2024 IEEE 63rd Conference on Decision and Control (CDC)
December 16-19, 2024. MiCo, Milan, Italy

Passivity Preserving Safety-Critical Control of Switched Systems

Qingkai Meng, Andreas Kasis, Marios M. Polycarpou

Abstract— **System safety refers to the property of the state trajectories to remain within some predefined set at all times. Integrating safety and stability offers significant advantages, such as resilience to disturbances and enhanced reliability and predictability. This paper combines control barrier functions and passivity methodologies, in the context of switched systems, to simultaneously ensure stability and safety. We derive conditions under which the passivity of switched systems is preserved under control barrier function-based switched safety-critical control. This enables the construction of a suitable design framework for observer-based output feedback controllers and switching laws, that achieve simultaneous stability and safety guarantees, without imposing any assumption on the safety of individual subsystems. Furthermore, we show that the simultaneous passivity and safety of interconnected switched systems under feedback and parallel configurations can be guaranteed by applying the developed conditions on each local switched subsystem. The applicability of the developed theoretical results is validated through a planar moving body example.**

I. Introduction

Motivation and literature review: Safety control aims to enable the reliable operation of systems by preventing system states from entering hazardous regions, such as maintaining a safe distance during cruise control in cars [1] and restricting the water levels of the tanks in water systems [2]. The reliable operation of dynamic systems is also related to their stability [3]. Combining safety and stability in systems offers significant advantages, such as resilience to disturbances and enhanced reliability and predictability. These properties are crucial for switched systems, which are composed of a discrete logic switching law and a series of continuous dynamic subsystems. Switched systems can effectively describe systems with multi-modal dynamics and/or parameter jumps and characterize a wide array of practical applications, such as power systems, robotics, communication networks, and process control [4]. Therefore, simultaneously ensuring the safety and stability of switched systems is of high importance.

Mathematically, ensuring the safety of a system involves constraining its states within a safe subset of the state space, satisfying the set forward invariance property [5]. As a general safety control technique, safe sets are characterized as super-level subsets in the state space by introducing control barrier functions (CBF) and forward invariance is established based on Lyapunov-like conditions [6]. Moreover, passivity-based control (PBC) enables guarantees on system stability by preventing energy accumulation without external excitation [7]. PBC offers robust solution approaches for designing controllers of a class of large-scale interconnected systems [8]–[11]. Specifically, it follows the principle of energy locality, where control signals are generated solely from local system state information, without reliance on global information or external inputs. This local control approach facilitates stability analysis based on local system components, avoiding the complexity of considering the overall system, and facilitating scalability and applicability to large-scale systems.

The stability of switched systems based on PBC was systematically studied in [12], where cross-supply rates are introduced to show the energy influence of the activated mode on the inactivated modes. Regarding the safety of switched systems based on CBF, the existing results in the literature are limited. In [13], multiple CBF and multiple Lyapunov functions are merged as a tool for analyzing the asymptotic stability of switched systems with guaranteed safety. In [14], the model reference adaptive method and multiple CBF are combined to deal with safety-critical control of switched systems.

The integration of passivity with safety-critical control holds promise for simultaneously achieving both stability and safety guarantees. Furthermore, this combination is particularly appealing as it could enable the derivation of scalable conditions, leveraging input-output properties, to ensure the safe and stable operation of large-scale systems. Some notable attempts along this direction are [15]–[17]. In [15], the authors first introduce a purely kinematic CBF, which integrates kinetic energy to minimize model dependence, and applies to both underactuated and fully-actuated systems. In [16], the CBF and energy-tank approaches are combined to guarantee the passivity of a mechanical system. In [17], the authors present conditions under which safety-critical control implemented with CBF preserves passivity for nonlinear affine systems. However, the above results cannot be directly extended to switched systems, since they involve coordinating passivity and safety under the influence of multiple switching modes. Ensuring the safety of switched systems while maintaining their passivity presents an intricate challenge, compounded by the existence of numerous storage functions and multi-CBF, which introduce both opportunities and complexities to the problem-solving process. To the

This paper was supported in part by funding from the European Research Council (ERC) under grant agreement No. 951424 (Water-Futures) and the European Union's Horizon 2020 research and Innovation programme under grant agreement No. 739551 (KIOS CoE); in part by National Natural Science Foundation of China under grant agreement No. 62303218, China Postdoctoral Science Foundation under grant agreement No. 2022M721607, and the Jiangsu Funding Program for Excellent Postdoctoral Talent under grant agreement No. 2023ZB573.

The authors are with the KIOS Research and Innovation Center of Excellence and the Department of Electrical and Computer Engineering, University of Cyprus, 1678, Nicosia, Cyprus. (meng.qingkai@ucy.ac.cy, kasis.andreas@ucy.ac.cy, mpolycar@ucy.ac.cy)

979-8-3503-1632-2/24/$31.00 ©2024 IEEE

authors' best knowledge, the passivity preserving safety-critical control of switched systems has not been explored in the literature.

Contribution: This paper explores conditions, under which both the passivity and safety properties of linear switched systems are guaranteed. Firstly, by considering the passivity of switched systems, determined by storage functions and supply/cross-supply rates, a safety control problem is formulated by introducing the desired reference switched model. Secondly, considering that only partial state information can be measured, a class of high-gain estimators and a class of passivity-related CBF are constructed, which are utilized for the design of switching laws. Thirdly, analytical conditions are provided that enable passivity guarantees through safety-critical control. These results are extended to feedback and parallel interconnected systems consisting of switched subsystems. For these cases, we show that the stability and safety of the overall system can be guaranteed if the local conditions can be verified for each subsystem. This lays the ground work for the development of suitable control schemes that simultaneously enable the stability and safety of large-scale switched systems. The presented analytical results are validated through numerical simulations that illustrate the effectiveness of the proposed approach by demonstrating the safety and stability of a considered switched system.

