2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

# Who Gets the Whip? How Supplier Diversification Influences Bullwhip Effect in a Supply Chain

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*Abstract*— **To navigate the evolving terrain of Supply Chains (SC), firms require new tools with broader applicability. Current research ignores the forest in favor of trees, with focal firms and serial networks assumed. This paper explicates a novel and scalable model for SC study at a broad level. We utilize the core of the model to observe the effect of structure and policy on demand disturbance in a SC as a whole. We find that complex structure alone does not effect change in disturbances; dynamic policy is necessary and sufficient for amplification. The model and simulations build to our main result: diversifying a firm's supplier-base can amplify disturbances more quickly.**

## I. Introduction

A supply chain (SC) is a network of firms that coordinate the flows of goods from raw materials (RM) to the end user, linked together through physical, information, and monetary flows. Given the massive expanse of the "global SC," that is, the network structure of every firm that participates in the global economy, the study of SCs typically assumes a focal firm. Because extant research has largely been interested in *intra*-firm processes [1], SCs are typically modelled as either single-product dyadic or serial networks [2][3]. However, this is not a picture of reality, as firms in a SC often have multiple suppliers and/or customers. Further, supply chains are complex adaptive systems, implying that they have emergent properties. Those emergent properties make the study of supply chain behavior incomplete at any level lower than the supply chain.

A common result in SCM literature is the presence of the bullwhip effect (BWE) [4] [5] [6]. The BWE is the observation that the variance of the demand signal increases as one traverses up the SC, and is broadly categorized as a behavior of the SC system. If an input signal is denoted $u(t)$, and an output signal $y(t)$, BWE states that $\frac{V[y(t)]}{V[u(t)]} > 1$, where $V[\cdot]$ is the variance operator. The work in [1] notes that there are two underlying drivers of the BWE: (1) demand distortion and (2) variance amplification. Demand distortion is when "orders to suppliers tend to have larger variance than sales to the buyer" [4], and variance amplification is when that "distortion propagates upstream in an amplified form" [4]. The literature has identified 19 managerial decisions, herein called 'policies,' that drive the BWE [3] [1]. Example policies include safety stock, order batching, shipping lag, etc. The BWE is a problem faced by approximately 2/3 of all firms [3]. Negative effects on firms stemming from BWE include higher cost of capital, increased inventory holding costs, and a decrease in profitability[3].

Controlling SCs requires both understanding system-wide behavior and knowing how network structure relates to that behavior. We seek to understand how two phenomena—managerial decision making (policies) and network structure—interact to affect the behavior of a SC. We introduce the theoretical notion that policies can act on the input, output, or state of a SC system, as well as its connected sub-systems (customer-supplier pairs). And it is those policies, not the structure directly, that are shown to have an influence on the BWE. The key connection is that some policies are limited by the SC structure. In that way, policy mediates the relationship between structure and behavior. This notion is given credence with results such as Li, et al. [7], who find that a production policy mediates the relationship between structure and behavior (BWE).

Of those papers that seek to understand the relationship between structure and behavior, the dominant research paradigm is to assume one of two types of structures: divergent and convergent (Figure 1). A convergent structure is one where a focal firm has multiple suppliers, and a divergent structure is one where a focal firm has multiple customers. According to [3], at the time of their writing, 28% of articles looking at the impact that structure has on a SC used a dyadic structure, 42% used a serial structure, and 17% used a divergent structure. There were no papers that looked at convergent structures.

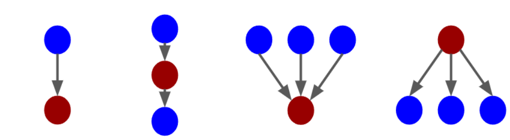


Fig. 1. Comparison between structures with a focal firm (red).

As can be seen, there is a need to model SC behavior in convergent structured SC. Literature has typically specified that the convergent structure is 'assembly' [3], meaning that a focal supplier provides a unique part that is assembled together into one good. There is another type of convergent structure, what we call 'diversification,' where a focal supplier purchases the same good from multiple unique suppliers. This is often done as a means

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to lower risk, similar to choosing a diverse investment portfolio in the financial markets. To distinguish between those two paradigms, we will denote 'convergent-assembly' and 'convergent-diversification' (CD). To our knowledge, our study is the first to analyze how a CD structure influences BWE in a supply chain.

