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Rational Basis Functions in Iterative Learning Control for Multivariable Systems

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Abstract— **Feedforward control with task flexibility for MIMO systems is essential to meet ever-increasing demands on throughput and accuracy. The aim of this paper is to develop a framework for data-driven tuning of rational feedforward controllers in iterative learning control (ILC) for noncommutative MIMO systems. A convex optimization problem in ILC is achieved by rewriting the nonlinear terms in the control scheme as a function of the previous feedforward parameters. A simulation study on an multivariable industrial printer shows that the developed framework converges and achieves significant better performance than direct application of the RBF algorithm using SK-iterations for SISO systems.**

I. INTRODUCTION

Feedforward control is essential for achieving performance, e.g., to meet ever-increasing demands on throughput and accuracy in high-tech systems [1]. The following requirements are identified. First, achieving accuracy requires data-driven tuning. Second, to ensure high throughput and accommodate industrial usage, task flexibility is crucial. Third, achieving performance in multivariable systems requires MIMO control.

High accuracy control for trial-invariant tasks is enabled by iterative learning control (ILC) [2] where a feedforward signal is learned iteratively. A specific class of ILC, norm-optimal iterative learning control (NOILC) [3], utilizes a cost function with past trial data and an approximate model of the system to optimize the next trial's feedforward. This approach is inherently applicable to multivariable systems, see, e.g., [4]. Multivariable aspects are also taken into account in frequency-domain ILC [5]. In both domains, a key assumption in ILC is made on a trial-invariant task and extrapolation of the feedforward signal to other tasks generally leads to a significant performance deterioration [6], hampering widespread industrial adoption.

The requirements on task flexibility has spurred the development of new task flexible ILC approaches. In [7], the task is divided into subtasks that are learned individually, restricting the task to consist of subtasks that are in the library. In [6], [8], [9], basis functions are introduced to enable extrapolation in ILC. Here, the feedforward signal is parameterized in terms of the task. In [8], polynomial basis functions are employed and can be interpreted as parameterized the feedforward signal using a finite impulse response (FIR) filter, see [6]. The optimization problems of the aforementioned polynomial basis functions methods in ILC retain the analytic solution of NOILC. However, the parameterization implies the system has a unit numerator, which is in many physical systems that are modeled by rational models, i.e., containing both poles and zeros, not the case. This leads to under-modeling and poor performance with respect to accuracy and extrapolation properties.

To improve the accuracy and extrapolation properties of ILC, rational feedforward for ILC is developed. In [10], an input shaping approach is developed using polynomial basis functions, which model both the denominator and numerator of the system, but is only focused on settling performance. In [9], rational basis functions (RBFs) for ILC are introduced, enabling high tracking accuracy and task flexibility for SISO systems. However, the analytic solution is lost as the optimization problem becomes nonconvex. In [9], an iterative solution based on Sanathanan-Koerner (SK) iterations, as introduced in [11], is presented for solving the optimization problem through a series of weighted least-squares problems. Other solutions include the use of an instrumental variables approach, see [12]. However, an extension towards MIMO is not evident, since all these approaches rely on the commutative property of SISO systems.

Although rational basis functions enhance the extrapolation properties of ILC algorithms, the RBF algorithm using SK-iterations for SISO systems cannot directly be applied to MIMO systems. This limitation arises because the derivation of the algorithm relies on the inherent commutative property of SISO systems. The aim of this paper is to develop a general framework for data-driven tuning of rational feedforward controllers in ILC for noncommutative MIMO systems.

The main contribution of this paper is a general framework for RBF in ILC for noncommutative MIMO systems. Moreover, connections to the RBF algorithm using SK-iterations for SISO systems [9] are established, as well as to polynomial basis functions [8].

The outline of this paper is as follows. In the next section the notation that is used in this paper is introduced. In Sec. II, the problem formulation is stated. The developed approach is presented in Sec. III and connections to the RBF algorithm for SISO systems are established. Then, in Sec. IV, a simulation study is presented. Sec. V contains conclusions

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and future work.