Notations. The distance between a point $x \in \mathbb{R}^n$ and the closed set $\mathcal{Y} \subset \mathbb{R}^n$ is defined as $\mathrm{dist}(x, \mathcal{Y}) = \inf_{y \in \mathcal{Y}} \|x-y\|$ where $\|\cdot\|$ is the standard 2-norm. The interior of a set $(\cdot)$ is denoted by int $(\cdot)$. A continuous function $\alpha : (0, \infty) \to (0, \infty)$ belongs to class $\mathcal{K}$, if it is strictly increasing and $\alpha(0) = 0$. A function $S : \mathbb{R}^n \to \mathbb{R}$ is called positive-definite on $\mathbb{R}^n$ if $S(0) = 0$ and $S(x) > 0$ for every non-zero $x \in \mathbb{R}^n$.

II. Problem Formulation

Consider a switched system of the form

$$\begin{aligned} \dot{x} &= A_\sigma x + B_\sigma u_\sigma, \\ y &= C_\sigma x, \end{aligned} \tag{1}$$

where σ is the switching signal taking values in

$$\sigma \in M = \{1, 2, \cdots, m\},$$

which is a right-continuous piece-wise constant function; $x \in \mathbb{R}^n$, $u_i \in \mathbb{R}^r$ and $y \in \mathbb{R}^r$ are the state, input and output vectors of the ith subsystem, respectively; A_i, B_i and C_i for each $i \in M$, are the system, input, and output matrices with appropriate dimensions. The switching signal σ can be characterized by the switching sequence

$$\Sigma = \{(i_0, t_0), (i_1, t_1), \cdots, (i_k, t_k), \cdots \mid i_k \in M, k \in \mathbb{Z}\}$$

in which t_0 is the initial time and i_0 the initial switching state. When $t \in [t_k, t_{k+1}), \sigma(t) = i_k$, that is, the i_kth subsystem is active. Therefore, the *state trajectory* $x(t)$ of the switched system (1) is defined as the solution $x_{i_k}(t)$ of the i_kth subsystem when $t \in [t_k, t_{k+1})$ with $x_{i_k}(t_k) = x(t_k)$. For any $j \in M$, let

$$\Sigma_j = \{t_{j_1}, t_{j_2}, \cdots, t_{j_n}, \cdots, \mid i_{j_q} = j, q \in \mathbb{Z}_+\}$$

be the sequence of switching times when the jth subsystem is switched ON and thus

$$\{t_{j_1+1}, t_{j_2+1}, \cdots, t_{j_n+1}, \mid i_{j_q} = j, q \in \mathbb{Z}_+\}$$

is the sequence of switching times when the jth subsystem is switched OFF.

The *safe set*, denoted by $\mathcal{S} \subset \mathbb{R}^n$, represents the region of the permissible system states.

Definition 1 ([6]). *A system is called* safe *if its state trajectory is always maintained in a prescribed safe set $\mathcal{S}$. Hence, the safety condition is satisfied if*

$$\forall x(0) \in \mathcal{S} \Rightarrow x(t) \in \mathcal{S}, \ \forall t > 0. \qquad \square$$

To represent the desired behaviors of the system, denoting the reference safe set by $\mathcal{S}_r \subseteq \mathcal{S}$, consider the following reference switched model

$$\dot{x}_r = A_{r\sigma} x_r, \ y_r = C_\sigma x_r, \tag{2}$$

where $x_r \in \mathbb{R}^n$ is the reference state, y_r is the reference output, and $A_{r\sigma}, C_\sigma$, for $\sigma \in M$, are reference system and output matrices. The only assumption about the trajectory is that $x_r(0) \in \mathcal{S}_r$. Naturally, depending on various practical application objectives, the form of x_r can vary, even being a static point, as long as its initial state is within the safe set $\mathcal{S}$ and there exists σ such that for all $t \geq 0$, $x_r(t) \in \mathcal{S}_r$. The error variable of x with respect to x_r is denoted by $e = x - x_r$.

With the above reference switched model, the following error system is obtained

$$\begin{aligned} \dot{e} &= \underbrace{A_{i_k} x - A_{r i_k} x_r}_{f^{cl}_{i_k}(e)} + B_{i_k} u_i \\ y_e &= C_{i_k} x - C_{i_k} x_r = C_{i_k} e, \ t \in [t_k, t_{k+1}), \end{aligned} \tag{3}$$

where $y_e \in \mathbb{R}^r$ denotes the output of the error system, and $f^{cl}_{i_k}(e)$ denotes the system function.

A. Passivity-based control

The following definition aims to facilitate the design.