We show through analytical and simulation results that, as compared to a purely serial SC, a more complex CD structure does not amplify the BWE if a static policy is chosen. However, the complex structure with a time-varying policy significantly amplifies the BWE. This is evidence that the relationship between structure and SC behavior is mediated by policies. We find intriguing indications that this relationship is further modified by the nature of the demand signal. We are encouraged that future studies on this effect can provide meaningful insights.

Finally, we present a generalized form of a SC model that future researchers can use to derive both analytic and numerical results. This generalized form of a SC allows for any structural configuration; expanding the ability to analyze SCs beyond a simple dyadic or serial structure. This generalized mathematical form of an entire SC starts to answer the call by researchers interested in SC-wide coordination research [8][9], which is anticipated to be enabled through blockchain technologies [10][11]. Any research agenda that is ultimately seeking to **control** the blockchain-enabled SC through coordination mechanisms must first understand how the entire system behaves as a result of individual manager decision making. We take first steps in doing so.

In summary, we make three primary contributions:

1) Section II provides a novel mathematical model of a SC with more complex structure. This general model can be adapted and built upon based on the particular structure and policies of interest.
2) We show in Theorem 1 the relationship between structure and policy, demonstrating that structure in and of itself does not necessarily amplify the BWE. Rather, it is the policy that amplifies the BWE. In fine, structure enables policy and policy enables BWE.
3) We explicate the conditions under which results on the BWE in serial SCs—a majority of extant research—are applicable to CD supply chains (Section **??**). These same conditions also enable results on BWE in CD supply chains to be performed on a simplified (i.e. serial) structure.

The paper is set out as follows: Section II details the generalized SC model, Section III provides the main results with discussion, Section IV provides the simulation results, and we conclude in Section V.

## II. Model

In this section we lay out a general form of a SC model, to aid researchers in future studies on the impact of structure and behavior. In a later section the model will be used in an agent-based simulation, verifying the above results and adding additional richness to the nature of the amplification effect.

The BWE is a result that is derived from the SC being viewed as an input-output (IO) relationship. However, IO relationships typically ignore the structure of the system (i.e. are 'black box'). Thus, we are interested in a class of models where structural relationships can be defined. Further, models of this class have state space representations, encoding the structure of the SC. We will now explicate this set of models.

### A. The $\mathscr{S}^G$ Model

Let $\mathscr{S}^G$ be the set of all models that are under our consideration. These systems are causal, time-invariant, and have memory. They are defined by the graph structure $G : (V,E)$, where edges $E$ represent the flow of information (e.g. demand signals) and material goods, & nodes $V$ represent firms.

*1) State Space Representation:* Let the vector $\mathbf{y}[t]$ represent the outputs of the system and $N^{RET}$ and $N^{RM}$ be the number of retail firms and raw material firms in the SC, respectively. The vector can be partitioned into two vectors: $\mathbf{y^{FG}}[t] \in N^{RET}$ are the outputs of physical finished goods (FG) to the end consumer, and $\mathbf{y^{DEM}}[t] \in N^{RM}$ are the demand signals from raw materials producers for "extraction". It represents the SC's demand for particular raw materials. Thus, $\mathbf{y}[t] = \begin{bmatrix}\mathbf{y^{FG}}[t]^T & \mathbf{y^{DEM}}[t]^T\end{bmatrix}^T$.

Let the vector $\mathbf{u}[t]$ represent the inputs into the system. The vector can be partitioned into two vectors: $\mathbf{u^{RM}}[t] \in N^{RM}$ are the inputs of physical raw materials, and $\mathbf{u^{DEM}}[t] \in N^{RET}$ are the demand signals from end consumers. A demand input is a random variable pulled from a distribution D, with mean $\mu$ and variance $\sigma^2$: $\mathbf{u^{DEM}}[t] \sim D(\mu, \sigma^2)$. The raw materials input $\mathbf{u^{RM}}[t]$ is equal to the demand signal output $\mathbf{y^{DEM}}[t]$ with some lag $\Delta$ (representing the process of extracting raw materials): $\mathbf{u^{RM}}[t] = \mathbf{y^{DEM}}[t-\Delta]$. Thus, $\mathbf{u} = \begin{bmatrix}\mathbf{y^{DEM}}[t-\Delta]^T & \mathbf{u^{DEM}}[t]^T\end{bmatrix}^T$.

