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Adaptive Safe Backstepping for Collaborative Levitation Control of Maglev Trains with Unknown Mass

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Abstract— **Maglev trains are levitated by magnetic forces to maintain a desired air gap between gudieway and magnets. Although every two magnets are mechanically coupled via a levitation bogie, the existing controllers are usually designed only for each *individual* magnet, which may result in unstable air gaps or levitation failures. Differently, this paper proposes an adaptive safe backstepping scheme to *collaboratively* control two magnets of the same bogie. In particular, safety constraints are introduced to adaptive backstepping control via quadratic programs, which ensures the air gap within a permissible range and significantly reduces overshoots. Finally, simulation results validate the effectiveness of the proposed collaborative controller.**

I. INTRODUCTION

Maglev trains have received great attention due to the superiorities of less noise, less friction, minor wear and low-cost maintenance [1]–[3]. In China, maglev train lines have been operated for decades, yet there are still some operational issues, including physical contact between the guideway and magnets as well as levitation failure, which reduce operational efficiency and pose a threat to operational safety [4]–[7]. These issues are closely relevant to the levitation control systems.

Levitation control is one of the crucial aspects of maglev technology. Regarding levitation controller implementation, proportional-integral-differential (PID) controllers are still the preferred option in commercial maglev trains due to their simple control structure [8], [9]. However, the individual PID control for each levitation point makes it difficult to satisfy the needs for coupled points' collaboration, since every two points on the same side are mechanically coupled [10]. Moreover, passengers getting on and off change the levitation load, and PID control with fixed parameters also cannot satisfy the mass variation of maglev trains. It deteriorates the levitation capabilities and may even cause physical contact or levitation failure. To compensate for the deficiency of PID with fixed parameters, some researchers focused on PID parameter tuning. For instance, Zhao et al. [11] adopted deep reinforcement learning technology to learn the real dataset and search for the optimal PID control gains and ensure the control stability of levitation systems. Zhang et al. [12] applied the stochastic approximation method to adjust the PID gains and conducted experimental verifications. However, it is hard to ensure the safety constraints of levitation systems with PID control, especially when the mass varies.

Motivated by the above observation, we propose a novel adaptive collaborative control law for the two coupled levitation points of maglev trains with unknown passenger loads. Because levitation control systems are typically safety-critical systems, they require a controlled levitation gap within a permissible range. However, there is limited scope for adjustment as the maximum gap range is only 0-20mm. This makes it susceptible to potential physical contact or levitation failure due to excessive regulation as passengers enter or exit the train.

Control barrier function (CBF) is a powerful tool to tackle control constraints and ensures system safety [13], which has been applied in a wide range of safety-critical systems, including adaptive cruise control [14], traffic merging [15] and bipedal robotic systems [16]. Inspired by the ideas of adaptive CBF [17], [18] and safe backstepping [19], the authors synthesize a novel adaptive law and safe backstepping CBF to formulate a novel levitation controller, which not only addresses coupled points collaboration of maglev trains with varying mass, but also guarantees the controlled error in a pleasant range. Moreover, the adaptive safe backstepping CBF significantly decreases the complexity of the design process rather than establishing a high-order CBF to guarantee the safety of the mutual coupling points.

The safe adaptive collaborative control enjoys two advantages: (a) it achieves the collaboration of the two levitation points considering mass variation; (b) it mediates the tracking performance and the controlled error constraints to guarantee the levitation errors within the practical levitation range if the mass changes, which avoid potential operational safety issues.

The remainder of this paper is given as follows. In Section II, the model of the coupled levitation system is described. Section III proposes the adaptive safe collaboration control based on backstepping. The effectiveness of the proposed scheme is verified through simulations in Section IV and conclusions are drawn in Section V.

