2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Predefined-Time Distributed Optimal Consensus for Euler–Lagrangian Systems Based on Dynamic Event-Triggered Mechanism

Feisheng Yang, Jiaming Liu and Qian Ma

***Abstract*— It is a challenging problem to achieve fast distributed optimal consensus for Euler–Lagrangian (EL) systems meanwhile economizing communication resources. To solve the problem, a novel predefined-time distributed optimal consensus strategy for EL systems is proposed by applying time-base generator (TBG) and dynamic event-triggered mechanism, which can reach the optimal consensus in a completely distributed manner. It is proven that the algorithm can converge in predefined time by Lyapunov energy function and Zeno behavior is avoided. A dynamic event-triggered method is designed which event-triggered thresholds are replaced by dynamic variables. The numerical simulation is given to show the effectiveness and superiority of the algorithm.**

I. INTRODUCTION

Recently, the distributed optimal consensus problem in the cyber-physical system (CPS) has received more and more attention from researchers. Distributed optimal consensus control plays an important role in resource allocation and cost optimization for sensor networks [1], power grids [2], and so on. Each agent has a local cost function, which cooperates to solve global optimization problems [3]. The Euler–Lagrangian (EL) system is a type of CPS which can denote a lot of nonlinear systems, such as robotic manipulators, spacecraft, and marine vessels [4].

Authors in [5]- [6] proposed a class of distributed coordination algorithms to solve network optimization problems. It may cause redundant communication problem when running continuous-time distributed optimal consensus protocols. Therefore, researchers in [7]- [8] presented event-triggered distributed optimal consensus protocols. A time-triggered algorithm and an event-triggered algorithm are raised to solve the optimization problem in [9]. In [10], authors studied a fully distributed optimal coordinated control protocol based on event-triggered mechanism for networked EL systems subject to unknown model parameters. Authors in [3] designed an event-triggered distributed optimal consensus control strategy for EL systems. In the existing distributed optimal consensus protocols for EL systems, the process of achieving consensus is often asymptotically or exponentially convergent. In order to reach the optimal consensus in finite time, some researchers raised the finite-time consensus algorithm [11] and the fixed-time consensus algorithm [12]. In [4], the fixed-time distributed coordination controllers for multiple EL systems are designed. A predefined-time distributed optimization method based on a time-base generator (TBG) is proposed in [13]. In [14], authors introduced predefined-time consensus to resource allocation.

Inspired by the above works, this article focuses on fast achieving distributed optimal consensus for EL systems with saving communication resources. In this paper, a dynamic event-triggered and predefined-time distributed optimal consensus algorithm for EL systems is proposed. Compared with the existing works, the main contributions of this paper are as follows: (1) To our best knowledge, it's the first time to achieve predefined-time distributed optimal consensus for EL systems by TBG method. (2) A novel event-triggered strategy is raised with internal dynamic variables being designed as event-triggered thresholds. (3) The proposed protocol has good privacy which can reach the optimal consensus of the EL system in a completely distributed mode.

Notation: $\|\cdot\|$ means the Euclidean norm of a vector or a matrix. $\mathbf{1}_n$ is a n-dimensional column vector with all elements being 1. $A \otimes B$ denotes the Kronecker product of A and B. I_m is $m \times m$ identity matrix.

II. PRELIMINARY

Communication topology in this paper can be described through an undirected and connected graph $\mathscr{G} = (\mathscr{N}, \mathscr{E}, \mathscr{A})$. $\mathscr{N} = \{1, \cdots, n\}$ represents the set of vertexes in the graph. $\mathscr{E}$ is the edge set, and $(i, j) \in \mathscr{E}$, $\mathscr{A} = [a_{ij}]_{n\times n}$ denotes adjacency matrix. The set included the neighbours of vertex i is denoted as $N_i = \{j \in \mathscr{N} | (i, j) \in \mathscr{E}\}$. If $(i, j) \notin N_i$, $a_{ij} = 0$, else $a_{ij} > 0$, besides, $a_{ii} = 0$. The Laplacian matrix of $\mathscr{G}$ is $\mathcal{L} = [l_{ij}]_{n\times n}$, where $l_{ij} = \sum_{i=1}^{n} a_{ij} (i = j)$, $l_{ij} = -a_{ij} (i \neq j)$.

Definition 1 ([14]): The function $\varrho(t)$ is TBG if the following conditions hold, (1) $\varrho(0) = 0$, $\dot{\varrho}(0) = 0$; (2) $\varrho(t) > 0$, $\dot{\varrho}(t) > 0$, when $t \in (0, t_f)$; (3) $\varrho(t) = 1$, $\dot{\varrho}(t) = 0$, when $t \geq t_f$, where $t_f < \infty$.

Definition 2 ([4]): The system converges at predefined time t_f. (1) $\lim\limits_{t\to t_f} \|q(t) - q^\star\| \leq a$; (2) $\|q(t) - q^\star\| \to 0, t > t_f$, where $q \in \mathbb{R}^m$, a is a small positive number.

Lemma 1 ([2]): For dynamics $\dot{p}(t) = -\vartheta(\chi(t) + 1)p(t)$, where $\vartheta > 0$, $\chi(t) = \frac{\dot{\varrho}(t)}{1-\varrho(t)+\epsilon}$, $0 < \epsilon < 1$, $\varrho(t)$ is the same as Definition 1, then $p(t)$ converges to $p(0)\left(\frac{\epsilon}{\epsilon+1}\right)^{\vartheta}$ at t_f.

Lemma 2 ([15]): For undirected graph $\mathscr{G}$, we have

$$x^{\mathrm{T}}\mathcal{L}x \leq \lambda_n(\mathcal{L})x^{\mathrm{T}}x,\ x^{\mathrm{T}}\mathcal{L}x \geq \lambda_2(\mathcal{L})x^{\mathrm{T}}x$$

where $\lambda_n(\mathcal{L})$ and $\lambda_2(\mathcal{L})$ denote the maximum eigenvalue and the minimum eigenvalue of $\mathcal{L}$, respectively.

Feisheng Yang and Jiaming Liu are with the School of Automation, Northwestern Polytechnical University, Xi'an City, 710129, China, also with Research & DevelopmentInstitute of Northwestern Polytechnical University in Shenzhen, SanhangScience & Technology Buliding, No.45th, Gaoxin South 9th Road, Nanshan District, Shenzhen City, 518063, China; Qian Ma is with Nanjing University of Science and Technology, Nanjing, Jiangsu 210094, China. yangfeisheng@nwpu.edu.cn.

Consider that a group of robots, each of them can be described by the EL equation.

$$M_i(q_i(t))\ddot{q}_i(t) + C_i(q_i(t), \dot{q}_i(t))\dot{q}_i(t) + G_i(q_i(t)) = \tau_i(t) \tag{1}$$

where $q_i, \dot{q}_i \in \mathbb{R}^m$ represent the generalized angle and angular velocity vectors, respectively. $M_i(q_i, \dot{q}_i) \in \mathbb{R}^{m\times m}$ denotes the symmetric positive-definite inertia matrix; $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{m\times m}$ means the Coriolis and centrifugal torque; $G_i(q_i) \in \mathbb{R}^m$ represents the gravitational torque vector; and $\tau_i \in \mathbb{R}^m$ denotes the control torque.

