effects such as delay and random packet dropouts. With only a few exceptions, for example, in [15], the authors investigate the suboptimal scheduling considering the effects of delay and packet dropouts while does not provide an explicit solution. In [16], the authors design the optimization-oriented event-triggered control for NCSs subject to delay. To author's knowledge, there is no study taking both communication delay and packet dropouts into consideration when designing the optimal event triggering and it lacks the performance analysis of network-effect-aware VoI-based scheduling.

Based on the above observation, we propose a VoI-based optimal scheduling scheme for the remote state estimation subject to delay and packet dropouts. In this study, the transmission delay is assumed to be known and constant, and the dropouts are assumed to be Bernoulli-distributed. The design objective is to minimize the long-term cost penalizing the communication rate and the estimation performance measured by the estimation error mean square. We employ the dynamic programming approach to solve the formulated sequential decision optimization problem and

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Siyi Wang and Sandra Hirche are with the Chair of Information-oriented Control (ITR), Technical University of Munich, Germany, {siyi.wang, hirche}@tum.de

obtain the metric representing the importance of the remote information update. As the resulting VoI metric is computationally expensive, a rollout algorithm is utilized to obtain its approximation. To further simplify the tradeoff analysis between estimation performance and communication rate, we employ the waiting strategy, which enables the scheduler to be aware of the remote status after a one-step delay. Then, by formulating the triggering intervals as an ergodic Markov chain on the finite space, the theoretical communication rate is calculated from their probability mass function. Finally, we provide the upper bound of the estimation performance under the designed VoI-based scheduling.

The remainder of this paper is structured as follows: Section II introduces the system model and problem statement. Section III presents the VoI-based scheduling policy and analyzes its performance. Section IV demonstrates the efficacy of the proposed scheduling policy by numerical simulations. Section V draws conclusions. Section VI provides supplementary proof.

Notations: In this study, let $\mathbb{R}$ and $\mathbb{R}^n$ denote the one-dimension and n-dimension real value sets, respectively. Let $\mathbf{E}[\cdot]$ and $\mathbf{E}[\cdot|\cdot]$ denote the expected value and the conditional expectation, respectively. Let $x \sim \mathcal{N}(\mu, C_x)$ denote Gaussian random variable x with mean μ and covariance matrix C_x. Let $\lceil a \rceil$ denote the ceiling function on $a \in \mathbb{R}$. Let w.p. denote the abbreviation of $\mathrm{with\ probability}$.

II. Preliminaries

We consider a resource-constrained system closed over the communication network as illustrated in Fig. 1. The event-based scheduler determines whether or not to send out the estimate produced by a local Kalman filter. Transmitted packets experience a τ-step delay and might be dropped with some probability. In addition, we assume that the remote receiver's status will be feedbacked to the scheduler side with a unit delay.

A. System model

Consider a discrete-time stochastic dynamical system:

$$\begin{aligned} x_{k+1} &= Ax_k + w_k \\ y_k &= Cx_k + v_k, \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^n$, $y_k \in \mathbb{R}^p$ are the state vector and the measurement, respectively. The system matrices are given by $A \in \mathbb{R}^{n\times n}$, $C \in \mathbb{R}^{p\times n}$, in which the pair (A, C) is observable. The process noise $w_k \in \mathbb{R}^n \sim \mathcal{N}(0, W)$ and the measurement noise $v_k \in \mathbb{R}^p \sim \mathcal{N}(0, V)$ are assumed to be independent identically distributed (i.i.d.) Gaussian noises with zero mean and positive semi-definite variances $W \in \mathbb{R}^n$ and $V \in \mathbb{R}^p$. The initial state $x_0 \sim \mathcal{N}(\bar{x}_0, R_0)$ is a random vector with the initial mean value $\bar{x}_0$ and the semi-definite covariance R_0, which is assumed to be statistically independent of the process noise w_k for all k.

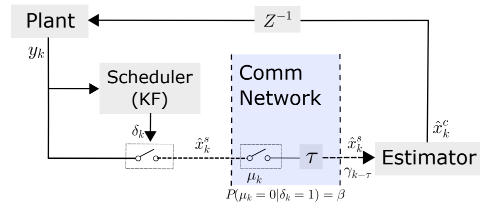

Fig. 1. NCS architecture

B. Network model

Generally, sending the estimate rather than the measurement enables the remote estimator encode more information [17]. Therefore, we assume that the local scheduler periodically accesses the local estimate generated by the Kalman filter and raw measurements (see Fig 1). The event-based scheduler decides whether or not to transmit the state estimate $\hat{x}_k^s$ through the communication network. Denote δ_k, $\mu_k \in \{0, 1\}$ as the transmission decision variable and the packet dropouts index at time k, respectively. To simplify notations, we denote the packet arrival index as $\gamma_k = \delta_k \mu_k$, which takes value from $\{0, 1\}$. Let $\gamma_k = 1$ denote that the packet triggered at time k will arrive after τ-step delay successfully, otherwise $\gamma_k = 0$.