II. THE MODEL OF COUPLED LEVITATION SYSTEM

Individual control for single levitation point degrades the control performance due to the strongly coupled structure. The coupled levitation system of maglev trains is shown in Fig. 1. The coupled levitation points require to be levitated

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collaboratively. In order to address two coupled electromagnets, we establish a new model for the two suspension points according to the mechanical structures, which fully reflects the rigid connection. The dynamics of two levitation points on the same side will be illustrated as follows.

To facilitate the system formulation and controller design, the following assumptions are adopted.

Assumption 1: The length between two coupled electromagnets is much larger than the levitation distances [20], i.e., $l \gg \max\{\delta_1(\mathrm{t}), \delta_2(\mathrm{t})\}$.

Remark 1: In real-world structure, the length of the connected structure is at the meter level, about $3\mathrm{m}$, while the controlled levitation distances are at the millimeter level, 8-10mm. It is verified in [10] that Assumption 1 is reasonable and practical. Furthermore, Assumption 1 indicates the following approximation equations $\cos\theta(t) \approx 1$, $\theta(t) \approx \sin\theta(t) = (\delta_2 - \delta_1)/l$, where $\theta(t)$ denote the pitch angle of the module around its centroid.

Assumption 2: The mass of the levitation system is piecewise constant.

Remark 2: The mass of an operational maglev train keeps constant between stations and only changes when passengers get on or off in the station.

Denote the vertical displacement of the centroid between two electromagnets by $d_{s_0}(t)$, then the positions of the electromagnets $\mathcal{A}$ and $\mathcal{B}$ are expressed as

$$\delta_1(t) = d_{s_0(t)} + \frac{l}{2}\theta(t), \tag{1a}$$

$$\delta_2(t) = d_{s_0(t)} - \frac{l}{2}\theta(t). \tag{1b}$$

According to Newton's second law and Euler's equation, the vertical and rotational dynamics are given as:

$$M\ddot{d}_{s_0}(t) = Mg - F_{\mathrm{mag}_1}(t) - F_{\mathrm{mag}_2}(t), \tag{2}$$

$$J\ddot{\theta}(t) = \frac{l}{2}(F_{\mathrm{mag}_2}(t)) - F_{\mathrm{mag}_1}(t)), \tag{3}$$

where M and g are the suspended mass and gravity acceleration, respectively; $F_{\mathrm{mag}_1}(t)$ and $F_{\mathrm{mag}_2}(t)$ denote the levitation forces of point $\mathcal{A}$ and $\mathcal{B}$, respectively; $J = Ml^2/12$ is the rotary inertia of the levitation module around its centroid. From (1)-(3), we have

$$M\ddot{\delta}_1(t) = Mg - \lambda_1 F_{\mathrm{mag}_1}(t) + \lambda_2 F_{\mathrm{mag}_2}(t), \tag{4a}$$

$$M\ddot{\delta}_2(t) = Mg + \lambda_2 F_{\mathrm{mag}_1}(t) - \lambda_1 F_{\mathrm{mag}_2}(t), \tag{4b}$$

where $\lambda_1 = Ml^2/(4J) + 1$ and $\lambda_2 = Ml^2/(4J) - 1$.

The magnetic forces generated by exciting currents are

$$F_{\mathrm{mag}_1}(t) = \frac{k_a i_1^2(t)}{\delta_1^2(t)}, F_{\mathrm{mag}_2}(t) = \frac{k_a i_2^2(t)}{\delta_2^2(t)}, \tag{5}$$

where $i_1(t)$ and $i_2(t)$ denote the exciting currents of point $\mathcal{A}$ and $\mathcal{B}$, respectively; $k_a = \mu_0 A N^2/4$; μ_0, A and N denote the permeability of air, the magnet pole area, the number of turns of the coil.

