2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

Least-Squares Composite Learning Backstepping Control With High-Order Tuners

Yongping Pan[1], Tian Shi[2], and Changyun Wen[3]

***Abstract*— Transient performance improvement in adaptive backstepping control is beneficial for the stability and robustness of control systems. In addition, parameter convergence in classical adaptive control is dependent on a stringent condition named persistent excitation (PE). This paper proposes a least squares-based composite learning backstepping control (LS-CLBC) strategy with high-order tuners for strict-feedback uncertain nonlinear systems such that exponential stability with parameter convergence is achieved under interval excitation (IE) or even partial IE that is strictly weaker than PE. In the LS-CLBC, the storage and forgetting of online historical data are determined by the exciting strength of a novel excitation matrix consisting of only active regressor channels, such that excitation information of regressor channels is exploited more effectively and efficiently to achieve parameter estimation. The learning rate is adjusted online based on LS and integrated into a high-order tuner to obtain the high-order time derivatives of parameter estimates. The closed-loop system is proven exponentially stable. Simulation results have demonstrated the superiority of the proposed approach.**

I. Introduction

The matching condition is a significant obstacle for adaptive control of nonlinear systems with uncertain parameters, and the adaptive backstepping technique with overparameterization is a precursor that relaxes this restriction by designing adaptive laws to tune virtual control inputs at each design step [1]. In [2], two modifications of the adaptive backstepping without overparameterization were developed, including tuning function and modular identifier approaches. The tuning function approach involves a virtual adaptive law that is constructed at each backstepping step to compensate for nonlinear dynamics, while the actual adaptive law is generated at the last step by all previous tuning functions [2, Ch. 4]. A major deficiency of the tuning function approach is the "explosion of complexity" caused by the repeated derivations of virtual control inputs. The modular identifier approach designs independent gradient estimators for unknown parameters to enhance control robustness, but the time derivatives of parameter estimates are treated as additive disturbances in the closed-loop system, which destroys the transient performance [2, Ch. 6].

In the modular identifier approach, removing the disturbing influence caused by the time derivatives of parameter estimates is beneficial for improving the adaptive control performance [3]–[6]. In [3], a direct adaptive backstepping control scheme combined with a linear filter was designed to counteract the transient process of parameter estimates resulting from their high-order time derivatives, where these time derivatives are obtained by high-order tuners. In [4]–[6], memory regressor extension (MRE) was applied to design an indirect adaptive backstepping scheme with a time-vary learning rate, but the learning rate relies on a normalization factor, and the high-order time derivatives of parameter estimates can be provided only under a constant learning rate.

Exponential parameter convergence is crucial for the robustness of adaptive control against disturbances but is commonly restricted by a stringent condition termed persistent excitation (PE) [7]. A natural way of relaxing excitation conditions is to exploit online historical data (OHD). This idea has inspired the emergence of composite learning, in which a generalized prediction error with OHD is applied to achieve exponential parameter convergence under a condition of interval excitation (IE) that is much weaker than PE [8]–[10]. Composite learning has been widely applied to uncertain nonlinear systems with successful applications to real-world robots [11]–[19]. A major feature of composite learning is that storing and forgetting OHD are driven by the minimum singular value (MSV) of an excitation matrix, which requires that all regressor channels are simultaneously activated in a certain instant.

This paper puts forward a least-squares composite learning backstepping control (LS-CLBC) approach with high-order tuners for strict-feedback uncertain nonlinear systems. In the LS-CLBC, the storage and forgetting of OHD are determined by the exciting strength of a novel excitation matrix consisting of only active regressor channels, which removes the restriction that all regressor channels must be activated simultaneously. A time-varying learning rate based on LS is introduced and integrated into a high-order tuner to avoid the normalization factor, and the high-order time derivatives of parameter estimates with a time-varying learning rate can be obtained directly. Exponential stability of the closed-loop system is established under IE or even partial IE. Simulations under IE and partial IE cases are carried out to verify the proposed approach.

Throughout this paper, $\mathbb{R}$, $\mathbb{R}^+$, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the spaces of real numbers, positive real numbers, real n-vectors and real $m \times n$-matrices, respectively, L_∞ denotes the space of bounded signals, $\sigma_{\min}(A)$ denotes the MSV of A, $\|\boldsymbol{x}\|$ denotes the Euclidean norm of $\boldsymbol{x}$, I denotes an identity matrix with a proper dimension, $\min\{\cdot\}$ and $\max\{\cdot\}$ denote the minimum and maximum operators, respectively, $\sup_{x\in S}\{f(x)\}$ $:= \{f(x)|f(y) \leq f(x), \forall y \in S\}$, and $\arg\max_{x\in S} f(x) :=$

*This work was supported in part by the Guangdong Provincial Pearl River Talents Program, China, under Grant 2019QN01X154, and by the Fundamental Research Funds for the Central Universities, Sun Yat-sen University, China, under Grant 23lgzy004 (*Corresponding author: Yongping Pan*).

[1]Y. Pan is with School of Advanced Manufacturing, Sun Yat-sen University, Shenzhen 518100, China panyongp@mail.sysu.edu.cn

[2]T. Shi is with School of Computer Science and Engineering, Sun Yat-sen University, Guangzhou 510006, China shit23@mail2.sysu.edu.cn

[3]C. Wen is with School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798 ecywen@ntu.edu.sg

$\{x \in S | f(y) \le f(x), \forall y \in S\}$ with $f : \mathbb{R} \mapsto \mathbb{R}$ and $S \subset \mathbb{R}$, where $A \in \mathbb{R}^{n\times n}$ and $\boldsymbol{x} \in \mathbb{R}^n$.

II. Problem Formulation

Consider a strict-feedback uncertain nonlinear system [2]

$$\begin{cases} \dot{x}_i = \boldsymbol{\varphi}_i^T(\boldsymbol{x}_i)\boldsymbol{\theta} + x_{i+1}, \\ \dot{x}_n = \boldsymbol{\varphi}_n^T(\boldsymbol{x})\boldsymbol{\theta} + \beta(\boldsymbol{x})u, \\ y = x_1 \end{cases} \tag{1}$$

with $i = 1, 2, \cdots, n-1$ and $\boldsymbol{x}_i := [x_1, x_2, \cdots, x_i]^T \in \mathbb{R}^i$, in which $\boldsymbol{x} := [x_1, x_2, \cdots, x_n]^T \in \mathbb{R}^n$ is a measurable state, $u \in \mathbb{R}^n$ is a control input, $y \in \mathbb{R}$ is a control output, $\boldsymbol{\theta} \in \mathbb{R}^N$ is an unknown parameter, $\boldsymbol{\varphi}_i : \mathbb{R}^i \to \mathbb{R}^N$ is a known and smooth regressor, $\beta : \mathbb{R}^n \to \mathbb{R}$ is a known control gain function that satisfies $|\beta(\boldsymbol{x})| \ge b_0, \forall \boldsymbol{x}$ with $b_0 \in \mathbb{R}^+$ being a constant [2], and N is the number of parameter elements. Let $L(s)$ be a stable filter in the transfer function form with a sufficiently large relative degree, where s is the complex argument of the Laplace transform. The following definitions are introduced for the subsequent analysis.