The EL system (1) aims to cooperatively achieve the distributed optimal consensus. The optimal consensus problem is described as follows.

$$\min_{z\in\mathbb{R}^m} f(z), \ \ f(z) = \sum_{i=1}^{n} f_i(z) \tag{2}$$

where $f(z)$ is the global cost function, $f_i(z) : \mathbb{R}^m \to \mathbb{R}$ is the local cost of each agent.

Assumption 1 ([3]): f_i is a differentiable and ω_i-strongly convex function, i.e., $(a-b)^T(\nabla f_i(a)-\nabla f_i(b)) \geq \omega_i\|a-b\|^2$, $\forall a, b \in \mathbb{R}^m$.

Assumption 2 ([6]): f_i is θ_i-Lipschitz, i.e., $\|\nabla f_i(a) - \nabla f_i(b)\| \leq \theta_i\|a - b\|$, $\forall a, b \in \mathbb{R}^m$, where $\theta_i > 0$.

III. MAIN RESULTS

In this section, the event-triggered and predefined-time distributed optimal consensus protocol for EL system (1) is proposed. The controller τ_i is designed as follows.

$$\begin{cases} \tau_i = C_i\dot{q}_i + G_i - kM_i(\chi(t)+1)\dot{q}_i - M_i(\chi(t)+1)^2\nabla f_i(q_i) \\ \quad -M_i(\chi(t)+1)^2 w_i - M_i(\chi(t)+1)^2\sum_{j\in N_i} a_{ij}\left(\bar{q}_i - \bar{q}_j\right) \\ \quad +M_i\dot{\chi}(t)\frac{\dot{q}_i(t)}{\chi(t)+1} \\ \dot{w}_i = (\chi(t)+1)\left(\sum_{j\in N_i} a_{ij}\left(\bar{q}_i - \bar{q}_j + \bar{\dot{p}}_i - \bar{\dot{p}}_j\right)\right) \\ \sum_{i=1}^{n} w_i(0) = 0 \end{cases} \tag{3}$$

where $\bar{q}_i$, $\bar{\dot{p}}_i$ denote $q\left(t^i_{k_a}\right)$ and $\dot{p}\left(t^i_{k_b}\right)$, respectively, $k \geq 1$, $\chi(t)$ is the same as the definition in Lemma 1. Combining (1) and (3), it yields

$$\begin{cases} \dot{q}_i(t) = (\chi(t)+1)p_i(t) \\ \dot{p}_i(t) = (\chi(t)+1)\Big(-kp_i(t) - \nabla f_i(t)\left(q_i(t)\right) - w_i(t) \\ \qquad -\sum_{j\in N_i} a_{ij}\left(\bar{q}_i(t) - \bar{q}_j(t)\right)\Big) \\ \dot{w}_i(t) = (\chi(t)+1)\Big(\sum_{j\in N_i} a_{ij}\big(\bar{q}_i(t) - \bar{q}_j(t) \\ \qquad +\bar{\dot{p}}_i(t) - \bar{\dot{p}}_j(t)\big)\Big) \\ \sum_{i=1}^{n} w_i(0) = 0. \end{cases} \tag{4}$$

The event-triggered instants are designed as follows

$$\begin{aligned} t^i_{k_a+1} &= \inf\left\{t : t > t^i_{k_a}, \kappa^a_i\|e_{a,i}(t)\|^2 \geq \eta^a_i(t)\right\} \\ t^i_{k_b+1} &= \inf\left\{t : t > t^i_{k_b}, \kappa^b_i\|e_{b,i}(t)\|^2 \geq \eta^b_i(t)\right\} \end{aligned} \tag{5}$$

where $\eta^a_i(0), \eta^b_i(0) > 0$, $\kappa^a_i > \frac{1-\delta^a_i}{\phi^a_i}$, $\kappa^b_i > \frac{1-\delta^b_i}{\phi^b_i}$, $e_{a,i}(t) = q_i(t^i_{k_a}) - q_i(t)$, $e_{b,i}(t) = \dot{p}_i(t^i_{k_b}) - \dot{p}_i(t)$. The dynamic equations for the internal dynamic variables $\eta^a_i(t)$ and $\eta^b_i(t)$ are designed as

$$\begin{aligned} \eta^a_i(t) &= \eta^a_i(0)e^{\int_0^t -\phi^a_i(\chi(s)+1) - \frac{\delta^a_i(\chi(s)+1)\|e_{a,i}(s)\|^2}{\eta^a_i(s)} ds} \\ \eta^b_i(t) &= \eta^b_i(0)e^{\int_0^t -\phi^b_i(\chi(s)+1) - \frac{\delta^b_i(\chi(t)+1)\|e_{b,i}(s)\|^2}{\eta^b_i(s)} ds} \end{aligned} \tag{6}$$

where the parameters satisfy $\eta^a_i(0), \eta^b_i(0) > 0$, $\kappa^a_i > \frac{1-\delta^a_i}{\phi^a_i}$, $\kappa^b_i > \frac{1-\delta^b_i}{\phi^b_i}$, then the dynamic variables $\eta^a_i(t), \eta^b_i(t) > 0$. $\varrho(t)$ and ϵ are the same as the definition of Definition 1. Take derivative of (6), we have

$$\begin{aligned} \dot{\eta}^a_i(t) &= -\phi^a_i\left(\chi(t)+1\right)\eta^a_i(t) - \delta^a_i(\chi(t)+1)\|e_{a,i}(t)\|^2 \\ \dot{\eta}^b_i(t) &= -\phi^b_i\left(\chi(t)+1\right)\eta^b_i(t) - \delta^b_i(\chi(t)+1)\|e_{b,i}(t)\|^2 \end{aligned} \tag{7}$$

Next, we will discuss the optimality of (4). Suppose that $(q^\star, p^\star, w^\star)$ are the equilibrium points of (4). When $t > t_f$, $\chi(t) = 0$, we have

$$\begin{cases} p^\star = 0 \\ -kp^\star - (L\otimes I_m)\,q^\star - w^\star - \nabla f\left(q^\star\right) = 0 \\ (L\otimes I_m)\left(q^\star + p^\star\right) = 0 \end{cases}$$

By premultiplying the equations above with $\mathbf{1}^{\mathrm{T}}_n \otimes I_m$, one has $-\sum_{i=1}^{n} w^\star_i - \sum_{i=1}^{n}\nabla f_i\left(q^\star_i\right) = 0$, $\sum_{i=1}^{n}\dot{w}_i(t) = 0$. According to $\sum_{i=1}^{n} w_i(0) = 0$, we have $\sum_{i=1}^{n} w^\star_i = 0$. Then $\sum_{i=1}^{n}\nabla f_i(q^\star_i) = 0$ can be concluded. Hence, $q^\star$ is the optimal solution of (2).