Now, the dynamics of the coupled levitation system is rewritten as first-order differential equations as (6). The

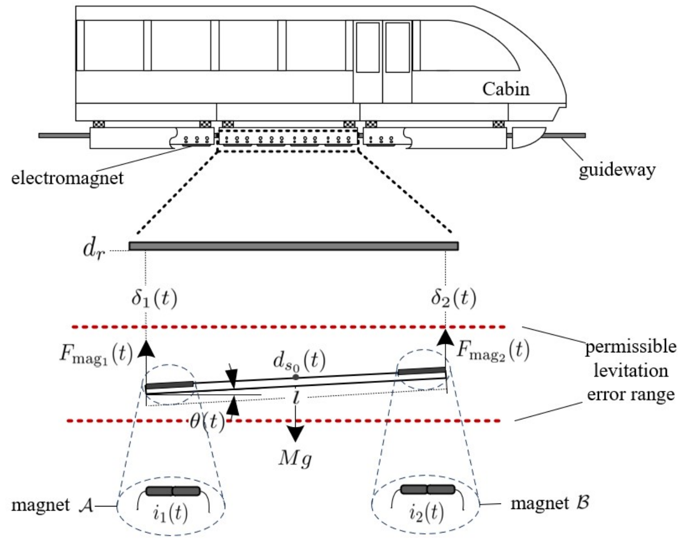

Fig. 1. A simplified diagram of coupled levitation systems.

control input of two levitation points is mutually coupled. If one levitation point adapts the control input, the other point is influenced.

$$\dot{z}_{11} = z_{21}, \tag{6a}$$

$$\dot{z}_{21} = g - 4\frac{k_a}{M}\frac{u_1}{z_{11}^2} + 2\frac{k_a}{M}\frac{u_2}{z_{12}^2}, \tag{6b}$$

$$\dot{z}_{12} = z_{22}, \tag{6c}$$

$$\dot{z}_{22} = g + 2\frac{k_a}{M}\frac{u_1}{z_{11}^2} - 4\frac{k_a}{M}\frac{u_2}{z_{12}^2}. \tag{6d}$$

where $\delta_1(t)$ and $\dot{\delta}_1(t)$ are the system states $z_{11}(t)$ and $z_{21}(t)$ of point $\mathcal{A}$. Similarly, $\delta_2(t)$ and $\dot{\delta}_2(t)$ are the system states $z_{12}(t)$ and $z_{22}(t)$ of point $\mathcal{B}$. Choose $i_1^2(t)$ and $i_2^2(t)$ as the control inputs $u_1(t)$ and $u_2(t)$, respectively. To facilitate the presentation, the time dependence will be omitted in the following without causing confusion. Meanwhile, the control inputs u_1 and u_2 satisfies the following constraint:

$$0 \le u_j \le i_{\max}^2, \quad j = 1, 2, \tag{7}$$

where $i_{\max}$ is the maximum of the permissible control current.

Additionally, the controlled levitation distance for each point needs to satisfy the constraint condition as follows:

$$\begin{aligned} |z_{11} - z_r| &\le \epsilon_{\min}, \\ |z_{12} - z_r| &\le \epsilon_{\min}, \end{aligned} \tag{8}$$

where z_r is the desired levitation distance and $\epsilon_{\min}$ is the maximal permissible controlled error bound, which is a practical need for levitation systems for operational specifications.

Remark 3: The coupled levitation system is a typical two-input-two-output system with mutually coupled dynamics, which calls for the collaboration of both two inputs and causes difficulties in controller design to maintain a safe levitation distance in a small adjustment space.

III. Adaptive Safe Collaboration Design Based on Backtepping

The purpose of this paper is to design an adaptive safe backstepping levitation controller to regulate the levitation distance between the electromagnets and guideways with unknown mass. The superiorities of the proposed controller at least include:

(a) The unknown mass, consisting of the bogie frame, the cabin and the passengers getting on and off, can be addressed, which enables the train to be levitated stably when the mass changes.

(b) The controller ensures the levitation tracking errors are within the permissible range when the parameter uncertainty exists in the system, avoiding physical contact between the guideway and magnets or levitation failure.

This section first proposes an adaptive collaborative controller based on backstepping to achieve the collaboration of the mutually coupled electromagnets without mass knowledge. Based on this controller, adaptive safe backstepping is formulated to construct safe constraints for electromagnets collaboration. To achieve the objective of this paper, the collaborative controller and safe constraints are synthesized by a quadratic program.