The scheduler makes transmission decisions according to the locally available information as follows:

$$\delta_k = \pi(\mathcal{I}_k^s) = \begin{cases} 1 & \text{transmission occurs} \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where $\mathcal{I}_k^s = \mathcal{I}_{k-1}^s \bigcup \{y_k, \hat{x}_k^s, \delta_{k-1}, \gamma_{k-\tau-1}\}$ denotes the local information set with the initial value $\mathcal{I}_0^s = \{y_0, \hat{x}_0^s, \delta_0\}$. Specifically, at time k, the Kalman filter accesses to the transmission decision by time k and the packet arrival status by time $k - \tau - 1$ due to the round trip time $\tau + 1$. The function π is the scheduling law. The local Kalman filter processes as follows:
i) prediction step:

$$\begin{aligned} \hat{x}_{k|k-1}^s &= A\mathbf{E}[x_{k-1}|\mathcal{I}_{k-1}^s] \\ \hat{e}_{k|k-1}^s &= x_k - \hat{x}_{k|k-1}^s = A\hat{e}_{k-1}^s + w_{k-1} \\ P_{k|k-1}^s &= \mathrm{cov}[\hat{e}_{k|k-1}^s|\mathcal{I}_{k-1}^s] = A^\top P_{k-1}^s A + W \end{aligned}$$

with the initial value $\hat{x}_{0|-1}$, $P_{0|-1} = X_0 \in \mathbb{R}^{n\times n}$,
ii) update step:

$$\hat{x}_k^s = \mathbf{E}[x_k|\mathcal{I}_k^s] = \hat{x}_{k|k-1}^s + K_k(y_k - C\hat{x}_{k|k-1}^s)$$

$$\hat{e}_k^s = x_k - \hat{x}_k^s = (I - K_k C)(A\hat{e}_{k-1}^s + w_{k-1}) - K_k v_k \tag{3}$$

$$P_k^s = \mathrm{cov}[\hat{e}_k^s|\mathcal{I}_{k-1}^s] = (I - K_k C)P_{k|k-1}^s \tag{4}$$

with $K_k = P_{k|k-1}^s C^\top (C P_{k|k-1}^s C^\top + V)^{-1}$. As the pair (A, C) is assumed to be observable, the covariance matrices $P_{k|k-1}^s$, P_k^s and K_k will converge to constant values $\tilde{P}^s$, P^s and K, respectively.

When $\delta_k = 1$, the packet dropouts is modeled as a Bernoulli process $\{\mu_k\}_k$ with the dropout probability $\beta = P[\mu_k = 0|\delta_k = 1]$. It implies that a transmitted packet will be dropped with a probability of β. To facilitate further analysis, define the elapsed time of the information update at the remote estimator by time k as $\eta_k = k - \max\{l-\tau|\gamma_{l-\tau} = 1, l \leq k\}$. It evolves as

$$\eta_{k+\tau} = (1-\gamma_k)(\eta_{k+\tau-1}+1) + \gamma_k \tau$$

with $\eta_k = 0$ for $k \in [0, \tau-1]$.

At the remote side, a linear estimator predicts the state x_k based on the received information as follows:

$$\hat{x}_k^{\varepsilon} = \mathbf{E}[x_k|\mathcal{I}_k^{\varepsilon}] = \begin{cases} A^{\tau}\hat{x}_{k-\tau}^{s} & \text{if } \gamma_{k-\tau} = 1 \\ A\hat{x}_{k-1}^{\varepsilon} & \text{otherwise,} \end{cases} \quad (5)$$

where $\mathcal{I}_k^{\varepsilon} = \mathcal{I}_{k-1}^{\varepsilon} \bigcup \{\hat{x}_{k-1}^{\varepsilon}, \hat{x}_{k-\eta_k}^{s}, \gamma_{k-\tau}\}$ is the information set at the remote side, with the initial value $\{\hat{x}_{0|-1}^{\varepsilon}\}$. Accordingly, we denote the remote estimation error as $\hat{e}_k^{\varepsilon} = x_k - \hat{x}_k^{\varepsilon}$, which is utilized to describe the estimation performance in the following sections.

C. Problem statement

In this study, we are interested in searching for the optimal scheduling policy π^* for the resource-aware remote estimation subject to delay and packet dropouts, which is mathematically described as

$$\min_{\pi} \Psi(\pi) = J(\pi) + \mathcal{R}(\pi), \quad (6)$$

where $J(\pi)$ is the estimation performance measured by

$$J(\pi) = \limsup_{N\to\infty} \frac{1}{N+1}\mathbf{E}\big[\sum_{k=0}^{N}(\hat{e}_k^{\varepsilon})^{\top}\Gamma\hat{e}_k^{\varepsilon}\big] \quad (7)$$

with Γ being a positive definite matrix. The function $\mathcal{R}(\pi)$ penalizes the average communication cost, is described by $\mathcal{R}(\pi) = \limsup_{N\to\infty}\frac{1}{N+1}\mathbf{E}\big[\sum_{k=0}^{N}\theta\delta_k\big]$ with the positive scalar θ being the unit transmission cost.

III. Main results

In this section, we are going to construct a VoI metric to efficiently schedule the data transmission for remote estimation subject to delay and packet dropouts. To reduce the computation burden when using dynamic programming approach to solve the sequential decision problem, a rollout algorithm will be used to obtain the VoI proxy (VoIP) function. In addition, we analytically characterize the performance of the VoIP-based scheduling policy with the waiting strategy.

A. VoI-based optimal scheduling

As the transmitted packets drop off with probability β, the packet arrival status $\{\gamma_{k-\tau},\ldots,\gamma_{k-1}\}$ are unknown for the local Kalman filter by time k. Therefore, we provide the expected value of $\eta_{k+\tau-1}$ from the perspective of the local estimator in the following lemma.