After the above discussion, the following result shows that the EL system (1) can achieve distributed optimal consensus at predesigned time t_f under controller (3).

Theorem 1: Assume that Assumptions 1-2 hold. Adopting controller (3), the two objectives can be achieved. 1) Zeno behavior can be excluded. 2) The optimal consensus of the EL system (1) can be reached at predefined-time t_f, i.e.,

$$\begin{aligned} &\lim_{t\to t_f}\|q - q^*\| < \sqrt{\frac{2}{k-1}V(0)}\left(\frac{\epsilon}{\epsilon+1}\right)^{\frac{\rho_2}{2\rho_1}} \\ &\|q - q^*\| < \sqrt{\frac{2}{k-1}V(0)}\left(\frac{\epsilon}{\epsilon+1}\right)^{\frac{\rho_2}{2\rho_1}}, \ t > t_f \\ &\lim_{t\to\infty}\|q - q^*\| = 0 \end{aligned}$$

where $\rho_1 = \max\{\sigma+1, \frac{k+1}{2}, \frac{1}{2\lambda_2(\mathcal{L})}+\sigma, \gamma_1, \gamma_2\}$, $\rho_2 = \min\{R_1, R_2, R_3, R_4, R_5, R_6\}$, $R_1 = 1 - \sigma\sigma_0 - \frac{1}{4\varepsilon_1}$, $R_2 = k - 1 - \theta(\frac{\theta}{4\omega}+1)$, $R_3 = \frac{4\omega^2}{\theta+4\omega} - 2\sigma\theta^2 - \frac{1}{4\varepsilon_2}$, $R_4 = \frac{\sigma}{4} - \frac{1}{4\varepsilon_3} - \frac{1}{4\varepsilon_4} - \frac{\sigma}{4\varepsilon_6}$, $R_5 = \gamma_1\phi^a_{\min} - \frac{1}{\kappa^a_{\min}}\left((\varepsilon_1+\varepsilon_2)\lambda^2_n(\mathcal{L}) + \varepsilon_3 - \gamma_1\delta^a_{\min}\right)$, $R_6 = \gamma_2\phi^b_{\min} - \frac{1}{\kappa^b_{\min}}\left(\varepsilon_4 + \sigma(\varepsilon_5+\varepsilon_6)\lambda^2_n(\mathcal{L}) - \gamma_2\delta^b_{\min}\right)$ and $\sigma_0 = k^2 + k + \frac{5}{4} + \lambda^2_n(\mathcal{L}) + \lambda_n(\mathcal{L}) + \frac{1}{4\varepsilon_5}$. Let $\delta^a_{\min} = \min\{\delta^a_i\}$, $\delta^b_{\min} = \min\{\delta^b_i\}$, $\omega = \min\{\omega_i\}$, $\theta = \max\{\theta_i\}$, $k \geq \theta\left(1+\frac{\theta}{4\omega}\right)+2+\frac{1}{4}\lambda_n(\mathcal{L})$, $z_1 = (\varepsilon_1+\varepsilon_2)\lambda^2_n(\mathcal{L})+\varepsilon_3-\gamma_1\delta^a_{\min} > 0$, $z_2 = \varepsilon_4 + \sigma(\varepsilon_5+\varepsilon_6)\lambda^2_n(\mathcal{L}) - \gamma_2\delta^b_{\min} > 0$, $\frac{\varepsilon_6(\varepsilon_4+\varepsilon_3)}{\varepsilon_3\varepsilon_4(\varepsilon_6-1)} < \sigma < \min\{\frac{1}{\sigma_0} - \frac{1}{4\varepsilon_1\sigma_0}, \frac{2\omega^2}{\theta^2(\theta+4\omega)} - \frac{1}{8\varepsilon_2\theta}\}$, $\gamma_1 > \frac{(\varepsilon_1+\varepsilon_2)\lambda^2_n(\mathcal{L})+\varepsilon_3}{\delta^a_{\min}+\kappa^a_{\min}\phi^a_{\min}}$ and $\gamma_2 > \frac{\sigma(\varepsilon_5+\varepsilon_6)\lambda^2_n(\mathcal{L})+\varepsilon_4}{\delta^b_{\min}+\kappa^b_{\min}\phi^b_{\min}}$. $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6 > 0$ can be arbitrarily set.

Proof: For the convenience of proof, (4) can be rewritten as

$$\begin{cases} \dot{q} = (\chi(t)+1)p \\ \dot{p} = (\chi(t)+1)(-kp - L(q+e_a) - \nabla f(q) - w) \\ \dot{w} = (\chi(t)+1)\left(L\left(q+e_a+\dot{p}+e_b\right)\right) \end{cases} \tag{8}$$

where $p = [p_1, p_2, \cdots, p_n]^{\mathrm{T}}$, $q = [q_1, q_2, \cdots, q_n]^{\mathrm{T}}$, $w = [w_1, w_2, \cdots, w_n]^{\mathrm{T}}$, $\dot{q} = [\dot{q}_1, \dot{q}_2, \cdots, \dot{q}_n]^{\mathrm{T}}$, $\dot{p} = [\dot{p}_1, \dot{p}_2, \cdots, \dot{p}_n]^{\mathrm{T}}$, $e_a = [e_{a,1}, e_{a,2}, \cdots, e_{a,n}]^{\mathrm{T}}$, $e_b = [e_{b,1}, e_{b,2}, \cdots, e_{b,n}]^{\mathrm{T}}$, $\nabla f(q) = [f_1(q_1), \cdots, f_n(q_n)]^{\mathrm{T}}$.

Define three auxiliary variables, $\zeta(t) = q(t) - q^\star$, $\xi(t) = p(t) - p^\star = p$, $\varpi(t) = w(t) - w^\star$. Set an orthogonal matrix $Q = (r, U)$, where $r = \frac{\mathbf{1}_n}{\sqrt{n}}$, $U \in \mathbb{R}^{n\times(n-1)}$, satisfying $\mathbf{1}_n^T U = \mathbf{0}_{n-1}$, $U^{\mathrm{T}}U = I_{n-1}$, $UU^{\mathrm{T}} = I_n - \frac{1}{n}\mathbf{1}_n\mathbf{1}_n^{\mathrm{T}}$.

$$\begin{aligned} \hat{\zeta} &= \left(Q^{\mathrm{T}} \otimes I_m\right)\zeta = \left[\hat{\zeta}_1^{\mathrm{T}}, \hat{\zeta}_{2:n}^{\mathrm{T}}\right]^{\mathrm{T}} \\ \hat{\xi} &= \left(Q^{\mathrm{T}} \otimes I_m\right)\xi = \left[\hat{\xi}_1^{\mathrm{T}}, \hat{\xi}_{2:n}^{\mathrm{T}}\right]^{\mathrm{T}} \\ \hat{\varpi} &= \left(Q^{\mathrm{T}} \otimes I_m\right)\varpi = \left[\hat{\varpi}_1^{\mathrm{T}}, \hat{\varpi}_{2:n}^{\mathrm{T}}\right]^{\mathrm{T}} \\ \hat{e}_a &= \left(Q^{\mathrm{T}} \otimes I_m\right)e_a = \left[\hat{e}_{a,1}^{\mathrm{T}}, \hat{e}_{a,2:n}^{\mathrm{T}}\right]^{\mathrm{T}} \\ \hat{e}_b &= \left(Q^{\mathrm{T}} \otimes I_m\right)e_b = \left[\hat{e}_{b,1}^{\mathrm{T}}, \hat{e}_{b,2:n}^{\mathrm{T}}\right]^{\mathrm{T}} \end{aligned}$$

