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# Designing Optimal Personalized Incentive for Traffic Routing using BIG Hype

Panagiotis D. Grontas, Carlo Cenedese, Marta Fochesato,
Giuseppe Belgioioso, John Lygeros, Florian Dörfler

***Abstract*— We study the problem of routing plug-in electric and conventional fuel vehicles on a city scale using incentives. In our model, commuters selfishly aim to minimize a local cost that combines travel time and the financial expenses of using city facilities, i.e., parking and service stations. The traffic authority can influence the commuters' routing choice via personalized discounts on parking tickets and on the energy price at service stations. We formalize the problem of optimally designing these monetary incentives to induce traffic decongestion as a large-scale bilevel game, where constraints arise at both levels due to the finite capacities of city facilities and incentives budget. Then, we develop an efficient scalable solution scheme with convergence guarantees based on *BIG Hype*, a recently-proposed hypergradient-based algorithm for bilevel games. Finally, we validate our approach via numerical simulations over the Anaheim's traffic network, showcasing its advantages in terms of traffic decongestion and scalability.**

## I. Introduction

The problem of effectively managing traffic in large urban areas is of paramount importance in modern society. The EU alone incurs an annual cost exceeding 267 billion euros [1] due to high traffic congestion in major cities, which is also responsible for broad environmental damages, since higher levels of traffic congestion lead to higher CO2 emissions [2]. A traditional approach to mitigate congestion consists in increasing road capacity or building alternative routes. Recently, the focused has been shifted towards "non-invasive" interventions such as tolling or incentives provision.

A popular concept within this area of research is *congestion pricing*, which dates back to [3] and proposes to heavily *toll* congested roads to influence commuters' routing choices – i.e., the Vehicle Routing Problem (VRP) – with the overall objective of decreasing traffic congestion. Over the years, a large body of research has addressed this topic, most of which studies the effect that a given pricing scheme produces in terms of network decongestion, see [4], [5] and references therein. Arguably of higher interest is the problem of computing an optimal set of incentives (or tolls) that maximizes traffic decongestion. In [3], the authors use a marginal congestion cost to design tolls, while in [6], the authors set tolls that minimize system inefficiency.

These classical results are difficult to generalize whenever the Traffic Authority (TA) has to meet some limitations, such as a finite budget for the incentives or localized interventions, viz. act only over a limited number of roads/facilities in the network. A natural extension to this constrained setup can be obtained using the paradigm of bilevel games, wherein the TA plays the role of the leader. When the TA intervention is limited to tolls, this bilevel game is generally known as *restricted network tolling problem*, and is inherently ill-posed and, in practice, tractable only for small traffic networks [7] or without optimality guarantees [8]. In [9], the authors propose a data-driven method based on the scenario approach to design robust tolls, while in [10], [11], the authors propose a multi-level optimization problem to compute optimal incentives focusing mostly on Plug-in Electric Vehicles (PEVs). Yet, optimality guarantees are missing, except for very simple scenarios [12]. In [13], [14] the authors study the impact of service stations and parking lots on the flow of commuters and show that incentive design for such facilities can (indirectly) influence the VRP.

In summary, designing tolls to influence the VRP is a well-studied topic. Yet, due to the inherent problem complexity and lack of scalable algorithms, designing incentives with limited budget resources and/or targeted interventions remains elusive. Moreover, the study on how discounts on facilities (rather than tolls) influence the VRP has been limited. In this work, we aim at bridging both these gaps by exploiting the potential of BIG Hype [15]. This novel algorithm, specifically designed for large-scale bilevel games, allows us to maintain the original problem complexity as well as to guarantee local optimality of the proposed interventions.

The main contributions of this paper are the following:

- We formulate a bilevel game that describes the VRP in an urban area where commuters park at predefined facilities (parking lots and charging stations), whereas the TA influences the commuters' routing by providing personalized discounts to access these facilities. We introduce constraints at both levels to model the facilities' capacity, the limits on the personalized discounts, and the predefined limited budget that the TA has allocated.
- We tailor BIG Hype [15] on this problem to obtain an efficient scalable algorithm with convergence guarantees to a profile of locally optimal personalized incentives.
- We validate the effectiveness of the proposed approach via numerical studies on the traffic network of the city of Anaheim, California. Our findings demonstrate the capability of the proposed approach in terms of both traffic decongestion and scalability, thereby highlighting its potential for addressing incentives design problems

All the authors are with Automatic Control Laboratory, Department of Electrical Engineering and Information Technology, ETH Zürich, Physikstrasse 3 8092 Zürich, Switzerland. (e-mails:{pgrontas, ccenedese, mfochesato, gbelgioioso, jlygeros, dorfler}@ethz.ch). This work was supported by NCCR Automation, a National Centre of Competence in Research, funded by the Swiss National Science Foundation (grant number 180545).

in real-world complex urban transportation networks.

## II. Problem formulation

In this section, we formalize the VRP in an urban area where a TA is able to affect the routing choice of a subset of the commuters, by providing discounts for specific parking lots, and for the electricity price at charging stations. Naturally, the former influences those commuters owning either a fuel vehicle (FV) or a PEV, while the latter only PEV owners. We consider facilities in which commuters leave their car for several hours during the day, e.g., the parking lots used during working hours, and assume that the goal of the TA is to alleviate traffic congestion. Nevertheless, the proposed formulation can readily accommodate other objectives, such as revenue maximization, see Remark 1. The commuters' goal is to minimize a combination of the travel time and the monetary cost of using parking lots and service stations. Only a subset of all commuters is reactive to the proposed discounts, and thus takes part in the VRP, while the rest is modelled as an exogenous demand contributing to the congestion of the road network and the city facilities.