Notation

Systems are assumed to be linear and time-invariant (LTI), discrete-time, n_i inputs, and n_o outputs. Discrete-time transfer functions are generally rational in the complex indeterminate z. Input signals are often tacitly assumed of length $n_iN \in \mathbb{Z}_+$ and output signals $n_oN \in \mathbb{Z}_+$. Let $h_k \in \mathbb{R}^{n_o \times n_i}$ with $k \in \mathbb{Z}_+$ be the Markov parameter at time-step k of the MIMO system $H(z)$ with state-space matrices A_d, B_d, C_d, D_d. For a given input $u \in \mathbb{R}^{n_iN}$, the output $y \in \mathbb{R}^{n_oN}$ is

$$y[k] = h_k \otimes u[k] = \sum_{l=0}^{\infty} h_l u[k-l], \quad h_k = \begin{cases} D_d, & k = 0 \\ C_d A_d^{k-1} B_d, & k \geq 1 \end{cases}. \quad (1)$$

Assuming $u[k] = 0$ for $k < 0$ and $k > N-1$ and zero initial conditions,

$$\underbrace{\begin{bmatrix} y^1[0] \\ \vdots \\ y^{n_o}[0] \\ y^1[1] \\ \vdots \\ y^{n_o}[N-1] \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} h_0 & 0 & \cdots & 0 \\ h_1 & h_0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ h_{N-1} & h_{N-2} & \cdots & h_0 \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} u^1[0] \\ \vdots \\ u^{n_i}[0] \\ u^1[1] \\ \vdots \\ u^{n_i}[N-1] \end{bmatrix}}_{u}, \quad (2)$$

where $u^q[k], y^p[k]$ denote the input and output for input and output direction number q, p, respectively, where $q = 1, \ldots, n_i$ and $p = 1, \ldots, n_o$, and $H \in \mathbb{R}^{n_oN \times n_iN}$ is a MIMO block-Toeplitz matrix. Let $\theta[i]$ denote the i-th element of the vector θ. The weighted 2-norm of a vector x is denoted as $\| x \|_W := \sqrt{x^\top W x}$, where W is a weighting matrix. W is positive-definite ($W \succ 0$) if and only if $x^\top W x > 0, \forall x \neq 0$ and positive semi-definite ($W \succeq 0$) if and only if $x^\top W x \geq 0, \forall x \neq 0$.

II. PROBLEM FORMULATION

A. Problem setup

Consider the closed-loop control scheme shown in Fig. 1 where $P \in \mathbb{R}^{n_oN \times n_iN}$ is an n_i-input and n_o-output system and $C \in \mathbb{R}^{n_iN \times n_oN}$ a feedback controller. The aim is to design the feedforward signal $f_j \in \mathbb{R}^{n_iN}$ such that the output $y_j \in \mathbb{R}^{n_oN}$ tracks the reference $r_j \in \mathbb{R}^{n_oN}$ as accurate as possible, where j denotes the trial or experiment index. The tracking error $e_j \in \mathbb{R}^{n_oN}$ in trial j follows from Fig. 1 and is given by

$$e_j = S r_j - S P f_j, \quad (3)$$

where $S = (I + PC)^{-1} \in \mathbb{R}^{n_oN \times n_oN}$ is the output sensitivity and $SP \in \mathbb{R}^{n_oN \times n_iN}$ the process sensitivity. The tracking error for trial $j+1$ is given by

$$e_{j+1} = S r_{j+1} - S P f_{j+1}. \quad (4)$$

Next, ILC is employed to design f_{j+1} that minimizes the tracking error e_{j+1}.

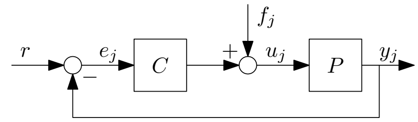


Fig. 1. Closed-loop control scheme with plant P with n_i-inputs and n_o-outputs, feedback controller C, and feedforward f_j.

B. Norm-optimal ILC

The goal in usual ILC is to design f_{j+1} to minimize the error in the next trial e_{j+1} using an approximate model of the system. Under assumption of a trial-invariant reference, any repetitive disturbance is compensated for in an iterative manner [2].

Assume a trial-invariant reference, i.e., $r_j = r_{j+1} = r$, by subtracting e_j in (3) from e_{j+1} in (4) the error propagation from trial j to $j+1$ is derived and is given by

$$e_{j+1} = e_j - SP\left(f_{j+1} - f_j\right). \quad (5)$$

The assumption on a trial-invariant reference is exploited in ILC to design f_{j+1}.

In norm-optimal ILC, the feedforward signal f_{j+1} is iteratively learned by minimization of the cost function in Def. 1.