So (8) can be written as follows

$$\begin{cases} \dot{\hat{\zeta}}_1 = (\chi(t)+1)\hat{\xi}_1 \\ \dot{\hat{\xi}}_1 = (\chi(t)+1)(-k\hat{\xi}_1 - \left(r^{\mathrm{T}} \otimes I_m\right)h) \\ \dot{\hat{\zeta}}_{2:n} = (\chi(t)+1)\hat{\xi}_{2:n} \\ \dot{\hat{\xi}}_{2:n} = (\chi(t)+1)\big(-k\hat{\xi}_{2:n} - \hat{\varpi}_{2:n} - \left(U^{\mathrm{T}} \otimes I_m\right)h \\ \qquad - \left(U^{\mathrm{T}}\mathcal{L}U \otimes I_m\right)\left(\hat{\zeta}_{2:n} + \hat{e}_{a,2:n}\right)\big) \\ \dot{\hat{\varpi}}_1 = \mathbf{0}_m \\ \dot{\hat{\varpi}}_{2:n} = (\chi(t)+1)\left(U^{\mathrm{T}}\mathcal{L}U \otimes I_m\right) \\ \qquad \times \left(\hat{\zeta}_{2:n} + \hat{e}_{a,2:n} + \hat{\xi}_{2:n} + \hat{e}_{b,2:n}\right) \end{cases} \tag{9}$$

where $h = \nabla f(\zeta + q^\star) - \nabla f(q^\star)$.

Construct Lyapunov energy function as $V(t) = V_1(t) + V_2(t) + V_3(t)$, each item of V is defined as follows.

$$\begin{cases} V_1 = \frac{1}{2}\left\|\hat{\xi}_1\right\|^2 + \hat{\xi}_1^{\mathrm{T}}\hat{\zeta}_1 + \frac{k}{2}\left\|\hat{\zeta}_1\right\|^2 + \frac{1}{2}\left\|\hat{\xi}_{2:n}\right\|^2 + \hat{\xi}_{2:n}^{\mathrm{T}}\hat{\zeta}_{2:n} \\ \qquad + \frac{k}{2}\left\|\hat{\zeta}_{2:n}\right\|^2 + \frac{1}{2}\hat{\varpi}_{2:n}^{\mathrm{T}}\left(\left(U^{\mathrm{T}}\mathcal{L}U\right)^{-1} \otimes I_m\right)\hat{\varpi}_{2:n}, \\ V_2 = \frac{\sigma}{2}\left(\hat{\xi}_{2:n} + \hat{\varpi}_{2:n}\right)^2 \\ V_3 = \gamma_1\sum_{i=1}^{n}\eta_i^a(t) + \gamma_2\sum_{i=1}^{n}\eta_i^b(t) \end{cases} \tag{10}$$

where $\gamma_1 > 0$, $\gamma_2 > 0$, $\sigma > 0$.

In light of (10) and Young's inequality $x^{\mathrm{T}}y \leq \frac{1}{4\varepsilon}x^{\mathrm{T}}x + \varepsilon y^{\mathrm{T}}y$ ($\varepsilon > 0$), it can be yielded that

$$\begin{aligned} V =& \frac{1}{2}\hat{\xi}_1^{\mathrm{T}}\hat{\xi}_1 + \hat{\xi}_1^{\mathrm{T}}\hat{\zeta}_1 + \frac{k}{2}\hat{\zeta}_1^{\mathrm{T}}\hat{\zeta}_1 + \frac{1}{2}\hat{\xi}_{2:n}^{\mathrm{T}}\hat{\xi}_{2:n} + \hat{\xi}_{2:n}^{\mathrm{T}}\hat{\zeta}_{2:n} + \frac{k}{2}\hat{\zeta}_{2:n}^{\mathrm{T}}\hat{\zeta}_{2:n} \\ &+ \frac{1}{2}\hat{\varpi}_{2:n}^{\mathrm{T}}\left((U^{\mathrm{T}}\mathcal{L}U)^{-1} \otimes I_m\right)\hat{\varpi}_{2:n} + \frac{\sigma}{2}\left(\hat{\xi}_{2:n} + \hat{\varpi}_{2:n}\right)^2 \\ &+ \gamma_1\sum_{i=1}^{n}\eta_i^a(t) + \gamma_2\sum_{i=1}^{n}\eta_i^b(t) \\ \leq& \|\hat{\xi}_1\|^2 + (\sigma+1)\|\hat{\xi}_{2:n}\|^2 + \frac{k+1}{2}\|\hat{\zeta}\|^2 + \gamma_1\sum_{i=1}^{n}\eta_i^a(t) \\ &+ \gamma_2\sum_{i=1}^{n}\eta_i^b(t) + \left(\frac{1}{2\lambda_2(\mathcal{L})} + \sigma\right)\|\hat{\varpi}_{2:n}\|^2 \\ \leq& \rho_1\mu \end{aligned} \tag{11}$$

where $\mu = \hat{\xi}^{\mathrm{T}}\hat{\xi} + \hat{\zeta}^{\mathrm{T}}\hat{\zeta} + \hat{\varpi}^{\mathrm{T}}\hat{\varpi} + \sum_{i=1}^{n}\eta_i^a(t) + \sum_{i=1}^{n}\eta_i^b(t)$, By virtue of the definition of V, we can conclude that

$$\frac{k-1}{2}\hat{\zeta}^{\mathrm{T}}\hat{\zeta} \leq V \leq \rho_1\mu \tag{12}$$