Definition 1 [20]: A bounded regressor $\Phi(t) \in \mathbb{R}^{N\times n}$ is of PE if there exist a constant $\sigma \in \mathbb{R}^+$ such that

$$F(t) := L(s)[\Phi\Phi^T] \ge \sigma I, \forall t \ge 0.$$

Definition 2 [21]: A bounded regressor $\Phi(t) \in \mathbb{R}^{N\times n}$ is of IE if there exist constants $\sigma, T_{\mathrm{e}} \in \mathbb{R}^+$ such that

$$F(t) := L(s)[\Phi\Phi^T] \ge \sigma I, t = T_{\mathrm{e}}.$$

Definition 3: A bounded regressor $\Phi(t) \in \mathbb{R}^{N\times n}$ is of partial IE, if there exist constants σ, $T_{\mathrm{e}} \in \mathbb{R}^+$ such that

$$F(t) := L(s)[\Phi_\zeta\Phi_\zeta^T] \ge \sigma I, t = T_{\mathrm{e}}.$$

where $\Phi_\zeta \in \mathbb{R}^{m\times n}$ is a sub-regressor constituted by some row vectors of Φ with $1 \le m < N$.

For convenience, a column $\boldsymbol{\phi}_i \in \mathbb{R}^n$ $(i = 1, 2, \cdots, N)$ of a regressor $\Phi^T(t) \in \mathbb{R}^{n\times N}$ is named as a channel. Thus, one has $\Phi^T(t) = [\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \cdots, \boldsymbol{\phi}_N]^T$. A channel $\boldsymbol{\phi}_i$ $(i = 1, 2, \cdots, m)$ of $\Phi^T(t)$ is named an *active channel* if $\boldsymbol{\phi}_i(t) \ne \mathbf{0}$, conversely termed an *inactive channel*. It should be noted that in Definition 3, Φ_ζ is constructed by all active channels, and therefore, Φ_ζ is of IE and Φ is of partial IE.

Let $y_{\mathrm{r}}(t) \in \mathbb{R}$ be a reference output generated by a reference model $y_{\mathrm{r}} = (a_0/a(s))[r(t)]$, where $a(s) := s^n + a_{n-1}s^{n-1} + \cdots + a_0$ is a monic Hurwitz polynomial, $r(t) \in \mathbb{R}$ is a bounded piecewise-continuous command signal, and $a_i \in \mathbb{R}$ $(i = 0, 1, \cdots, n-1)$ are certain constants. The above reference model permits the implementation of the time derivatives of y_{r} up to the *n*th order used in the control design. This paper aims to design a suitable adaptive control strategy for the system (1) to guarantee closed-loop exponential stability and parameter convergence under the lack of the PE condition.

III. Modular Adaptive Backstepping Design

Define an output tracking error $e_1 := x_1 - y_{\mathrm{r}}$ and virtual tracking errors $e_i := x_i - \alpha_{i-1} - y_{\mathrm{r}}^{(i-1)}$ $(i = 2, \cdots, n)$, where α_1, $\alpha_i \in \mathbb{R}$ are virtual control inputs given by [4]

$$\begin{aligned} &\alpha_1(x_1, \hat{\boldsymbol{\theta}}, y_{\mathrm{r}}) = -c_1e_1 - d_1e_1 - \boldsymbol{\phi}_1^T\hat{\boldsymbol{\theta}}, \\ &\alpha_i(\boldsymbol{x}_i, \Theta_{i-1}, Y_{i-1}^{\mathrm{r}}) = -e_{i-1} - (c_i + d_i)e_i - \boldsymbol{\phi}_i^T\hat{\boldsymbol{\theta}} \\ &+ \sum_{k=1}^{i-1}\left[\frac{\partial\alpha_{i-1}}{\partial x_k}x_{k+1} + \frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol{\theta}}^{(k-1)}}\hat{\boldsymbol{\theta}}^{(k)} + \frac{\partial\alpha_{i-1}}{\partial y_{\mathrm{r}}^{(k-1)}}y_{\mathrm{r}}^{(k)}\right] \end{aligned} \tag{2}$$

with $\Theta_{i-1} := [\hat{\boldsymbol{\theta}}, \dot{\hat{\boldsymbol{\theta}}}, \cdots, \hat{\boldsymbol{\theta}}^{(i-1)}] \in \mathbb{R}^{N\times i}$ and $Y_{i-1}^{\mathrm{r}} := [y_{\mathrm{r}}, \dot{y}_{\mathrm{r}}, \cdots, y_{\mathrm{r}}^{(i-1)}]^T \in \mathbb{R}^{N\times i}$, in which $\boldsymbol{\phi}_1 := \boldsymbol{\varphi}_1$, $\boldsymbol{\phi}_i := \boldsymbol{\varphi}_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\boldsymbol{\varphi}_k \in \mathbb{R}^N$ are new regressors, $d_1 := \kappa_1\|\boldsymbol{\phi}_1\|^2$, $d_i := \kappa_i\|\boldsymbol{\phi}_i\|^2 \in \mathbb{R}^+$ are damping terms, and c_1, c_i, κ_1, $\kappa_i \in \mathbb{R}^+$ are control gain parameters. The control law u derived in the final backstepping step is given by

$$u = \frac{1}{\beta(\boldsymbol{x})}(\alpha_n(\boldsymbol{x}, \Theta_{n-1}, Y_{n-1}^{\mathrm{r}}) + y_{\mathrm{r}}^{(n)}). \tag{3}$$

