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Power Purchase Agreements with Renewables: Optimal Timing and Design

Zuguang Gao, Nur Sunar, and John R. Birge

Abstract— **In this paper, we design a power purchase agreement (PPA) where the firm agrees to make a certain transfer payment to the renewable generator, and the generator invests that payment to build new renewable energy facilities. The firm will then have access to all electricity generation from the new facilities for a long-term period. The firm dynamically decides when to start the PPA and the transfer payment based on the evolving market conditions. The firm's objective is to maximize its long-term discounted benefit (total savings) from signing the PPA. We mathematically formulate the firm's decision problem as an optimal stopping problem and provide analytical solutions. We also provide insights into how the firm's optimal investment capacity, expected savings, and the expected total new generation change with respect to different problem parameters.**

I. INTRODUCTION

As climate change becomes a global challenge, there has been increasing interest in the study of renewable energy integration and management [1]–[18]. A power purchase agreement (PPA) is a contractual agreement between a firm (buyer) and a renewable energy generator (seller) [19]. PPAs provide more financial certainty to both the buyer and the seller, thus removing a significant roadblock to building new renewable facilities [20]. In 2022, global renewable PPA volume was 36.7GW, which is 18% higher than the 2021 figure, and the volume of total renewable PPAs signed by corporations between 2008 and 2022 exceeded the entire energy generation capacity of France [21]. As the number of new renewable energy deals continue to grow, it is becoming more important than ever to have better-designed PPAs that not only financially benefit the firm but also provide incentives for more investment in renewable energy facilities, and make the firm's electricity consumption more eco-friendly.

Toward this goal, in this paper, we design a PPA where the firm agrees to make a certain transfer payment to the renewable generator, and the generator invests that payment to build new renewable energy facilities, such as solar photovoltaics (PVs) and/or wind turbines. The firm will then have access to all electricity generation from the new facilities for a long-term period (e.g., 20 years). The firm may dynamically decide when to start the PPA on an ongoing basis, based on the evolving market conditions, and the transfer payment is also specified by the firm. The firm's objective is to maximize its long-term discounted benefit (total savings) from signing the PPA.

We mathematically formulate the firm's decision problem as an optimal stopping problem and provide analytical solutions. In this work, we characterize how renewable energy production characteristics (which are determined by the weather and the conversion efficiency of PVs, the length of the PPA, and the renewable investment cost) affect the firm's optimal PPA. We conclude that with an increased site efficiency or with an increased length of the PPA, the firm will optimally sign a PPA earlier, with a smaller new renewable capacity, and the firm attains higher expected value. The expected total new generation from the PPA may increase or decrease with the site efficiency, depending on the variation and efficiency of PV generations. Due to space limits, all proofs are omitted but can be found in an extended version of the paper [22].

II. POWER PURCHASE AGREEMENT MODEL

A *firm* must fulfill its uncertain residual electricity demand - excess electricity demand unmet by existing sources (generation or contracts) - at any time $t \geq 0$. This *residual demand* is represented by a stochastic process, $\mathbb{U} := \{U_t, t \geq 0\}$. The firm can satisfy its residual demand by procuring electricity from the wholesale market at the wholesale market price. The firm also has the option to sign a long-term power purchase contract with a *new* renewable generator at any t. We call this contract a *renewable power purchase agreement* (renewable PPA). Such a contract may fulfill all or part of the firm's residual demand, while the remaining residual demand may still be met from the wholesale market. We now detail each of these procurement options.

A. Renewable Power Purchase Agreement

If the firm decides to sign a renewable power purchase agreement at $t = \tau$, the firm makes a transfer payment of C to the renewable generator at the beginning of the PPA. This transfer payment is a decision variable to the firm and can be interpreted as the firm's total discounted payments over the contract duration of $T > 0$. In exchange of this payment, a new renewable energy facility becomes operational, and the firm owns the entire electricity production of this facility over the contract duration $[\tau, \tau + T]$. The new renewable facility has a lifespan of $\hat{T} \geq T$. This is in line with practice; the lifespan of a solar farm can be as long as 35 years whereas it is rare to find renewable PPAs of that length [23]. The production from one unit of renewable facility capacity

Z. Gao is with the Paul Merage School of Business, University of California, Irvine, Irvine, CA 92697, USA. z.gao@uci.edu
N. Sunar is with the Kenan–Flagler Business School, University of North Carolina at Chapel Hill, Chapel Hill, NC 27599, USA. Nur_Sunar@kenan-flagler.unc.edu
J. R. Birge is with the University of Chicago Booth School of Business, Chicago, IL 60637, USA. john.birge@chicagobooth.edu

follows a stochastic process represented by $\{Q_t, t \geq \tau\}$ where Q_t is a random variable with mean μ_Q and standard deviation σ_Q for $t > 0$.