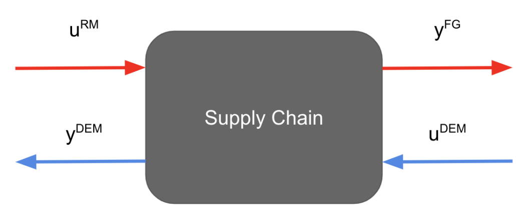


Fig. 2. 'Black Box' Model of a SC

The states of the system, $\mathbf{x}[t]$, can also be partitioned. The first partition is raw material inventories $\mathbf{x^{RM}}[t] \in \mathbb{R}^N$, the second partition is FG inventories $\mathbf{x^{FG}}[t] \in \mathbb{R}^N$, and the third partition are the emergent states of the system $\mathbf{x^{EMRG}}[t] \in \mathbb{R}^E$ where $N$ represents the total number of firms in the system and $E$ represents the number of emergent states.

Putting it together, the vector of states for models in $\mathscr{S}^G$ for a given time period $t$ is:

$$\mathbf{x}[t] = \begin{bmatrix}\mathbf{x^{FG}}[t]^T & \mathbf{x^{RM}}[t]^T & \mathbf{x^{EMRG}}[t]^T\end{bmatrix}^T \tag{1}$$

There are $2N+E$ states in the system per time period, while the size of the input and output vectors are dependent on the types of firms in the system.

There are four categories ('echelons') of firms considered in our model [12] [13]. The first are raw materials producers. These are by necessity at the "top" of the SC; these firms handle the demand and supply of raw materials to the system. These firms do not have any other firms as suppliers. The second and third type of firms are manufacturers and distributors. The fourth are retailers, which by necessity are at the "bottom" of the SC; these firms handle the demand and supply of FG for the SC system. Retailers do not have any firms as customers.

Using those definitions, the size of the output vector $\mathbf{y}[t]$ is $N_{RM}+N_{RET}$; there are as many demand outputs as there are raw material firms, and as many Fg outputs as there are retail firms. The size of the input vector $\mathbf{u}[t]$ is similarly $N_{RM}+N_{RET}$; there are as many demand inputs as there are retail firms and as many raw materials inputs as there are raw material firms.

The larger system (the 'SC') is composed of subsystems, representing firms. Each firm has its own policies, and they do not have access to any other firm's policies. Therefore, the general form of the system dynamics in state space form is:

$$\begin{bmatrix} \mathbf{x}_1[t+1] \\ \vdots \\ \mathbf{x}_N[t+1]) \end{bmatrix} = \begin{bmatrix} g_1(\mathbf{x}_1[t]) \\ \vdots \\ g_N(\mathbf{x}_N[t]) \end{bmatrix} + \begin{bmatrix} k_1(\mathbf{u}_1[t]) \\ \vdots \\ k_N(\mathbf{u}_N[t]) \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} \mathbf{y}_1[t] \\ \vdots \\ \mathbf{y}_N[t] \end{bmatrix} = \begin{bmatrix} h_1(\mathbf{x}_1[t]) \\ \vdots \\ h_N(\mathbf{x}_N[t]) \end{bmatrix}, \tag{3}$$

with $g_i$, $h_i$, and $k_i$ representing firm $i$'s 'state policy', 'output policy', and 'input policy' respectively (see Section II-C). Equations 2 and 3 show that each firm in the SC have their own unique $g$, $h$ and $k$.

### B. Customer-Supplier Relationships: Building Blocks of the System

The fundamental building blocks of models in the set $\mathscr{S}^G$ are customer-supplier relationships. Define a firm as an input-output relationship, and denote the inputs and outputs into a particular firm with a subscript numeral $i$: $\mathbf{u}_i[t] = \begin{bmatrix} u_i^{RM}[t] & u_i^{DEM}[t] \end{bmatrix}^T$ and $\mathbf{y_i}[t] = \begin{bmatrix} y_i^{FG}[t] & y_i^{DEM}[t] \end{bmatrix}^T$. Each firm is assumed to have only one type of raw material good and one type of FG[1].

Now consider two firms, $i=1$ (the customer) and $i=2$ (the supplier) in a customer-supplier relationship:

$$\mathbf{u}_1[t] = \begin{bmatrix} y_2^{FG}[t] & u_1^{DEM}[t] \end{bmatrix}^T \tag{4}$$

$$\mathbf{u}_2[t] = \begin{bmatrix} u_2^{RM}[t] & y_1^{DEM}[t] \end{bmatrix}^T \tag{5}$$

$$\mathbf{y}_1[t] = \begin{bmatrix} y_1^{FG}[t] & y_1^{DEM}[t] \end{bmatrix}^T \tag{6}$$

$$\mathbf{y}_2[t] = \begin{bmatrix} y_2^{FG}[t] & y_2^{DEM}[t] \end{bmatrix}^T, \tag{7}$$

taking into account that the raw material output of firm 1 is the FG input for firm 2, and the demand output of firm 1 is the demand input for firm 2. The states for these firms mirror the states for the larger SC system: $\mathbf{x}_i[t] = \begin{bmatrix} x_i^{FG}[t] & x_i^{RM}[t] & \mathbf{x_i^{EMRG}}[t] \end{bmatrix}$.