Definition 1 (Cost function of norm-optimal ILC)**.** *The cost function for norm-optimal ILC for MIMO systems is given by*

$$\mathcal{J}_j(f_{j+1}) := \left\|\hat{e}_{j+1}\right\|_{W_e}^2 + \left\|f_{j+1}\right\|_{W_f}^2 + \left\|f_{j+1} - f_j\right\|_{W_{\Delta f}}^2 \quad (6)$$

where $\hat{e}_{j+1} = e_j - \widehat{SP}\left(f_{j+1} - f_j\right)$ *is the model-based error propagation equivalent to* (5)*,* $\widehat{SP} \in \mathbb{R}^{n_oN \times n_iN}$ *a model of the process sensitivity, and* f_j *and* e_j *are measured signals of iteration* j*,* $W_e \in \mathbb{R}^{n_oN \times n_oN}$ *a symmetric positive-definite weighting matrix, and* $W_f, W_{\Delta f} \in \mathbb{R}^{n_iN \times n_iN}$ *symmetric positive semi-definite weighting matrices.*

The feedforward f_{j+1}^* that minimizes this cost function is given by

$$f_{j+1}^* = \underset{f_{j+1}}{\operatorname{argmin}} \, \mathcal{J}_j(f_{j+1}). \quad (7)$$

The solution to (7) can be computed analytically since $\mathcal{J}_j(f_{j+1})$ of (6) is quadratic in f_{j+1}, see, e.g., [4]. Norm-optimal ILC has significant performance improvements that are enabled by non-causal filter operations in time-domain, see [2]. However, as a consequence of the assumption on a trial-invariant reference r, the feedforward signal f_{j+1} is optimal in (6) for this specific reference r. Hence, changing the reference r results in non-optimal performance since the term $S r_{j+1}$ in (4) is not canceled anymore. To mitigate this effect, basis functions are introduced to enable extrapolation capabilities in ILC.

C. Rational basis functions ILC

To enhance the extrapolation capabilities in ILC, basis functions are introduced, see, e.g., [8], [9], [12].

Now, the feedforward signal is parameterized as

$$f_j = F(\theta_j)r \tag{8}$$

where $F(\theta_j)$ are the basis functions and $\theta_j \in \mathbb{R}^{n_\theta}$ are the feedforward parameters. From (3) and substitution of (8) it follows that

$$e_j = S\left(I - PF(\theta_j)\right)r. \tag{9}$$

Hence, minimization of the tracking error $e_j = 0$ for all r is achieved for $F(\theta_j) = P^{-1}$.

In this paper, $F(\theta_j)$ consists of rational basis functions, see Def. 2, since typical physical systems are modeled using rational models that contain both poles and zeros, a rational F allows for full description of the plant inverse P^{-1} to obtain perfect tracking.

Definition 2 (Rational basis functions for MIMO systems)**.** *The MIMO rational feedforward parameterization is defined using a right matrix fraction description (RMFD), see [13], and is given by*

$$F(z, \theta_j) = A(z, \theta_j)B^{-1}(z, \theta_j) \tag{10}$$

where $A(z, \theta_j) \in \mathbb{R}^{n_i \times n_o}[z]$ and $B(z, \theta_j) \in \mathbb{R}^{n_o \times n_o}[z]$ are polynomial matrices with real coefficients. These polynomial matrices are affinely parameterized with respect to the parameters $\theta_j \in \mathbb{R}^{n_\theta \times 1}$ using a set of basis functions $\{\xi_i(z)\}_{i=1}^{n_\theta}$, where $\xi_i(z) \in \mathbb{R}^{(n_i+n_o)n_o \times 1}[z]$, such that

$$\operatorname{vec}\left(\begin{bmatrix} A(z, \theta_j) \\ B(z, \theta_j) \end{bmatrix}\right) = \sum_{i=1}^{n_\theta} \xi_i(z)\theta_j[i] + \xi_0(z), \tag{11}$$

corresponding to a full-polynomial form, see, e.g., [14], where $\xi_0(z) \in \mathbb{R}^{(n_i+n_o)n_o \times 1}[z]$ is a polynomial independent of θ that constraints the parameterization, e.g., constraining the denominator polynomial to be monic, to guarantee the rational structure of $F(\theta_j)$ is well defined for all θ.

Note that in case $B(z, \theta_j) = I$, the feedforward parameterization becomes $F(z, \theta_j) = A(z, \theta_j)$ which is linear in the parameters and recovers the polynomial basis functions parameterizations, i.e., FIR parametrization, in [8], [15]. Moreover, note that $F(z, \theta_j)$ of (10) is nonlinear in the parameters θ_j due to $B^{-1}(\theta_j)$.