The renewable generator incurs an investment cost of $I(k)$ to start a new renewable facility of capacity $k > 0$. Following the common formulation in the literature, we consider a linear investment cost, i.e., $I(k) = bk$ where b is a positive constant (see, e.g., [10], [24] that consider linear investment cost as we do. In practice, businesses typically prioritize growth rather than profit in their early years. Consistent with this fact, in our formulation, if the renewable PPA is signed, the renewable generator uses the entire transfer payment C to maximize the new facility capacity. Thus, the capacity of the new renewable facility is K such that $I(K) = C$.

B. Wholesale Electricity Market

Let the total demand for electricity in the wholesale market be $\{D_t, t \geq 0\}$, which follows a geometric Brownian motion (GBM), i.e., $dD_t = \mu_D D_t dt + \sigma_D D_t dW_t$, with the assumption that $\mu_D > \sigma_D^2/2$. The GBM process of energy demand has been commonly assumed in literature [25], [26].

If the new renewable facility of capacity K becomes operational at time τ as a result of a PPA, it remains operational until $\tau + \hat{T}$, and generates an amount of $\hat{Q}_t := KQ_t$ at each $t \in [\tau, \tau + \hat{T}]$. Then, the wholesale market demand in excess of the renewable facility's generation, i.e., *net wholesale electricity demand*, becomes $N_t := D_t - KQ_t$ for $t \in [\tau, \tau + \hat{T}]$. However, whenever the renewable facility is absent or non-operational, the net wholesale electricity demand is $N_t := D_t$. In practice, the wholesale market price for electricity increases with the net wholesale market demand, and various studies consider a linear relationship between them (see, e.g., [9]) In line with the literature, in our setting, the wholesale market price at t is $p_t = \theta N_t$ where θ is a positive constant.

Our analysis will eventually require comparing scenarios with and without a renewable PPA. For clarity, p_t^N (respectively, p_t^Y) represents the wholesale market price at time t when a renewable PPA is never (respectively, ever) signed. Then, because $p_t = \theta N_t$, we have

$$p_t^Y = \begin{cases} \theta(D_t - KQ_t) & t \in [\tau, \tau + \hat{T}], \\ \theta D_t & t \notin [\tau, \tau + \hat{T}], \end{cases} \tag{1}$$

and $p_t^N = \theta D_t$ for $t > 0$.

C. Firm's Problem

Consider a firm that needs to satisfy an uncertain residual electricity demand, i.e., the excess demand that is not met by existing generation sources or power contracts. The firm's residual demand at each time instant t is given by $\{U_t, t \geq 0\}$. We assume that $U_t = \alpha D_t$ where α being a constant in $[0, 1]$. The residual demand can be satisfied by procuring electricity from the wholesale market at the wholesale market price. The firm also has the option to make a long term capacity contract with a new renewable generator, which gives the firm access to all electricity produced by the renewable generator for T length of time (years). Such a contract may fulfill all or part of the residual demand, while the remaining residual demand may still be satisfied from the wholesale market.

Let λ_d be the discount rate of uninvested cash (real interest rate). Throughout the rest of this paper, we assume that $\lambda_d > 2\mu_D + \sigma_D^2$.

Without a PPA, the firm's total cost, starting from any time τ and discounted to time τ, of procuring electricity from the wholesale market is given by

$$\int_{\tau}^{\infty} e^{-\lambda_d(s-\tau)} p_s^N U_s ds. \tag{2}$$

With a PPA starting from τ and lasts till $\tau + T$, the firm incurs the following expected discounted cost, discounted to $t = \tau$, to meet its residual electricity demand during $[\tau, \infty)$:

$$\int_{\tau}^{\tau+T} e^{-\lambda_d(s-\tau)} p_s^Y \left[U_s - \hat{Q}_s\right]^+ ds + \int_{\tau+T}^{\tau+\hat{T}} e^{-\lambda_d(s-\tau)} p_s^Y U_s ds + \int_{\tau+\hat{T}}^{\infty} e^{-\lambda_d(s-\tau)} p_s^Y U_s ds + C. \tag{3}$$