We model the transportation network as a strongly connected digraph $\mathcal{G} := (\mathcal{V}, \mathcal{E})$, where the nodes $\mathcal{V}$ represent intersections or points of interest, while the edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ correspond to the roads connecting them. Given an edge $\varepsilon \in \mathcal{E}$, we denote $\varepsilon_-, \varepsilon_+ \in \mathcal{V}$ its starting and ending node, respectively. The charging stations and parking lots located at certain nodes of the network are described by the sets $\mathcal{C}^{\mathrm{c}} \subseteq \mathcal{V}$ and $\mathcal{C}^{\mathrm{p}} \subseteq \mathcal{V}$, respectively, with possible nonempty intersection $\mathcal{C}^{\mathrm{c}} \cap \mathcal{C}^{\mathrm{p}} \neq \emptyset$. Further, we introduce $\mathcal{C} = \mathcal{C}^{\mathrm{c}} \cup \mathcal{C}^{\mathrm{p}}$, $n_e := |\mathcal{E}|$, $n_v := |\mathcal{V}|$, $n_{\mathrm{c}} := |\mathcal{C}^{\mathrm{c}}|$, and $n_{\mathrm{p}} := |\mathcal{C}^{\mathrm{p}}|$.

### A. Lower Level: Routing and Charging/ Parking Game

We group the commuters into $N$ classes of similar characteristics, referred to as *agents* and indexed by $i \in \mathcal{N} := \{1, \dots, N\}$. Each agent $i \in \mathcal{N}$ represents a population of PEVs or FVs composed of $P_i$ vehicles sharing the same origin and destination nodes, denoted by $(o_i, d_i) \in \mathcal{V} \times \mathcal{V}$, with $o_i \neq d_i$. Each commuter aims at reaching $d_i$, but must park its vehicle at a node $j \in \mathcal{C}$ and, if $d_i \notin \mathcal{C}$, complete its trip by travelling from $j$ to $d_i$. Each agent $i$ seeks to determine the fraction of vehicles that will park at the lot $\ell \in \mathcal{C}^{\mathrm{p}}$, denoted by $g_i^{\mathrm{p},\ell} \in [0, 1]$. If $i$ is composed of PEVs, the fraction that will charge at each station $j \in \mathcal{C}^{\mathrm{c}}$, is denoted by $g_i^{\mathrm{c},j} \in [0, 1]$. If the vehicles in $i$ are FVs, then $g_i^{\mathrm{c},j} = 0$, for all $j \in \mathcal{C}^{\mathrm{c}}$. For conciseness, we let $g_i^{\mathrm{c}} := (g_i^{\mathrm{c},j})_{j \in \mathcal{V}} \in \mathbb{R}^{n_v}$, where $g_i^{\mathrm{c},j} = 0$ if $j \notin \mathcal{C}^{\mathrm{c}}$. Similarly, we define $g_i^{\mathrm{p}} := (g_i^{\mathrm{p},\ell})_{\ell \in \mathcal{V}} \in \mathbb{R}^{n_v}$, and the collective vectors $\boldsymbol{g}^{\mathrm{c}} := (g_i^{\mathrm{c}})_{i \in \mathcal{N}}$ and $\boldsymbol{g}^{\mathrm{p}} := (g_i^{\mathrm{p}})_{i \in \mathcal{N}}$.

Further, the routing choice of agent $i$ is modelled via the variable $\phi_i^{\varepsilon} \in [0, 1]$, that represents the percentage of vehicles traversing road $\varepsilon \in \mathcal{E}$, so $\phi_i := (\phi_i^{\varepsilon})_{\varepsilon \in \mathcal{E}}$, $\boldsymbol{\phi} := (\phi_i)_{i \in \mathcal{N}}$. The consistency of the resulting vehicles flow is obtained by imposing, at each node $v \in \mathcal{V}$, the following constraints:

$$\sum_{\varepsilon:\varepsilon_+ = v} \phi_i^{\varepsilon} - \sum_{\varepsilon:\varepsilon_- = v} \phi_i^{\varepsilon} = \begin{cases} -1, & v = o_i, \\ g_i^{\mathrm{c},j} + g_i^{\mathrm{p},j}, & \text{otherwise} \end{cases}. \quad (1)$$

The goal of agent $i \in \mathcal{N}$ is to choose $g_i^{\mathrm{c}}$, $g_i^{\mathrm{p}}$, and $\phi_i$ so as to minimize a multi-objective cost function $f_i$ that combines travel time, charging/parking cost, and last-mile cost.

The travel time of agent $i \in \mathcal{N}$ is given by

$$f_i^{\mathrm{t}} := \eta_i \textstyle\sum_{\varepsilon \in \mathcal{E}} P_i \phi_i^{\varepsilon} t_{\varepsilon}(\sigma_{\varepsilon}(\boldsymbol{\phi})), \quad (2)$$

where $\eta_i$ is the (monetary) value of time, $\sigma_{\varepsilon}(\boldsymbol{\phi}) := \sum_{i \in \mathcal{N}} P_i \phi_i^{\varepsilon}$ is the aggregate vehicles flow on road $\varepsilon$, and $t_{\varepsilon}(\cdot)$ is the latency on $\varepsilon$. Similarly to [13], [14], we derive our latency function from that used by the Bureau of Public Roads, yielding the following affine term

$$t_{\varepsilon}(\sigma_{\varepsilon}(\boldsymbol{\phi})) := a_{\varepsilon} + b_{\varepsilon}(h_{\varepsilon} + \sigma_{\varepsilon}(\boldsymbol{\phi})), \quad (3)$$

where $a_{\varepsilon}, b_{\varepsilon}$ are positive constants, while $h_{\varepsilon}$ represents the flow of vehicles that are not reactive to the discounts or traffic conditions and it is assumed fixed.