*a) The Special Case of the Raw Materials Firm:* As inputs for raw material firms are delayed outputs, there are no possible suppliers to these firms [14]. The customer-supplier relationship is otherwise the same for raw materials firms, as long as they are the supplier.

*b) The Special Case of the Retail Firm:* As outputs for retail firms are sold to the end consumer, there are no possible firm customers to these firms [14]. The customer-supplier relationship is otherwise the same for retail firms, as long as they are the customer.

### C. Policies

Policies can be thought of as the decision processes that managers in firms have to make. We have already introduced the three categories of policies, 'state policies' ($g_i$), 'output policies' ($h_i$), and 'input policies' ($k_i$).

Each firm's independent choice of policy, be it input, output, or state, has a causal influence on the BWE in the (sub-)system. That is, BWE arises because managers in each firm in a SC are making decisions with varying levels of information and coordination with other firms. The result of this, alongside simple manager preference, is that each firm has unique input, state, and output policies. These categories do not need be a strict partition.

*1) State Policies $g_i$:* We can decompose those policies further to get an insight into the underlying mechanics. State policies are the primary drivers of the demand-distortion portion of BWE. They are *intra*-firm processes that directly affect the states of the system $x_i^{FG}$ and $x_i^{RM}$. For example, after the state policy $p_i$ is chosen, it adjusts $x_i^{RM}$ (decrease) and $x_i^{FG}$ (increase) in each time period. Some examples include how many (1) raw materials (RM) to hold as safety stock, $\alpha_i(\cdot)$, (2) FG to produce, $p_i(\cdot)$, and (3) FG to ship, $s_i^Q(\cdot)$.

*2) Output Policies $h_i$:* Output policies are the primary drivers of the variance amplification portion of the BWE. These are *inter*-firm processes that directly change the nature of the output signal $\mathbf{y}_i$. Examples include how (1) many RM to order, $o_i^Q(\cdot)$, (2) to price FG, $\phi_i(\cdot)$, (3) many FG to ship, $s_i^Q(\cdot)$, who to (4) order RM from, $o_i^A(\cdot)$, and (5) ship to, $s_i^A(\cdot)$.

*3) Input Policies $k_i$:* The final category of policies, input, are *intra*-firm processes that aim to predict (e.g. forecasting), directly modify (e.g. rejecting shipment at the door), feed-forward (e.g. setting $y_i^{DEM}[t]$ equal to $u_i^{DEM}[t]$), or otherwise directly influence the input *before* they are added to the states. Examples are how to (1) forecast incoming demand, $d_i(\cdot)$, (2) forecast incoming supply, $\delta_i(\cdot)$, (3) conduct marketing, $m(\cdot)$, and (4) Which shipments to accept or reject, $r_i(\cdot)$.

See Figure 3 for a visualization on how policies interact with signals. Thus, we have for a given firm $i$ the pair of

[1]Sometimes referred to in the literature as an 'intermediate good' if it requires further processing.

equations:

$$\mathbf{x}_i[t+1] = g_i(\mathbf{x}_i[t]; \alpha_i[t], \ldots) + k_i(\mathbf{u}_i[t]; d_i[t], \ldots) \quad (8)$$

$$\mathbf{y}_i[t] = h_i(\mathbf{x}_f[t]; o_i^A[t], \ldots). \quad (9)$$

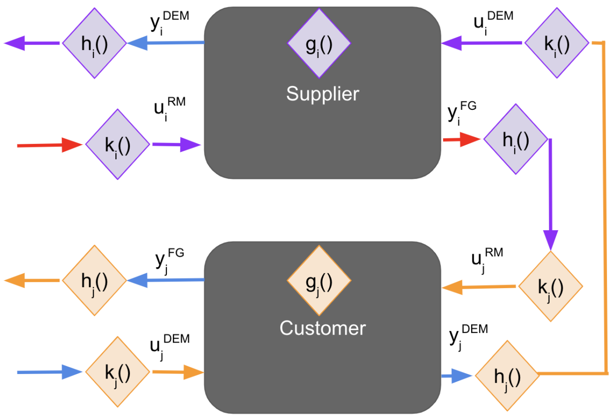


Fig. 3. Customer-Supplier Relationships showing modifications on incoming/outgoing signals. Purple represents a signal changed by the policies of firm $i$ (supplier) and orange represents signals changed by firm $j$ (customer). The state policies $g_i(\cdot)$ are placed showing they only effect the states directly.