Now, evaluating (6) for (8) with (10) results in the following cost function for RBF ILC.

Definition 3 (Cost function for rational basis functions ILC)**.** *The cost function for RBF ILC for MIMO systems is given by*

$$\mathcal{J}_j(\theta_{j+1}) := \left\|\hat{e}_{j+1}(\theta_{j+1})\right\|_{W_e}^2 + \left\|F(\theta_{j+1})r\right\|_{W_f}^2 + \left\|F(\theta_{j+1})r - f_j\right\|_{W_{\Delta f}}^2 \tag{12}$$

where $\hat{e}_{j+1}(\theta_{j+1}) = e_j - \widehat{SP}\left(F(\theta_{j+1})r - f_j\right)$ is the model-based error propagation equivalent to (4) *with* (8)*, $\widehat{SP}$ a model of the process sensitivity, and f_j and e_j are measured signals of iteration j, W_e a symmetric positive-definite weighting matrix, and $W_f, W_{\Delta f}$ symmetric positive semi-definite weighting matrices.*

The cost function (12) is nonlinear in θ_{j+1} for $F(\theta_{j+1})$ as in Def. 2, leading to the considered problem in this paper.

D. Problem definition

The considered problem in this paper is to determine the optimal feedforward parameters in Def. (2), i.e., determine

$$\theta_{j+1}^* = \underset{\theta_{j+1}}{\operatorname{argmin}}\, \mathcal{J}_j(\theta_{j+1}), \tag{13}$$

with $\mathcal{J}_j(\theta_{j+1})$ as defined in Def. 3.

The RBF algorithm using SK-iterations for the SISO case, see [9], solves the nonlinear optimization problem through a series of weighted least-squares problems. However, this methodology relies on the commutative property of SISO systems and is therefore not directly applicable for general MIMO systems. The proposed approach for RBF in ILC for MIMO systems without relying on the commutative property is presented in the next section.

III. MIMO RATIONAL BASIS FUNCTIONS ILC

In this section, the main contribution of this paper, which is to solve (13) for mutlivariable systems, is presented.

A. Recasting the error propagation

Consider the closed-loop control scheme depicted in Fig. 1 with feedforward f_j. Now, by substitution of the feedforward signal (8) with parameterization $F(z, \theta_j)$ of (10) defined in Def. 2, the control-scheme of Fig. 2(a) is obtained. Here, the dependency of θ_j in $A_j = A(\theta_j)$ and $B_j = B(\theta_j)$ is omitted for brevity. As a result,

$$e_j = Sr - SPA_jB_j^{-1}r. \tag{14}$$

Note that e_j is nonlinear in parameters θ_j due to the term B_j^{-1}, generally resulting in the nonconvex optimization problem of (12) in Def. 3. Next, this nonlinear term is circumvented.

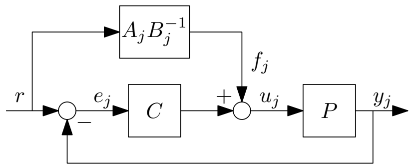
The key idea in the developed approach is to rewrite the control scheme with RBFs for trial $j+1$ as Fig. 2(b) and replace the unknown nonlinear term B_{j+1}^{-1} with the known term B_j^{-1} to obtain Fig. 2(c). By doing so, the approximate error for trial $j+1$ becomes linear in the parameters θ_{j+1}. The error $\hat{e}_{j+1}$ in Fig. 2(c) is

$$\tilde{e}_{j+1} = SB_{j+1}B_j^{-1}r - SPA_{j+1}B_j^{-1}r, \tag{15}$$

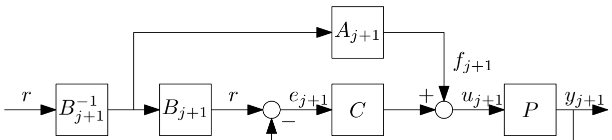
and clearly, for converged parameters, i.e., $\theta_{j+1} = \theta_j = \theta$, the error (14) with index $j+1$ is recovered.