With the PPA, the firm owns all electricity produced by the renewable generator during $[\tau, \tau + T)$, and when $\hat{Q}_t > U_t$, the firm can sell $\hat{Q}_t - U_t$ back, again at the wholesale market price. Therefore, when the PPA is signed at time τ, the firm will earn the following revenue from selling:

$$\int_{\tau}^{\tau+T} e^{-\lambda_d(s-\tau)} p_s^Y \left[\hat{Q}_s - U_s\right]^+ ds. \tag{4}$$

Thus, the firm's expected total *net* discounted cost, with a PPA signed at τ, is given by

$$(3) - (4) = \int_{\tau}^{\tau+T} e^{-\lambda_d(s-\tau)} p_s^Y \left[U_s - \hat{Q}_s\right] ds + \int_{\tau+T}^{\tau+\hat{T}} e^{-\lambda_d(s-\tau)} p_s^Y U_s ds + \int_{\tau+\hat{T}}^{\infty} e^{-\lambda_d(s-\tau)} p_s^Y U_s ds + C. \tag{5}$$

Note that $\hat{T}$ is the lifespan of the renewable facilities, and thus $N_t = D_t$ for $t > \tau + \hat{T}$ with and without the PPA, which results in $p_t^Y = p_t^N$ for $t \in (\tau + \hat{T}, \infty)$. The firm's savings (discounted to time τ) from signing a PPA with transfer amount C at time τ is then

$$(2) - (5) = \int_{\tau}^{\tau+T} e^{-\lambda_d(s-\tau)} p_s^N U_s ds + \int_{\tau+T}^{\tau+\hat{T}} e^{-\lambda_d(s-\tau)} p_s^N U_s ds - \int_{\tau}^{\tau+T} e^{-\lambda_d(s-\tau)} p_s^Y \left[U_s - \hat{Q}_s\right] ds - \int_{\tau+T}^{\tau+\hat{T}} e^{-\lambda_d(s-\tau)} p_s^Y U_s ds - C. \tag{6}$$

Note that p_s^Y can be viewed as a function of K, and thus (6) is a function of C and K. The firm's decisions include dynamically choosing a time τ to start the PPA, as well as choosing a transfer C, or equivalently, choosing an investment capacity K (since $C = bK$). The firm's objective is to maximize its discounted expected saving from a PPA:

$$\max_{(\tau,C)} \mathbb{E}\left[e^{-\lambda_d \tau} \cdot (6)\right] \quad \text{s.t. } I(K) = C. \tag{7}$$

Note that from (7), the firm has to decide both C, the investment amount, and τ, the starting time of the PPA. On one hand, the firm's optimal decision on C, given any starting time τ, is such that (6) is maximized. On the other hand, the firm dynamically decides τ, based on the evolving process of D_t, to maximize its discounted expected saving, assuming that C is chosen optimally.

III. Analysis and Solution to the Power Purchase Agreement Model

In this section, we derive the optimal solution to the PPA model and provide a complete analysis on the properties of the optimal investment capacity and the firm's expected savings.

A. Signing PPA at any given time τ

To solve the PPA model, we need to find the firm's optimal decisions on whether/when to sign a PPA and on the transfer amount C. We will formulate the firm's dynamic decision on whether to sign a PPA (at each time instance) as an optimal stopping problem. Before that, however, we first study in this subsection how much the firm could save if it signs the PPA at an arbitrary given time τ, which builds the foundation for the optimal stopping problem that we present in the next subsection.

When the firm signs a PPA at time τ, its savings (6) are optimized by choosing an optimal transfer C. Correspondingly, C/b units of renewable facilities will be built. In the following lemma, we derive the firm's expected saving if a PPA is signed at time τ.

Lemma 1. *If a PPA is signed at time τ, and the transfer payment satisfies $C = bK$ where K is the amount of new renewable facilities built, then, the firm's expected saving from this PPA, discounted to time τ, is given by*

$$K\frac{\theta\mu_Q D_\tau}{\lambda_d-\mu_D}\left[1-e^{(\mu_D-\lambda_d)T}\right]+K\frac{\alpha\theta\mu_Q D_\tau}{\lambda_d-\mu_D}\left[1-e^{(\mu_D-\lambda_d)\hat{T}}\right] - K^2\frac{\theta\left(\sigma_Q^2+\mu_Q^2\right)}{\lambda_d}\left[1-e^{-\lambda_d T}\right]-bK. \tag{8}$$

Lemma 1 provides the expected savings of the firm in terms of D_τ, the total demand of the wholesale market at time τ, and K, the capacity of new renewable energy facilities. Since the firm chooses $C = bK$, these savings can be further optimized over K. In the following proposition, we present the optimal capacity and savings of the firm.