*4) SC Structure:* The structure (graph) of the SC modifies the allowable values for output policies. A SC where every firm has no more than one supplier (customer) is limited to *whom* they can ship to (order from): they only have one choice! The 'whom' part of outgoing policies are partitions on a 1-norm (e.g. how much to produce, $\|p_i(\cdot)\|_1$). It can also be viewed as a weighting vector $\omega$ of size $\mathbb{R}^{N_i^{SUP}}$ that acts on the signal $\mathbf{y}_i[t]$, where $N_i^{SUP}$ are the number of suppliers for firm $i$. In this way, the structure of the SC directly determines the size of the weighting vector $\omega$.

The astute reader will notice the lack of financial signals going into and out of the SC (sub-)system(s). Indeed, financial considerations are a key driver of managerial policies, and need to be taken into account. We believe the simplest form of the model has two signals: demand and supply. All other factors, such as information, pricing, etc. are accounted for within the policies (input or output). For example, the choice of price for a finished good (an input policy) affects the incoming demand. That said, the model is general enough that additional signals can be added, not only financial but even environmental signals (e.g. carbon emissions).

With a generalized model of a SC, we now turn to a specific result on the output policy $s_i^A(\cdot)$.

## III. Results

For our main results, we only consider how the policy $s_i^A(\cdot)$ influences BWE. A manager could choose from a wide range of policies $s_i^A(\cdot)$. The policy could be as simple as setting a fixed proportion that is purchased from each supplier. We call this a fixed (time-invariant) policy, and denote it with $y_i = s_i^A(\cdot) \triangleq \lambda$. Every other type of policy is therefore a varying policy: $y_i = s_i^A(\cdot)[t] \triangleq \Lambda[t]$.

Let $\mathbf{y}_i^{DEM}[t]$ be the outgoing demand signal of size $|\omega|$ from a focal firm and $\mathbf{u}_i^{DEM}[t]$ be the incoming demand signal for the same focal firm.

**Theorem 1.** *Hold all other policies as time invariant, allowing only $s_i^A$ to be time-varying. Then,*

$$V(\mathbf{u}_i^{DEM}[t]) \leq V(\mathbf{y}_i^{DEM}[t]), \quad (10)$$

*with equality iff a constant policy $s_i^A = \lambda$ is chosen. Furthermore, for any varying policy $s_i^A(\cdot)[t]$, $V(\mathbf{y}_i^{DEM}[t])$ strictly increases in the number of suppliers $N$ to firm $i$.*

*Proof:* Let $Y$ be a demand signal drawn from distribution $D$, and $\mathbf{Y}$ be that same signal with a partition determined by policy $s_i^A(\cdot)$. Finally, let $\mathbf{Y}_\lambda$ be the signal partitioned using a constant policy $\lambda$, and $\mathbf{Y}_\Lambda$ be the signal partitioned using a varying policy $\Lambda$. We first prove equality.

Assume a time-varying signal $Y_\lambda$ and scalar values $\lambda_{i=1:N}$, representing the scaled versions of that signal, with $\sum_\lambda = 1$. Define $\lambda_N$ to be $(1-\lambda_1-\ldots-\lambda_N)$. The variance of the outgoing signal is $V[\lambda_1 Y_\lambda + \ldots + \lambda_N Y_\lambda] = V[\lambda_1 Y_\lambda] + \ldots + V[\lambda_N Y_\lambda]$, which equals $V[Y_\lambda] + \lambda_1^2 V[Y_\lambda] - \lambda_1^2 V[Y_\lambda] + \ldots + \lambda_N^2 V[Y_\lambda] - \lambda_N^2 V[Y_\lambda]$, making $= V[Y_\lambda] = V[Y]$.