Definition 2 ([12]). *A system in the form* (3) *is said to be* strictly passive *under the switching law σ if there exist positive-definite continuous functions $W_1(e), W_2(e), \cdots, W_m(e)$, called storage functions, with the property that for some constants $\bar{k}_i, \underline{k}_i$,*

$$\underline{k}_i \|e\|^2 \leq W_i(e) \leq \bar{k}_i \|e\|^2, \ i \in M, \ \forall e \in \mathbb{R}^n, \tag{4}$$

locally, integrable functions $\omega^i_i(u_i, y_e) = u_i^\top y_e - \theta_i y_e^\top y_e, i \in M, \theta_i \geq 0$, called supply rates, *and locally integrable functions $\omega^i_j(e, u_i, y_e, t) = \varphi^i_j(e)(u_i^\top y_e - \theta_i y_e^\top y_e)$ for some positive definite continuous functions $\varphi^i_j(e)$, $1 \leq i, j \leq m, i \neq j$, called* cross-supply rates*, such that*

i) $$\begin{aligned} &W_{i_k}(e(t)) - W_{i_k}(e(s)) \\ &\leq \int_s^t \omega^{i_k}_{i_k}(u_{i_k}, C_{i_k} e(\tau)) d\tau \\ &k = 0, 1, 2 \cdots, \ t_k \leq s \leq t < t_{k+1}, \end{aligned}$$

ii) $$W_j(e(t)) - W_j(e(s)) \le \int_s^t \omega_j^{i_k}(x(\tau), u_{i_k}, C_{i_k}e(\tau), \tau)d\tau$$
$$j \neq i_k,\ k = 0,1,2\cdots,\ t_k \le s \le t < t_{k+1}.$$

iii) *There exist $u_i(t)$, such that $\omega_i^i(u_i, C_ie) \le 0, \forall t \ge 0$.* □

According to the relation between passivity and stability, the following corollary is obtained.

Corollary 1 (Thm 3.7, [12])**.** *If the system* (3) *is strictly passive, then, the origin is asymptotically stabilized by any controller $u_i, i \in M$, of the form*

$$u_i(t) = \alpha_i(e,t)$$

satisfying $\alpha_i(0,t) \equiv 0$ and $u_i^\top C_ie \le 0$. □

For simplicity, the following assumption is given.

Assumption 1. *System* (2) *is asymptotically stable at the origin for any switching law $\sigma(t)$.* □

This is a mild condition since $A_{ri}, i \in M$ are defined through design. For example, x_r can be a constant point, when $A_{ri} = 0, \forall i \in M$, that is independent of the switching law.

The PBC objective for system (1) is to find bounded output feedback laws $u_i(e(t)) = v_i(e(t), t)$ satisfying $v_i(0,t) \equiv 0$ and switching law $\sigma(t)$ such that the closed-loop system

$$\begin{aligned} \dot{e}(t) &= f_{i_k}^{cl}(e(t)) + B_{i_k}u_{i_k}(e(t)), \\ y_e(t) &= C_{i_k}e(t),\ t \in [t_k, t_{k+1}), \end{aligned} \tag{5}$$

is passive with respect to some closed-loop storage functions W_{i_k}, and the input-output pair (u_{i_k}, y_e). The natural dissipation, denoted by $d_p(e)$, represents the process by which energy within the system is naturally dissipated without any external intervention. It should be noted that, for a strictly passive switched system, the natural dissipation is reduced into

$$0 \le \theta_i\varphi_j^i(e)C_i^\top eC_ie \le d_p(e) := -L_{f_i^{cl}(e)}W_j,\ e \in \Omega_i$$

and

$$L_{B_i}W_j = \varphi_j^i(e)C_i^\top e,\ e \in \Omega_i,$$

where $L_{B_i}W_j$ and $L_{f_i^{cl}(e)}W_j$ denotes the Lie derivative of W_j along B_i and $f_i^{cl}(e)$ respectively. Moreover, $\cup_{i=1}^m \Omega_i = \mathbb{R}^n$, int $(\Omega_i \cap \Omega_j) = \emptyset$, $i \neq j$.

We consider a controller operating under incomplete state information, i.e., constrained to output feedback. This requires observability, as stated in the subsequent assumption.

Assumption 2. *The system* (3) *is asymptotically zero-state detectable. That is, for any $\theta > 0$, there exists $\delta > 0$, such that when $\|y_e(t+s)\| < \delta$ holds for some $t \ge 0$, $0 \le s \le \Delta$ and $\Delta > 0$, we have $\|e(t)\| < \theta$.* □

B. Problem statement

The problem considered in this paper is stated below.

Problem 1: For the system (1), design a switching law σ and construct controllers $u_i, i \in M$, for individual subsystems, such that when Assumptions 1-2 hold, then

i) The error system (3) is asymptotically stable at the origin;
ii) The system state trajectory x of (1) always remains within the safe set $\mathcal{S}$;
iii) The switching law and control input rely only on system and observer outputs. □

Objective i) aims to achieve the stability of the error system. Objective ii) requires the switched system's state trajectories to always remain within the safety set to ensure the system's safety. Achieving these objectives requires the design of feedback controllers and switching laws merely based on measurable output and system structural information, as objective iii) stated.

III. Passivity preserving safety-critical control

In this section, we investigate conditions under which the passivity of (3) is preserved while simultaneously ensuring the safety of system (1). Since the states of systems (1) and (2) cannot be directly accessed, we design the following observers using the output and known system structure information

$$\begin{aligned} \dot{\hat{x}} &= A_{i_k}\hat{x} + B_{i_k}u_i + L_{i_k}(y - C_{i_K}\hat{x}), \\ \dot{\hat{x}}_r &= A_{ri_k}\hat{x}_r + L_{ri_k}(y_r - C_{i_K}\hat{x}_r),\ t \in [t_{i_k}, t_{i_{k+1}}), \end{aligned} \tag{6}$$

where observer gains L_i and $L_{ri}, i = 1,2,\cdots,m$, are matrices such that $(A - L_iC_i)$ and $(A_r - L_{ri}C_i)$ are stable.