Proposition 1. *Suppose that the firm signs the PPA at time $\tau > 0$. Then, the newly added capacity of renewable energy facilities because of the PPA is*

$$\widetilde{K}(D_\tau) := \frac{\left[\frac{\theta\mu_Q D_\tau}{\lambda_d-\mu_D}\left[1+\alpha-e^{(\mu_D-\lambda_d)T}-\alpha e^{(\mu_D-\lambda_d)\hat{T}}\right]-b\right]^+}{2\frac{\theta\left(\sigma_Q^2+\mu_Q^2\right)}{\lambda_d}\left[1-e^{-\lambda_d T}\right]}, \tag{9}$$

and the firm's optimal expected savings from the PPA are

$$S(D_\tau) := \begin{cases} \dfrac{\left[\frac{\theta\mu_Q D_\tau}{\lambda_d-\mu_D}\left[1+\alpha-e^{(\mu_D-\lambda_d)T}-\alpha e^{(\mu_D-\lambda_d)\hat{T}}\right]-b\right]^2}{4\frac{\theta\left(\sigma_Q^2+\mu_Q^2\right)}{\lambda_d}\left[1-e^{-\lambda_d T}\right]}, \\ \quad \text{if } D_\tau \ge \dfrac{b(\lambda_d-\mu_D)}{\theta\mu_Q\left[1+\alpha-e^{(\mu_D-\lambda_d)T}-\alpha e^{(\mu_D-\lambda_d)\hat{T}}\right]}, \\ 0, \ \text{if } D_\tau < \dfrac{b(\lambda_d-\mu_D)}{\theta\mu_Q\left[1+\alpha-e^{(\mu_D-\lambda_d)T}-\alpha e^{(\mu_D-\lambda_d)\hat{T}}\right]}. \end{cases} \tag{10}$$

From Proposition 1, we see that the firm would choose a capacity $K > 0$, resulting in a positive expected saving S, if and only if $D_\tau \ge \frac{b(\lambda_d-\mu_D)}{\theta\mu_Q\left[1+\alpha-e^{(\mu_D-\lambda_d)T}-\alpha e^{(\mu_D-\lambda_d)\hat{T}}\right]}$. In other words, when the investment cost b is high enough, i.e., $b > \frac{D_\tau\theta\mu_Q\left[1+\alpha-e^{(\mu_D-\lambda_d)T}-\alpha e^{(\mu_D-\lambda_d)\hat{T}}\right]}{\lambda_d-\mu_D}$, then the firm's optimal investment capacity would be zero, meaning that the cost is too high for the firm to make any investment.

Next, we are interested in how the optimal capacity and the savings change with respect to the production process (described by μ_Q and σ_Q) and T, the length of the PPA. The results are summarized as Proposition 2.

Proposition 2. *The optimal renewable capacity and the firm's optimal expected savings change with respect to different problem parameters as follows.*

1) *There exists a unique threshold $\bar{\mu}$ such that $\frac{\partial\widetilde{K}(D_\tau)}{\partial\mu_Q} < 0$ if and only if $\mu_Q > \bar{\mu}$.*
2) *There exists a unique threshold $\hat{\mu}$ such that $\frac{\partial S(D_\tau)}{\partial\mu_Q} > 0$ if and only if $\mu_Q > \hat{\mu}$.*
3) *If $\frac{\theta\mu_Q D_\tau\alpha}{\lambda_d-\mu_D}\left[1-e^{(\mu_D-\lambda_d)\hat{T}}\right] > b$, then, there exists a threshold T_1 such that when $T < T_1$, we have that $\frac{\partial\widetilde{K}(D_\tau)}{\partial T} < 0$. If $\frac{\theta\mu_Q D_\tau\alpha}{\lambda_d-\mu_D}\left[1-e^{(\mu_D-\lambda_d)\hat{T}}\right] \le b$, then there exists a threshold T_2 such that when $T < T_2$, we have that $\frac{\partial\widetilde{K}(D_\tau)}{\partial T} > 0$.*
4) *If $\frac{\theta\mu_Q D_\tau\alpha}{\lambda_d-\mu_D}\left[1-e^{(\mu_D-\lambda_d)\hat{T}}\right] \ne b$, then, there exists a threshold T_3 such that when $T < T_3$, we have that $\frac{\partial S(D_\tau)}{\partial T} < 0$. If $\frac{\theta\mu_Q D_\tau\alpha}{\lambda_d-\mu_D}\left[1-e^{(\mu_D-\lambda_d)\hat{T}}\right] \ne b$, then, there exists a threshold T_4 such that when $T < T_4$, we have that $\frac{\partial S(D_\tau)}{\partial T} > 0$.*
5) $\frac{\partial\widetilde{K}(D_\tau)}{\partial\sigma_Q} < 0$, $\frac{\partial S(D_\tau)}{\partial\sigma_Q} < 0$.