Applying (2), (3) and $x_i = \alpha_{i-1} + e_i + y_{\mathrm{r}}^{(i-1)}$ into (1), one obtains a closed-loop error system

$$\dot{\boldsymbol{e}} = A\boldsymbol{e} + \Phi^T\tilde{\boldsymbol{\theta}} \tag{4}$$

with $\boldsymbol{e} := [e_1, e_2, \cdots, e_n] \in \mathbb{R}^n$, where $\Phi := [\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \cdots, \boldsymbol{\phi}_n] \in \mathbb{R}^{N\times n}$ is a full regressor, and

$$A = \begin{bmatrix} -c_1 - d_1 & 1 & \cdots & 0 \\ -1 & -c_2 - d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & -c_n - d_n \end{bmatrix}.$$

Commonly, one can utilize a direct adaptive estimation law

$$\dot{\hat{\boldsymbol{\theta}}} = -\Gamma_0\Phi\boldsymbol{e} \tag{5}$$

where $\Gamma_0 \in \mathbb{R}^{N\times N}$ is a positive-definite constant learning rate matrix. The control law (3) with (5) guarantees exponential stability of the closed-loop system (4) under PE [2]. However, the direct adaptive scheme is generally infeasible in practice as Φ contains inaccessible high-order time derivatives $\hat{\boldsymbol{\theta}}^{(i)}$ $(i = 1, 2, \cdots, n-1)$. Besides, its parameter convergence depends on PE, which requires that the reference trajectory y_{r} includes considerably rich spectral information all the time.

IV. Composite Learning Backstepping Control

Noting that (4) can be rewritten into $\boldsymbol{e} = W(s)[\Phi^T\tilde{\boldsymbol{\theta}}]$ with $W(s) := (sI - A)^{-1}$, one has

$$\Phi_{\mathrm{f}}^T\boldsymbol{\theta} = \boldsymbol{e}(t) + W(s)[\Phi^T\hat{\boldsymbol{\theta}}] \tag{6}$$

with $\Phi_{\mathrm{f}} := W(s)[\Phi]$. Define a modeling error

$$\boldsymbol{\varepsilon}(t) := \boldsymbol{e}(t) + W(s)[\Phi^T\hat{\boldsymbol{\theta}}] - \Phi_{\mathrm{f}}^T\hat{\boldsymbol{\theta}} \tag{7}$$

which implies $\boldsymbol{\varepsilon} = \Phi_{\mathrm{f}}^T\tilde{\boldsymbol{\theta}}$. Let an excitation matrix be

$$Q(t) := L(s)[\Phi_{\mathrm{f}}\Phi_{\mathrm{f}}^T]. \tag{8}$$

Multiplying (6) by $\Phi_{\rm f}$ and applying $L(s)$ and (8) yields

$$Q(t)\boldsymbol{\theta} = L(s)[\Phi_{\rm f}\Phi_{\rm f}^T]\boldsymbol{\theta} := \boldsymbol{\psi}(t). \tag{9}$$

It is assumed that there exist certain constants $T_{\rm e}$, $\sigma \in \mathbb{R}^+$ such that the IE condition $Q(T_{\rm e}) \geq \sigma I$ in Definition 2 holds. Define a generalized prediction error

$$\boldsymbol{\xi}(t) := \begin{cases} \boldsymbol{\psi}(t_{\zeta,\rm e}) - Q(t_{\zeta,\rm e})\hat{\boldsymbol{\theta}}(t), & t < T_{\rm e} \\ \boldsymbol{\psi}(t_{\rm e}) - Q(t_{\rm e})\hat{\boldsymbol{\theta}}(t), & t \geq T_{\rm e} \end{cases} \tag{10}$$

with $t_{\zeta,\rm e} := \arg\max_{\varsigma\in[0,t]}\sigma_{\min}(Q_\zeta(\varsigma))$, $Q_\zeta(t) := L(s)[\Phi_{{\rm f},\zeta}\Phi_{{\rm f},\zeta}^T]$, and $t_{\rm e} := \arg\max_{\varsigma\in[T_{\rm e},t]}\sigma_{\min}(Q(\varsigma))$, in which $\Phi_{{\rm f},\zeta}^T$ is a sub-regressor composed of some columns $\boldsymbol{\phi}_{{\rm f}k_i}$ of $\Phi_{\rm f}^T$ satisfying $\|\boldsymbol{\phi}_{{\rm f}k_i}\| > 0$, i.e., $\Phi_{{\rm f},\zeta}^T = [\boldsymbol{\phi}_{{\rm f}k_1}, \boldsymbol{\phi}_{{\rm f}k_2}, \cdots, \boldsymbol{\phi}_{{\rm f}k_{N_\zeta}}]$ with $1 \leq k_i \leq N$ and $i = 1, 2, \cdots, N_\zeta < N$. Design a high-order tuner under the LS-based composite learning scheme:

$$\dot{\hat{\boldsymbol{\theta}}} = \begin{cases} \Gamma(t_{\zeta,\rm e})\boldsymbol{\xi}(t), & t < T_{\rm e} \\ \Gamma(t_{\rm e})\boldsymbol{\xi}(t), & t \geq T_{\rm e} \end{cases} \tag{11a}$$

$$\dot{\Gamma} = -\Gamma(t)sL(s)[\Phi_{\rm f}\Phi_{\rm f}^T]\Gamma(t), \Gamma(0) = \Gamma_0 > 0 \tag{11b}$$

and definite the current maximal exciting strength

$$\sigma_{\rm c}(t) := \begin{cases} \sigma_{\min}(Q(t_{\zeta,\rm e})), t < T_{\rm e} \\ \sigma_{\min}(Q(t_{\rm e})), \ \ t \geq T_{\rm e} \end{cases} \tag{12}$$

where $\Gamma \in \mathbb{R}^{N\times N}$ is a positive-definite learning rate matrix.

To generate the time derivatives of $\hat{\boldsymbol{\theta}}$, one commonly sets $L(s) = b_0/b(s)$, in which $b(s) = s^\rho + b_{\rho-1}s^{\rho-1} + \cdots + b_0$ is a monic Hurwitz polynomial with $\rho \geq n-1$. Thus, the time derivatives of $\hat{\boldsymbol{\theta}}$ and Γ in (11) up to the order $n-1$ are implemented physically by a direct differentiation scheme [22]. The high-order time derivatives of $\hat{\boldsymbol{\theta}}$ can be calculated by

$$\hat{\boldsymbol{\theta}}^{(k+1)} = \sum_{i=0}^{k} C_k^i \Gamma^{(k-i)} \left(\boldsymbol{\psi}^{(i)} - \sum_{j=0}^{i} C_i^j Q^{(i-j)} \hat{\boldsymbol{\theta}}^{(j)} \right), \tag{13}$$

$$\Gamma^{(k+1)} = -\sum_{i=0}^{k} C_k^i \Gamma^{(k-i)} \left(\sum_{j=0}^{i} C_i^j Q^{(j+1)} \Gamma^{(i-j)} \right) \tag{14}$$

with $\hat{\boldsymbol{\theta}}^{(i)}(0) = \mathbf{0}$, where $C_k^i = k!/(i!(k-i)!)$ are the binomial coefficients with $0 \leq i \leq k$ and $1 \leq k \leq n-2$. The following theorem establishes the stability result of this study.