Taking derivative on V, one has

$$\begin{aligned} \dot{V}_1 =& (\chi(t)+1)\Big(-(k-1)\|\hat{\xi}_1\|^2 - (k-1)\|\hat{\xi}_{2:n}\|^2 \\ &- (\hat{\zeta}_{2:n}^{\mathrm{T}} + \hat{\xi}_{2:n}^{\mathrm{T}})(U^{\mathrm{T}}\mathcal{L}U \otimes I_m)(\hat{\zeta}_{2:n} + \hat{e}_{a,2:n}) \\ &+ \varpi_{2:n}^{\mathrm{T}}(\hat{e}_{a,2:n} + \hat{e}_{b,2:n}) - (\xi+\zeta)^{\mathrm{T}}h\Big) \end{aligned} \tag{13}$$

$$\begin{aligned} \dot{V}_2 =& \sigma(\chi(t)+1)\Big(-\|\hat{\varpi}_{2:n}\|^2 - k\left\|\hat{\xi}_{2:n}\right\|^2 - (k+1)\hat{\varpi}_{2:n}^{\mathrm{T}}\hat{\xi}_{2:n} \\ &- \hat{\xi}_{2:n}^{\mathrm{T}}\left(U^{\mathrm{T}} \otimes I_m\right)h - \hat{\varpi}_{2:n}^{\mathrm{T}}\left(U^{\mathrm{T}} \otimes I_m\right)h \\ &+ \hat{\xi}_{2:n}^{\mathrm{T}}\left(U^{\mathrm{T}}\mathcal{L}U \otimes I_m\right)\hat{\xi}_{2:n} + \hat{\varpi}_{2:n}^{\mathrm{T}}\left(U^{\mathrm{T}}\mathcal{L}U \otimes I_m\right)\hat{\xi}_{2:n} \\ &+ (\xi_{2:n}^{\mathrm{T}} + \varpi_{2:n}^{\mathrm{T}})(U^{\mathrm{T}}\mathcal{L}U \otimes I_m)\hat{e}_{b,2:n}\Big) \end{aligned} \tag{14}$$

$$\begin{aligned} \dot{V}_3 =& (\chi(t)+1)\Big(-\gamma_1\sum_{i=1}^{n}\phi_i^a\eta_i^a(t) - \gamma_1\sum_{i=1}^{n}\delta_i^a\|e_{a,i}(t)\|^2 \\ &- \gamma_2\sum_{i=1}^{n}\phi_i^b\eta_i^b(t) - \gamma_2\sum_{i=1}^{n}\delta_i^b\|e_{b,i}(t)\|^2\Big) \end{aligned} \tag{15}$$

Inequality (15) can be organized into the following form.

$$\begin{aligned} \dot{V}_1 =& (\chi(t)+1)\big(-\operatorname{col}\left(\hat{\zeta}_{2:n}, \hat{\xi}_{2:n}\right)^{\mathrm{T}}(F \otimes I_m)\operatorname{col}\left(\hat{\zeta}_{2:n}, \hat{\xi}_{2:n}\right) \\ &- (k_0+1)\left\|\hat{\xi}_{2:n}\right\|^2 - (k-1)\left\|\hat{\xi}_1\right\|^2 - (\xi+\zeta)^{\mathrm{T}}h \\ &- (\hat{\zeta}_{2:n}^{\mathrm{T}} + \hat{\xi}_{2:n}^{\mathrm{T}})(U^{\mathrm{T}}\mathcal{L}U \otimes I_m)\hat{e}_{a,2:n} + \varpi_{2:n}^{\mathrm{T}}(\hat{e}_{a,2:n} + \hat{e}_{b,2:n})\big) \end{aligned} \tag{16}$$

where $F = \begin{pmatrix} U^{\mathrm{T}}\mathcal{L}U & \frac{1}{2}U^{\mathrm{T}}\mathcal{L}U \\ \frac{1}{2}U^{\mathrm{T}}\mathcal{L}U & (k-k_0-2)I_{n-1} \end{pmatrix}$ and $k_0 = \theta\left(1 + \frac{\theta}{4\omega}\right)$. By the Schur Complement Lemma and

$(k-k_0-2)\,I_{n-1}-\frac{1}{4}U^{\mathrm{T}}\mathcal{L}U\geq 0$, F is nonnegative definite. Since ∇f_i is θ-Lipschitz and ω_i-strongly convex, we have $-\xi^{\mathrm{T}}h\leq\theta\left(\frac{\theta}{4\omega}+1\right)\|\hat{\xi}\|^2+\theta\frac{\omega}{\theta+4\omega}\|\hat{\zeta}\|^2$ and $-\zeta^{\mathrm{T}}h\leq-\omega\zeta^{\mathrm{T}}\zeta$. By virtue of Young's inequality, we have

$$-\hat{\xi}_{2:n}^{\mathrm{T}}(U^{\mathrm{T}}\mathcal{L}U\otimes I_m)\hat{e}_{a,2:n}\leq\frac{1}{4\varepsilon_1}\|\hat{\xi}_{2:n}\|^2+\varepsilon_1\lambda_n^2(\mathcal{L})\|\hat{e}_{a,2:n}\|^2$$

$$-\hat{\zeta}_{2:n}^{\mathrm{T}}(U^{\mathrm{T}}\mathcal{L}U\otimes I_m)\hat{e}_{a,2:n}\leq\frac{1}{4\varepsilon_2}\|\hat{\zeta}_{2:n}\|^2+\varepsilon_2\lambda_n^2(\mathcal{L})\|\hat{e}_{a,2:n}\|^2$$

$$\hat{\varpi}_{2:n}^{\mathrm{T}}\hat{e}_{a,2:n}\leq\frac{1}{4\varepsilon_3}\|\hat{\varpi}_{2:n}\|^2+\varepsilon_3\|\hat{e}_{a,2:n}\|^2$$

$$\hat{\varpi}_{2:n}^{\mathrm{T}}\hat{e}_{b,2:n}\leq\frac{1}{4\varepsilon_4}\|\hat{\varpi}_{2:n}\|^2+\varepsilon_4\|\hat{e}_{b,2:n}\|^2$$

Thus, one has

$$\begin{aligned}\dot{V}_1\leq&(\chi(t)+1)\Big(-(k_0+1)\left\|\hat{\xi}_{2:n}\right\|^2-(k-1)\left\|\hat{\xi}_1\right\|^2\\&-(\xi+\eta)^{\mathrm{T}}h-(\hat{\zeta}_{2:n}^{\mathrm{T}}+\hat{\xi}_{2:n}^{\mathrm{T}})(U^{\mathrm{T}}\mathcal{L}U\otimes I_m)\hat{e}_{a,2:n}\\&+\varpi_{2:n}^{\mathrm{T}}(\hat{e}_{a,2:n}+\hat{e}_{b,2:n})\big)\\\leq&(\chi(t)+1)\Big(-\big(1-\frac{1}{4\varepsilon_1}\big)\left\|\hat{\xi}_{2:n}\right\|^2-(\omega-\theta\frac{\omega}{\theta+4\omega})\|\hat{\zeta}\|^2\\&-\big(k-1-\theta(\frac{\theta+4\omega}{4\omega})\big)\left\|\hat{\xi}_1\right\|^2+\frac{1}{4\varepsilon_2}\|\hat{\zeta}_{2:n}\|^2\\&+(\frac{1}{4\varepsilon_3}+\frac{1}{4\varepsilon_4})\|\hat{\varpi}_{2:n}\|^2+\big((\varepsilon_1+\varepsilon_2)\lambda_n^2(\mathcal{L})\\&+\varepsilon_3\big)\|e_{a,2:n}\|^2+\varepsilon_4\|e_{b,2:n}\|^2\Big)\end{aligned}\tag{17}$$