From the first item of Proposition 2, we see that $\widetilde{K}(D_\tau)$ first increases with μ_Q, then after $\mu_Q > \bar{\mu}$, the investment capacity starts to decrease. On the other hand, $S(D_\tau)$ first stays 0, then after $\mu_Q > \hat{\mu}$, the firm's saving starts to increase. The change of $\widetilde{K}(D_\tau)$ and $S(D_\tau)$ with respect to T are less tractable, but we obtain their limiting behaviors as $T \to 0$. Finally, as the variance of production σ_Q increases, both $\widetilde{K}(D_\tau)$ and $S(D_\tau)$ decrease.

B. Optimal time to sign PPA - dynamic decision

We now come back to the original model where the firm needs to dynamically decide $\tau \ge 0$, the time to sign a PPA, and the transfer amount C to the renewable energy generator. We will use x as a generic notation to represent the realization of the total electricity demand in the wholesale market. Let $V(x)$ be the firm's expected saving if it optimally chooses the time to sign the PPA and the transfer payment C given the initial demand realization is x. Recall that optimizing with respect to C is equivalent to optimizing with respect to K, and the optimal saving at the stopping time is already given by (10). Therefore, the firm's value function can be written as the following optimal stopping problem:

$$V(x) = \max_{\tau\ge 0}\mathbb{E}\left[e^{-\lambda_d\tau}S(D_\tau) \mid D_0 = x\right], \tag{11}$$

where we recall that τ is the time to sign a PPA (or the starting time of the PPA, or the time the firm stops waiting).

We assume and later verify that $V(x)$ is twice continuously differentiable and nonnegative. From (10), we know that $S(x)$ is continuous, nonnegative, and monotone. The decision to start or not to start a PPA at any time t when the realization $D_t = x$ depends on the comparison of $V(x)$ and $S(x)$. If $V(x) > S(x)$, the optimal τ^* that solves (11) is strictly positive, and it is more beneficial for the firm to wait, since the expected value of waiting is higher than the value of starting a PPA immediately. The set $\{x \mid V(x) > S(x)\}$ is called the *continuation region*. If $V(x) = S(x)$, then $\tau^* = 0$, and it is optimal for the firm to start the PPA immediately with the expected saving $S(x)$. The set $\{x \mid V(x) = S(x)\}$ is called the *stopping region*. We next have the following lemma on the characterization of $V(x)$.

Lemma 2. *The value function satisfies the following Hamilton–Jacobi–Bellman (HJB) equation:*

$$V(x) = \max\left\{S(x),\ \frac{1}{\lambda_d}\mu_D x V'(x) + \frac{1}{2\lambda_d}\sigma_D^2 x^2 V''(x)\right\}. \quad (12)$$

Lemma 2 verifies that when the firm stops waiting, $V(x) = S(x)$, and in the continuation region, we have

$$V(x) = \frac{1}{\lambda_d}\mu_D x V'(x) + \frac{1}{2\lambda_d}\sigma_D^2 x^2 V''(x). \quad (13)$$

The firm's decision to continue waiting or to stop waiting (and start a PPA) only depends on the realized total market demand. As the firm waits, we have that $S(x) < \frac{1}{\lambda_d}\mu_D x V'(x) + \frac{1}{2\lambda_d}\sigma_D^2 x^2 V''(x)$. The market demand continues to evolve while the firm is waiting, until x reaches some x_* such that $S(x_*) = \frac{1}{\lambda_d}\mu_D x_* V'(x_*) + \frac{1}{2\lambda_d}\sigma_D^2 x_*^2 V''(x_*)$, at which point the firm would stop waiting. The set $\left\{x \mid V(x) = S(x) = \frac{1}{\lambda_d}\mu_D x V'(x) + \frac{1}{2\lambda_d}\sigma_D^2 x^2 V''(x)\right\}$ is called the *optimal stopping boundary*. Then, at x_* in the optimal stopping boundary, we have

$$V(x_*) = S(x_*), \quad (14a)$$
$$V'(x_*) = S'(x_*), \quad (14b)$$

where (14a) is the *value matching condition* and (14b) is the *smooth pasting condition*.