Next we prove the inequality. Define $\Lambda_N$ to be $(1-\Lambda_1-\ldots-\Lambda_N)$, all time varying signals. So, $V[Y_\Lambda] = V[\Lambda_1 Y_\Lambda] + \ldots + V[\Lambda_N Y_\Lambda] + 2\sum_{i\neq j}^N Cov(\Lambda_i Y_\Lambda, \Lambda_j Y_\Lambda)$. Remembering that $\Lambda_N \triangleq (1-\Lambda_1-\ldots-\Lambda_N)$ gives us $V[\Lambda_1 Y_\Lambda + \ldots + \Lambda_N Y_\Lambda] = V[Y_\Lambda] + V[\Lambda_1 Y_\Lambda] - V[\Lambda_1 Y_\Lambda] + \ldots + V[\Lambda_N Y_\Lambda] - V[\Lambda_N Y_\Lambda] + 2\sum_{i\neq j}^N Cov(\Lambda_i Y_\Lambda, \Lambda_j Y_\Lambda) = V[Y_\Lambda] + 2\sum_{i\neq j}^N Cov(\Lambda_i Y_\Lambda, \Lambda_j Y_\Lambda)$

Note that $Y_\Lambda$ is a signal that is partitioned into $N$ scaled signals. Thus, all covariances for these scaled signals are necessarily positive. This means that the right hand side is equal to $V[Y_\Lambda]$ plus a positive number, $V[Y] < V[Y_\Lambda] + 2\sum_{i\neq j}^N Cov(\Lambda_i Y_\Lambda, \Lambda_j Y_\Lambda)$. Using the fact that $Y = Y_\Lambda$, we have $V[Y_\lambda] < V[Y_\Lambda]$.

Finally, we prove the last statement. As was shown in Equation III, for any number of firms $N$, the variance in the SC is equal to the variance of the signal $Y$ plus a number of covariance terms. All of those covariances are scaled versions of the signal $Y$, and therefore are positive. Thus, N suppliers: $\sum_{i\neq j}^{(N-1)} 2Cov(\Lambda_i Y_\Lambda, \Lambda_j Y_\Lambda) + \sum_{i=1}^{(N-1)} 2Cov(\Lambda_i Y_\Lambda, \Lambda_N Y_\Lambda)$.

As this shows, each additional supplier added will add $(N-1)$ Covariance terms. $\square$

## IV. Simulation

We realize specific functional forms of the policies and run an agent-based simulation[2]. We run the simulation for 500 time periods, with a 200 time period 'calibration period.' We repeat each simulation 1,000 times and average the results. End consumer demand is set to be a draw from a random distribution, $u_i^{DEM} \sim N(100, 1)$. We set up three experimental conditions. For Condition 1, we make no adjustments to demand. For Conditions 2 and 3, we implement exogenous demand shocks. In Condition 2, there is a single demand shock up of $2x$ normal demand. In condition 3, there is a

[2]Our simulation is set up very similarly to [3], but with the notable exception that returns are not allowed.

single demand shock down of 0.2$x$ normal demand. We do not allow returns or charge-backs (orders cannot be negative), and assume any unmet demand in a particular time period is lost (demand does not carry over to the next time period). Each firm in the simulation makes one type of good, requires only one type of good to enter production, and outputs one type of good from production. All suppliers to a firm produce substitute goods (i.e. have a CD structure). Our model induces BWE with the presence of delays $\Delta$ (shipping or production) and safety stock $\alpha$. Initial simulation results verify that BWE is present in our model. The realizations of the policies are now explicated.

### A. Policies

Unless stated otherwise, all policies are uniform and unchanging across time and firms.

*1) Policy Realizations:*

*a) Production function $p_i(\cdot)$:* $(\mathbb{R}^{|u_i^{RM}|+1} \to \mathbb{R})$. The lesser of raw materials or demand input.

$$p_i(\mathbf{u}_i^{RM}[t], x_i^{RM}[t-1]) \triangleq \min(\|\mathbf{u}_i^{DEM}[t]\|_1, x_i^{RM}[t-1]) \quad (11)$$

*b) Shipment function $s_i^Q(\cdot)$:* $(\mathbb{R} \to \mathbb{R})$. The amount shipped is amount ordered, or the available finished goods. The amount shipped is always the amount produced.

$$s_i^Q(\mathbf{u}_i^{RM}[t], x_i^{RM}[t-1]) \triangleq p_i(\cdot) \quad (12)$$

*c) Safety Stock function $\alpha_i(\cdot)$:* $(\mathbb{R} \to \mathbb{R})$. Each non-manufacturing firm has safety stock equal to 0.1x expected demand. The manufacturing firms (2 and 3) have different alphas for heterogeneity in the sim.