A. Adaptive Collaborative Control

In the following, we establish the collaborative controller for the coupled electromagnets via backstepping. To address the unknown mass, we construct a novel Lyapunov function and give the adaptive law.

Define the errors and virtual errors of the two levitation points as follows:

$$\begin{aligned} e_{1j} &= z_{1j} - z_r, \\ e_{2j} &= z_{2j} - \alpha_{1j}, \end{aligned} \tag{9}$$

where z_r is the desired levitation distance, $j = 1, 2$ is the notation of levitation points. α_{1j} is the virtual control law for the j-th levitation point.

Step 1: The Lyapunov function is chosen as follows:

$$V_1 = \sum_{j=1}^{2} \frac{1}{2} e_{1j}^2, \tag{10}$$

Differentiating both sides of V_1, we have

$$\dot{V}_1 = \sum_{j=1}^{2} e_{1j}\dot{e}_{1j} = \sum_{j=1}^{2} e_{1j}(\alpha_{1j} + e_{2j}), \tag{11}$$

Design the virtual control law as

$$\alpha_{1j} = \bar{\alpha}_{1j} = -k_1 e_{1j}, \tag{12}$$

where $k_1 > 0$ is the control parameter. Then, one has $\dot{V}_1 = -2k_1 V_1$.

Step 2: Choose the Lyapunov function as follows:

$$V_2 = V_1 + \sum_{j=1}^{2} \frac{1}{2} M z_{11}^2 z_{12}^2 e_{2j}^2 + \frac{1}{2\Gamma_l} \tilde{M}_l^{\,2}, \tag{13}$$

where $\tilde{M}_l = \hat{M}_l - M$ is the estimation error of the unknown mass, Γ_l is the adaptive gain to be designed. Differentiating both sides of V_2, we have

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \sum_{j=1}^{2} M z_{11}^2 z_{12}^2 e_{2j}\dot{e}_{2j} + \sum_{j=1}^{2} M z_{11} z_{21} z_{12}^2 e_{2j}^2 \\ &\quad + \sum_{j=1}^{2} M z_{11}^2 z_{12} z_{22} e_{2j}^2 + \frac{1}{\Gamma_l}\tilde{M}_l\dot{\hat{M}}_l \\ &= -2k_1 V_1 + \sum_{j=1}^{2} M z_{11}^2 z_{12}^2 e_{2j}\bigg(g - 4\frac{k_a}{M}\frac{u_j}{z_{1j}^2} + 2\frac{k_a}{M}\frac{u_{3-j}}{z_{1(3-j)}^2} \\ &\quad - \dot{\alpha}_1\bigg) + \sum_{j=1}^{2} M(z_{11} z_{21} z_{12}^2 + z_{11}^2 z_{12} z_{22}) e_{2j}^2 + \frac{1}{\Gamma_l}\tilde{M}_l\dot{\hat{M}}_l \\ &= -2k_1 V_1 + \sum_{j=1}^{2} e_{2j}(-4k_a z_{1(3-j)}^2 u_j + 2k_a z_{1j}^2 u_{3-j}) \\ &\quad + M\sum_{j=1}^{2}\beta_j + \frac{1}{\Gamma_l}\tilde{M}_l\dot{\hat{M}}_l, \end{aligned} \tag{14}$$

where $\beta_j = z_{11}^2 z_{12}^2 e_{2j}(g - \dot{\alpha}_1) + (z_{11} z_{21} z_{12}^2 + z_{11}^2 z_{12} z_{22}) e_{2j}^2$.