In light of Young's inequality and Assumption 2,

$$\hat{\xi}_{2:n}^{\mathrm{T}}(U^{\mathrm{T}}\mathcal{L}U\otimes I_m)\hat{e}_{b,2:n}\leq\frac{1}{4\varepsilon_5}\|\hat{\xi}_{2:n}\|^2+\varepsilon_5\lambda_n^2(\mathcal{L})\|\hat{e}_{b,2:n}\|^2$$

$$\hat{\varpi}_{2:n}^{\mathrm{T}}(U^{\mathrm{T}}\mathcal{L}U\otimes I_m)\hat{e}_{b,2:n}\leq\frac{1}{4\varepsilon_6}\|\hat{\varpi}_{2:n}\|^2+\varepsilon_6\lambda_n^2(\mathcal{L})\|\hat{e}_{b,2:n}\|^2$$

$$-\xi_{2:n}^{\mathrm{T}}(U^{\mathrm{T}}\otimes I_m)h\leq\frac{1}{4}\|\xi_{2:n}\|^2+\theta^2\|\zeta\|^2$$

$$-\hat{\varpi}_{2:n}^{\mathrm{T}}(U^{\mathrm{T}}\otimes I_m)h\leq\frac{1}{4}\|\hat{\varpi}\|^2+\theta^2\|\zeta\|^2$$

$$(k+1)\hat{\varpi}_{2:n}^{\mathrm{T}}\hat{\xi}_{2:n}^{\mathrm{T}}\leq\frac{1}{4}\|\varpi_{2:n}\|^2+(k+1)^2\|\hat{\xi}_{2:n}\|^2$$

$$\hat{\varpi}_{2:n}^{\mathrm{T}}(U^{\mathrm{T}}\mathcal{L}U)\hat{\xi}_{2:n}^{\mathrm{T}}\leq\frac{1}{4}\|\varpi_{2:n}\|^2+\lambda_n^2(\mathcal{L})\|\hat{\xi}_{2:n}\|^2$$

Therefore, one has

$$\begin{aligned}\dot{V}_2\leq&\sigma(\chi(t)+1)\Big(-(\frac{1}{4}-\frac{1}{4\varepsilon_6})\left\|\hat{\varpi}_{2:n}\right\|^2+2\theta^2\|\hat{\zeta}\|^2\\&+\sigma_0\left\|\hat{\xi}_{2:n}\right\|^2+(\varepsilon_5+\varepsilon_6)\lambda_n^2(\mathcal{L})\|\hat{e}_{b,2:n}\|^2\Big)\end{aligned}\tag{18}$$

where $\sigma_0=k^2+k+\frac{5}{4}+\lambda_n^2(\mathcal{L})+\lambda_n(\mathcal{L})+\frac{1}{4\varepsilon_5}$.

After the above discussion, it can be yielded that

$$\begin{aligned}\dot{V}\leq&(\chi(t)+1)\Big(-\big(1-\sigma\sigma_0-\frac{1}{4\varepsilon_1}\big)\|\hat{\xi}_{2:n}\|^2\\&-\big(k-1-\theta(\frac{\theta}{4\omega}+1)\big)\|\hat{\xi}_1\|^2-\big(\frac{4\omega^2}{\theta+4\omega}-2\sigma\theta^2\\&-\frac{1}{4\varepsilon_2}\big)\|\hat{\zeta}\|^2-\big(\frac{\sigma}{4}-\frac{1}{4\varepsilon_3}-\frac{1}{4\varepsilon_4}-\frac{\sigma}{4\varepsilon_6}\big)\|\hat{\varpi}_{2:n}\|^2\\&+\big((\varepsilon_1+\varepsilon_2)\lambda_n^2(\mathcal{L})+\varepsilon_3-\gamma_1\delta_{\min}^a\big)\|\hat{e}_a\|^2\\&+\big(\varepsilon_4+\sigma(\varepsilon_5+\varepsilon_6)\lambda_n^2(\mathcal{L})-\gamma_2\delta_{\min}^b\big)\|\hat{e}_b\|^2\\&-\gamma_1\sum_{i=1}^n\phi_i^a\eta_i^a-\gamma_2\sum_{i=1}^n\phi_i^b\eta_i^b\Big)\end{aligned}\tag{19}$$

When the event is not triggered, $\|e_a\|^2<\frac{1}{\kappa_{\min}^a}\sum_{i=1}^n\eta_i^a(t)$ and $\|e_b\|^2<\frac{1}{\kappa_{\min}^b}\sum_{i=1}^n\eta_i^b(t)$ where $\kappa_{\min}^a=\min\{\kappa_i^a\}$, $\kappa_{\min}^b=\min\{\kappa_i^b\}$, we have

$$\begin{aligned}\dot{V}<&(\chi(t)+1)\Big(-R_1\|\hat{\xi}_{2:n}\|^2-R_2\|\hat{\xi}_1\|^2-R_3\|\hat{\zeta}\|^2\\&-R_4\|\hat{\varpi}_{2:n}\|^2-R_5\sum_{i=1}^n\eta_i^a-R_6\sum_{i=1}^n\eta_i^b\Big)\\\leq&-\rho_2(\chi(t)+1)\mu\end{aligned}\tag{20}$$

On the basis of (12), we can get that $\dot{V}<-\frac{\rho_2}{\rho_1}(\chi(t)+1)V$. In light of Lemma 1, we can yield that $V(t_f)=\left(\frac{\epsilon}{\epsilon+1}\right)^{\frac{\rho_2}{\rho_1}}V(0)$. Combining with (12), it can be yielded and $\|\zeta(t)\|^2\leq\frac{2}{k-1}V$. Consequently, $\|\zeta(t_f)\|=\sqrt{\frac{2}{k-1}V(0)}\left(\frac{\epsilon}{\epsilon+1}\right)^{\frac{\rho_2}{2\rho_1}}$. When $t>t_f$, $\chi(t)=0$, we can derive that $\dot{V}<-\frac{\rho_2}{\rho_1}V$. Therefore, we have $V<V(0)e^{-\frac{\rho_2}{\rho_1}t}$, which implies that $\|\zeta(t)\|\to 0$ ($\|q(t)-q^*\|\to 0$) with an exponential convergence rate when $t>t_f$. The proof of predefined-time convergence ends.

Next, the Zeno-free behavior of the algorithm will be proven. According to the definition of $e_{a,i}(t)$, $\dot{e}_{a,i}(t)=-\dot{q}_i(t)$, we have $e_{a,i}(t)=-\int_{t_k^i}^t\dot{q}_i(t)dt$.