The charging cost is given by

$$f_i^{\mathrm{c}} := \textstyle\sum_{j \in \mathcal{C}^{\mathrm{c}}} q_i g_i^{\mathrm{c},j} (\overline{c}_j^{\mathrm{c}} - c_j^{\mathrm{c},i}), \quad (4)$$

where $q_i$ is the amount of electricity that all PEVs in class $i$ purchase to fully charge their batteries, $\overline{c}_j^{\mathrm{c}} > 0$ is the base price of electricity at station $j \in \mathcal{C}^{\mathrm{c}}$, whereas $c_j^{\mathrm{c},i} \geq 0$ is the discount provided for class $i$ in station $j$ by the TA. We stress that $c_j^{\mathrm{c},i}$ is a design variable of the TA. Among the PEVs in each population $i \in \mathcal{N}$, a small percentage $\overline{g}_i^{\mathrm{c}} \in [0, 1]$ is required to charge during the day due to an initial low state of charge, yielding the local constraint

$$\mathbf{1}^{\top} g_i^{\mathrm{c}} \geq \overline{g}_i^{\mathrm{c}}. \quad (5)$$

Similarly, the cost of parking is given by

$$f_i^{\mathrm{p}} := \textstyle\sum_{j \in \mathcal{C}^{\mathrm{p}}} g_i^{\mathrm{p},j} (\overline{c}_j^{\mathrm{p}} - c_j^{\mathrm{p},i}), \quad (6)$$

where the price of parking at $j \in \mathcal{C}^{\mathrm{p}}$ and the associated discount are denoted by $\overline{c}_j^{\mathrm{p}}$, $c_j^{\mathrm{p},i}$, respectively. We denote the stacked vector of all discounts by $\boldsymbol{c} := (c^i)_{i \in \mathcal{N}} \in \mathbb{R}^m$ with $c^i := (c_j^i)_{j \in \mathcal{C}}$, $c_j^i := (c_j^{\mathrm{c},i}, c_j^{\mathrm{p},i})$, and $m := N(n_{\mathrm{c}} + n_{\mathrm{p}})$.

Finally, the last mile cost, namely, the cost for travelling from the charging station/parking $j \in \mathcal{C}$ (where the vehicle is parked) to their final destination $d_i$, is given by

$$f_i^{\mathrm{lm}} := \eta_i \| g_i^{\mathrm{p}} + g_i^{\mathrm{c}} - \hat{g}_i \|_W^2, \quad (7)$$

where $\hat{g}_i \in \mathbb{R}^{n_v}$ is a basis vector of all 0 entries except for its $d_i$-th component that is equal to 1, and $W$ is a diagonal matrix with positive elements. The weight in $W_{jj}$ represents the discomfort that agent $i$ faces to reach $d_i$ from $j \in \mathcal{C}$. There are several ways to model $W$ that depend both on the city's structure, and on the different means of transportation used to cover the last-mile trip. Hereafter, we assume that the discomfort is proportional to the time required to move from $j$ to $d_i$ in free-flow conditions via the shortest path. Notice that from (3), it follows that the time required to travel through road $\varepsilon \in \mathcal{E}$ in free-flow conditions is $a_{\varepsilon} > 0$. Moreover, we impose $W_{d_i d_i} = \epsilon > 0$, where $\epsilon$ is a small scalar, to ensure $W \succ 0$.

To guarantee that vehicles are able to access their selected facility, we limit the amount of vehicles of each class $i$ that can use the charging stations via the following constraints:

$$P_i g_i^{\mathrm{c},j} \leq \delta_i^{\mathrm{c},j}, \quad \forall j \in \mathcal{C}^{\mathrm{c}} \tag{8}$$

where $\delta_i^{\mathrm{c},j} > 0$ denotes the maximum number of slots allocated a priori for agent $i$ by the TA. Further, we impose similar constraints also for the parking facilities.

We compactly denote the decision variable of each agent $i$ by $y_i := (\phi_i, g_i^{\mathrm{c}}, g_i^{\mathrm{p}}) \in \mathbb{R}^{n_i}$, and its local feasible set by

$$\mathcal{Y}_i := \{y_i \in [0,1]^{n_i} \,|\, (1), (5), (8) \text{ hold}\}, \tag{9}$$

where $n_i := n_e + 2n_v$. Further, we let $\boldsymbol{y} := ((y_i)_{i\in\mathcal{N}}) \in \mathcal{Y}$ be the collective strategy profile of all agents, where $\mathcal{Y} := \prod_{i\in\mathcal{N}} \mathcal{Y}_i \subseteq \mathbb{R}^n$ is the global feasible set and $n := \sum_{i\in\mathcal{N}} n_i$.

Overall, each agent $i \in \mathcal{N}$ is faced with the following optimization problem:

$$\underset{y_i \in \mathcal{Y}_i}{\text{minimize}} \quad f_i(\boldsymbol{c}, y_i, \boldsymbol{\sigma}(\boldsymbol{y})), \tag{$\mathcal{P}_i$}$$

where $f_i := f_i^{\mathrm{t}} + f_i^{\mathrm{c}} + f_i^{\mathrm{p}} + f_i^{\mathrm{lm}}$, and $\boldsymbol{\sigma}(\boldsymbol{y}) := ((\sigma_\varepsilon)_{\varepsilon\in\mathcal{E}})$ is the aggregate vehicles' flow over $\mathcal{G}$. Notably, $f_i$ depends on the local variable $y_i$ as well as the aggregative quantity $\boldsymbol{\sigma}(\boldsymbol{y})$. Thus, the collection of inter-dependent problems $\mathcal{L}(\boldsymbol{c}) := \{(\mathcal{P}_i)\}_{i\in\mathcal{N}}$ constitutes an aggregative game [16], parametrized by the discounts $\boldsymbol{c}$ designed by the TA.