***Lemma** 1:* Incorporating with the triggering decisions $\{\delta_{k-\tau},\ldots,\delta_{k-1}\}$, the value of $\eta_{k+\tau-1}$ is given as

$$\eta_{k+\tau-1} = \begin{cases} \eta_{k-1}+\tau & \text{w.p. } \mathcal{F}_{\tau} \\ \tau + i & \text{w.p. } \delta_{k-i-1}(1-\beta)\mathcal{F}_i, \\ & i \in \{1,\ldots,\tau-1\} \\ \tau & \text{w.p. } \delta_{k-1}(1-\beta) \end{cases} \quad (8)$$

with $\mathcal{F}_i = \beta^{\sum_{r=1}^{i}\delta_{k-r}}$ denoting the probability that no packet arrived at the remote side during $t \in [k-i, k-1]$.

Proof: The potential value set of $\eta_{k+\tau-1}$ is categorized into the following three classes. For the third line of (8), it occurs only when $\delta_{k-1} = 1$, and this transmitted packet arrives at the remote side with the probability of $1-\beta$. Therefore, we have $\mathrm{P}(\gamma_k = 1|\delta_k = 1) = \delta_{k-1}(1-\beta)$. Second, for the case of $\eta_{k+\tau-1} = \tau + i$, it occurs only when $\gamma_{k-i-1} = 1$ and $\gamma_{k-i} = \cdots = \gamma_{k-1} = 0$. Incorporate the transmission decisions $\{\delta_{k-i-1},\ldots,\delta_{k-1}\}$ to represent the information freshness at the remote estimator by time $k-1$, we obtain the second line of (8). Taking $i=1$ as an example, in this case, $\mathcal{F}_1 = \beta^{\delta_{k-1}}$. According to (8), the occurrence probability of $\eta_{k+\tau-1} = \tau+1$ is

$$\mathrm{P}(\eta_{k+\tau-1} = \tau+1) = \begin{cases} 0 & \text{if } \delta_{k-2} = 0 \\ (1-\beta)\beta & \text{if } \delta_{k-1} = \delta_{k-2} = 1 \\ \beta^2 & \text{otherwise.} \end{cases}$$

Finally, for the case of $\eta_{k+\tau-1} = \eta_{k-1}+\tau$, it implies that no information update arrives during time $[k, k+\tau-1]$, and the remote side keeps the previous data packet. As a result, we obtain F_{τ} as its occurrence probability. ■

In the following, we are going to parameterize the error function under the network-induced delay and packet dropouts. The innovation $\hat{e}_k^s$ in (3) evolves as

$$\hat{e}_k^s + \xi_{k-1} = A\hat{e}_{k-1}^s + w_{k-1} \quad (9)$$

with the random variable $\xi_k = KC(A\hat{e}_k^s + w_k) + Kv_{k+1} \in \mathbb{R}^n$. Note that the estimate $\hat{x}_{k-\tau}^{\varepsilon}$ is available for the scheduler with the feedback signal $\gamma_{k-\tau-1}$, thus we can obtain ξ_k without the access to process and measurement noise realizations w_k and v_k. The remote estimation error depending on the transmission arrival status $\gamma_{k-\tau}$ is written as

$$\begin{aligned}\hat{e}_k^{\varepsilon} = &(1-\gamma_{k-\tau})(A\hat{e}_{k-1}^{\varepsilon} + w_{k-1}) \\ &+ \gamma_{k-\tau}(A^{\tau}\hat{e}_{k-\tau}^{s} + \sum_{r=1}^{\tau}A^{r-1}w_{k-r}).\end{aligned} \quad (10)$$

Denote e_k as the mismatch between the local estimate $\hat{x}_k^s$ and the remote estimate $\hat{x}_k^{\varepsilon}$, i.e.,

$$e_k = \hat{x}_k^s - \hat{x}_k^{\varepsilon} = \hat{e}_k^{\varepsilon} - \hat{e}_k^{s} \quad (11)$$

Substituting the local (9) and remote estimation error (10) into the the random process e_k, we have

$$\begin{aligned} e_k &= (1-\gamma_{k-\tau})Ae_{k-1} + \gamma_{k-\tau}\sum_{r=2}^{\tau}A^{r-1}\xi_{k-r} + \xi_{k-1} \\ &= \sum_{r=1}^{\eta_{k+\tau}}A^{r-1}\xi_{k+\tau-r} \end{aligned} \quad (12)$$

We are going to present the main result on the network-effect-aware VoI-based scheduling for the remote estimation.

***Theorem** 1:* Consider the optimization problem (6) for system dynamics (1). The optimal scheduling policy minimizing the optimization problem (6) is given by

$$\delta_k^* = \pi^*(\mathcal{I}_k^s) = \begin{cases} 1 & \text{if } \mathrm{VoI}_k > 0 \\ 0 & \text{otherwise,} \end{cases} \quad (13)$$

where VoI_k is the VoI at time k, expressed as

$$\text{VoI}_k = -\theta + (1-\beta)\epsilon_k^\top \Gamma \epsilon_k + \rho_k \tag{14}$$

with $\epsilon_k = \sum_{r=\tau+1}^{\eta_{k+\tau-1}+1} A^{r-1}\xi_{k+\tau-r}$ and the residual term $\rho_k = \mathbf{E}[V_{k+1}|\mathcal{I}_k^s, \delta_k = 0] - \mathbf{E}[V_{k+1}|\mathcal{I}_k^s, \delta_k = 1]$, where the expression of the cost-to-go function V_{k+1} is

$$V_{k+1} = \mathbf{E}[\sum_{t=k+1}^{N-\tau} g(e_{t+\tau}, \delta_t)|\mathcal{I}_t^s] \tag{15}$$

with the stage cost $g(e_{t+\tau}, \delta_t) = \theta\delta_t + e_{t+\tau}^\top \Gamma e_{t+\tau}$, for $t \in [0, N-\tau]$.