Theorem 1. Consider the system (1) driven by the adaptive control law (3) with (11). If there exist suitably large control parameters c_1 to c_n and κ_1 to κ_n, the closed-loop system (4) with (11) has stability in the sense that:

- All closed-loop signals are of L_∞ on $t \in [0, \infty)$;
- $\boldsymbol{e}$, $\tilde{\boldsymbol{\theta}} \to \mathbf{0}$ exponentially on $t \in [T_{\rm e}, \infty)$ if IE in Definition 2 holds for some constants T_e, $\sigma \in \mathbb{R}^+$;
- $\boldsymbol{e}$, $\tilde{\boldsymbol{\theta}}_\zeta \to \mathbf{0}$ exponentially on $t \in [T_{\zeta,\rm e}, \infty)$ if partial IE in Definition 3 holds for some constants $T_{\zeta,\rm e}$, $\sigma \in \mathbb{R}^+$, where $\tilde{\boldsymbol{\theta}}_\zeta$ denotes a parameter estimation error that is corresponding to the sub-regressor $\Phi_{{\rm f},\zeta}$.

Proof: First, choose a Lyapunov function candidate

$$V(\boldsymbol{e}, \tilde{\boldsymbol{\theta}}) = \underbrace{\boldsymbol{e}^T\boldsymbol{e}/2}_{V_1} + \underbrace{\tilde{\boldsymbol{\theta}}^T\tilde{\boldsymbol{\theta}}/2}_{V_2} \tag{15}$$

Differentiating V_1 in (15) with respect to t yields

$$\dot{V}_1 = \boldsymbol{e}^T\dot{\boldsymbol{e}}/2 + \dot{\boldsymbol{e}}^T\boldsymbol{e}/2.$$

Applying (4) to the above result, one obtains

$$\dot{V}_1 = \boldsymbol{e}^T(A + A^T)\boldsymbol{e}/2 + \boldsymbol{e}^T\Phi^T\tilde{\boldsymbol{\theta}}$$

As $A + A^T = -\text{diag}(c_1 + d_1, c_2 + d_2, \cdots, c_n + d_n)$, one has

$$\dot{V}_1 \leq \sum_{i=1}^{n}(-c_i e_i^2 - \kappa_i\|\boldsymbol{\phi}_i\|^2 e_i^2 + e_i\boldsymbol{\phi}_i^T\tilde{\boldsymbol{\theta}}).$$

Applying Young's inequality $2\boldsymbol{a}^T\boldsymbol{b} - \|\boldsymbol{a}\|^2 \leq \|\boldsymbol{b}\|^2$ with $\boldsymbol{a} = \sqrt{\kappa_i}e_i\boldsymbol{\phi}_i$ and $\boldsymbol{b} = \tilde{\boldsymbol{\theta}}/(2\sqrt{\kappa_i})$ to the above expression yields

$$\dot{V}_1 \leq \sum_{i=1}^{n}(-c_i e_i^2 + \|\tilde{\boldsymbol{\theta}}\|^2/(4\kappa_i)) \leq -\boldsymbol{e}^T C\boldsymbol{e} + \|\tilde{\boldsymbol{\theta}}\|_\infty^2/(4\kappa_0)$$

where $C := \text{diag}(c_1, c_2, \cdots, c_n)$, $\|\tilde{\boldsymbol{\theta}}\|_\infty := \sup_{t\geq 0}\{\|\tilde{\boldsymbol{\theta}}(t)\|\}$, and $\kappa_0 := (\sum_{i=1}^{n} 1/\kappa_i)^{-1}$. Also, one has $\Gamma(t)Q(t) \geq 0$ and

$$\dot{V}_2 \leq -\tilde{\boldsymbol{\theta}}^T\Gamma(t)Q(t)\tilde{\boldsymbol{\theta}} \leq 0, \forall t \geq 0 \tag{16}$$

which implies that $0 \leq V_2(t) \leq V_2(0)$, $\forall t \geq 0$. Thus, $\tilde{\boldsymbol{\theta}} \in L_\infty$ and $\|\tilde{\boldsymbol{\theta}}\|_\infty < \infty$. Furthermore, one gets

$$\dot{V}_1 \leq -\boldsymbol{e}^T C\boldsymbol{e} + \bar{d} \leq -k_{\rm e}V_1 + \bar{d}$$

where $k_{\rm e} := 2\lambda_{\min}(C) \in \mathbb{R}^+$ and $\bar{d} := \|\tilde{\boldsymbol{\theta}}\|_\infty^2/(4\kappa_0) \in \mathbb{R}^+$. Solving the above inequality yields [23, Lemma A.3.2]

$$V_1(t) \leq (V_1(0) - \bar{d})e^{-k_{\rm e}t} + \bar{d}. \tag{17}$$

Accoridng to (16) and (17), the closed-loop system is stable in the sense of $\boldsymbol{e}$, $\tilde{\boldsymbol{\theta}} \in L_\infty$ implying $\boldsymbol{x}$, $\hat{\boldsymbol{\theta}}$, Φ, $u \in L_\infty$. The update of Γ in (11b) drives $\dot{\Gamma} \leq 0$ and $\Gamma(t) \leq \Gamma_0$. Thus, all closed-loop signals are of L_∞ on $t \in [0, \infty)$.