### B. Upper Level: Personalized Incentive Design

The TA chooses the personalized incentives $\boldsymbol{c}$ with the goal of minimizing traffic congestion in key areas of the network, modelled via the Total Travel Time (TTT) function

$$\varphi(\boldsymbol{c}, \boldsymbol{\sigma}(\boldsymbol{y})) := \sum_{\varepsilon\in\mathcal{E}_D} (h_\varepsilon + \sigma_\varepsilon(\boldsymbol{\phi}))(a_\varepsilon + b_\varepsilon(h_\varepsilon + \sigma_\varepsilon(\boldsymbol{\phi}))), \tag{10}$$

where $\mathcal{E}_D \subseteq \mathcal{E}$ is the subset of the road network that the TA wants to decongest, e.g., the city center. It is important to highlight that, the dependence of $\varphi$ on $\boldsymbol{c}$ is implicit, i.e., the discounts $\boldsymbol{c}$ shape the traffic pattern $\boldsymbol{\phi}$ which, in turn, determines the TTT over $\mathcal{E}_D$.

*Remark 1:* The problem formulation described above can easily be modified to address other scenarios in which the TA is replaced by a private entity, e.g., the owner of the facilities, that aims at maximizing the attained revenue. In this case, (10) should be replaced by $\tilde{\varphi} = -\sum_{i\in\mathcal{N}}(f_i^{\mathrm{c}} + f_i^{\mathrm{p}})$. □

To avoid large discrepancies in facility prices across vehicle classes, we restrict the personalized discounts to the set $\mathcal{D} := \{\boldsymbol{c} \in \mathbb{R}^m \,|\, c_j^i \in [0,\, \theta_j^i \bar{c}_j^i]\}$, where $\theta_j^i \in (0,1)$. Further, we assume that the TA has to compensate the facilities for the provided discounts, but can only spend a limited budget $C > 0$. Concretely, the TA's budget constraint reads as

$$C^{\mathrm{b}}(\boldsymbol{c}, \boldsymbol{y}) := \sum_{i\in\mathcal{N}} \left( \sum_{j\in\mathcal{C}^{\mathrm{c}}} q_i g_i^{\mathrm{c},j} c_i^{\mathrm{c},j} + \sum_{\ell\in\mathcal{C}^{\mathrm{p}}} P_i g_i^{\mathrm{c},\ell} c_i^{\mathrm{c},\ell} \right) \leq C. \tag{11}$$

For computational reasons, we model (11) as a soft constraint by means of the smooth penalty function $\varphi^{\mathrm{b}}(\boldsymbol{c}, \boldsymbol{y}) := \max\left\{C^{\mathrm{b}}(\boldsymbol{c}, \boldsymbol{y}) - C, 0\right\}^2$. Hence, the TA's objective becomes

$$\varphi_{\mathrm{TA}}(\boldsymbol{c}, \boldsymbol{y}) := \varphi(\boldsymbol{c}, \boldsymbol{\sigma}) + \mu\varphi^{\mathrm{b}}(\boldsymbol{c}, \boldsymbol{y}),$$

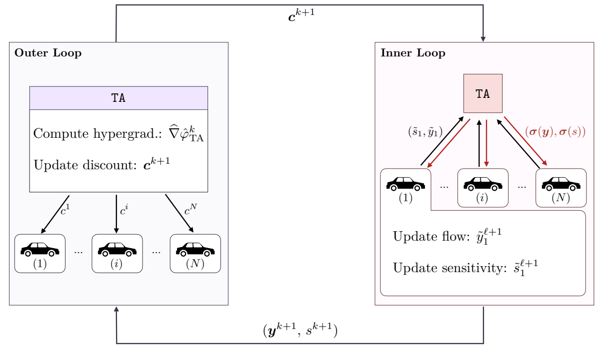


Fig. 1: Schematic representation of the outer and inner loop in Alg. 1 used to compute the optimal discount $\boldsymbol{c}^\star$.

where $\mu > 0$ is a positive parameter.

### C. Bilevel Game Formulation

Overall, our considered traffic model operates as follows: the TA sends the personalized incentives $\boldsymbol{c}$ to the commuters, which respond by updating their routing preferences that are a solution to the parametric aggregative game $\mathcal{L}(\boldsymbol{c})$.

A relevant solution concept for $\mathcal{L}(\boldsymbol{c})$ is the Nash equilibrium that corresponds to a strategy profile where no agent can reduce its cost by unilaterally deviating from it.

*Definition 1:* Given $\boldsymbol{c}$, a strategy profile $\boldsymbol{y}^\star \in \mathcal{Y}$ is a Nash equilibrium (NE) of $\mathcal{L}(\boldsymbol{c})$ if, for all $i \in \mathcal{N}$:

$$f_i(\boldsymbol{c}, y_i^\star, \boldsymbol{\sigma}(\boldsymbol{y}^\star)) \leq f_i(\boldsymbol{c}, y_i, \boldsymbol{\sigma}((y_i, \boldsymbol{y}_{-i}^\star))), \ \forall y_i \in \mathcal{Y}_i. \quad \square$$

In our setting, $\boldsymbol{y}^\star$ is an NE of $\mathcal{L}(\boldsymbol{c})$ if and only if it is a solution to a specific Variational Inequality (VI) problem [17, Prop. 1.4.2], namely, if it satisfies:

$$F(\boldsymbol{c}, \boldsymbol{\sigma}(\boldsymbol{y}^\star))^\top (\boldsymbol{y} - \boldsymbol{y}^\star) \geq 0, \quad \forall \boldsymbol{y} \in \mathcal{Y}, \tag{12}$$

where $F(\boldsymbol{c}, \boldsymbol{\sigma}(\boldsymbol{y})) := (\nabla_{y_i} f_i(\boldsymbol{c}, y_i, \boldsymbol{\sigma}(\boldsymbol{y})))_{i\in\mathcal{N}}$ is the so-called Pseudo-Gradient (PG) mapping. We denote $\mathrm{SOL}(F(\boldsymbol{c}, \cdot), \mathcal{Y})$ the set of solutions to (12). Next, we prove existence and uniqueness of an NE.