Proof: By the tower property of the conditional expectation, we have

$$\begin{aligned}\mathbf{E}[(\hat{e}_{k+d}^\varepsilon)^\top \Gamma \hat{e}_{k+d}^\varepsilon] &= \mathbf{E}[\mathbf{E}[(\hat{e}_{k+d}^\varepsilon)^\top \Gamma \hat{e}_{k+d}^\varepsilon|\mathcal{I}_k^s]] \\ &= \mathbf{E}[e_{k+d}^\top \Gamma e_{k+d}] + \text{tr}(\Gamma P_{k+d}^s).\end{aligned}$$

The second equality establishes as $\mathbf{E}[\hat{e}_{k+d}^\varepsilon|\mathcal{I}_k^s] = e_{k+d}$ and $\text{cov}[\hat{e}_{k+d}^\varepsilon|\mathcal{I}_k^s] = P_{k+d}^s$. Note that the term $\text{tr}(\Gamma P_{k+d}^s)$ are independent of decision variables δ_k, for $k \in [0, N-\tau]$. Moreover, for $k \in [0, \tau)$, the error e_k is independent of the triggering policy as no packet arrives at the remote estimator before time instant τ due to the transmission delay. Thus, the original optimization objective (6) is reduced to (15).

Expand (15) and apply the minimization operator to it, we have

$$\begin{aligned}V_k^* &= \min_{\delta_k, \delta_{k+1}, \dots} \{\theta\delta_k + e_{k+\tau}^\top \Gamma e_{k+\tau} + \mathbf{E}[V_{k+1}|\mathcal{I}_k^s] \mid \mathcal{I}_k^s]\} \\ &= \min_{\delta_k}\{\theta\delta_k + e_{k+\tau}^\top \Gamma e_{k+\tau} + \mathbf{E}[V_{k+1}^*|\mathcal{I}_k^s] \mid \mathcal{I}_k^s]\}.\end{aligned} \tag{16}$$

Invoking of the error mismatch expression (12), $\mathbf{E}[e_{k+\tau}^\top \Gamma e_{k+\tau}|\mathcal{I}_k^s, \delta_k]$ is further written as

$$\begin{aligned}&\mathbf{E}[e_{k+\tau}^\top \Gamma e_{k+\tau}|\mathcal{I}_k^s, \delta_k] \\ &= \mathbf{E}[(1-\delta_k(1-\beta))(Ae_{k+\tau-1} + \xi_{k+\tau-1})^\top \Gamma \\ &\quad (Ae_{k+\tau-1} + \xi_{k+\tau-1}) \\ &\quad + \delta_k(1-\beta)\big(\sum_{r=1}^{\tau} A^{r-1}\xi_{k+\tau-r}\big)^\top \Gamma \sum_{r=1}^{\tau} A^{r-1}\xi_{k+\tau-r}] \\ &= \mathbf{E}[(Ae_{k+\tau-1} + \xi_{k+\tau-1})^\top \Gamma (Ae_{k+\tau-1} + \xi_{k+\tau-1}) \\ &\quad - \delta_k(1-\beta)\epsilon_k^\top \Gamma \epsilon_k]\end{aligned} \tag{17}$$

with ϵ_k defined in (14). The first equality establishes as the packet dropout status is independent of the random variable ξ_k. Inserting (17) into (16), the cost-to-go function (16) under the minimizer δ_k^* is obtained as (13). ■

Remark 1: The classic VoI expression is $\text{VoI}_k = e_k^\top \Gamma e_k - \theta + \rho_k$ [11], where the error variable e_k is as in (12). It can be observed that the network-effect-aware VoI in (14) contains Gaussian noise realizations since the latest triggering, which is more informative than the classic one. Additionally, a larger communication delay leads to increasing information freshness and more frequent triggering, while the awareness of packet dropouts probability will make the packet less valuable.

Remark 2: By leveraging the structural analysis of event-triggered control subject to delay and packet dropouts in [15], the optimal scheduling-based remote estimation can be easily extended to the optimal event-triggered control problem.

The residual term ρ_k is interpreted as the deviation of the cost-to-go under different scheduling decisions δ_k. To reduce the computation burden and facilitate the implementation of the VoI metric, we will eliminate the ρ_k using a rollout algorithm. In the following, we use a rollout algorithm as the baseline policy and evaluate the cost-to-go under the designed scheduling policy.

Lemma 2: Consider the optimization problem (6) for system dynamics (1), the rollout-based scheduling policy $\tilde{\pi}$

$$\tilde{\delta}_k = \tilde{\pi}(\mathcal{I}_k^s) = \begin{cases} 1 & \text{if } \text{VoIP}_k > 0 \\ 0 & \text{otherwise,} \end{cases} \tag{18}$$

where $\text{VoIP}_k = -\theta + (1-\beta)\epsilon_k^\top \Gamma \epsilon_k$ denotes the approximation of the VoI at time k, is suboptimal, and outperforms a periodic triggering policy.