It remains to find the x_* in the optimal stopping boundary, such that the firm would optimally start a PPA whenever D_t first reaches x_*. In the following proposition, we formalize the optimal policy and specify x_* by solving the differential equation (13) subject to the boundary conditions (14).

Proposition 3. *Suppose that the firm dynamically chooses when to sign a PPA with a renewable energy generator. For the firm, it is optimal to sign the PPA at* $\tau^* = \inf\{t \geq 0 \mid D_t = x_*\}$, *where* x_* *is the demand threshold and is given by*

$$x_* = \frac{b\omega_+(\lambda_d - \mu_D)}{(\omega_+ - 2)\theta\mu_Q\left[1 + \alpha - e^{(\mu_D - \lambda_d)T} - \alpha e^{(\mu_D - \lambda_d)\hat{T}}\right]}, \quad (15)$$

where

$$\omega_+ = \frac{\sigma_D^2 - 2\mu_D + \sqrt{(2\mu_D - \sigma_D^2)^2 + 8\sigma_D^2\lambda_d}}{2\sigma_D^2} > 2. \quad (16)$$

Proposition 3 states that the firm's optimal policy is to start the PPA when the total market demand first reaches x_*, which is a given explicitly by (15). We also note that the obtained x_* is positive. This can be seen by noting that $\mu_D < \lambda_d$, $e^{(-\lambda_d + \mu_D)T} < 1$, and $\omega_+ > 2$.

Next, we have the following corollary, which provides explicit expressions for the optimal invested capacity and the firm's optimal expected saving, as well as distribution of the waiting time before starting the PPA (for a given initial demand).

Corollary 1. *From Proposition 3 and Proposition 1, we can obtain the optimal additional capacity and the optimal expected savings:*

$$K^* = \frac{\frac{b}{\omega_+ - 2}}{\frac{\theta(\sigma_Q^2 + \mu_Q^2)}{\lambda_d}\left[1 - e^{-\lambda_d T}\right]}, \quad (17)$$

$$S(x_*) = \frac{\left[\frac{b}{\omega_+ - 2}\right]^2}{\frac{\theta(\sigma_Q^2 + \mu_Q^2)}{\lambda_d}\left[1 - e^{-\lambda_d T}\right]}, \quad (18)$$

with ω_+ *as given in* (16). *Moreover, the value function in the continuation region is given by*

$$V(x) = \frac{\frac{\left[\frac{b}{\omega_+ - 2}\right]^2}{\frac{\theta(\sigma_Q^2 + \mu_Q^2)}{\lambda_d}\left[1 - e^{-\lambda_d T}\right]}}{\left(\frac{b\omega_+(\mu_D - \lambda_d)}{(\omega_+ - 2)\theta\mu_Q\left[1 + \alpha - e^{(\mu_D - \lambda_d)T} - \alpha e^{(\lambda_d - \mu_D)\hat{T}}\right]}\right)^{\omega_+}} \cdot x^{\omega_+}. \quad (19)$$

Furthermore, let the initial demand be some $D_0 < x_*$, *then, the optimal time to sign a PPA follows the inverse Gaussian distribution* $\mathrm{IG}\left(\frac{\ln\left(\frac{x_*}{D_0}\right)}{\mu_D - \sigma_D^2/2}, \left(\frac{\ln\left(\frac{x_*}{D_0}\right)}{\sigma_D}\right)^2\right)$, *with a mean* $\frac{\ln\left(\frac{x_*}{D_0}\right)}{\mu_D - \sigma_D^2/2}$.

Following Corollary 1, we are interested in how K^*, $V(x)$, and $\mathbb{E}[\tau^*]$ change with respect to the production process parameters μ_Q and σ_Q as well as T, the length of the PPA. The results are summarized as Proposition 4.

Proposition 4. *Under the optimal policy, the newly added renewable capacity* K^*, *the firm's optimal expected savings* $S(x_*)$, *the value function* $V(x)$, *and the expected stopping time* $\mathbb{E}[\tau^*]$ *change as the following with respect to different problem parameters.*

$$\frac{\partial K^*}{\partial \mu_Q} < 0, \qquad \frac{\partial K^*}{\partial \sigma_Q} < 0, \qquad \frac{\partial K^*}{\partial T} < 0, \quad (20a)$$

$$\frac{\partial V(x)}{\partial \mu_Q} > 0, \qquad \frac{\partial V(x)}{\partial \sigma_Q} < 0, \quad (20b)$$

$$\frac{\partial \mathbb{E}[\tau^*]}{\partial \mu_Q} < 0, \qquad \frac{\partial \mathbb{E}[\tau^*]}{\partial \sigma_Q} = 0, \qquad \frac{\partial \mathbb{E}[\tau^*]}{\partial T} < 0. \quad (20c)$$