*Lemma 1:* For any fixed profile of incentives $\boldsymbol{c} \in \mathcal{D}$, the parametric game $\mathcal{L}(\boldsymbol{c})$ admits a unique NE. □

*Proof:* See Appendix A. ■

In view of Lemma 1, we can define the single-valued parameter-to-NE mapping $\boldsymbol{y}^\star(\cdot) : \boldsymbol{c} \mapsto \mathrm{SOL}(F(\boldsymbol{c}, \cdot), \mathcal{Y})$, and use to it express the TA's incentives design problem as the following single-leader multi-follower Stackelberg game:

$$\underset{\boldsymbol{c} \in \mathcal{D}}{\text{minimize}} \quad \varphi_{\mathrm{TA}}(\boldsymbol{c}, \boldsymbol{y}^\star(\boldsymbol{c})) =: \hat{\varphi}_{\mathrm{TA}}(\boldsymbol{c}), \tag{13}$$

where the dependence of $\varphi_{\mathrm{TA}}$ on $\boldsymbol{y}^\star(\cdot)$ highlights that the TA anticipates the rational response of the commuters to $\boldsymbol{c}$.

## III. Optimal incentive design via BIG Hype

The implicit nature of $\boldsymbol{y}^\star(\cdot)$ renders $\hat{\varphi}_{\mathrm{TA}}$ non-smooth and non-convex [15, §IV]. A globally optimal solution to (13) can be theoretically found by recasting the problem as mixed-integer program and using off-the-self solvers [18]. However, the resulting computational complexity would drastically increase with the problem size [15, §V-B.3], making this approach unsuitable in realistic scenarios. Therefore, we focus instead on first-order methods that can exploit the inherent distributed structure of (13) to give a locally optimal solution in a computationally efficient manner.

In this work, we employ the algorithm presented in [15], called BIG Hype (Best Intervention in Games using Hypergradients) and originally developed for a wider class of bilevel games. It requires weak assumptions on the upper level objective and utilizes readily-implementable update rules. Further, it preserves the distributed structure of the agents' problem making it efficient even for a large number of agents, see Section IV. In its core, BIG Hype uses projected-gradient descent to obtain a local solution of (13). Informally, the gradient of $\hat{\varphi}_{\text{TA}}$, referred to as the *hypergradient*, can be expressed using the chain rule as

$$\nabla\hat{\varphi}_{\text{TA}}(\boldsymbol{c}) = \nabla_{\boldsymbol{c}}\varphi_{\text{TA}}(\boldsymbol{c}, \boldsymbol{y}^\star(\boldsymbol{c})) + \mathbf{J}\boldsymbol{y}^\star(\boldsymbol{c})^\top \nabla_{\boldsymbol{y}}\varphi_{\text{TA}}(\boldsymbol{c}, \boldsymbol{y}^\star(\boldsymbol{c})).$$

To compute $\nabla\hat{\varphi}_{\text{TA}}(\boldsymbol{c})$, the TA requires knowledge of $\boldsymbol{y}^\star(\boldsymbol{c})$ as well as its Jacobian $\mathbf{J}\boldsymbol{y}^\star(\boldsymbol{c})$, which is known as the *sensitivity* and, intuitively, represents how the commuters react to a marginal change in the discounts $\boldsymbol{c}$.

*Remark 2:* Technically, BIG Hype computes a *conservative gradient* of $\hat{\varphi}_{\text{TA}}$, denoted by $\boldsymbol{\mathcal{J}}\hat{\varphi}_{\text{TA}}$, which generalizes the gradient for non-smooth non-convex functions [19]. □

The proposed hierarchical traffic-shaping scheme, obtained by deploying BIG Hype on (13), is summarized in Alg. 1, and consists of two nested loops, that we describe in the following, aided by Figure 1. The TA sends the current personalized discount $c^i$ to each agent $i$ computed in the outer loop iteration $k$. Then, for each inner loop iteration $\ell$, see Alg. 2, the commuters estimate their routing and facility choice $\tilde{y}_i^{\ell+1} \in \mathbb{R}^{n_i}$ along with their sensitivity $\tilde{s}_i^{\ell+1} \in \mathbb{R}^{n_i \times m}$ using the aggregative quantities $\boldsymbol{\sigma}(\tilde{\boldsymbol{y}}^\ell)$, $\boldsymbol{\sigma}(\tilde{s}^\ell)$, which are broadcast at the end of each iteration $\ell$ by the TA. The inner loop terminates once the estimates are sufficiently accurate. Then, in the outer loop, the TA gathers the approximate NE and its sensitivity, $\boldsymbol{y}^{k+1}$ and $s^{k+1}$, which are used to update $\boldsymbol{c}$ via a projected hypergradient step.

*Remark 3:* The sensitivity update step in Alg. 2 requires each agent $i$ to compute the auxiliary matrices $S_{1,i}$, $S_{2,i}$, and $S_{3,i}$, that store the partial Jacobian of the mapping $\mathbb{P}_{\mathcal{Y}_i}[y_i - \gamma F(\boldsymbol{c}, \boldsymbol{\sigma}(\boldsymbol{y}))]$ with respect to $\boldsymbol{c}$, $y_i$, and $\boldsymbol{\sigma}$, respectively. This computation is non-trivial as it requires differentiating through the projection operator $\mathbb{P}_{\mathcal{Y}_i}$ that projects on the set $\mathcal{Y}_i$. Moreover, these local sensitivity updates only require the knowledge of the aggregate sensitivity $\boldsymbol{\sigma}(s)$, which can be broadcast by the TA while maintaining the agents' preferences private. □

Next, we establish convergence of Alg. 1 to a critical point[1] of (13) under appropriate choices of the step sizes $\{\alpha^k\}_{k\in\mathbb{N}}, \gamma$ and the tolerance sequence $\{\sigma^k\}_{k\in\mathbb{N}}$.