$$\alpha_i(\cdot) \triangleq \alpha_i = \begin{cases} 0.05 \text{ if } i = 2 \\ 0.15 \text{ if } i = 3 \\ 0.10 \text{ otherwise} \end{cases} \quad (13)$$

*d) Ordering function $o_i^Q(\cdot)$:* $(\mathbb{R}^{2+2\cdot|\mathbf{u}_i[t]|} \to \mathbb{R})$. The amount ordered needs to take into account the desired safety stock, size of prior unreceived orders, inventory on hand, and expected demand. Define a helper function that provides the expected demand from time $t+1$ to $t+\gamma$, where $\gamma$ is the amount of time it takes for ordered product to be received:

$$E[\mathbf{u}_i^{DEM}(t+1:t+\gamma)] \triangleq \gamma \cdot \|\mathbf{u}_i^{DEM}[t] + \mathbf{u}_i^{DEM}[t-1]\|_1 \quad (14)$$

$$\begin{aligned} o_i^Q(&\alpha_i, \gamma, \mathbf{u}_i^{DEM}[t], \mathbf{u}_i^{DEM}[t-1], x_i^{RM}[t]) \\ &\triangleq \max(0, E[\mathbf{u}_i^{DEM}(t+1:t+\gamma)] \cdot \frac{1+\alpha_i}{\gamma} - \\ &\qquad x_i^{RM}[t] - \sum_{\tau=1}^{\gamma} o_i^Q[t-\tau]) \end{aligned} \quad (15)$$

For our simulation we set $\gamma = 2$.

*e) Demand function $d_i(\cdot)$:* $(\mathbb{R}^{2\cdot|\mathbf{u}_i^{DEM}[t]|} \to \mathbb{R})$. We implement demand smoothing [3] averaging over the previous two time periods:

$$z_i(\mathbf{u}_i^{DEM}[t], \mathbf{u}_i^{DEM}[t-1]) \triangleq \frac{\mathbf{u}_i^{DEM}[t] + \mathbf{u}_i^{DEM}[t-1]}{2} \quad (16)$$

*f) Shipping function $s_i^A(\cdot)$:* $(\mathbb{R}^{2+|\mathbf{u}_i^{FG}[t]|} \to \mathbb{R}^{|\mathbf{u}_i^{FG}[t]|})$. If there are fewer FG than there are total orders, the amount of FG is shipped equally to each customer.

$$\begin{aligned} &s_i^A(N^{CUST}, \mathbf{u}_i^{FG}[t-\gamma], x_i^{FG}[t]) \triangleq \frac{x_i^{FG}[t]}{N^{CUST}} \forall i, \\ &\quad \text{if } x_i^{FG}[t] < \|\mathbf{u}_i^{FG}[t-\gamma]\|_1 \text{ and } \mathbf{u}_i^{DEM}[t-\gamma], \text{ otherwise.} \end{aligned} \quad (17)$$

*2) Ordering function $o_i^A(\cdot)$:* $(\mathbb{R}^{1+2\cdot|\mathbf{u}_i^{RM}[t]|} \to \mathbb{R}^{|\mathbf{u}_i^{RM}[t]|})$. The choice of policy for whom to order from, $o_i^A(\cdot)$, can be either time-varying or time-invariant. We use Markowitz optimization [15]—a time varying policy—in each time period as $o_i^A(\cdot)$.

Consider a focal firm where each direct supplier in their 'diversified supplier portfolio' provides substitute goods. The buying manager at the firm is tasked with determining a stochastic vector $\omega$, where the $i$th component $\omega_i$ represents the weighting on the demand signal sent to supplier $i$. A random variable $r_i$ represents the returns of the supplier. which can be anything from on-time ship percentage to the negative of price. Assume that the expected value $e_i := \mathbb{E}(r_i)$ is known to the manager for all $i$.

In Markowitz optimization[15], the manager's goal is to minimize the risk (covariance) associated with her decision $\omega$ while maintaining a minimum value of expected portfolio performance $\mu \geq 0$. The covariance matrix $C = [c_{ij}] \in \mathbb{R}^{N_i^{SUP} \times N_i^{SUP}}$ is such that $c_{ij} = \mathbb{E}[(r_i - e_i)(r_j - e_j)]$. At each time $t$, both the firm and each supplier have the ability to adjust their policies. The firm will adjust the decision vector $\omega$; the supplier $i$ will adjust some aspect of $r_i$ based on the previous $\omega_i$. We assume that the supplier has the ability to update the expected value $e_i$ at each time step: for instance, via contract. At time step $t$ the firm manager's decision can be formulated as the following quadratic programming problem. Since $C$ is positive semi-definite it makes (18) convex optimization.