By extending the results presented in [18], if $x(0)$ and $x_r(0)$ are known, with bounded input u_i, one can select L_i and L_{ri} such that $x - \hat{x}$ and $x_r - \hat{x}_r$ asymptotically converge to zero, and $\|x - \hat{x}\| \le \lambda$, $\|x_r - \hat{x}_r\| \le \lambda_r$ for arbitrary $\lambda > 0$ and $\lambda_r > 0$, for all $t \ge 0$. In this paper, we assume that $x_r(0)$ and $x(0)$ are within the safe set and that the distances from them to the safety boundaries $\hat{\varepsilon}(0) := \text{dist}(x(0), \mathbb{R}/\mathcal{S})$ and $\hat{\varepsilon}_r(0) := \text{dist}(x_r(0), \mathbb{R}/\mathcal{S}_r)$ are known. Moreover, we assume that the initial estimated errors $\|x(0) - \hat{x}(0)\|$ and $\|x_r(0) - \hat{x}_r(0)\|$ are known. They are summarized in the following assumption.

Assumption 3. *By using estimators* (6)*, there exist $\lambda \le \hat{\varepsilon}(0)$ and $\lambda_r \le \hat{\varepsilon}_r(0)$, such that $\|x - \hat{x}\| \le \lambda$ and $\|x_r - \hat{x}_r\| \le \lambda_r$, for all $t \ge 0$.* □

Denote the error under estimation by $\hat{e} = \hat{x} - \hat{x}_r$. Hereafter, the safe set $\mathcal{S}_i$ with respect to subsystem i is built as the super-level set of a continuously differentiable function $l_i(t, \hat{e}) : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}$. Let

$$\begin{aligned} \Omega_i &= \{\hat{e} \in \mathbb{R}^n : W_i(\hat{e}) - W_j(\hat{e}) \le 0, \forall j \in M\}, i \in M, \\ \tilde{\Omega}_{ij} &= \{\hat{e} \in \mathbb{R}^n : W_i(\hat{e}) - W_j(\hat{e}) = 0\}, j \neq i, i,j \in M, \end{aligned} \tag{7}$$

be the state space partitions of the error system. It is reasonable to select $W_i, i \in M$, such that when $\hat{e} \neq 0$, if $W_i(\hat{e}) = W_j(\hat{e})$, then there exists no $k \in M \setminus \{i,j\}$ that satisfies $W_i(\hat{e}) = W_k(\hat{e})$. This implies $\tilde{\Omega}_{ij} \cap \tilde{\Omega}_{ik} = \emptyset, j \neq k$, for all $\hat{e} \neq 0$. Without the safety consideration, the state-dependent switching trajectories would be designed to satisfy

$$\sigma(t) = \begin{cases} j, & \text{if } \sigma(t^-) = i \text{ and } \hat{e}(t) \in \tilde{\Omega}_{ij}, \\ \sigma(t^-), & \text{otherwise}, \end{cases} \tag{8}$$

where $t^- := \lim_{\epsilon \to 0} t - |\epsilon|$ denotes the time instant immediately before t. Let

$$\mathcal{S}_i = \{\hat{e} \in \mathbb{R}^n : l_i(t, \hat{e}) > 0\},\ i \in M, \\ \tilde{\mathcal{S}}_{ij} = \mathcal{S}_i \cap \mathcal{S}_j,\ i, j \in M, i \neq j, \tag{9}$$

with

$$l_i(t, \hat{e}) = \underline{k}(\varepsilon(t) - 2\lambda)^2 - W_i(\hat{e}), \tag{10}$$

be the CBF for error system (3) where $\underline{k} = \min_{i \in M}\{\underline{k}_i\}$ and $\varepsilon(t) := \mathrm{dist}(\hat{x}_r(t), \mathbb{R}^n/\mathcal{S})$.

Remark 1. *According to* (4), (9), *and* (10), *for the fixed $i \in M$, the CBF $l_i(t, \hat{e})$ is used to judge whether system state $x(t)$ belongs to the safe set. In fact, if $l_i(t, \hat{e}) > 0$, then $\underline{k}((\varepsilon - 2\lambda)^2 - \|\hat{e}\|^2) > 0$ from $W_i(\hat{e}) \geq \underline{k}_i\|\hat{e}\|^2$. It follows that $\varepsilon(t) - 2\lambda \geq \|\hat{e}\|$ and thus*

$$\|e\| \leq \|\hat{e}\| + \lambda \leq \varepsilon(t) - \lambda \leq \mathrm{dist}(x_r, \mathbb{R}^n/\mathcal{S}),$$

since $\|e\| = \|x - \hat{x} + \hat{x} - x_r\| \leq \|\hat{e}\| + \lambda$. Therefore, the distance between the state and reference state is less than the minimum distance between x_r from the boundary of safe set $\mathcal{S}$, and state x is always within the ball region with radius ε and centered at x_r. This means that when $l_i(t, \hat{e}) > 0$, the system is always safe. □

The following assumptions are given to facilitate the design.