*Proposition 1:* Let $\{\alpha^k\}_{k\in\mathbb{N}}$ be non-negative, non-summable and square-summable, let $\{\sigma^k\}_{k\in\mathbb{N}}$ be non-negative and satisfy $\sum_{k=0}^{\infty}\alpha^k\sigma^k < \infty$, and let $\gamma$ be sufficiently small. Then, any limit point of the sequence $\{\boldsymbol{c}^k\}_{k\in\mathbb{N}}$ generated by Alg. 1 is a critical point of (13).

*Proof:* See Appendix B. ■

[1]Any point $\boldsymbol{c} \in \mathcal{D}$ that satisfies $0 \in \boldsymbol{\mathcal{J}}\hat{\varphi}_{\text{TA}}(\boldsymbol{c}) + \mathrm{N}_{\mathcal{D}}(\boldsymbol{c})$ is called a *critical point* of (13), where $\mathrm{N}_{\mathcal{D}}$ denotes the normal cone of $\mathcal{D}$.

ALGORITHM 1. Hierarchical Traffic Shaping

**Parameters**: Step sizes $\{\alpha^k\}_{k\in\mathbb{N}}$, tolerances $\{\sigma^k\}_{k\in\mathbb{N}}$.
**Initialization**: $k \leftarrow 0,\ \boldsymbol{c}^k \in \mathcal{D},\ \boldsymbol{y}^k \in \mathbb{R}^n,\ s^k \in \mathbb{R}^{m\times n}$.
**Iterate until convergence:**

```
(TA)
```

Compute inexact hypergradient:
$$\widehat{\nabla}\hat{\varphi}^k_{\text{TA}} = \nabla_{\boldsymbol{c}}\varphi_{\text{TA}}(\boldsymbol{c}^k, \boldsymbol{y}^k) + (s^k)^\top \nabla_{\boldsymbol{y}}\varphi_{\text{TA}}(\boldsymbol{c}^k, \boldsymbol{y}^k)$$
Update discounts:
$$\boldsymbol{c}^{k+1} = \mathbb{P}_{\mathcal{D}}[\boldsymbol{c}^k - \alpha^k \widehat{\nabla}\hat{\varphi}^k_{\text{TA}}]$$
Send $\boldsymbol{c}^{k+1}$ to `(PEVs/FVs)`

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(TA+PEVs/FVs)
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Estimate flows and sensitivity:
$$(\boldsymbol{y}^{k+1},\ s^{k+1}) = \textbf{Inner Loop}(\boldsymbol{c}^{k+1},\ \boldsymbol{y}^k,\ s^k,\ \sigma^k)$$
$k \leftarrow k+1$

Any local minimum of $\hat{\varphi}_{\text{TA}}$ is a critical point [19, Prop. 1]. However, the set of critical points can also include spurious points, such as saddle points or local maxima, although such occurrences are infrequent in practical scenarios.

## IV. Numerical Simulations

We deploy our proposed traffic-shaping scheme on a modified version of the Anaheim city dataset [20]. For demonstration purposes, we only considered a sub-network of Anaheim consisting of 50 nodes and 118 edges with some additional edges to ensure path connectivity.

The parameters $a_\varepsilon, b_\varepsilon, h_\varepsilon$ are derived from Anaheim's original non-linear data via linearization as described in [14]. We let $\eta_i = 30\,[\$/\text{h}]$ for all agents, while $q_i$ is drawn uniformly at random in the interval $[20, 60]$. We consider two charging stations located at nodes 0 and 14, hence $\mathcal{C}^{\text{c}} = \{0, 14\}$. The associated prices are set to $\bar{c}^{\text{c}}_0 = 0.35\,[\$/\text{kWh}]$ and $\bar{c}^{\text{c}}_{14} = 0.3\,[\$/\text{kWh}]$. Parking lots are located in nodes 8 and 20, hence $\mathcal{C}^{\text{p}} = \{8, 20\}$, and the prices are $\bar{c}^{\text{p}}_8 = 17\,\$$ and $\bar{c}^{\text{p}}_{20} = 20\,\$$. The TA discounts are restricted to $c_i^{\text{c},j} \in [0, 0.2]$, for all $i \in \mathcal{N}, j \in \mathcal{C}^{\text{c}}$, whereas $c_i^{\text{p},j} \in [0, 5]$, for all $i \in \mathcal{N}, j \in \mathcal{C}^{\text{p}}$. The access rate of class $i$ to facility $j$ is set to $\delta_i^{c,j} = 0.75\frac{P_i}{n_{\text{c}}}$ and $\delta_i^{p,j} = 0.75\frac{P_i}{n_{\text{p}}}$ for all $i \in \mathcal{N}, j \in \mathcal{C}$.

We consider 20 classes of PEVs and 20 classes of FVs that amount to 5% and 15% of the total vehicles for their respective type, ensuring a policy penetration rate of 20%. Given the total vehicles $n_{\text{veh}} = 242584$ on the network, each class size is computed as $P_i = \rho_i\frac{n_{\text{veh}}}{20}$, where $\rho_i = 0.05$ if $i$ is composed of PEVs and $\rho_i = 0.20$ otherwise. For each PEV class, the minimum fraction of vehicles that need to charge, $\bar{g}_i^c$, is randomly selected.