Now, the error propagation from trial j to $j+1$ is derived from (15) in four steps. First, introduce $x_j = B_j^{-1}r$, according to Fig. 2(c). Second, there exists, according to Def. 2, a $\Psi_{x_j}^A \in \mathbb{R}^{n_iN \times n_{\theta^A}}$, $\Psi_{0,x_j}^B \in \mathbb{R}^{n_oN \times 1}$, and $\Psi_{x_j}^B \in \mathbb{R}^{n_oN \times n_{\theta^B}}$ such that $A(\theta_{j+1})x_j = \Psi_{x_j}^A\theta_{j+1}^A$ and $B(\theta_{j+1})x_j = \Psi_{0,x_j}^B + \Psi_{x_j}^B\theta_{j+1}^B$. Third, introduce $\theta_{j+1} = \theta_j + \theta_\Delta$, where θ_j is known from the last iteration j and θ_Δ is the unknown parameter to be optimized. By applying the above three steps, the error can be written as

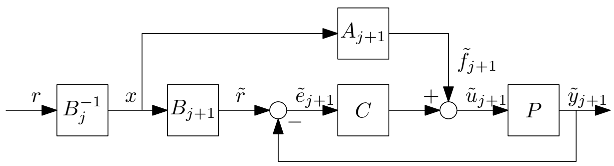
$$\tilde{e}_{j+1} = S\Psi_{0,x_j}^B + S\Psi_{x_j}^B\left(\theta_j^B + \theta_\Delta^B\right) - SP\Psi_{x_j}^A\left(\theta_j^A + \theta_\Delta^A\right). \tag{16}$$



(a) Closed-loop RBF control scheme.



(b) Rewritten closed-loop RBF control scheme for iteration $j+1$.



(c) Rewritten closed-loop RBF control scheme where B_{j+1}^{-1} is replaced with B_j^{-1} to achieve a convex optimization problem.

Fig. 2. Three different representations of the closed-loop control schemes with rational basis functions where $A_j = A(\theta_j)$ and $B_j = B(\theta_j)$ for brevity.

Fourth, the terms that are dependent on θ_j reduce to e_j in (14) after substitution of $x_j = B^{-1}(\theta_j)r$ in those terms. Then, the error propagation reduces to

$$\tilde{e}_{j+1} = e_j + S\Psi^B_{x_j}\theta^B_\Delta - SP\Psi^A_{x_j}\theta^A_\Delta \tag{17}$$

$$= e_j - \left[SP\Psi^A_{x_j}, -S\Psi^B_{x_j}\right]\theta_\Delta, \tag{18}$$

where $\theta_\Delta = \begin{bmatrix}\theta^A_\Delta \\ \theta^B_\Delta\end{bmatrix}$. This derivation follows along the lines of [10]. Next, the optimization problem and its solution is derived.

B. Optimization problem and solution

Next, the optimization problem, as defined in Def. 3, is rewritten using (18) as function of θ_Δ. For simplicity of presentation the weights W_f and $W_{\Delta f}$ in (12) are set to zero and $W_e = I$. Note that the solution can easily be extended to include W_e, W_f, and $W_{\Delta f}$ in case of model uncertainties and presence of trail-varying disturbances. The resulting cost function is as follows:

$$\mathcal{J}_j(\theta_\Delta) := \Big\|e_j - \underbrace{\left[\widehat{SP}\Psi^A_{x_j}, -\hat{S}\Psi^B_{x_j}\right]}_{\Phi_{x_j}}\theta_\Delta\Big\|^2 \tag{19}$$

where $e_j - \Phi_{x_j}\theta_\Delta$ is the model-based error propagation equivalent to (18), where Φ_{x_j} is a function of x_j, the model of the output sensitivity $\hat{S}$, and the model of the process sensitivity $\widehat{SP}$.

Now, the solution to the optimization problem presented in (13) for cost function (19) with θ_Δ is derived. The optimum of the cost function is given by a left matrix inverse of Φ_{x_j} and is

$$\theta^*_{j+1} = \theta_j + \left(\Phi^\top_{x_j}\Phi_{x_j}\right)^{-1}\Phi^\top_{x_j}e_j. \tag{20}$$

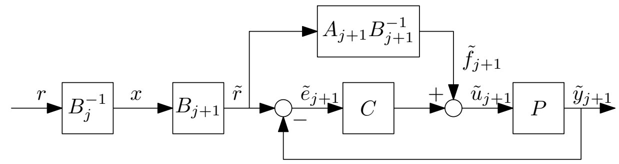


Fig. 3. Closed-loop RBF control scheme of Fig. 2(c) rewritten to recover the RBF algorithm using SK-iterations for commutative systems.