Second, consider the control problem under IE on $t \in [T_{\rm e}, \infty)$. From (11b) we first obtain

$$d(\Gamma^{-1}(t))/dt = -\Gamma^{-1}(t)\dot{\Gamma}\Gamma^{-1}(t) = sL(s)[\Phi_{\rm f}\Phi_{\rm f}^T]. \tag{18}$$

Integrating both sides of (18) over $[0, t]$ and applying (8) yields $\Gamma^{-1} - \Gamma_0^{-1} = Q$. Noting $Q(T_{\rm e}) \geq \sigma I$, one obtains

$$\begin{aligned} &\Gamma(t_{\rm e})Q(t_{\rm e}) \geq I - \Gamma_0^{-1}\Gamma(T_{\rm e}) \\ &\geq I - \Gamma_0^{-1}(\sigma I + \Gamma_0^{-1})^{-1} = \sigma(\sigma I + \Gamma_0)^{-1} := \Psi_0. \end{aligned} \tag{19}$$

Differentiating V in (15) with respect to t yields

$$\dot{V} \leq -\boldsymbol{e}^T C\boldsymbol{e} + \tilde{\boldsymbol{\theta}}^T\tilde{\boldsymbol{\theta}}/(4\kappa_0) - \tilde{\boldsymbol{\theta}}^T\Gamma(t_{\rm e})Q(t_{\rm e})\tilde{\boldsymbol{\theta}}. \tag{20}$$

Applying (19) with $t \geq T_{\rm e}$ to (20), one obtains

$$\begin{aligned} \dot{V} \leq &- \boldsymbol{e}^T C\boldsymbol{e} - \tilde{\boldsymbol{\theta}}^T(I - I/(4\kappa_0) - \Gamma_0^{-1}\Gamma(T_{\rm e}))\tilde{\boldsymbol{\theta}} \\ \leq &- \boldsymbol{e}^T C\boldsymbol{e} - \tilde{\boldsymbol{\theta}}^T(\Psi_0 - I/(4\kappa_0))\tilde{\boldsymbol{\theta}}. \end{aligned}$$

Choosing suitably large parameters κ_i to ensure $\lambda_{\min}(\Psi_0 - I/(4\kappa_0)) > 0$, one obtains

$$\dot{V} \leq -\lambda_{\min}(C)\boldsymbol{e}^T\boldsymbol{e} - \lambda_{\min}(\Psi_0 - I/(4\kappa_0))\tilde{\boldsymbol{\theta}}^T\tilde{\boldsymbol{\theta}} \leq -k_{\rm s}V$$

with $k_{\rm s} := 2\min\{\lambda_{\min}(C), \lambda_{\min}(\Psi_0 - I/(4\kappa_0))\}$, which implies that the closed-loop system (4) with (11) has exponential stability with $\boldsymbol{e}, \tilde{\boldsymbol{\theta}} \to \mathbf{0}$ on $t \in [T_{\rm e}, \infty)$.

Third, consider the control problem under partial IE. For convenience, let $\hat{\boldsymbol{\theta}}_\zeta \in \mathbb{R}^{N_\zeta}$ denotes a parameter estimate that is corresponding to the sub-regressor $\Phi_{\mathrm{f},\zeta}$. From Definition 3, there exist constants $T_{\zeta,\mathrm{e}}, \sigma \in \mathbb{R}^+$ such that

$$Q_\zeta(T_{\zeta,\mathrm{e}}) := L(s)[\Phi_{\mathrm{f},\zeta}\Phi_{\mathrm{f},\zeta}^T] \geq \sigma I.$$

Therefore, one obtains $Q(T_{\zeta,\mathrm{e}}) \geq \sigma I^*$, where I^* is a diagonal matrix with $I^*_{ii} = \begin{cases} 1, & i \leq N_\zeta \\ 0, & i > N_\zeta \end{cases}$ and

$$I - \Gamma_0^{-1}\Gamma(T_{\zeta,\mathrm{e}}) \geq \sigma(\sigma I^* + \Gamma_0)^{-1} I^* \geq \sigma^* I^*$$

with $\sigma^* := \lambda_{\min}(\sigma(\sigma I^* + \Gamma_0)^{-1}) \in \mathbb{R}^+$ and $\Gamma_\zeta(T_{\zeta,\mathrm{e}})Q_\zeta(T_{\zeta,\mathrm{e}}) \geq \sigma^* I$, where $\Gamma_\zeta \in \mathbb{R}^{N_\zeta \times N_\zeta}$ is a learning rate matrix corresponding to $\hat{\boldsymbol{\theta}}_\zeta$. Consider a Lyapunov function candidate

$$V_\zeta(\boldsymbol{e}, \tilde{\boldsymbol{\theta}}_\zeta) = \boldsymbol{e}^T\boldsymbol{e}/2 + \tilde{\boldsymbol{\theta}}_\zeta^T\tilde{\boldsymbol{\theta}}_\zeta/2. \tag{21}$$

Differentiating V_ζ in (21) with respect to t yields

$$\begin{aligned}\dot{V}_\zeta \leq & \boldsymbol{e}^T(A + A^T)\boldsymbol{e}/2 + \boldsymbol{e}^T\Phi_\zeta^T\tilde{\boldsymbol{\theta}}_\zeta - \tilde{\boldsymbol{\theta}}_\zeta^T\Gamma_\zeta(T_{\zeta,\mathrm{e}})Q_\zeta(T_{\zeta,\mathrm{e}})\tilde{\boldsymbol{\theta}}_\zeta \\ \leq & \sum_{i=1}^n(-c_ie_i^2 - \kappa_i\|\boldsymbol{\phi}_{\zeta,i}\|^2e_i^2 + e_i\boldsymbol{\phi}_{\zeta,i}^T\tilde{\boldsymbol{\theta}}_\zeta) - \sigma^*\tilde{\boldsymbol{\theta}}_\zeta^T\tilde{\boldsymbol{\theta}}_\zeta.\end{aligned}$$

Applying Young's inequality $2\boldsymbol{a}^T\boldsymbol{b} - \|\boldsymbol{a}\|^2 \leq \|\boldsymbol{b}\|^2$ with $\boldsymbol{a} = \sqrt{\kappa_i}e_i\boldsymbol{\phi}_{\zeta,i}$ and $\boldsymbol{b} = \tilde{\boldsymbol{\theta}}_\zeta/(2\sqrt{\kappa_i})$ to the above result yields

$$\begin{aligned}\dot{V}_\zeta \leq & \sum_{i=1}^n(-c_ie_i^2 + \|\tilde{\boldsymbol{\theta}}_\zeta\|^2/(4\kappa_i)) - \sigma^*\tilde{\boldsymbol{\theta}}_\zeta^T\tilde{\boldsymbol{\theta}}_\zeta \\ = & -\boldsymbol{e}^TC\boldsymbol{e} - (\sigma^* - 1/(4\kappa_0))\tilde{\boldsymbol{\theta}}_\zeta^T\tilde{\boldsymbol{\theta}}_\zeta\end{aligned}$$

where C and κ_0 are defined before. Choose suitably large parameters κ_i to get $\sigma^* - 1/(4\kappa_0) \in \mathbb{R}^+$ such that

$$\dot{V}_\zeta \leq -k_\zeta V_\zeta, t \in [T_{\zeta,\mathrm{e}}, \infty)$$

with $k_\zeta := 2\min\{\lambda_{\min}(C), \sigma^* - 1/(4\kappa_0)\} \in \mathbb{R}^+$. This implies that the closed-loop system (4) with (11) has exponential stability with $\boldsymbol{e}, \tilde{\boldsymbol{\theta}}_\zeta \to \boldsymbol{0}$ on $t \in [T_{\zeta,\mathrm{e}}, \infty)$. ■

Remark 1. From Theorem 1, one gets that the proposed LS-CLBC achieves exponential stability of the closed-loop system (4) with (11) in the presence of IE or partial IE, which implies that it owns robustness against perturbations resulting from external disturbances and measurement noise, where some rigorous proofs can be referred to [7], [20].