Assumption 4. *The signal $\varepsilon(t)$ is continuously differentiable along trajectories of* (2) *and satisfies $|\dot{\varepsilon}| \leq (\underline{\gamma}/2)\varepsilon(t)$ when $\dot{\varepsilon} < 0$, for some $\underline{\gamma} > 0$.* □

Assumption 5. *For the system* (3)*, it holds that $\mathcal{S}_r \cap \Omega_i \neq \emptyset$, $\forall i \in M$. Moreover, $\hat{e}(0) \in \mathrm{int}(\mathcal{S}_i \cap \Omega_i)$ for some $i \in M$.* □

Assumption 4 can be achieved when $\mathcal{S}$ is a convex and compact set. Assumption 5 requires that each state space partition subset intersects with the reference safe set $\mathcal{S}_r$ and thus intersects with the safe set $\mathcal{S}$. When the initial state is within the safe set, according to Remark 1 and Assumption 3, it follows that the second part of Assumption 5 is always satisfied when selecting $W_i, i \in M$ properly. Moreover, according to the definition of $l_i, i \in M$, it follows that $\tilde{\mathcal{S}}_{ij} \neq \emptyset$ since $\tilde{\Omega}_{ij} \cap \mathcal{S}_r \neq \emptyset$ can be deduced by the first part of Assumption 5. Under Assumption 5, the initial value of the switching law would be $\sigma(0) = i$ for some $i \in M$ satisfying $\hat{e}(0) \in \mathrm{int}(\Omega_i \cap \mathcal{S}_i)$. For state space partitions (7) and CBF $l_i(t, \hat{e})$ the switching trajectories are designed to satisfy

$$\sigma(t) = \begin{cases} j, & \text{if } \sigma(t^-) = i \text{ and } \hat{e}(t) \in \{\tilde{\Omega}_{ij} \cap \tilde{\mathcal{S}}_{ij}\}\backslash\{0\}, \\ \sigma(t^-), & \text{otherwise.} \end{cases} \tag{11}$$

Note that, CBF $l_i(t, \hat{e})$ as well as safe sets $\mathcal{S}_i$ with respect to subsystem i may be time-varying, since $\varepsilon(t)$ is time-varying when $x_r(t)$ is dynamic. Therefore, the switching law (11) may be both time and state dependent. The following theorem states that both stability and safety can be guaranteed by using the above safety-critical PBC scheme.

Theorem 1. *Consider systems* (1) *and* (2)*, let Assumptions 1-5 hold, and controllers $u_i, i \in M$, and switching law* (8) *ensure that error system* (3) *satisfy conditions i)-iii) in Definition 2. Then, if the switching law* (11) *is implemented with the state partition* (7) *and safe sets* (9)*, the closed-loop system* (3) *with the controllers $u_i, i \in M$, is asymptotically stable and safe.* □

Proof: The proof is split into two parts: the proof of passivity for the overall switched system (3) (verification of conditions in Definition 2) and the proof of safety for the closed-loop system (verification of $l_{\sigma(t)} \geq 0$ for all $t \geq 0$).

According to Assumption 1 and Corollary 1, one has that the reference trajectory $x_r(t)$ is always within the reference safe set $\mathcal{S}_r$. Construct a new storage function $V_i(\hat{e})$, $i = 1, 2, \cdots, m$, as $V_i(\hat{e}) = \frac{W_i(\hat{e})}{l_i(t,\hat{e})}$. When the ith subsystem is activated, taking the derivative of $V_i(\hat{e})$ along (5) and using condition i) in Definition 2 yields that

$$\dot{V}_i(\hat{e}) \leq \frac{l_i(t, \hat{e}) + W_i}{l_i^2(t, \hat{e})}(u_i^\top C_i\hat{e} - \theta_i C_i^\top \hat{e} C_i \hat{e}) - \frac{2\underline{k}_i\varepsilon^\top\dot{\varepsilon}}{l_i^2(t, \hat{e})}W_i. \tag{12}$$

From Assumption 2, it follows that $\|y\| \leq \zeta_i(t)\|\hat{e}\|$, for some positive definite function $\zeta_i(t)$. Moreover, using condition iii) in Definition 2 yields that there exists some positive constant ω_i such that $u_i^\top C_i\hat{e} - \theta_i C_i^\top \hat{e} C_i\hat{e} \leq -\omega_i\|y\|^2$. As a result, (12) can be rewritten as

$$\dot{V}_i(\hat{e}) \leq -\gamma_i V_i(\hat{e}) - \gamma_i V_i^2(\hat{e}) - \frac{2\underline{k}_i\varepsilon^\top\dot{\varepsilon}}{l_i^2(t, \hat{e})}W_i(\hat{e}), \tag{13}$$

where $\gamma_i = \sup_{t \in [t_{i_k}, t_{i_{k+1}})}\{\omega_i\underline{k}_i\zeta_i(t)\}$.

When $\dot{\varepsilon} \geq 0$, one has, $\forall t \in [t_k, t_{k+1})$,

$$V_{i_k}(\hat{e}(t)) \leq e^{-\gamma_{i_k}(t - t_k)}V_{i_k}(\hat{e}(t_k)). \tag{14}$$

Therefore, for the new storage function $V_{i_k}(\hat{e}(t))$, there exists an equivalent control input $u_i' = -\gamma' C_{i_k}\hat{e}$, satisfying condition i) in Definition 2. Similarly, for $j \neq i_k$, in the case one has

$$\begin{aligned} \dot{V}_j(\hat{e}(t)) \leq & -\frac{l_j(t, \hat{e}) + W_j}{l_j^2(t, \hat{e})}\zeta_i\varphi_j^{i_k}(\hat{e})\underline{k}_j W_j(\hat{e}(t)) \\ = & -\gamma_j^{i_k}(\hat{e})V_j(\hat{e}(t_k)) \end{aligned} \tag{15}$$

and thus condition ii) in Definition 2 is satisfied. Moreover, since the new input u_i' is in the proportional form of the original one u_i, condition iii) of Definition 2 is naturally established.