Design the adaptive law as follows:

$$\dot{\hat{M}}_l = \Gamma_l \sum_{j=1}^{2} \beta_j. \tag{15}$$

Design the adaptive collaborative controller respectively for levitation point $\mathcal{A}$ and $\mathcal{B}$ as

$$\begin{aligned} u_{a1} &= -\frac{\Phi_1 + 2\Phi_2}{6k_a z_{11}^2}, \\ u_{a2} &= -\frac{2\Phi_1 + \Phi_2}{6k_a z_{12}^2}, \end{aligned} \tag{16}$$

where

$$\begin{aligned} \Phi_1 &= -k_2 e_{21} - e_{11} + \hat{M}_l z_{11}^2 z_{12}^2(\dot{\alpha}_{11} - g) \\ &\quad - \hat{M}_l z_{11}^2 z_{21} z_{12}^2 e_{21} - \hat{M}_l z_{11}^2 z_{12} z_{22} e_{21}, \\ \Phi_2 &= -k_2 e_{22} - e_{12} + \hat{M}_l z_{11}^2 z_{12}^2(\dot{\alpha}_{21} - g) \\ &\quad - \hat{M}_l z_{11}^2 z_{21} z_{12}^2 e_{22} - \hat{M}_l z_{11}^2 z_{12} z_{22} e_{22}, \end{aligned}$$

with $k_2 > 0$ is the control parameter.

Let u_1 and u_2 be u_{a1} and u_{a2}, respectively. $\dot{V}_2$ is transformed into:

$$\begin{aligned} \dot{V}_2 &= -k_2\sum_{j=1}^{2} e_{2j}^2 - k_1\sum_{j=1}^{2} e_{1j}^2 \\ &\le -kV_2, \end{aligned} \tag{17}$$

where $k = \min\{2k_1, 2k_2\}$.

Hence, the control inputs u_{a1} and u_{a2} collaborates through Φ_1 and Φ_2 and achieve asymptotic stability of both levitation points. We refer to the inputs u_{a1} and u_{a2} as the nominal controller in the following section.

B. Adaptive Safe Collaboration

To maintain the levitation error of each levitation point within a permissible range, we propose adaptive safe backstepping and establish safe constraints for the levitation points based on the previous adaptive collaborative design. The main result is presented in Theorem 1, which illustrates the invariance of the permissible levitation range.

Denote the maximal permissible levitation error by $\epsilon_{\min}$. Before establishing the constraints w.r.t. input u_1 and u_2, we first introduce a modification term ε_j into the virtual law as

$$\alpha_{1j} = \bar{\alpha}_{1j} + \varepsilon_j. \tag{18}$$

The function of ε_j is a compromise between the adaptive tracking and safe constraints. Meanwhile, a relaxation term $\eta_j\psi_j(e_j)$ is added to the derivative of Lyapunov function $\dot{V}_1$, i.e., $\dot{V}_1 \leq -2k_1V_1 + \sum_{j=1}^{2}\eta_j\psi(e_j)$, where η_j is a design parameter and the bump function ψ refers to [21]. Therefore, the modification term ε_j should satisfy

$$\sum_{j=1}^{2} e_{1j}\varepsilon_j \leq \sum_{j=1}^{2}\eta_j\psi_j(e_{1j}). \tag{19}$$

Rather than constructing a direct high-order CBF, we design two CBFs respectively for each order of the system to guarantee tracking errors of both points within $\epsilon_{\min}$. The CBFs are constructed as follows

$$h_{1j} = \epsilon_{\min}^2 - e_{1j}^2, \tag{20}$$

$$h_{2j} = h_{1j} - \frac{1}{2\mu}Mz_{11}^2z_{12}^2e_{2j}^2 - \frac{1}{2\Gamma_b}\tilde{M}_b^{\,2}, \tag{21}$$

where $\tilde{M}_b = \hat{M}_b - M$, $\hat{M}_b$ is the estimation of M for safe constraints.