Note that similarly to ILC, each iteration θ^*_{j+1} is computed using the update law based on θ_j and e_j. Next, the algorithm is presented in Alg. 1.

Algorithm 1 MIMO RBF ILC algorithm

1: **Set:** $\Psi^A, \Psi^B_0, \Psi^B, r, j = 0$, and $\theta_0 = 0$.
2: **while** θ_j not converged **do**
3: Determine $f_j = A(\theta_j)B^{-1}(\theta_j)r$.
4: Perform experiment with r and f_j and measure e_j, as in Fig. 2(a).
5: Determine $x_j = B^{-1}(\theta_j)r$.
6: Compute $\theta^*_{j+1} = \theta_j + \left(\Phi^\top_{x_j}\Phi_{x_j}\right)^{-1}\Phi^\top_{x_j}e_j$.
7: $j \rightarrow j+1$.
8: **end while**
9: **Output:** $\theta = \theta_j$

The algorithm is related to the SK-iterations that are often used in system identification, see [11], [16], however, instead of solving a series of weighted least-squares problems offline, a series of experiments j are performed till convergence. Typically, algorithms employing SK-iterations have good convergence properties and are insensitive to local optima [17], but as presented in [12], the stationary point of the algorithm is not necessarily an optimum and will be analyzed in future work.

C. Recover RBF algorithm for SISO systems

The RBF algorithm using SK-iterations for SISO systems presented in [9] can be recovered as a special case. First, note that Fig. 3 can be obtained by rewriting Fig. 2(c). Second, indeed, the error of Fig. 3 is equivalent to the error derived in (15), and is rewritten into

$$\tilde{e}_{j+1} = \left(S - SPA_{j+1}B^{-1}_{j+1}\right)B_{j+1}B^{-1}_j r. \tag{21}$$

Now, for SISO systems or commutative MIMO systems, i.e., systems for which $SPA_{j+1}B^{-1}_j = B^{-1}_j B_{j+1}SPA_{j+1}B^{-1}_{j+1}$ and $SB_{j+1}B^{-1}_j = B^{-1}_j B_{j+1}S$, this error can be expressed as

$$\tilde{e}_{j+1} = B^{-1}_j B_{j+1}\left(Sr - SPA_{j+1}B^{-1}_{j+1}r\right), \tag{22}$$

which is equivalent to the term used in the weighted cost function of the SISO solution presented in [9] if $k_{\max} = 1$, i.e., 1 SK-iteration each trial j.

This leads to an experimentally more efficient approach for SISO and commutative MIMO systems compared to the developed approach. However, it requires that the system commutes which does not hold for MIMO systems in general. This is further illustrated in the next section.

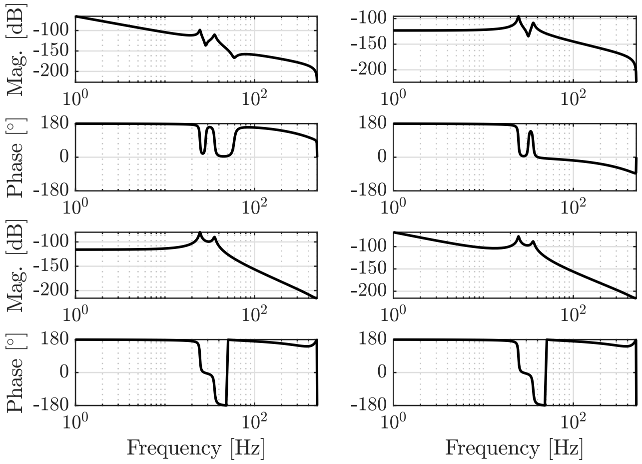


Fig. 4. Bode diagram of the $P(z)$ used in simulation and ILC design. Clearly, the MIMO systems has substantial interaction between axis and is not symmetric.

IV. SIMULATION STUDY

In this section, a simulation study is performed to show the performance and convergence properties of the developed approach in comparison to the RBF algorithm using SK-iterations for SISO systems that relies on the commutative property.

A. System description

The simulations are performed using a model and controller of an industrial multivariable flatbed printer, see [5]. Here, the outputs are the translation of the gantry x [m] and rotation of the carriage φ [rad], and the inputs are the force F_x [N] and the torque T_φ [Nm]. In Fig. 4, the Bode diagram of the system is depicted and clearly shows the substantial interaction present in the MIMO system and shows that the system is not symmetric.