Proof: Let $\bar{\pi} = \{\bar{\delta}_0, \bar{\delta}_1, \dots\}$ be a periodic scheduling policy with a period of p. We choose $\tilde{\pi} = \{\delta_k, \bar{\delta}_{k+1}, \bar{\delta}_{k+2}, \dots\}$ as the baseline policy for the rollout algorithm to deal with ρ_k. Under the same communication delay and packet dropouts probability, we have $\mathbf{E}[V_{k+1}|\mathcal{I}_k^s, \delta_k = 0, \bar{\delta}_{k+1}, \dots] = \mathbf{E}[V_{k+1}|\mathcal{I}_k^s, \delta_k = 1, \bar{\delta}_{k+1}, \dots]$. Thus, it results in $\rho_k = 0$ using the rollout algorithm, and the VoIP function is obtained as (18).

The next is to prove that the obtained VoIP-based scheduling policy outperforms a periodic triggering policy. Let the cost-to-go function under VoIP-based and periodic triggering policies be $\tilde{V}_k$ and $\bar{V}_k$, respectively. We need to prove $\tilde{V}_k \le \bar{V}_k$, which enables $\Psi(\tilde{\gamma}) \le \Psi(\bar{\gamma})$ establish. Assume that the claim holds for $k+1$, we have

$$\begin{aligned}\tilde{V}_k &= \mathbf{E}[\theta\tilde{\delta}_k + e_{k+\tau}^\top \Gamma e_{k+\tau} + \tilde{V}_{k+1} \mid \mathcal{I}_k^s] \\ &\le \mathbf{E}[\theta\tilde{\delta}_k + e_{k+\tau}^\top \Gamma e_{k+\tau} + \bar{V}_{k+1} \mid \mathcal{I}_k^s] \\ &\le \mathbf{E}[\theta\bar{\delta}_k + e_{k+\tau}^\top \Gamma e_{k+\tau} + \bar{V}_{k+1} \mid \mathcal{I}_k^s] = \bar{V}_k,\end{aligned}$$

The first inequality follows from the induction hypothesis, and the second inequality follows from the definition of the sub-optimal triggering policy $\tilde{\gamma}$. ■

B. Performance characterization

Consider for the special case $\{\tilde{\delta}_{k-\tau} = \dots = \tilde{\delta}_{k-1} = 0\}$, we have $\eta_{k+\tau-1} = \eta_{k-1} + \tau$. In this case, the local scheduler has full knowledge of the remote estimation. In order to simplify the tradeoff analysis, we implement a waiting strategy to further simplify performance characterization. To achieve it, we introduce a variable s_k to denote the availability of a communication network. The communication channel is available when $s_k = 0$, otherwise $s_k = 1$. The network availability status is denoted as

$$s_{k+1} = \begin{cases} \tau & \text{if } \tilde{\delta}_k = 1 \;\&\; s_k = 0 \\ s_k - 1 & \text{if } \tilde{\delta}_k = 0 \;\&\; s_k > 0 \\ 0 & \text{if } \tilde{\delta}_k = 0 \;\&\; s_k = 0 \end{cases} \tag{19}$$

with $s_0 = 0$. Specifically, the waiting strategy (19) implies that the local scheduler has received feedback from the remote estimator about transmitted packets status before making decisions.

To simplify notations, we define the elapsed time since the last triggering by time k as $t_k = k - \max\{l|\delta_l = 1, l < k\}$. Impose a time-out interval T on the triggering policy (18), we obtain the scheduling policy with waiting strategy as follows:

$$\check{\delta}_k = \check{\pi}(\mathcal{I}_k^s, s_k) = \begin{cases} 1 & \text{if } t_k = T \\ \tilde{\delta}_k & \text{else if } s_k = 0 \\ 0 & \text{otherwise.} \end{cases} \tag{20}$$

Therefore, the triggering intervals $\{t_k\}_k$ is restricted within a finite state space $\mathcal{S} := \{1, \ldots, T\}$. Note that the inter-event intervals will be not less than the round trip time $\tau + 1$, we require $T > \tau + 1$ to guarantee that the time-out restriction will not conflict with the waiting strategy.

In the following, we prove that the scheduling policy (20) outperforms a periodic triggering policy.

Lemma 3: Consider the optimization problem (6) for system dynamics (1), the scheduling policy $\check{\pi}$ with waiting strategy as in (20) is suboptimal, and outperforms a periodic triggering policy.

Proof: Let the cost-to-go function under triggeing policy (20) be $\check{V}_k$. The same as Lemma 2, we need to prove $\check{V}_k \leq \bar{V}_k$. Assume that the claim holds for $k+1$, we have

$$\begin{aligned} \check{V}_k &= \mathbf{E}[\theta\check{\delta}_k + e_{k+\tau}^\top \Gamma e_{k+\tau} + \check{V}_{k+1} \mid \mathcal{I}_k^s] \\ &\leq \mathbf{E}[\theta\check{\delta}_k + e_{k+\tau}^\top \Gamma e_{k+\tau} + \bar{V}_{k+1} \mid \mathcal{I}_k^s] \\ &\leq \mathbf{E}[\theta\bar{\delta}_k + e_{k+\tau}^\top \Gamma e_{k+\tau} + \bar{V}_{k+1} \mid \mathcal{I}_k^s] = \bar{V}_k, \end{aligned}$$

The remaining proof is the same as in Lemma 2. ■

In this section, we are going to characterize the performance of the VoIP-based scheduling policy (20) for remote estimation subject to delay and packet dropouts. Before presenting the main result, we have the following lemmas.