Theorem 1: Consider system (6). Assume Assumptions 1 and 2 hold, and $h_{1j}(0) > 0$ and $h_{2j}(0) > 0$. Given the adaptive law

$$\dot{\hat{M}}_b = \frac{\Gamma_b}{\mu}\beta_j, \tag{22}$$

and ε_j satisfying

$$2e_{1j}\varepsilon_j \leq \gamma h_{1j} + 2k_1e_{1j}^2, \tag{23}$$

and the controller satisfying

$$\boldsymbol{u} \in S_b = \{\boldsymbol{u} \in R^2 | \frac{2k_a}{\mu}e_{2j}\boldsymbol{b}_j\boldsymbol{u} \leq \Xi_j\}, \tag{24}$$

where $\gamma > 0$ is a design parameter, $\boldsymbol{u} = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^{\mathrm{T}}$, $b_1 = \begin{bmatrix} -2z_{12}^2 & z_{11}^2 \end{bmatrix}$, $b_2 = \begin{bmatrix} z_{12}^2 & -2z_{11}^2 \end{bmatrix}$, and

$$\begin{aligned}\Xi_j = {} & \gamma(h_{1j} - \frac{1}{2\mu}(\hat{M}_b + \bar{\tilde{M}}_b)z_{11}^2z_{12}^2e_{2j}^2 - \frac{1}{2\Gamma_b}\bar{\tilde{M}}_b^2) \\ & - 2e_{1j}z_{2j} - \frac{1}{\mu}\hat{M}_b\beta_j,\end{aligned}$$

$\bar{\tilde{M}}_b$ is the estimation error bound of M, both levitation points will never violate the maximal permissible levitation errors.

Proof: Differentiating (20) gets

$$\dot{h}_{1j} = -2e_{1j}\dot{e}_{1j} = -2e_{1j}z_{2j}. \tag{25}$$

In the backstepping process, z_{2j} is designed to track α_{1j}. When $z_{2j} = \alpha_{1j}$, one has

$$\dot{h}_{1j} = -2e_{1j}(-k_1e_{1j} + \varepsilon_j), \tag{26}$$

Because the modification term ε_j satisfies (23), it further gets

$$\dot{h}_{1j} \geq -\gamma h_{1j}, \tag{27}$$

Moreover, the derivative of (21) is

$$\begin{aligned}\dot{h}_{2j} = {} & \dot{h}_{1j} - \frac{1}{\mu}e_{2j}\dot{e}_{2j} - \frac{1}{\mu}M(z_{11}z_{21}z_{12}^2 + z_{11}^2z_{12}z_{22})e_{2j}^2 \\ & - \frac{1}{\Gamma_b}\tilde{M}_b\dot{\hat{M}}_b \\ = {} & \dot{h}_{1j} - \frac{1}{\mu}e_{2j}Mz_{11}^2z_{12}^2(g - 4\frac{k_a}{M}\frac{u_j}{z_{1j}^2} + 2\frac{k_a}{M}\frac{u_{3-j}}{z_{1(3-j)}^2} \\ & - \dot{\alpha}_{1j}) - \frac{1}{\mu}M(z_{11}z_{21}z_{12}^2 + z_{11}^2z_{12}z_{22})e_{2j}^2 - \frac{1}{\Gamma_b}\tilde{M}_b\dot{\hat{M}}_b \\ = {} & \dot{h}_{1j} + \frac{4k_a}{\mu}e_{2j}z_{1(3-j)}^2u_j - \frac{2k_a}{\mu}e_{2j}z_{1j}^2u_{3-j} \\ & - \frac{1}{\mu}e_{2j}M\beta_j - \frac{1}{\Gamma_b}\tilde{M}_b\dot{\hat{M}}_b.\end{aligned} \tag{28}$$

Substituting (22) into (30), it comes to

$$\begin{aligned}\dot{h}_{2j} = {} & \dot{h}_{1j} + \frac{4k_a}{\mu}e_{2j}z_{1(3-j)}^2u_j - \frac{2k_a}{\mu}e_{2j}z_{1j}^2u_{3-j} \\ & - \frac{1}{\mu}e_{2j}\hat{M}_b\beta_j.\end{aligned} \tag{29}$$