B. Simulation setup

The developed approach and the RBF algorithm for SISO systems that relies on the commutative property are applied in simulation.

The feedforward parameterization $F(z, \theta_j)$ is selected according to Def. 2 and is as follows. Let $\xi(z) = \frac{1-z^{-1}}{T_s}$, the polynomial matrices $A(z, \theta_j)$ and $B(z, \theta_j)$ are given by

$$A_{p,q}(z, \theta_j) = \xi^A_{0,p,q}(z) + \sum_{i=1}^{n^A_{\theta,p,q}} \xi^{i-1}(z)\theta^A_{j,p,q}[i] \tag{23}$$

$$B_{p,q}(z, \theta_j) = \xi^B_{0,p,q}(z) + \sum_{i=1}^{n^B_{\theta,p,q}} \xi^{i}(z)\theta^B_{j,p,q}[i] \tag{24}$$

where p denotes the output direction number and q the input direction number, the polynomial $\xi^A_{0,p,q}(z) = 0, \forall p, q$, the polynomial $\xi^B_{0,p,q}(z) = 1 \forall p = q$ to make $B(\theta_j)$ monic, and $n^A_{\theta,p,q}$ and $n^B_{\theta,p,q}$ denote the number of basis for A and B in

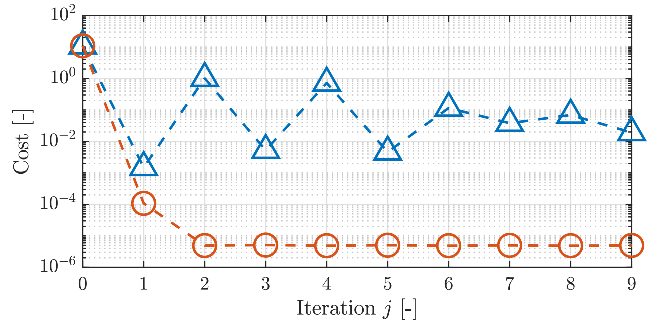


Fig. 5. Cost per iteration j for the RBF algorithm for SISO systems (-△-) and the developed MIMO RBF approach (-⊖-). Clearly, the RBF algorithm for SISO systems does not necessarily converge.

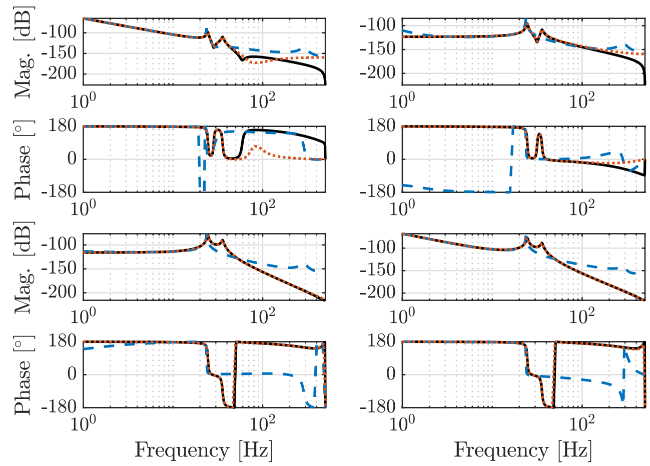


Fig. 6. Bode diagram of the plant $P(z)$ (—) and of the inverse feedforward filters $F^{-1}(z, \theta_{10})$ in trial $j = 10$ achieved with the RBF algorithm for SISO systems (- -) and the developed approach (·····). The feedforward filter of the developed approach fits the resonance and anti-resonance peaks of the plant significantly better, allowing enhanced performance.

output direction p and input direction q, respectively. Here,

$$n^A_\theta = \begin{bmatrix} 7 & 6 \\ 6 & 7 \end{bmatrix}, \quad n^B_\theta = \begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}, \tag{25}$$

hence, $n_\theta = 36$. Note that $\theta^A_{j,p,q}[i]$ is a specific selection of θ_j that correspond to output and input direction p, q in A, respectively, and similarly holds for B.

Both ILC approaches perform 10 ILC trials, and the RBF algorithm for SISO systems performs $k_{\max} = 19$ SK-iterations after each trial. Noteworthy, the polynomial matrices A and B and the system are noncommutative.