Remark 2. The proposed LS-CLBC has several distinctions compared to the MRE-based adaptive backstepping control (MRE-ABC) in [6]: 1) The learning rate Γ in (11b) is adjusted online such that all elements in $\hat{\boldsymbol{\theta}}$ converge with approximately the same speed, and the high-order time derivatives $\hat{\boldsymbol{\theta}}^{(i)}$ in (13) with a time-varying Γ can be implemented, whereas the MRE-ABC only provides a version with a constant Γ to calculate $\hat{\boldsymbol{\theta}}^{(i)}$; 2) the storage and forgetting of OHD guarantees that the exciting strength σ_c in (12) is monotonously non-decreasing, which is beneficial for the exponential stability and robustness of the closed-loop system (4) with (11), but σ_c in the MRE-ABC may not be monotonously non-decreasing.

V. Simulation Illustrations

Consider a Van der Pol oscillator described by [24]

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3 + \boldsymbol{\varphi}_2^T(\boldsymbol{x}_2)\boldsymbol{\theta}, \\ \dot{x}_3 = u, \\ y = x_1 \end{cases} \tag{22}$$

with $\boldsymbol{\theta} = [0.5, -1, 1]^T$ and $\boldsymbol{\varphi}_2(\boldsymbol{x}_2) = [-x_1, x_2, -x_1^2x_2]$, so $\boldsymbol{\varphi}_1(x_1), \boldsymbol{\varphi}_3(\boldsymbol{x}) = \boldsymbol{0}$. the reference trajectories y_r, $\dot{y}_\mathrm{r}$, $\ddot{y}_\mathrm{r}$, and $\dddot{y}_\mathrm{r}$ are generated by the reference model in Sec. II with $a_0 = 1$ and $a(s) = s^3 + 2s^2 + 2s + 1$. The CLBC law (2) with (3) specified for (22) given as follows:

$$\begin{cases} \alpha_1 = -c_1e_1, \\ \alpha_2 = -e_1 - (c_2 + d_2)e_2 - \boldsymbol{\phi}_2^T\hat{\boldsymbol{\theta}} + c_1(\dot{y}_\mathrm{r} - x_2), \\ \alpha_3 = -e_2 - (c_3 + d_3)e_3 - \boldsymbol{\phi}_3^T\hat{\boldsymbol{\theta}} \\ \quad + \frac{\partial\alpha_2}{\partial x_1}x_2 + \frac{\partial\alpha_2}{\partial x_2}x_3 + (\frac{\partial\alpha_2}{\partial\hat{\boldsymbol{\theta}}})^T\dot{\hat{\boldsymbol{\theta}}} + \frac{\partial\alpha_2}{\partial y_\mathrm{r}}\dot{y}_\mathrm{r} + \frac{\partial\alpha_2}{\partial\dot{y}_\mathrm{r}}\ddot{y}_\mathrm{r}, \\ u = \alpha_3 + y_\mathrm{r}^{(3)}, \end{cases}$$

where the parameter estimate $\hat{\boldsymbol{\theta}}$ is updated by (11) with

$$\Phi = [\boldsymbol{\varphi}_1, \boldsymbol{\phi}_2, \boldsymbol{\phi}_3]$$

in which $\boldsymbol{\phi}_2$, $\boldsymbol{\phi}_3$, and partial derivatives of α_2 are expressed by

$$\begin{aligned} \boldsymbol{\phi}_2 = & \boldsymbol{\varphi}_2, \ \boldsymbol{\phi}_3 = -\frac{\partial\alpha_2}{\partial x_2}\boldsymbol{\varphi}_2, \\ \frac{\partial\alpha_2}{\partial x_1} = & -1 - c_1(c_2 + d_2) - 2\kappa_2\boldsymbol{\varphi}_2^T\frac{\partial\boldsymbol{\varphi}_2}{\partial x_1}e_2 - (\frac{\partial\boldsymbol{\varphi}_2}{\partial x_1})^T\hat{\boldsymbol{\theta}}, \\ \frac{\partial\alpha_2}{\partial x_2} = & -(c_1 + c_2 + d_2) + 2\kappa_2\boldsymbol{\varphi}_2^T\frac{\partial\boldsymbol{\varphi}_2}{\partial x_2}e_2 - (\frac{\partial\boldsymbol{\varphi}_2}{\partial x_2})^T\hat{\boldsymbol{\theta}}, \\ \frac{\partial\alpha_2}{\partial\hat{\boldsymbol{\theta}}} = & -\boldsymbol{\phi}_2, \frac{\partial\alpha_2}{\partial y_\mathrm{r}} = 1 + (c_2 + d_2)c_1, \frac{\partial\alpha_2}{\partial\dot{y}_\mathrm{r}} = c_1 + c_2 + d_2. \end{aligned}$$

For simulations, we choose the control parameters $\Gamma_0 = 25I$, $\hat{\boldsymbol{\theta}}(0) = \boldsymbol{0}$, $c_i = 1$, and $\kappa_i = 0.1$ with $i = 1, 2, 3$, and the stable filter $L(s) = 6/(s^2 + 5s + 6)$ in (8). Gaussian white noise with 0 mean and 0.001 standard derivation is added to the measurement of the states x_1, x_2, and x_3. The classical higher-order tuner-based adaptive backstepping control (HOT-ABC) in [3] is selected as a baseline, where the shared parameters are set to be the same values for fair comparisons.

Case 1: Tracking with IE. Consider a tracking problem under IE, where the command signal r is generated by

$$r(t) = 1 + \sin t$$

and the initial state $\boldsymbol{x}(0) = [1, 0, 0]^T$. Performance comparisons of the two controllers are depicted in Fig. 1. It is observed that the tracking error e_1 by the proposed LS-CLBC converges to 0 after running 12 s [see Fig. 1(a)], and the estimation error $\tilde{\boldsymbol{\theta}}$ converges to $\boldsymbol{0}$ at about 15 s [see Fig. 1(b)]. Also, the control inputs u by the two controllers are comparable [see Fig. 1(c)]. The tracking and estimation performances of the LS-CLBC are much better than those of the HOT-ABC [see Figs. 1(a)-(b)], because the exciting strength σ_c of the LS-CLBC keeps a high level throughout [see Fig. 1(d)].