Since $\boldsymbol{u} \in S_b$, we have

$$\begin{aligned}\dot{h}_{2j} \geq {} & -\gamma\big(h_{1j} - \frac{1}{2\mu}(\hat{M}_b + \bar{\tilde{M}}_b)z_{11}^2z_{12}^2e_{2j}^2 - \frac{1}{2\Gamma_b}\bar{\tilde{M}}_b^2\big) \\ \geq {} & -\gamma h_{2j}.\end{aligned} \tag{30}$$

Using Definition 3 in [13], it can be concluded that h_{2j} is a control barrier function, and (27) and (30) imply $h_{1j} > 0$ and $h_{2j} > 0$. ■

Remark 4: The selection of ε_j can be calculated by Gaussian weighted centroid functions in [19] to get smooth values.

Remark 5: Set membership identification provides a method to estimate the range of unknown parameters, which is monotonously decreasing. Projecting $\hat{M}_b$ on the converging range gets the estimation of bound $\bar{\tilde{M}}_b$.

C. Adaptive Safe Synthesis

To achieve asymptotic stability of both levitation points in the safe sense, it necessitates the synthesis of the adaptive collaborative controller and safe set given in Theorem 1. The optimization-based controller is synthesized through a quadratic program as follows:

$$\mathbf{u}^* = \underset{\mathbf{u}\in \mathrm{R}^2}{\arg\min}\, \|\mathbf{u} - \mathbf{u_a}\|^2 \tag{31}$$

$$\begin{aligned}\text{s.t.} \quad & \tfrac{2k_a}{\mu}e_{2j}\boldsymbol{b}_j\boldsymbol{u} \leq \Xi_j,\ j = 1, 2, \\ & 0 \leq u_j \leq i_{\max}^2, j = 1, 2,\end{aligned} \tag{32}$$

where $\boldsymbol{u_a} = \begin{bmatrix} u_{a1} & u_{a2} \end{bmatrix}^{\mathrm{T}}$.

After the optimization, adaptive safe control for the coupled electromagnets is achieved by deviating from $\mathbf{u_a}$ with the minimal distance and satisfying the safe set.

IV. Simulation Results

To demonstrate the effectiveness of the proposed control for collaborative contol of maglev trains considering unknown mass variation, a series of simulations are conducted. The parameters of maglev trains are $N = 320$, $A = 0.0235\text{m}^2$, $M = 1750\text{kg}$, $\mu_0 = 4\pi\times10^{-7}$, $g = 9.8\text{m/s}^2$ and $l = 3.1\text{m}$. The desired levitation distance is set to 10mm. The initial conditions of two levitation points are set to $z_{11} = 0.007\text{m}$, $z_{21} = 0\text{m/s}$, $z_{12} = 0.013\text{m}$ and $z_{22} = 0\text{m/s}$.

To verify the proposed adaptive safe collaboration strategy, the adaptive collaborative control are conducted for comparison. The parameters of both controllers are presented in Table I. The permissible controlled error comforms to the practical constraints, i.e., $\epsilon_{\min} = 0.004\text{m}$. If the controlled error exceeds $\epsilon_{\min}$, it may cause physical contact or levitation failure.

TABLE I
CONTROLLER PARAMETERS

Adaptive Safe Collaboration					
Parameters	γ	η_1	η_2	μ	Γ_b
Value	20	100	100	80000	0.005
Adaptive Collaborative Control					
Parameters	k_1	k_2	Γ_l	–	–
Value	10	50000	0.000005	–	–