Lemma 4: The sequence of random variables $\{t_k\}_k$ is an ergodic Markov chain on the state space $\mathcal{S}$. Its transition probability $\mathbf{P} \in \mathbb{R}^{T\times T}$ is given as

$$\mathbf{P}_{ij} = \begin{cases} 1 & \text{if } j = i+1, i = \{1, \ldots, \tau\}, \\ & \text{or } i = T, j = 1 \\ p_{i,1} & \text{if } j = 1, i = \{\tau+1, \ldots, T-1\} \\ 1 - p_{i,1} & \text{if } j = i+1, i = \{\tau+1, \ldots, T-1\} \\ 0 & \text{otherwise,} \end{cases}$$

where its components are $\mathbf{P}_{ij} = \mathrm{P}(t_{k+1} = j | t_k = i)$ for some instants k, and $p_{i,j} \in (0, 1)$ are some positive scalars.

Proof: Firstly, we prove that the random variable $\{t_k\}_k$ is a Markov chain. According to the triggering law (20) and VoIP expression given in (18), we define the admissible region of ϵ_k as $\mathcal{M} := \{\epsilon_k \in \mathbb{R}^n | \|\epsilon_k\| \leq \theta\}$. We have

$$\begin{aligned} &\mathrm{P}\left(t_{k+1} \mid t_k, t_{k-1}, \cdots, t_0\right) \\ =& \int_{\mathcal{M}} \mathrm{P}\left(t_{k+1}, \epsilon_k \mid t_k, t_{k-1}, \cdots, t_0\right) \mathrm{d}\epsilon_k \\ =& \int_{\mathcal{M}} \mathrm{P}\left(t_{k+1} \mid \epsilon_k, t_k, t_{k-1}, \cdots, t_0\right) \\ &\times \mathrm{P}\left(\epsilon_k \mid t_k, t_{k-1}, \cdots, t_0\right) \mathrm{d}\epsilon_k \\ =& \int_{\mathcal{M}} \mathrm{P}\left(t_{k+1} \mid \epsilon_k, t_k\right) \mathrm{P}\left(\epsilon_k \mid t_k\right) \mathrm{d}\epsilon_k \\ =& \int_{\mathcal{M}} \mathrm{P}\left(t_{k+1}, \epsilon_{k+1} \mid t_k\right) \mathrm{d}\epsilon_{k+1} = \mathrm{P}\left(t_{k+1} \mid t_k\right), \end{aligned}$$

The third equality establishes as t_{k+1} is determined by triggering policy (20) based on ϵ_k and t_k, the random variable ϵ_k depends on $\eta_{k+\tau-1}$ as in (14), and $\eta_{k+\tau-1} = t_k + \tau - 1$. In addition, as the probability density function of the random variable ξ_k in the triggering condition (13) is continuous, one can easily verify that the Markov chain $\{t_k\}_k \in \mathcal{S}$ is irreducible, aperiodic, with all states being positive recurrent, and thus ergodic. According to the scheduling policy (20), the scheduler will not transmit the data before receiving feedback signal $\gamma_{k-\tau}$, we have $p_{i,i+1} = 1$, for $i = \{1, \ldots, \tau\}$. In addition, $p_{T,1} = 1$ as T is the time-out interval. The computation of components $p_{i,1}$, for $i = \{\tau+1, \ldots, T-1\}$, is presented in the Appendix. ■

The following lemma calculates the occurrence probability of the triggering intervals in the long run.

Lemma 5: Consider the system dynamics (1) under the VoIP-based triggering policy (20) with the time-out interval T. Denote $q_i = \mathrm{P}(t_k = i)$ as the stationary distribution of $t_k = i$, for some k and all $i \in \mathcal{S}$, which is given as

$$q_i = \begin{cases} r & \text{if } i = \{1, \ldots, \tau+1\} \\ r\prod_{j=\tau+1}^{i-1}(1 - p_{j,1}) & \text{if } i = \{\tau+2, \ldots, T\}, \end{cases} \tag{21}$$

where r denotes the average triggering rate is as follows:

$$r = \frac{1}{\tau + 1 + \sum_{i=\tau+2}^{T}\prod_{j=\tau+1}^{i-1}(1 - p_{j,1})}. \tag{22}$$

Proof: Give the waiting strategy (19), we have $\eta_{k+\tau-1} = t_{k-1} + \tau - 1$ for some k. The random variable ϵ_k in the scheduling policy (18) is therefore simplified as $\epsilon_k = \sum_{r=1}^{t_{k-1}} A^{r+\tau-1}\xi_{k-r}$. We use the notation $\epsilon(t_k = i)$ to highlight that the random variable ϵ_k is a function of t_k. Let $\mathbb{I}$ denotes the indicator function. By Birkhoff's Ergodic Theorem [18], the equality

$$r = \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\check{\delta}_k = \mathbf{E}[\mathbb{I}_{(1-\beta)(A^\tau\epsilon(\tau+1))^\top\Gamma A^\tau\epsilon(\tau+1)\geq\theta}]$$

establishes almost surely. Stack the stationary distribution q_i into a vector $\vec{q} = [q_1, \cdots, q_T]$. The stationary distribution q_i is calculated with $\vec{q} = \vec{q}\mathbf{P}$ and $\vec{q}\mathbf{1}_T = 1$, where $\mathbf{1}_T$ denotes the T-dimension vector with all entries being 1. ■

In the following, we are going to compute the expected average cost of the optimization objective defined in (6). Random packet dropouts make it difficult for the local scheduler to predict the information freshness at the remote estimator. Therefore, we provide the upper bound of the expected estimation performance. Define a function $h(X) = A^\top XA + W$ and denote h^i as the i-th function composition of h. The covariance of the remote estimation error at time k is $h^{\eta_k}(P^s)$, where the matrix P^s is the covariance of the local estimation error $\hat{e}_k^s$ as in (4).