To demonstrate the results of Proposition 4, we also numerically show the changes of K^* and $V(x)$ with respect to μ_Q, σ_Q, and T. In all numerical studies in this paper, we choose the following "default" parameters (i.e., the non-varying parameters are set to these values when making the plots): $\mu_D = 0.001$, $\sigma_D = 0.015$, $\lambda_d = 0.015$, $\alpha = 0.004$, $b = 300$, $\theta = 4 \times 10^{-14}$, $\mu_Q = 2000$, $\sigma_Q = 80$, $T = 20$, $\hat{T} = 50$, $D_0 = 4 \times 10^{12}$. With these numbers, Figure 1 shows how the optimal capacity K^* changes with respect to μ_Q and T; Figure 2 shows how the value function $V(x)$ changes with respect to μ_Q and T, when $x = D_0$.

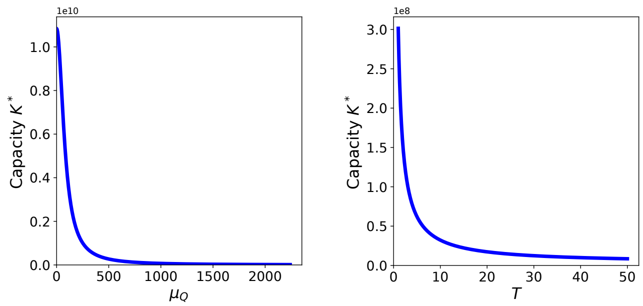

Fig. 1: Numerical illustration of how the optimal capacity K^* changes with respect to μ_Q and T.

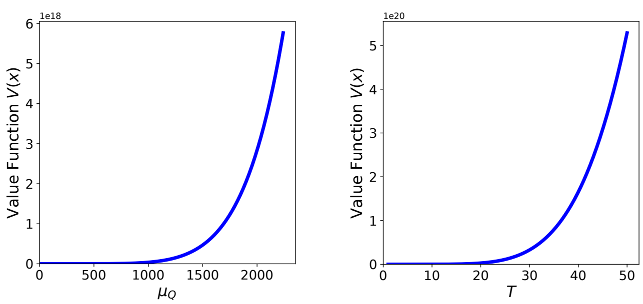

Fig. 2: Numerical illustration of how the value function $V(x)$ changes with respect to μ_Q and T.

Proposition 4 conveys several messages. First, as shown in (20a) and illustrated in Figure 1, the firm's optimal investment capacity decreases with respect to μ_Q, σ_Q, and T. As the mean production per unit of renewable facility increases, the capacity of newly built renewable facilities is smaller, since the firm may now reach the optimal generation from a smaller amount of facilities. This optimal amount of capacity also decreases with more variance on the generation, since the instability of the generation would likely make the firm benefit less from the renewable facilities. When the length of the PPA is longer, the firm would also have fewer new facilities, as the firm is benefiting for a longer term from each unit of renewable facility.

Second, as shown in (20b) and illustrated in Figure 2, the value function $V(x)$ increases with μ_Q, and decreases with the variance of production σ_Q. While the change of the value function with respect to the length of the PPA is not analytically tractable, it is intuitive that, with all other conditions fixed, the longer the length of the PPA, the more benefit the firm gets. Thus, the value function is higher with a longer PPA, which is also consistent with the numerical studies.

Moreover, as shown in (20c), the expected waiting time before the firm starts a PPA decreases with a higher mean production level μ_Q, or with a longer length T, but the expected waiting time does not change with respect to the variance σ_Q.

We next consider the effect of varying the investment cost, i.e., changing the parameter b in $I(k) = bk$. The following proposition summarizes how the optimal capacity K^* and the total new generation due to the PPA, $K^* \int_{\tau^*}^{\tau^*+\hat{T}} Q_t dt$, change with respect to b.

Proposition 5. *When $D_0 < x_*$, under the firm's optimal PPA, increasing the investment cost parameter b results in*

- *a strictly larger capacity for the new renewable facility and*
- *a strictly larger total new renewable energy output $K^* \int_{\tau^*}^{\tau^*+\hat{T}} Q_t dt$ with probability 1.*

Otherwise when $D_0 \geq x_*$, such an increase in b reduces the added renewable energy capacity and production with probability 1.