$$\begin{aligned} \omega_C := \min_{\omega} \quad & \omega^T C \omega \\ \text{s.t.} \quad & \mathbf{1}^T \omega = 1, \; e(t)^T \omega \geq \mu, \; \omega \geq 0 \end{aligned} \quad (18)$$

### B. Scenarios

The structure of the SC used in the simulation has three different scenarios: 'baseline' for a serial SC, 'structural' for a SC with a CD structure but with static policy $\lambda$, and 'policy' for a SC with a CD structure and a varying policy $\Lambda[t]$. In all three scenarios there are three echelons (customer-supplier pairs). Echelon 1 is for the retailer-distributor relationship. Echelon 2 is for the distributor-manufacturer relationship. Echelon 3 is for the manufacturer-raw materials relationship. For scenario 1 (baseline), we have a SC with a serial structure, such that there is only one firm in each echelon category, making $N = 4$. For scenarios 2 (structural) and 3 (policy), we have a SC with a diamond structure, such that there are two firms in the echelon category 'manufacturing', with $N = 5$. In scenario 2 there is a fixed policy $o_i^A(\cdot)$ such that the distributor orders 50% of supplies from each manufacturer each time period. In scenario 3 the ordering policy $o_i^A(\cdot)$ is a Markowitz optimization at each time period to determine the value $\omega_1$, with the result that $\omega_1$ is ordered from one manufacturer and $1-\omega_1$ is ordered from the other at a given time period.

### C. Simulation Results

The simulation outcome reported in Table I show strong support for our main results. The variances are reported as a proportion of the baseline condition, such that for each condition, baseline = 1. A value greater than 1 in either the structural or policy scenarios indicates that the BWE increased relative to baseline, and vice versa. The presence of policy $o_i^A(\cdot)$—all else equal—amplifies the bullwhip for the entire SC. However, all else *not* being equal in the presence of shocks, we find more complex amplification/dampening effects. In the presence of any shock, the BWE demonstrates more erratic behavior. More work will need to be done to uncover the underlying cause.

| BWE Condition | Base | Structural | Policy |
|---|---|---|---|
| No Shock | 1 | 0.81 | 8.64 |
| Shock Up | 1 | 1.06 | 0.70 |
| Shock Down | 1 | 0.91 | 0.19 |
| Average | 1 | 1.2 | 13.59 |

TABLE I

RATIOS OF VARIANCES OF DEMAND SIGNALS, AVERAGE ACROSS THREE CONDITIONS. $j$ REPRESENTS THE RETAIL FIRM AND $i$ REPRESENTS THE RAW MATERIAL FIRM.

## V. CONCLUSION

In this paper we proved that a structural change in a SC does *not* necessarily amplify the BWE. **Because the relationship between BWE and structure is mediated by policy, it is only when a structural change has an associated varying ordering policy is the BWE amplified.** Conversely, a structural change with an associated fixed ordering policy—all else equal—does not amplify the BWE.

Our analytical results show that (1) previous work done on the BWE that find results in a serial SC are **also applicable to SCs with a CD structure and a time-invariant ordering policy**, (2) future work looking at BWE in CD SC with an associated time-invariant ordering policy can **simplify calculations using a serial SC** (while the converse is *not* true), (3) a CD supply chain and a time-varying ordering policy—all else equal—**will strictly increase BWE**, and (4) the rate at which the **BWE is amplified increases exponentially with each additional supplier**.

These results have implications for managers. A risk-averse manager might want to diversify their supplier portfolio [16][17]. A manager might pursue this avenue if she were concerned about a particular supplier being unreliable, for example. However, **diversifying the portfolio can have some unintended consequences**, as we show. Thus, a manager will need to weigh the benefits (lowered risk profile coming from multiple suppliers) with the potential costs (increased demand volatility associated with adding more suppliers). Further, if a more complex structure is deemed necessary (e.g. needing more than one supplier for a single good), **removing managerial choice can mitigate the BWE amplification**. One such mitigation is negotiating long-term purchase contracts.

Our hope is this research paper will contribute to opening the door to the study of SC-level behaviors. Within that sphere, more needs to be done to explore how complex SC structures interact with managerial decision making to influence these behaviors. There is also ample space to explore additional SC behaviors, such as how structure impacts economic value, production idle time, excess inventory, etc. Given the far-reaching impact that SC disruptions have on the entire population, we firmly believe this is a research area worth investing in.

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