### A. Impact of the discounts budget

We investigate the TTT reduction attained by our algorithm as a function of the available budget. We consider

ALGORITHM 2. Inner Loop

**Parameters**: step size $\gamma$.
**Input**: $\boldsymbol{c}, \boldsymbol{y}, s, \sigma$.
**Define**: $h_i(\boldsymbol{c}, y_i, \boldsymbol{\sigma}(\boldsymbol{y})) := \mathbb{P}_{\mathcal{Y}_i}[y_i - \gamma F(\boldsymbol{c}, \boldsymbol{\sigma}(\boldsymbol{y}))]$
**Initialization**: $\ell \leftarrow 0,\ \tilde{\boldsymbol{\sigma}}(\boldsymbol{y}^\ell) = \boldsymbol{\sigma}(\boldsymbol{y}),\ \boldsymbol{\sigma}(\tilde{s}^\ell) = \boldsymbol{\sigma}(s)$,
$\zeta = 0$, $(\forall i \in \mathcal{N})\, S_{1,i} = \mathbf{J}_1 h_i(\boldsymbol{c}, \tilde{y}_i^\ell, \boldsymbol{\sigma}(\tilde{\boldsymbol{y}}^\ell))$,
$S_{2,i} = \mathbf{J}_2 h_i(\boldsymbol{c}, \tilde{y}_i^\ell, \boldsymbol{\sigma}(\tilde{\boldsymbol{y}}^\ell)),\ S_{3,i} = \mathbf{J}_3 h_i(\boldsymbol{c}, \tilde{y}_i^\ell, \boldsymbol{\sigma}(\tilde{\boldsymbol{y}}^\ell))$
**Iterate**
`(PEVs/FVs)` $\forall i \in \mathcal{N}$ (in parallel)
Update flow profile:
$\tilde{y}_i^{\ell+1} = h_i(\boldsymbol{c}, \tilde{y}_i^\ell, \boldsymbol{\sigma}(\tilde{\boldsymbol{y}}^\ell))$
If $\zeta = 1$:
$S_{1,i} = \mathbf{J}_1 h_i(\boldsymbol{c}, \tilde{y}_i^{\ell+1}, \boldsymbol{\sigma}(\tilde{\boldsymbol{y}}^{\ell+1}))$,
$S_{2,i} = \mathbf{J}_2 h_i(\boldsymbol{c}, \tilde{y}_i^{\ell+1}, \boldsymbol{\sigma}(\tilde{\boldsymbol{y}}^{\ell+1}))$,
$S_{3,i} = \mathbf{J}_3 h_i(\boldsymbol{c}, \tilde{y}_i^{\ell+1}, \boldsymbol{\sigma}(\tilde{\boldsymbol{y}}^{\ell+1}))$
Else:
Do not update $S_{1,i}, S_{2,i}, S_{3,i}$
Update sensitivity:
$\tilde{s}_i^{\ell+1} = S_{2,i}\tilde{s}_i^\ell + S_{3,i}\boldsymbol{\sigma}(\tilde{s}^{\ell+1}) + S_{1,i}$
`(TA)`
Gather: $\tilde{\boldsymbol{y}}^{\ell+1} := (y_i^{\ell+1})_{i\in\mathcal{N}}$, $\tilde{s}^{\ell+1} := (\tilde{s}_i^{\ell+1})_{i\in\mathcal{N}}$
If $\|\tilde{\boldsymbol{y}}^{\ell+1} - \tilde{\boldsymbol{y}}^\ell\| \geq \sigma$: $\zeta = 1$
Else: $\zeta = 0$
Broadcast: $\boldsymbol{\sigma}(\tilde{\boldsymbol{y}}^{\ell+1})$, $\boldsymbol{\sigma}(\tilde{s}^{\ell+1})$, $\zeta$
$\ell \leftarrow \ell + 1$
Until $\max\{\|\tilde{\boldsymbol{y}}^\ell - \tilde{\boldsymbol{y}}^{\ell-1}\|, \|\tilde{s}^\ell - \tilde{s}^{\ell-1}\|\} \leq \sigma$
**Output**: $\bar{\boldsymbol{y}} = \tilde{\boldsymbol{y}}^\ell, \bar{s} = \tilde{s}^\ell$.

both personalized and uniform discounts, where the latter correspond to providing the same discount to all agents that access a particular facility. In Table I, we present the TTT reduction as a percentage of the TTT without intervention. We observe that for large budgets of 20k \$ both personalized and uniform incentives perform similarly, and are able to provide sensible decongestion of the network, around $3\%$. The magnitude of these results is in line with those obtained in other works using incentives for traffic decongestion. For example, in an experiment performed in Lee county almost 17M \$ has been put in place to achieve a traffic reduction of around $5\%$ [21]. Further, we note that the optimization problem with personalized incentives includes fewer constraints than the one with uniform incentives. It might therefore appear contradictory that with a 20k \$ budget the latter yields a better outcome. This behavior, however, is a result of the local optimality of the generated solution. One way to alleviate this is by experimenting with different algorithm parameters and initializations. As the available budget decreases, personalized incentives outperform uniform incentives due to the more efficient and targeted allocation of resources.

| Discounts | 5k \$ | 10k \$ | 20k \$ |
|---|---|---|---|
| Uniform | $0\%$ | $1.3\%$ | $3.2\%$ |
| Personalized | $1.1\%$ | $1.8\%$ | $3.1\%$ |

TABLE I: Percentage of TTT reduction for different budgets.