When the system is switched to subsystem i_{k+1} at the switching instant t_{k+1}, using the switching law (11), it follows that $W_{i_{k+1}}(\hat{e}(t_{k+1})) = W_{i_k}(\hat{e}(t_{k+1}))$, and thus

$$\begin{aligned} l_{i_{k+1}}(t_{k+1}, \hat{e}(t_{k+1})) &= \underline{k}(\varepsilon(t_{k+1}) - 2\lambda)^2 - W_{i_{k+1}}(\hat{e}(t_{k+1})) \\ &= \underline{k}(\varepsilon(t_{k+1}^-) - 2\lambda)^2 - W_{i_k}(\hat{e}(t_{k+1}^-)) \\ &= l_{i_k}(t_{k+1}^-, \hat{e}(t_{k+1}^-)). \end{aligned} \tag{16}$$

This means the values of the CBF at the instances just before and after the switch $k+1$ are equal and thus the safety can be guaranteed using the designed switching law. According to the definition of $V_i(\hat{e}(t))$, (14) and (16), it follows that

$$V_{\sigma(t)}(\hat{e}(t)) \le e^{-\gamma_{i_k}(t-t_k)} V_{i_k}(\hat{e}(t_k)) = e^{-\gamma_{i_k}(t-t_k)} V_{i_k}(\hat{e}(t_k^-)) \\ \le e^{-\underline{\gamma}(t-t_0)} V_{i_0}(\hat{e}(t_0)),$$

where $\underline{\gamma} = \min_{i \in M}\{\gamma_i\}$. Therefore, the safety function $l_{\sigma(t)}(t, \hat{e}(t)) > 0$ for all $t \ge 0$, since $V_i(\hat{e}), i \in M$ are positive-definite functions, $\hat{e}(0) \in \mathcal{S}_i$, and the maximum of $V_{\sigma(t)}(\hat{e}(t))$ is a positive constant.

When $\varepsilon < 0$, there are two cases.

Case 1: $W_i(\hat{e}) \ge (2\underline{k}\varepsilon(t)|\dot{\varepsilon}|/\gamma_i), i \in M$. One can deduce (14), as well as (15), from (13). Thus, conditions in Definition 2 are satisfied and $l_{\sigma(t)}(t, \hat{e}(t)) > 0$.

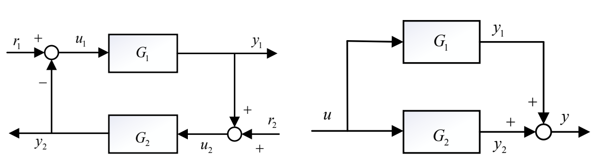
Case 2: $W_i(\hat{e}) < (2\underline{k}\varepsilon(t)|\dot{\varepsilon}|/\gamma_i), i \in M$. Combining the definition of $l_i(t, \hat{e}(t))$ with Assumption 4, it follows that $l_i(t, \hat{e}(t)) > \underline{k}\varepsilon^2(t) - ([2\underline{k}\varepsilon|\dot{\varepsilon}(t)|]/\gamma_i) \ge 0$. Thus, the state is in the safe set when each subsystem i is activated.

From the above inequality and Assumption 3, one has that there exists a $\mathcal{K}$ class function $\alpha(\cdot)$ such that $\|e\|^2 \le \|\hat{e}\|^2 + \|x - \hat{x}\|^2 + \|x_r - \hat{x}_r\|^2 \le -\alpha(\|e\|)$. Hence, the error system (3) is asymptotically stable. ■

Theorem 1 demonstrates that for a strictly passive switched system relying on state-dependent switches, it is not necessary to alter the control inputs to guarantee safety. Instead, applying state and time-dependent switching laws associated with control barrier functions ensures the strict passivity of the original system while guaranteeing system safety. This constructive result allows switched systems to simultaneously satisfy stability and safety requirements without the need for control redesign. Safety modifications can be achieved by building upon existing passivity results. This solves Problem 1 by merging the techniques of PBC and control barrier functions to simultaneously enable the safety and stability of switched systems through output feedback.

IV. Passivity Preserving Safety-critical Control of Interconnected switched systems

Consider two switched systems G_1, G_2 connected in the form of feedback interconnection, as shown in Fig. 1(a). The overall system has input $r(t) = [r_1(t), r_2(t)]^\top$ and output $y(t) = [y_1(t), y_2(t)]^\top$. The following result illustrates the passivity and safety of the overall system.



(a) (b)

Fig. 1. Feedback and parallel interconnection of two systems.

Theorem 2. *Consider two strictly passive and safe switched systems in the form of* (3)*, with inputs* u_1*,* u_2 *and outputs* y_1*,* y_2*, controllers such that* (5) *is passive and switching law described by* (11)*. Then, if Assumption 5 is satisfied for each subsystem,* $u_1 = r_1 - y_2$ *and* $u_2 = r_2 + y_1$*, then the system with input* $r = [r_1; r_2]$ *and output* $y = [y_1; y_2]$ *is strictly passive and safe.* □

Proof: By each system being passive and safe, there exist $V_{i_k}(e) = \frac{W_{i_k}(e)}{l_{i_k}(e)}$ and $V_{\hat{i}_k}(e') = \frac{W_{\hat{i}_{k'}}(e')}{l_{\hat{i}_{k'}}}$ such that the following inequalities hold, for $t \in [t_k, t_{k+1})$ and $t \in [t_{k'}, t_{k'+1})$.

$$\dot{V}_{i_k}(e) \le u_{i_k}^{'\top}(e) C_{i_k} e - \theta_{i_k}(C_{i_k} e)^\top C_{i_k} e,$$
$$\dot{V}_{\hat{i}_{k'}}(e') \le u_{\hat{i}_{k'}}^{'\top} C_{\hat{i}_{k'}} e' - \theta_{\hat{i}_{k'}}(e')(C_{\hat{i}_{k'}} e')^\top C_{\hat{i}_{k'}} e'.$$