***Theorem** 2:* Consider the optimization objective (6) for the system dynamics (1). Under the scheduling policy (20), the upper bound of the expected average cost is given by

$$\Psi_{\text{up}}(\breve{\pi}) = \frac{1}{\eta_{\text{ave}}} \sum_{i=1}^{\eta_{\text{ave}}} \text{tr}(\Gamma h^i(P^s)) + r\theta, \tag{23}$$

where $\eta_{\text{ave}} = \lceil \frac{1}{r(1-\beta)} \rceil + \tau$ represents the upper bound of the average information freshness of the remote estimator.

Proof: First, taking the expectation of the expected cost (6), we have

$$\Psi(\breve{\pi}) = \limsup_{N\to\infty} \frac{1}{N+1} \mathbf{E}[\sum_{k=0}^{\tau} (\hat{e}_k^\varepsilon)^\top \Gamma \hat{e}_k^\varepsilon + \sum_{k=0}^{N} \theta \breve{\delta}_k] + \sum_{k=\tau+1}^{N} \text{tr}(\Gamma h^{\eta_k}(P^s)). \tag{24}$$

The first term $\limsup_{N\to\infty} \frac{1}{N+1}\mathbf{E}[\sum_{k=0}^{\tau} (\hat{e}_k^\varepsilon)^\top \Gamma \hat{e}_k^\varepsilon]$ converges to zero when the time horizon T approaches infinity. The average communication cost is given as $\limsup_{N\to\infty} \frac{1}{N+1}\mathbf{E}[\sum_{k=0}^{N} \theta\breve{\delta}_k] = r\theta$, where r is calculated from Lemma 5.

The event-based scheduling policy (20) outperforms a pure offline policy, e.g., the periodic triggering policy, as proved in Lemma 3. Thus, we upperbound the estimation performance under the scheduling policy (20) by the cost under the periodic triggering policy with the same average communication rate. The average arrival rate between the local scheduler and the remote estimator is $r(1-\beta)$. Adding up the network-induced delay, we obtain η_{ave} as the average information freshness at the remote estimator. The expected estimation performance under the periodic triggering policy is give as $\frac{1}{\eta_{\text{ave}}} \sum_{i=1}^{\eta_{\text{ave}}} \text{tr}(\Gamma h^i(P^s))$. In combination of (24), we obtain (23). ■

IV. Simulation Result

In this section, we will illustrate the effectiveness of our proposed VoI-based scheduling approach through a numerical simulation. The stability analysis is provided in [15], which requires $\beta < \frac{1}{\|A\|_2^{2\tau+2}}$. Thus, we select a scalar linear system with system matrices $A = 0.9$ and $C = 1$. The covariance matrices of process noise and measurement noise are $W = 0.01$ and $V = 0.01$, respectively. The weighting factor in the estimation performance penalty function (6) is chosen as $\Gamma = 1$. The time horizon is chosen as $N = 500$.

In Fig. 2, we compare tradeoffs between estimation performance and transmission rate of the system (1) under the classic VoIP-based scheduling policy, the network effect-aware VoIP-based scheduling policies (18) and (20), and periodic scheduling policies, respectively. The empirical average triggering rate is defined as $\frac{1}{N+1}\sum_{t=0}^{N} \breve{\delta}_k$ and the

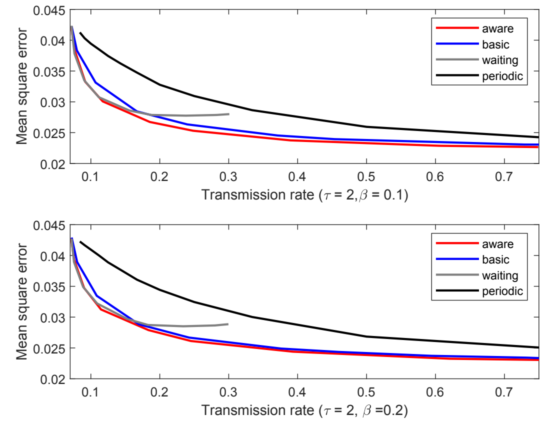

Fig. 2. Tradeoffs between estimation performance and communication rate.

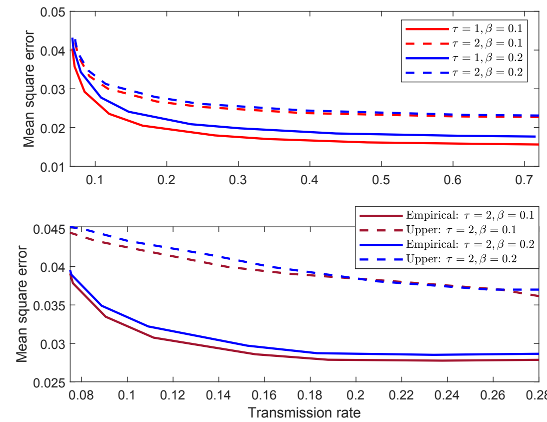

Fig. 3. From top to button: Tradeoffs between estimation performance and communication rate w.r.t different network effects; empirical estimation performance of scheduling policy (20) and its upper bound.

estimation performance is measured by the average mean square error $\frac{1}{N+1}\sum_{k=0}^{N} \|\hat{e}_k^\varepsilon\|^2$. Monte Carlo simulations run 2000 trials. It can be observed that the network-effect-aware VoIP-based scheduling policy (18) outperforms the remaining scheduling policies, including the classic VoI-based scheduling and periodic scheduling policies.