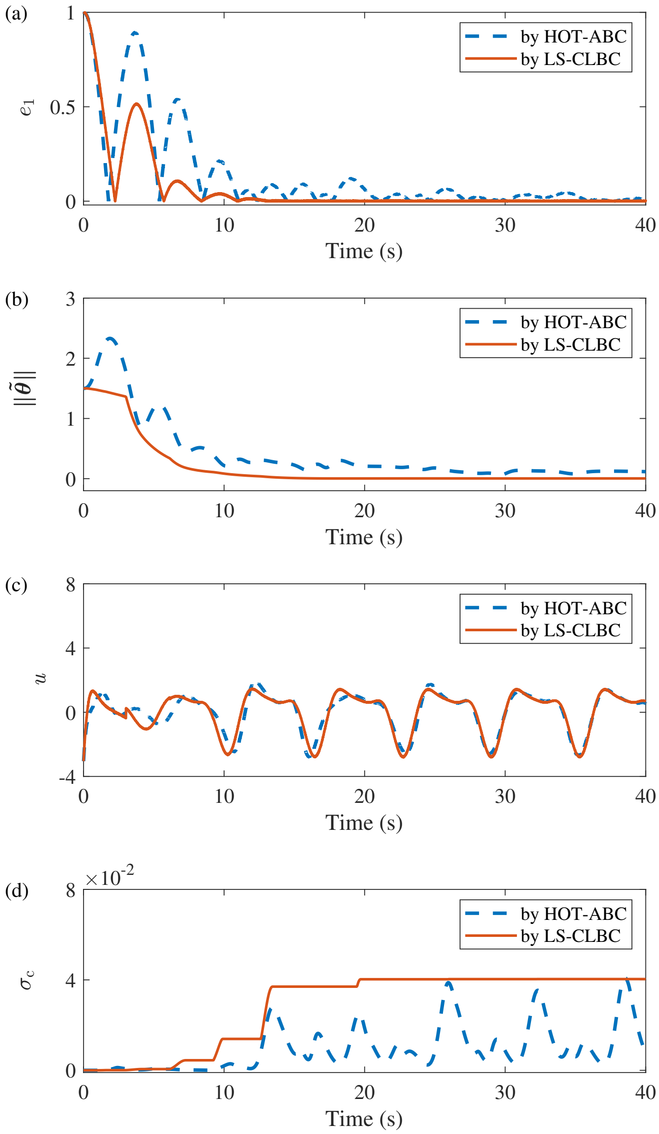

Fig. 1. Performance comparisons of two controllers for the regulation problem under the IE condition. (a) The tracking errors e_1. (b) The estimation errors $\|\tilde{\boldsymbol{\theta}}\|$. (c) The control inputs u. (d) The exciting strengths $\sigma_{\rm c}$.

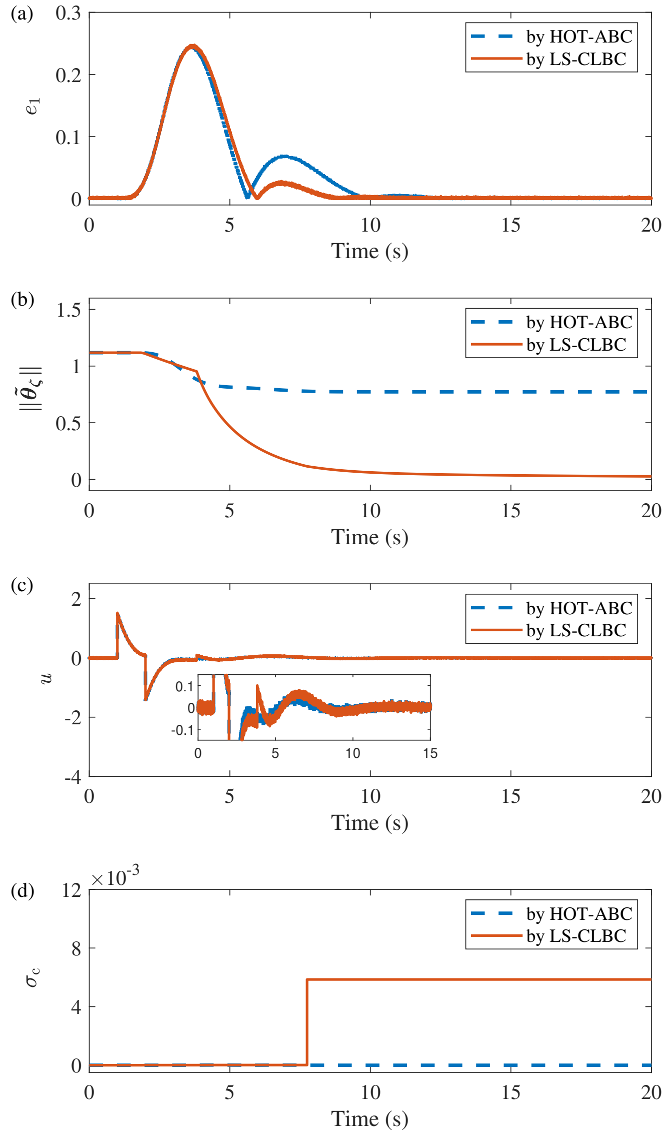

Fig. 2. Performance comparisons of two controllers for the regulation problem under the partial IE condition. (a) The tracking errors e_1. (b) The estimation errors $\|\tilde{\boldsymbol{\theta}}_\zeta\|$. (c) The control inputs u. (d) The exciting strengths $\sigma_{\rm c}$.