Define $M_1=\max\{\|p_i\|\}$. Since $\varrho(t)\in[0,1]$, one has $\|\chi(t)+1\|\leqslant\frac{s+\epsilon}{\epsilon}$ where $s=\max\{\dot{\varrho}(t)\}$. Thus, $\|e_{a,i}(t)\|\leq\frac{s+\epsilon}{\epsilon}M_1\left(t-t_{k_a}^i\right)$. When the event is triggered, $\|e_{a,i}(t_{k_a+1}^i)\|^2\geq\frac{1}{\kappa_i^a}\eta_i^a(t_{k_a+1}^i)$. In light of $\frac{\epsilon}{1+\epsilon}\leq 1-\frac{\varrho(t)}{1+\epsilon}\leq 1$, one has

$$\|e_{a,i}(t_{k_a+1}^i)\|>\sqrt{\frac{\eta_i^a(0)}{\kappa_i^a}e^{\int_0^t-\phi_i^a(\chi(s)+1)-\frac{\delta_i^a(\chi(t)+1)\|e_{a,i}(s)\|^2}{\eta_i^a(s)}ds}}$$

Define $\Delta_1=t_{k_a+1}^i-t_{k_a}^i$. When $t=t_{k_a+1}^i$, $\|e_{a,i}\left(t_{k_a+1}^i\right)\|\leq\frac{s+\epsilon}{\epsilon}M_1\Delta_1$, one has

$$\Delta_1>\frac{\epsilon}{M_1(s+\epsilon)}\sqrt{\frac{\eta_i^a(0)}{\kappa_i^a}e^{\int_0^{t_{k_a+1}^i}-\phi_i^a(\chi(s)+1)-\frac{\delta_i^a(\chi(t)+1)\|e_{a,i}(s)\|^2}{\eta_i^a(s)}ds}}$$

Similarly, $\dot{e}_{b,i}(t) = -\dot{p}_i(t)$, so one has $e_{b,i}(t) = -\int_{t_k^i}^{t} \dot{p}_i(t)dt$. By virtue of $\dot{p}_i(t) = (\chi(t)+1)\Big(-kp_i - \nabla f_i(q_i) - w_i(t) - \sum_{j\in N_i} a_{ij}\left(\bar{q}_i - \bar{q}_j\right)\Big)$. Define $M_2 = \max\{\|\dot{p}_i\|\} = kM_1 + M_3 + M_4 + M_5$, where $M_3 = \max\{\nabla f_i(q_i)\}$, $M_4 = \max\{w_i(t)\}$, $M_5 = \max\{\sum_{j\in N_i} a_{ij}\left(\bar{q}_i - \bar{q}_j\right)\}$. When $t > t_f$, $q_i \to 0_m$, $w_i \to 0_m$, $\nabla f_i(q_i) \to 0$, so M_1, M_2, M_3, M_4 and M_5 exist. Hence, $\|e_{b,i}(t)\| \le \frac{s+\epsilon}{\epsilon} M_2\left(t - t_{k_b}^i\right)$. Define $\Delta_2 = t_{k_b+1}^i - t_{k_b}^i$. When the event is triggered, $\|e_{b,i}(t_{k_b+1}^i)\|^2 \ge \frac{1}{\kappa_i^b}\eta_i^b(t_{k_b+1}^i)$, we have

$$\Delta_2 > \frac{\epsilon}{M_2(s+\epsilon)}\sqrt{\frac{\eta_i^b(0)}{\kappa_i^b} e^{\int_0^{t_{k_b+1}^i} -\phi_i^b(\chi(s)+1) - \frac{\delta_i^b(\chi(t)+1)\|e_{b,i}(s)\|^2}{\eta_i^b(s)} ds}}$$

Therefore, the Zeno behavior of algorithm (4) is excluded. The whole proof ends.

IV. SIMULATION

In this section, the numerical simulations are given. The system dynamics are given by

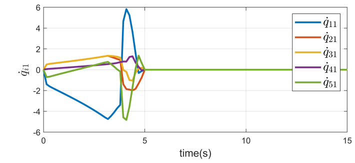
$$\begin{bmatrix} M_{11,i} & M_{12,i} \\ M_{21,i} & M_{22,i} \end{bmatrix}\begin{bmatrix} \ddot{q}_{i1} \\ \ddot{q}_{i2} \end{bmatrix} + \begin{bmatrix} C_{11,i} & C_{12,i} \\ C_{21,i} & C_{22,i} \end{bmatrix}\begin{bmatrix} \dot{q}_{i1} \\ \dot{q}_{i2} \end{bmatrix} = \begin{bmatrix} \tau_{i1} \\ \tau_{i2} \end{bmatrix}$$

where $M_{11,i} = b_{i1} + 2b_{i2}\cos(q_{i2})$, $M_{12,i} = b_{i3} + b_{i2}\cos(q_{i2})$, $M_{21,i} = b_{i3} + 2b_{i2}\cos(q_i2)$, $M_{22,i} = b_{i3}$, $C_{11,i} = -b_{i2}\sin(q_{i2})\dot{q}_{i2}$, $C_{12,i} = -b_{i2}\sin(q_{i2})(\dot{q}_{i1} + \dot{q}_{i2})$, $C_{21,i} = b_{i2}\sin(q_{i2})\dot{q}_{i1}$, $C_{22,i} = 0$, $i = 1, 2, \cdots, 5$. Select the parameters $k = 20$, $\epsilon = 10^{-17}$, $\sigma = 0.001$, $\varepsilon_1 = 5$, $\varepsilon_2 = 3$, $\varepsilon_3 = \varepsilon_4 = 2500$, $\varepsilon_5 = 3$, $\varepsilon_6 = 10$, $\gamma_1 = 10000$, $\gamma_2 = 1000$. The parameters of the event-triggered function are designed as $\delta_i^a = 0.1$, $\delta_i^b = 0.1$, $\phi_i^a = 1$, $\phi_i^b = 2$, $\kappa_i^a = 2$, $\kappa_i^b = 3$, $\eta_i^a(0) = \eta_i^b(0) = 10^6$. $f_i(q_i) = d_i\|q_i(t) - P_i\|^2$, $d_1 = d_2 = d_3 = d_4 = 1$, $d_5 = 2$. Set the initial value as $q_1(0) = [5, 6]^{\mathrm{T}}$, $q_2(0) = [2, 6]^{\mathrm{T}}$, $q_3(0) = [-4, 2]^{\mathrm{T}}$, $q_4(0) = [-5, -4]^{\mathrm{T}}$, $q_5(0) = [3, -3]^{\mathrm{T}}$, $P_1 = [3, 2]^{\mathrm{T}}$, $P_2 = [4, -2]^{\mathrm{T}}$, $P_3 = [-2, -4]^{\mathrm{T}}$, $P_4 = [-6, -1]^{\mathrm{T}}$, $P_5 = [-3, 2]^{\mathrm{T}}$. So $\theta = 4$, $\omega = 2$. The adjacency matrix $\mathscr{A}$ is set as follows.

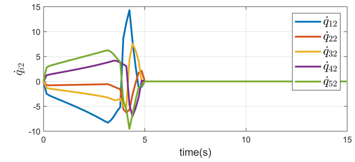
$$\mathscr{A} = \begin{pmatrix} 0 & 2 & 1 & 0 & 4 \\ 2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Therefore, $\lambda_5(\mathcal{L}) = 10.2446$, $\lambda_2(\mathcal{L}) = 0.7005$. After calculation, $R_1 = 0.4135$, $R_2 = 13$, $R_3 = 1.218$, $R_4 = 2.5\times10^{-5}$, $R_5 = 8.0682081\times10^3$, $R_6 = 9.6592\times10^2$. Choose $t_f = 5$s. The TBG function $\varrho(t)$ is defined as follows.