In the top subfirgure of Fig. 3, we depict the tradeoff between estimation performance and communication rate with VoIP-based scheduling policy (18) under different network effects. It can be observed that the performance decreases with the increasing network effects. The button subfigure of Fig. 3 shows that the derived theoretical value (23) successfully bounds the simulated average cost.

V. Conclusions

In this paper, we investigated the online scheduling for resource-aware remote state estimation in the presence of network-induced delay and packet dropouts. A network-effect-aware VoI metric is constructed to facilitate efficient communication between the sensor and the remote estimator. We prove that the designed scheduling outperforms other scheduling policies, including the scheduling based on the classic VoI unaware of the network effects. We implement and analyze the performance of scheduling based on this network-effect-aware VoI metric. The communication rate and the upper bound of the estimation performance are characterized analytically under the designed scheduling policy. Finally, numerical simulations demonstrate the efficacy of the metric-based scheduling policy.

VI. Appendix

Given the inter-event time t_k, the variable ϵ_k in the scheduling policy (18) is simplified as $\epsilon_k = \sum_{r=1}^{t_{k-1}} A^{r+\tau-1}\xi_{k-r}$, where $\mathrm{cov}[\xi_k|\mathcal{I}_k^s] = KC\tilde{P}^s$ with $\tilde{P}^s$ being the covariance of prediction error $\hat{e}^s_{k|k-1}$. Denote $\Pi := KC\tilde{P}^s$ and $\Pi_i = h^{i-1}(\Pi)$. Consider the error vector $\vec{\epsilon}(l) = [\epsilon(\tau+1)^\top \ \cdots \ \epsilon(\tau+l)^\top]^\top \in \mathbb{R}^l$, for $l \in \{1,\dots,T-\tau\}$, which is a random vector having the multivariate normal distribution with zero mean and the covariance matrix $\Sigma(l)$ with components $\Sigma_{ij}(l)$ as follows: for $i,j \in \{1,\dots,l\}$,

$$\Sigma_{ij}(l) = \begin{cases} A^\tau \Pi_{i+\tau}(A^\tau)^\top & \text{if } i = j, \\ A^\tau \Pi_{i+\tau}(A^{j+\tau-1})^\top & \text{if } j > i, \\ \Sigma_{ji}^\top(l) & \text{otherwise.} \end{cases}$$

Note that the negative information effect is not explored in this study, which generally leads to significantly more involved filtering algorithms, see [19]. Define the admission region of $\vec{\epsilon}(l)$ as $\mathcal{U}_l := \{\vec{\epsilon}(l) \in \mathbb{R}^l | \bigcap_{r=1}^{l} \epsilon(\tau+l)^\top \Gamma \epsilon(\tau+l) \leq \frac{\theta}{1-\beta}\}$, for $l = \{1,\dots,T-\tau\}$.

Lemma 6: The components $p_{l+\tau,1}$, for $l \in \{1,\dots,T-\tau-1\}$ of the transition matrix $\mathbf{P}$ are given as

$$p_{l+\tau,1} = \begin{cases} \mathcal{P}_i & l = 1 \\ 1 - \frac{\mathcal{P}_{l+1}}{\mathcal{P}_l} & l \in \{2,\dots,T-\tau-1\} \end{cases}$$

with $\mathcal{P}_l = \frac{1}{\sqrt{2\pi\Sigma(l)}} \int_{\mathcal{U}_l} \exp(-\frac{1}{2}\varsigma_l^\top(\Sigma(l))^{-1}\varsigma_l)\mathrm{d}\varsigma_l$ and $\varsigma_l \in \mathbb{R}^l$, for $l \in \{1,\dots,T-\tau-1\}$.

Proof: The transition probability $p_{l+\tau,1}$ is calculated based on the VoIP-based scheduling polic, as in (18). We classify the calculation into the following two categories.

1) The event $\{\check{\delta}_{k-i+1} = \cdots = \check{\delta}_{k-1} = 0, \check{\delta}_{k-i} = 1\}$ is equivalent to $\{t_{k-1} = i\}$, for some k. For $l = 1$, $p_{\tau+1,1}$ implies that the event $\{\check{\delta}_k = 1, t_{k-1} = \tau\}$, for some k occurs with the probability $p_{\tau+1,1} = \Pr(\epsilon(\tau+1) \in \mathcal{U}_{\tau+1}) = \mathcal{P}_1$.

2) For $l = \{2,\dots,T-\tau-1\}$, the conditional transition probability $p_{l+\tau,1}$ refers that the event $\{\check{\delta}_k = 1, t_{k-1} = l+\tau-1\}$ occurs with the probability

$$\begin{aligned} p_{l+\tau,1} &= \mathrm{P}(\check{\delta}_k = 1 | t_{k-1} = l+\tau-1) \\ &= \frac{\mathrm{P}(\check{\delta}_k = 1, t_{k-1} = l+\tau-1)}{\mathrm{P}(t_{k-1} = l+\tau-1)} \\ &= 1 - \frac{\mathrm{P}(t_k = l+\tau)}{\mathrm{P}(t_{k-1} = l+\tau-1)}. \end{aligned}$$

The event $t_k = l+\tau$, for $l = \{1,\dots,T-\tau\}$ and some k, occurs with the probability $\mathrm{P}(t_k = l+\tau) = \mathrm{P}(\vec{\epsilon}(l) \in \mathcal{U}_l)$. ■

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