In the meanwhile, $\forall j \in M, j \ne i_k, \hat{j} \in \hat{M}, \hat{j} \ne \hat{i}_{k'}$, there exist $\gamma_j^{i_k}(e) \ge 0, \gamma_{\hat{j}}^{\hat{i}'_k}(e') \ge 0$, such that

$$\dot{V}_j \le -\gamma_j^{i_k}(e) V_j, \dot{V}_{\hat{j}} \le -\gamma_{\hat{j}}^{\hat{i}'_k}(e') V_{\hat{j}}.$$

Define $V_{i\hat{i}} = V_i + V_{\hat{i}}$ and $\gamma_{j\hat{j}}^{i_k \hat{i}_{k'}} = \min\left\{\gamma_j^{i_k}, \gamma_{\hat{j}}^{\hat{i}_{k'}}\right\}$. Note that $u_1^\top y_1 + u_2^\top y_2 = (r_1 - y_2)^\top y_1 + (r_2 + y_1)^\top y_2 = r_1^\top y_1 + r_2^\top y_2 = r^\top y$. Repeating the process in the proof of Theorem 1 yields

$$\dot{V}_{i_k \hat{i}_{k'}} \le r^\top y - \theta_{i_k \hat{i}_{k'}} y^\top y, \dot{V}_{j\hat{j}} \le -\gamma_{j\hat{j}}^{i_k \hat{i}_{k'}} V_{j\hat{j}}.$$

Therefore, conditions in Definition 2 are verified and similarly, $l_i, l_{\hat{i}} \ge 0$ can be proven. ■

A similar conclusion can be obtained for two switched systems interconnected in parallel, as shown in Fig. 1(b).

Theorem 3. *Consider two strictly passive and safe switched systems in the form of* (3)*, with inputs* u_1, u_2 *and outputs* y_1*,* y_2*, passive controllers* (5) *and switching law* (11)*. Then, if Assumption 5 is satisfied for each subsystem, and* $u_1 = u_2$*, the system with input* $u = u_1 = u_2$ *and output* $y = y_1 + y_2$ *is strictly passive and safe.* □

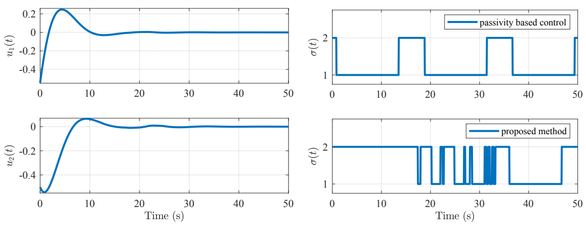
It should be stressed that the main difficulties to control large-scale network systems with local passivity include local and global coordination, system complexity and coupling effect, computational complexity, robustness and coordination of distributed control. However, our results may pave the way towards obtaining simultaneous stability and safety guarantees in large-scale interconnected switched systems.

V. Simulation

In this section, we consider a planar moving body with two switching dynamics. The state consists of coordinates in the horizontal and vertical directions, and the control inputs are the velocities in these two directions. When the horizontal velocity is directly controlled, the vertical velocity is inversely proportional to the horizontal coordinate; when the vertical velocity is directly controlled, the horizontal velocity is proportional to the vertical coordinate. Our control task is to return the moving body from its initial position to the origin without exceeding the safety boundary. Therefore, the dynamics in the form of (1) can be described as

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}, B_1 = [0\ 1]^\top, B_2 = [1\ 0]^\top,$$
$$C_1 = [1\ 0], C_2 = [0\ 1].$$

The reference trajectory is set at the origin with static dynamics. The safe set is selected as $\mathcal{S} = \{x \in \mathbb{R}^2 \mid x_1^2 + x_2^2 < 7.84\}$ while $\mathcal{S}_r = \mathcal{S}$, and the initial states as $x = [1.8, 2]^\top$. In addition, the initial errors between the states and boundaries of safe sets are set to $\hat{\varepsilon}(0) = 0.2$ and $\hat{\varepsilon}_r(0) = 4$. The estimator (6) is designed with $L_1 = L_{r1} = [5\ 7]^\top$, $L_2 = L_{r_2} = [-1.5\ 1.5]^\top$, and $\hat{x}(0) = [1.65, 1.8]^\top$.



(a) (b)

Fig. 2. Control input signals $u_i, i = 1, 2$, and switching law signal $\sigma(t)$ of the PBC and proposed controller.

To achieve the passivity preserving safety-critical based control for the above system, select $W_1(\hat{e}) = \hat{e}_1^2 + \hat{e}_2^2 + 0.5\hat{e}_1\hat{e}_2$, $W_2(\hat{e}) = 1.5\hat{e}_1^2 + \hat{e}_2^2 - 0.5\hat{e}_1\hat{e}_2$, with $\hat{e}_i = \hat{x}_i - \hat{x}_{ri}, i = 1, 2$. Thus, $l_i, i = 1, 2$, are given in the form of (10) with $\underline{k} = 1$. The passive controllers are designed as $u_1 = -0.1\hat{x}_1 - 0.2\hat{x}_2$ and $u_2 = -0.5\hat{x}_1 + 0.2\hat{x}_2$. One can see that the two controllers satisfy the conditions in Definition 2 and thus the nominal closed-loop switched system is strictly passive. The switching law in the form of (11) is employed. To demonstrate the effectiveness of our approach in terms of safety, we selected a PBC controller without considering safety (i.e., using the switching law (8)) as a reference.