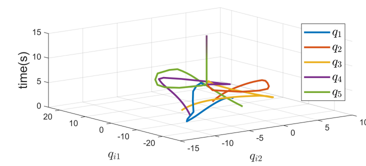
$$\varrho(t) = \begin{cases} \frac{10}{5^6}t^6 - \frac{24}{5^5}t^5 + \frac{15}{5^4}t^4, & 0 \le t \le t_f \\ 1, & t > t_f \end{cases}$$



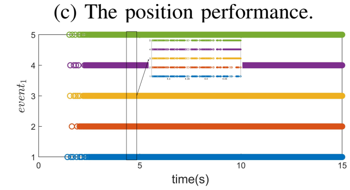
(a) The velocity performance of q_{i1}.



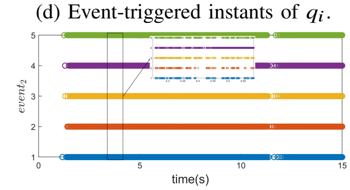
(b) The velocity performance of q_{i2}.



(c) The position performance.



(d) Event-triggered instants of q_i.



(e) Event-triggered instants of $\dot{q}_i$.

Fig. 1: Effectiveness test of the algorithm (4).

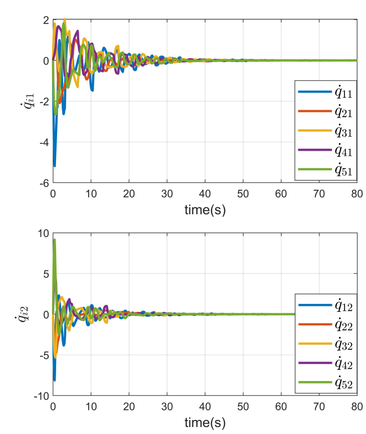


Fig. 2: Simulation of the algorithm in [7].

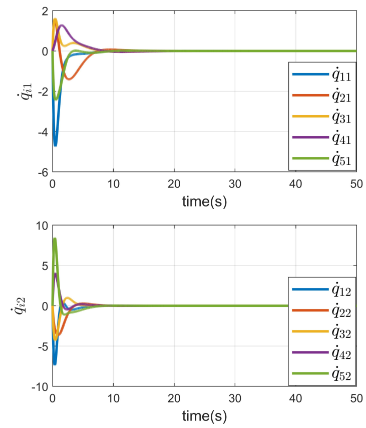


Fig. 3: Simulation of the algorithm in [3].

The simulation results of effectiveness test are shown in Fig. 1. From Fig. 1 (a)-(c), the position and the velocity under controller (3) can converge at predefined-time 5s. The simulation results of the comparison algorithms are displayed in Fig. 2 and Fig. 3, which converge at about 50s and 20s, respectively. The proposed algorithm converges significantly faster. The event-triggered instants are exhibited in Fig. 1 (d)-(e).

V. CONCLUSIONS

This paper studies the distributed optimal consensus problem for EL systems. To fast solve the problem meanwhile saving communication resources, a predefined-time and dynamic event-triggered distributed optimal consensus protocol for EL systems is raised. It is verified that the algorithm can converge at predesigned time and Zeno behavior is excluded. Ultimately, numerical simulations demonstrate the effectiveness and superiority of the proposed algorithm

ACKNOWLEDGMENTS

This work was supported partially by National Natural Science Foundation of China (62073269), Key Research and Development Program of Shaanxi (2022GY-244), Aeronautical Science Foundation of China (2020Z034053002), and Natural Science Foundation of Chongqing, China (CSTB2022NSCQ-MSX0963), Guangdong Basic and Applied Basic Research Foundation (2023A1515011220).

REFERENCES

[1] Adrian N. Bishop, Barış Fidan, Brian D.O. Anderson, Kutluyıl Doğançay, and Pubudu N. Pathirana. Optimality analysis of sensor-target localization geometries. *Automatica*, 46(3):479–492, 2010.

[2] Zhijun Guo and Gang Chen. Distributed dynamic event-triggered and practical predefined-time resource allocation in cyber–physical systems. *Automatica*, 142:110390, 2022.

[3] Qing Wang, Jie Chen, Bin Xin, and Xianlin Zeng. Distributed optimal consensus for euler–lagrange systems based on event-triggered control. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 51(7):4588–4598, 2021.

[4] Tao Xu, Zhisheng Duan, Zhiyong Sun, and Guanrong Chen. Distributed fixed-time coordination control for networked multiple euler–lagrange systems. *IEEE Transactions on Cybernetics*, 52(6):4611–4622, 2022.

[5] Solmaz S. Kia, Jorge Cortés, and Sonia Martínez. Distributed convex optimization via continuous-time coordination algorithms with discrete-time communication. *Automatica*, 55:254–264, 2015.

[6] Yanqiong Zhang, Zhenhua Deng, and Yiguang Hong. Distributed optimal coordination for multiple heterogeneous euler–lagrangian systems. *Automatica*, 79:207–213, 2017.

[7] Yanqiong Zhang, Zhenhua Deng, and Yiguang Hong. Event-triggered design for multi-agent optimal consensus of euler–lagrangian systems. *Kybernetika*, 53:179–194, 2017.

[8] Xinlei Yi, Lisha Yao, Tao Yang, Jemin George, and Karl H. Johansson. Distributed optimization for second-order multi-agent systems with dynamic event-triggered communication. In *2018 IEEE Conference on Decision and Control (CDC)*, pages 3397–3402, 2018.

[9] Ngoc-Tu Tran, Yan-Wu Wang, Xiao-Kang Liu, Jiang-Wen Xiao, and Yan Lei. Distributed optimization problem for second-order multi-agent systems with event-triggered and time-triggered communication. *Journal of the Franklin Institute*, 356(17):10196–10215, 2019.

[10] Yi Huang and Ziyang Meng. Fully distributed event-triggered optimal coordinated control for multiple euler–lagrangian systems. *IEEE Transactions on Cybernetics*, 52(9):9120–9131, 2022.

[11] Zongyu Zuo. Nonsingular fixed-time consensus tracking for second-order multi-agent networks. *Automatica*, 54:305–309, 2015.

[12] Gang Chen and Zhiyong Li. A fixed-time convergent algorithm for distributed convex optimization in multi-agent systems. *Automatica*, 95:539–543, 2018.

[13] J. Armando Colunga, Carlos R. Vázquez, Héctor M. Becerra, and David Gómez-Gutiérrez. Predefined-time consensus using a time base generator (tbg). *IFAC-PapersOnLine*, 51(13):246–253, 2018. 2nd IFAC Conference on Modelling, Identification and Control of Nonlinear Systems MICNON 2018.

[14] Zhijun Guo and Gang Chen. Predefined-time distributed optimal allocation of resources: A time-base generator scheme. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 52(1):438–447, 2022.

[15] R. Olfati-Saber and R.M. Murray. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Transactions on Automatic Control*, 49(9):1520–1533, 2004.