C. Simulation results

The results of the simulation are shown in Fig. 5-8. The cost function per iteration, i.e., $\|e_j\|^2$, in Fig. 5 shows that the developed approach converges fast to a significantly lower cost than the RBF algorithm for SISO systems. Noteworthy, the RBF algorithm for SISO systems does not necessarily converges if more iterations would be performed. This indicates that wrongfully relying on the commutative property

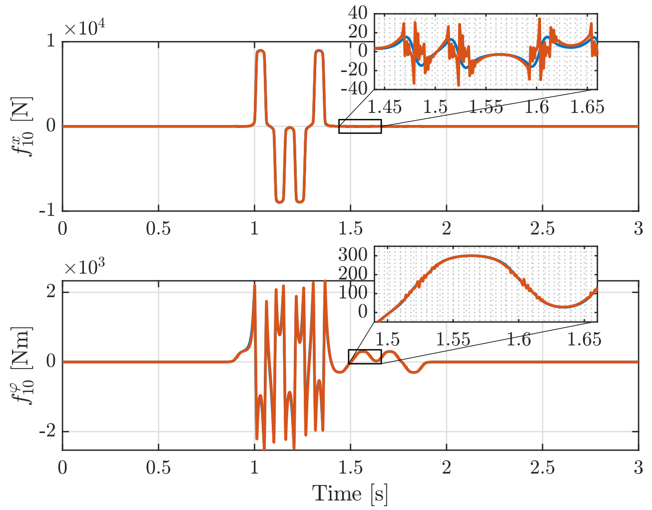


Fig. 7. Feedforward signals in trial $j = 10$ of the RBF algorithm for SISO systems (—) and the developed MIMO RBF approach (—). Noteworthy, in contrast to the RBF algorithm for SISO systems, the feedforward signal of the developed approach compensates for dynamics that are seen in Fig. 6.

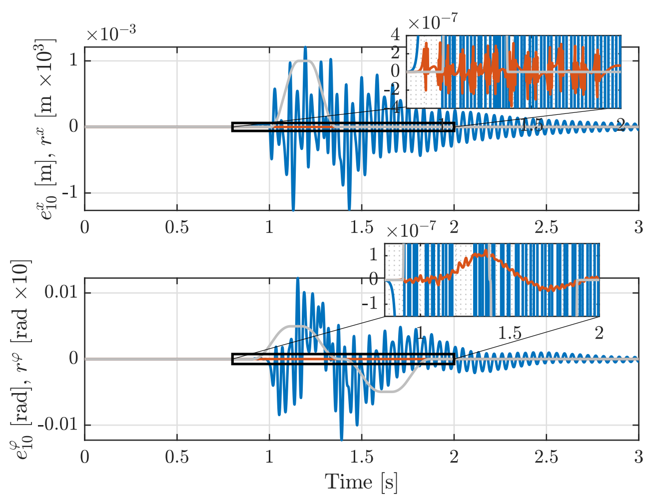


Fig. 8. Error signals in trial $j = 10$ of the RBF algorithm for SISO systems (—) and the developed MIMO RBF approach (—), for the scaled references trajectories r^x, r^φ (—), indicate a significant performance gain for the developed approach with the same feedforward parameterization.

of the system can lead to loss of convergence or to a lower achieved cost, hence [9] can not be directly applied to general MIMO systems.

The resulting Bode diagrams of $F^{-1}(z, \theta_{10})$ are shown in Fig. 6 and shows that the developed approach clearly achieves a near-perfect model inverse, see (9). The RBF algorithm for SISO systems is unable to fit the resonances, resulting in poor performance.

The time-domain performance for $j = 10$ is depicted in Fig. 8 for the feedforward signals shown in Fig. 7. The feedforward signal of the developed approach shows contributions related to the resonances shown in Fig. 6, which lead to high tracking performance. The poor performance of the RBF algorithm for SISO systems is caused by the poorly fit resonance, seen in $F^{-1}(z, \theta_{10})$ of Fig. 6.

V. CONCLUSIONS AND FUTURE WORKS

The developed framework enables data-driven tuning of rational feedforward controllers for general MIMO systems that are not necessarily commutative. In particular, by rewriting the ILC error using the nonlinear terms as a function of the feedforward parameters of the last experiment, the rational basis functions can be learned. A simulation study conducted on an multivariable industrial printer shows that the developed framework converges and achieves significantly better performance than the RBF algorithm for SISO systems that relies on the commutation property.

Future work focuses on the influence of model-mismatch, analysis of the convergence properties, and extending the framework with an iterative offline solution to avoid the need for more experiments.

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