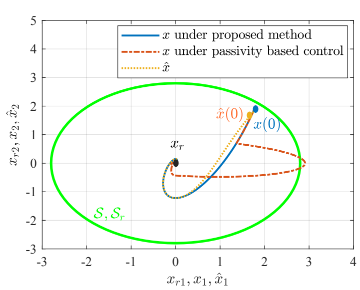


Fig. 3. Phase portrait trajectories of (x_{r1}, x_{r2}), $(x_1(t), x_2(t))$ under the proposed method, $(\hat{x}_1(t), \hat{x}_2(t))$, and $(x_1(t), x_2(t))$ under the PBC. The green ellipsoid depicts the boundary of the safe set $\mathcal{S}$ of state x.

Figures. 2(a) and 2(b) show the effect of the PBC controller (5) and the response curve of switching law (11). The phase portrait trajectories of system states are drawn in Fig. 3. It can be observed that during the convergence process, the state of the system with the PBC method exceeds the safety boundary, while the method proposed in this paper consistently ensures that the system's state remains within the safety set and thus solves objective ii) in Problem 1. Since the above objectives are achieved by using output feedback, objective iii) is also satisfied. Therefore, both passivity and safety are guaranteed under output feedback, which verifies the effectiveness of the proposed method.

VI. Conclusions

We present conditions under which safety-critical control implemented with CBF preserves passivity of switched systems. A new type of CBF is introduced by extending storage functions, and a switching law depending on both time and state is designed to achieve passivity and simultaneously deduce safety and stability. For interconnected systems consisting of switched subsystems in the form of feedback or parallel configurations, both passivity and safety of the overall system can be guaranteed when each subsystem is controlled by the proposed control scheme. Simulation results verify the effectiveness of the proposed theoretical analysis.

References

[1] L. Yu, and R. Wang, R, "Researches on adaptive cruise control system: A state of the art review," *Proc. IMechE. Part D J. Automob. Eng.*, vol. 236 no. 2-3, pp. 211–240, 2022.

[2] Q. Meng, S. Vrachimis, and M. Polycarpou. "Fault-tolerant safe control for water networks: An interconnected switched system approach." *IFAC-PapersOnLine*, vol. 58, no. 4, pp. 294-299, 2024.

[3] D. R. Merkin, *Introduction to the theory of stability*, Vol. 24, Springer Science & Business Media, 2012, pp. 5–19.

[4] D. Liberzon, *Switching in systems and control*, Vol. 190. Boston: Birkhauser, 2003.

[5] F. Blanchini, "Set invariance in control," *Automatica*, vol. 35, no. 11, pp. 1747–1767, 1999.

[6] A. D. Ames, X. Xu, X, J. W. Grizzle, and P. Tabuada, "Control barrier function based quadratic programs for safety critical systems," *IEEE Trans. Autom. Control*, vol. 62, no. 8, pp. 3861–3876, 2016.

[7] N. Chopra, and M. W. Spong, *Passivity-based control of multi-agent systems. In Advances in robot control*. Berlin, Heidelberg: Springer, 2006, pp. 107-–134.

[8] M. Arcak, "Passivity as a design tool for group coordination," *IEEE Trans. Autom. Control*, vol. 52, no. 8, pp. 1380–1390, 2007.

[9] A. Kasis, E. Devane, C. Spanias, and I. Lestas, "Primary frequency regulation with load-side participation—Part I: Stability and optimality," *IEEE Trans. Power Syst.*, vol. 32, no. 5, pp. 3505-3518, 2016.

[10] A. Kasis, N. Monshizadeh, E. Devane, and I. Lestas, "Stability and optimality of distributed secondary frequency control schemes in power networks," *IEEE Trans. Smart Grid*, vol. 10, no. 2, pp. 1747-1761, 2017.

[11] H. Zakeri, and P. J. Antsaklis, "Passivity measures in cyberphysical systems design: An overview of recent results and applications," *IEEE Control Syst. Mag.*, vol. 42, no. 2, pp. 118–130, 2022.

[12] J. Zhao, and D. J. Hill, "Dissipativity theory for switched systems," *IEEE Trans. Autom. Control*, vol. 53, no. 4, pp. 941–953, 2008.

[13] Q. Liu, and L. Long, "Asymptotic stability with guaranteed safety for switched nonlinear systems: a multiple barrier functions method," *IEEE Trans. Syst., Man, Cybern.: Syst.*, vol. 52, no. 6, pp. 3581–3590, 2021.

[14] C. Huang, and L. Long, "Safety-critical model reference adaptive control of switched nonlinear systems with unsafe subsystems: A state-dependent switching approach," *IEEE Trans. Cybern.*, vol. 53, no. 10, pp. 6353–6362, 2023

[15] A. Singletary, S. Kolathaya, and A. D. Ames, "Safety-critical kinematic control of robotic systems," *IEEE Contr. Syst. Lett.*, 2022, pp. 139—144.

[16] B. Capelli, C. Secchi, and L. Sabattini, "Passivity and control barrier functions: Optimizing the use of energy," *IEEE Robot. Autom. Lett.*, vol. 7, no. 2, pp. 1356—1363, 2022.

[17] F. Califano, "Passivity-preserving safety-critical control using control barrier functions," *IEEE Contr. Syst. Lett.*, vol. 7, pp. 1742–1747, 2023.

[18] M. Yousefi, K. van Heusden, I. M. Mitchell, and G. A. Dumont, "Output-feedback safety-preserving control," *2017 Amer. Control Conf.*, pp. 2550-2555, 2017.