A numerical illustration of Proposition 5 is given in Figure 3. Effectively, the total new renewable generation due to the PPA also decreases. When $D_0 < x_*$, the firm waits to sign a PPA. As the investment cost increases, the firm delays the PPA to sign it at a larger x_*. Since the total demand is now higher, the wholesale market price is also higher, which gives the firm more motivation to invest for a larger renewable energy capacity. In summary, when the current total demand is lower than the threshold x_*, reducing the investment cost for renewable energy is effective in shortening the firm's time to sign a renewable PPA, but it also reduces the new renewable capacity investment.

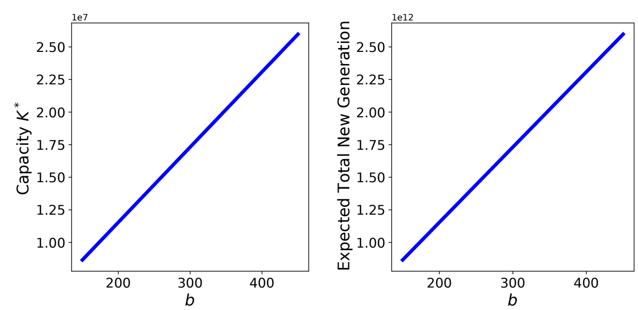

Fig. 3: Numerical illustration of how the optimal capacity and the expected total new generation change with respect to b when $D_0 < x_*$.

Lastly, we look at the total expected generation from the new capacities due to the PPA. The results are summarized as Proposition 6.

Proposition 6. *Under the firm's optimal PPA, the total expected generation from the newly added renewable capacity over its lifespan, $\mathbb{E}\left[K^* \int_{\tau^*}^{\tau^*+\hat{T}} Q_t dt\right]$, changes as follows with respect to different problem parameters.*

$$\frac{\partial \mathbb{E}\left[K^* \int_{\tau^*}^{\tau^*+\hat{T}} Q_t dt\right]}{\partial \mu_Q} \begin{cases} \geq 0 & \text{if } \mu_Q \leq \sigma_Q \\ < 0 & \text{if } \mu_Q > \sigma_Q \end{cases}, \tag{21a}$$

$$\frac{\partial \mathbb{E}\left[K^* \int_{\tau^*}^{\tau^*+\hat{T}} Q_t dt\right]}{\partial \sigma_Q} < 0, \quad \frac{\partial \mathbb{E}\left[K^* \int_{\tau^*}^{\tau^*+\hat{T}} Q_t dt\right]}{\partial T} < 0. \tag{21b}$$

We also show (21a) numerically in Figure 4. Proposition 6 implies that when μ_Q is relatively small compared with σ_Q, i.e., the coefficient of variation σ_Q/μ_Q is greater than 1, the expected total generation from the new renewable facility increases with respect to μ_Q; when μ_Q is relatively large compared with σ_Q, i.e., the coefficient of variation σ_Q/μ_Q is smaller than 1, the expected total generation from the new renewable facility decreases with respect to μ_Q. The total expected generation decreases with σ_Q and T, which follows directly from (20a).

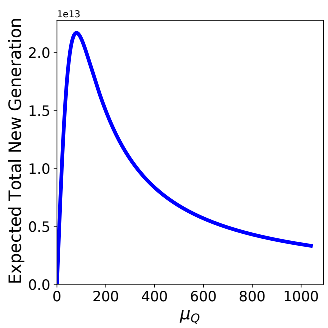

Fig. 4: Numerical illustration of the change of expected total new generation with respect to μ_Q.

IV. CONCLUSIONS

In this paper, we have proposed a power purchase agreement (PPA) model between the firm and a new renewable generator. We have formulated the firm's dynamic decision problem on when to start the PPA of a certain length and how much to invest (transfer) as an optimal stopping problem. We have defined the value function, derived the HJB equation, and solved the optimal policy for the firm. We have concluded that with an increased site efficiency μ_Q or with an increased length of the PPA T, the firm will optimally sign a PPA earlier, with a smaller capacity of new renewable facilities. The optimal capacity and the total new renewable generation may increase or decrease with the investment cost b, depending on the initial total demand. Moreover, the expected total new generation from the PPA increases (resp. decreases) with μ_Q, if the coefficient of variation σ_Q/μ_Q is greater (resp. smaller) than 1.

Some future directions we are currently pursuing are the consideration of an additional discount on renewable technology investment costs and the investigation of the effect of tax credits.

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