Case 2: Regulation with partial IE. Consider a regulation problem under partial IE with the command signal r given by

$$r(t) = \begin{cases} 1.5, & 1 < t \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

and the initial state $\boldsymbol{x}(0) = \mathbf{0}$. Performance comparisons of the two controllers are exhibited in Fig. 2. Note that $\tilde{\boldsymbol{\theta}}_\zeta = [\tilde{\theta}_1, \tilde{\theta}_2]^T$ is a partial estimation error that corresponds to the active channels. The tracking performance by the proposed LS-CLRC is still better than those of the HOT-ABC [see Fig. 2(a)]. The HOT-ABC performs much worse for partial parameter convergence [see Fig. 2(b)] since the exciting strength $\sigma_{\rm c}$ is 0 from the beginning to the end [see Fig. 2(d)]. In contrast, the LS-CLBC exhibits the convergence of the partial estimation error $\tilde{\boldsymbol{\theta}}_\zeta$ to $\mathbf{0}$ rapidly after 5 s, where $\sigma_{\rm c}$ in (12) is monotonic non-decreasing and keeps a high level after 8 s [see Fig. 2(d)] due to the storage of OHD. Also, the control inputs u by the two controllers in this case are comparable [see Fig. 2(c)]. The above results imply that: 1) The proposed method can ensure closed-loop exponential stability under the weakened IE or even parietal IE condition; 2) the storage and forgetting of OHD are beneficial for parameter convergence.

VI. Conclusions

This paper has presented a feasible adaptive backstepping control strategy named LS-CLBC for strict-feedback uncertain nonlinear systems, where exponential stability of the closed-loop system with parameter convergence is achieved under the IE or even partial IE condition. Simulations have verified that the proposed LS-CLBC has superior parameter estimation and control performances compared to the classical HOT-ABC. Further work would focus on the rigorous robustness analysis of the proposed approach in theory.

References

[1] I. Kanellakopoulos, P. V. Kokotović, and A. S. Morse, "Systematic design of adaptive controllers for feedback linearizable systems," *IEEE Trans. Autom. Control*, vol. 36, no. 11, pp. 1241–1253, Nov. 1991.

[2] M. Krstić, I. Kanellakopoulos, and P. V. Kokotović, *Nonlilnear and Adaptive Control Design*. New York, NY: Wiley, 1995.

[3] V. O. Nikiforov and K. V. Voronov, "Adaptive backstepping with a high-order tuner," *Automatica*, vol. 37, no. 12, pp. 1953–1960, Dec. 2001.

[4] D. N. Gerasimov, L. Liu, and V. O. Nikiforov, "Adaptive backstepping control with fast parametric convergence for a class of nonlinear systems," in *Proc. Eur. Control Conf.*, Naples, Italy, 2019, pp. 3432–3437.

[5] D. N. Gerasimov and A. V. Pashenko, "Robust adaptive backstepping control with improved parametric convergence," *IFAC-PapersOnLine*, vol. 52, no. 29, pp. 146–151, Dec. 2019.

[6] V. Nikiforov, D. Gerasimov, and A. Pashenko, "Modular adaptive backstepping design with a high-order tuner," *IEEE Trans. Autom. Control*, vol. 67, no. 5, pp. 2663–2668, May 2022.

[7] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. Englewood Cliffs, NJ, USA: Prentice Hall, 1996.

[8] Y. Pan, T. Sun, and H. Yu, "Composite learning from adaptive dynamic surface control," *IEEE Trans. Autom. Control*, vol. 61, no. 9, pp. 2603–2609, Sep. 2016.

[9] Y. Pan, J. Zhang, and Y. H., "Model reference composite learning control without persistency of excitation," *IET Control Theory Appl.*, vol. 10, no. 16, pp. 1963–1971, Oct. 2016.

[10] Y. Pan, T. Sun, Y. Liu, and H. Yu, "Composite learning from adaptive backstepping neural network control," *Neural Netw.*, vol. 95, pp. 134–142, Nov. 2017.

[11] K. Guo, Y. Pan, and H. Yu, "Composite learning robot control with friction compensation: A neural network-based approach," *IEEE Trans. Ind. Electron.*, vol. 66, no. 10, pp. 7841–7851, Dec. 2019.

[12] K. Guo, Y. Pan, D. Zheng, and H. Yu, "Composite learning control of robotic systems: A least squares modulated approach," *Automatica*, vol. 111, Art. No. 108612, 2020.

[13] X. Liu, Z. Li, and Y. Pan, "Preliminary evaluation of composite learning tracking control on 7-DoF collaborative robots," *IFAC-PapersOnLine*, vol. 54, no. 14, pp. 470–475, 2021.

[14] ——, "Experiments of composite learning admittance control on 7-DoF collaborative robots," in *Proc. Int. Conf. Intell. Robot. Appl.*, Yantai, China, 2021, pp. 532–541.

[15] D. Huang, C. Yang, Y. Pan, and L. Cheng, "Composite learning enhanced neural control for robot manipulator with output error constraints," *IEEE Trans. Ind. Informat.*, vol. 17, no. 1, pp. 209–218, Dec. 2021.

[16] K. Guo, Y. Liu, B. Xu, Y. Xu, and Y. Pan, "Locally weighted learning robot control with improved parameter convergence," *IEEE Trans. Ind. Electron.*, vol. 69, no. 12, pp. 13 236–13 244, Dec. 2022.

[17] Y. Pan, K. Guo, T. Sun, and M. Darouach, "Bioinspired composite learning control under discontinuous friction for industrial robots," *IFAC-PapersOnLine*, vol. 55, no. 12, pp. 85–90, Aug. 2022.

[18] B. Lai, Z. Li, W. Li, C. Yang, and Y. Pan, "Homography-based visual servoing of eye-in-hand robots with exact depth estimation," *IEEE Trans. Ind. Electron.*, to be published, DOI: 10.1109/TIE.2023.3277072.

[19] K. Guo and Y. Pan, "Composite adaptation and learning for robot control: A survey," *Annu. Rev. Control*, vol. 55, pp. 279–290, Dec. 2023.

[20] S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence and Robustness*. Englewood Cliffs, NJ, USA: Prentice Hall, 1989.

[21] G. Kreisselmeier and G. Rietze-Augst, "Richness and excitation on an interval-with application to continuous-time adaptive control," *IEEE Trans. Autom. Control*, vol. 35, no. 2, pp. 165–171, Feb. 1990.

[22] D. N. Gerasimov and V. O. Nikiforov, "On key properties of the Lion's and Kreisselmeier's adaptation algorithms," *IFAC-PapersOnLine*, vol. 53, no. 2, pp. 3773–3778, 2020.

[23] J. A. Farrel and M. M. Polycarpou, *Adaptive Approximation Based Control: Unifying Neural, Fuzzy and Traditional Adaptive Approximation Approaches*. Hoboken, NJ, USA: Wiley, 2006.

[24] M. Krstić, "On using least-squares updates without regressor filtering in identification and adaptive control of nonlinear systems," *Automatica*, vol. 45, no. 3, pp. 731–735, Mar. 2009.