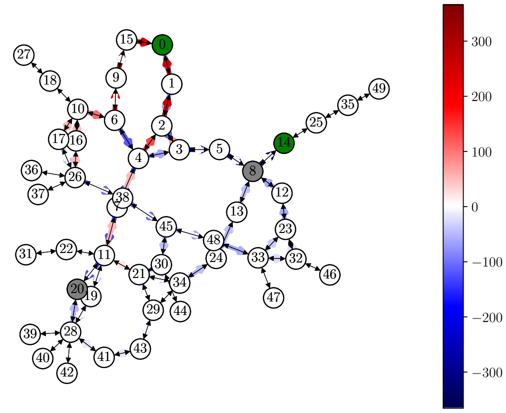


Fig. 2: Difference of the flows of PEV and FV with TA intervention $\phi^\star$ (i.e., discounts via BIG Hype) and without TA intervention $\bar{\phi}$ (i.e., NE under no discounts). Charging stations and parking lots are denoted by green and gray nodes respectively.

To showcase the effect of the TA's discounting policy on the routing game, we illustrate in Figure 2 the difference in the flow of *controllable* vehicles, i.e., PEV and FV, with and without TA intervention, considering a budget of 5k \$ and $\mathcal{E}_D = \mathcal{E}$. In this specific setting, the TA favours discounts in the facility at node 0 to redirect vehicles towards that node and decongest the areas around the remaining facilities, resulting in a decrement of the TTT by 76 h every day.

### B. Scalability with respect to the network size

Next, we explore the scalability of our proposed scheme by considering sub-networks of Anaheim of increasing size, starting from $n_v = 50$ to $n_v = 400$. We consider the computational cost of the TA's updates, in the outer loop, and the agents' updates, in the inner loop. Specifically, assuming a distributed implementation, the inner loop cost corresponds to the maximum computation time among all the agents. In Figure 3, we present both computational costs as a function of the number of nodes in the network.

To assess the overall computation time, we highlight that the number of outer loop iterations required for convergence is typically in the order of hundreds. Moreover, we can employ large tolerances for the inner loop iteration as in [15, §V], which allow running few inner iterations (usually only one) for each outer loop iteration.

## V. CONCLUSION

Smart incentives on the price of energy at service stations and on the price for accessing parking facilities are a viable option for the traffic authorities to promote traffic decongestion. Compared to tolling, the effect is limited due to the voluntary nature of such policies and their indirect effect on the commuters' routing preference. To compute the

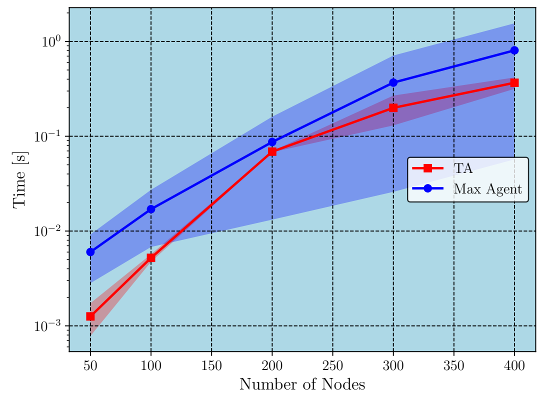


Fig. 3: Computation time of the inner and outer loops vs network size. The solid lines are the average and the shaded areas represent $\pm 1$ the standard deviation over 100 iterations.

optimal set of discounts, BIG Hype produces highly scalable solutions that can be applied to networks of large dimension.

The proposed model can be extended in many directions, for example the traffic authority can be endowed with the ability to impose tolls, yielding to a general formulation that subsumes the restricted network tolling problem.

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## APPENDIX

### A. Proof of Lemma 1

A sufficient condition for existence and uniqueness of an NE is that the PG $F(\boldsymbol{c},\cdot)$ is strongly monotone [17, Th. 2.3.3(b)]. Since $F(\boldsymbol{c},\cdot)$ is an affine mapping, strong monotonicity is equivalent to $\mathbf{J}F(\boldsymbol{c},\cdot)$ being positive definite [17, Th. 2.3.2(c)]. Observe that the functions $f_i^{\mathrm{t}}$ and $f_i^{\mathrm{c}}$, $f_i^{\mathrm{p}}$, $f_i^{\mathrm{lm}}$ depend only on $\boldsymbol{\phi}$ and $g_i^{\mathrm{c}}$, $g_i^{\mathrm{p}}$, respectively. Therefore, after applying an appropriate permutation we can express $\mathbf{J}F(\boldsymbol{c},\cdot)$ as $M = \mathrm{diag}(M_\phi, M_g)$, where $M_\phi := \mathbf{J}_\phi F(\boldsymbol{c},\cdot)$ and $M_g := \mathbf{J}_g F(\boldsymbol{c},\cdot)$. In the proof of [13, Lem. 1] it is shown $M_y \succ 0$, provided that $t_\varepsilon(\cdot)$ is an affine function. Moreover, $M_g \succ 0$ because $f_i^{\mathrm{c}} + f_i^{\mathrm{p}} + f_i^{\mathrm{lm}}$ is a strongly convex quadratic that depends only on $g_i$, for all $i \in \mathcal{N}$. Thus, $M_\phi$ and $M_g$ are positive definite implying that $M \succ 0$, as desired. □

### B. Proof of Proposition 1

To prove the claim, we will verify that Assumption 1 and Standing Assumptions (SAs) $1-4$ in [15] are satisfied by our model, and then invoke [15, Th. 2]. Notice that the $f_i$'s and $\mathcal{Y}_i$'s readily satisfy SA 1. For SA 2, we showed in the proof of Lemma 1 that $F(\boldsymbol{c},\cdot)$ is strongly monotone for any $\boldsymbol{c}$. Uniform strong monotonicity and Lipschitz continuity follow from the fact the $F$ is affine in $(\boldsymbol{c},\boldsymbol{y})$. SAs 3 and 4 hold since $F$ and $\varphi_{\mathrm{TA}}$ are semialgebraic and, thus, definable. □