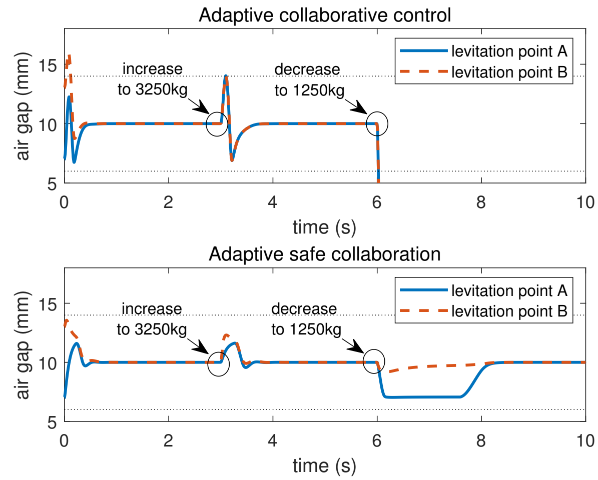

Fig. 2. Levitation distance of the mutually coupled electromagnets with the mass variation under two control schemes.

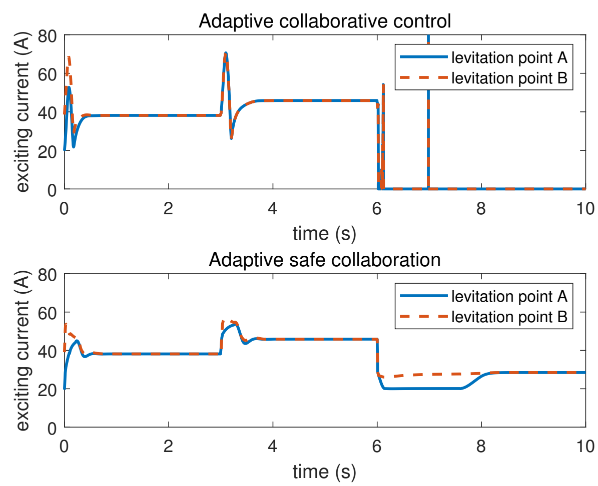

Fig. 3. Control input of each electromagnet with the mass variation under two control schemes.

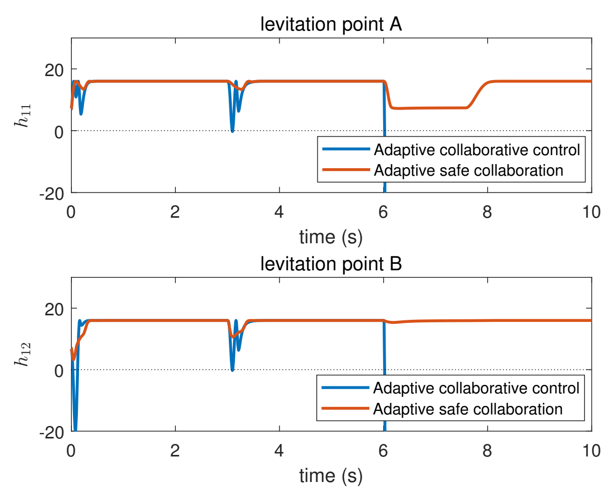

Fig. 4. Value of control barrier function h_{1j}.

Fig. 2 presents the levitation distance evolution of the two mutually coupled suspension points with the mass variation under two control schemes. In the transient process, the adaptive safe collaboration strategy maintains the levitation distance of both points, while the adaptive collaborative controller exceeds the permissible error range. When the system mass increase by 1000kg, the former controller approaches the error boundary but the proposed safe one keeps away from the boundary. Unfortunately, when the mass decrease to 1250kg, the former one fails to sustain the stability. Clearly, the proposed adaptive safe collaboration strategy outperforms a pure adaptive collaborative controller in addressing mutually coupled electromagnets.

From Fig. 3, the adaptive safe collaboration control outputs a stable current when the mass changes, while the input signal of the adaptive collaborative controller presents drastic changes and even cannot sustain stability.

Fig. 4 depicts the evolution of the control barrier function for two levitation points. It shows that $h_{1j} \geq 0$ holds under the adaptive safe controller, while the pure adaptive collaborative control cannot satisfy this condition.

Fig. 5 shows that the estimated boundaries of the system mass for the control barrier function via set membership identification.

V. Conclusions

Considering that the levitation system of maglev trains is mutually coupled and safety-critical, we have proposed

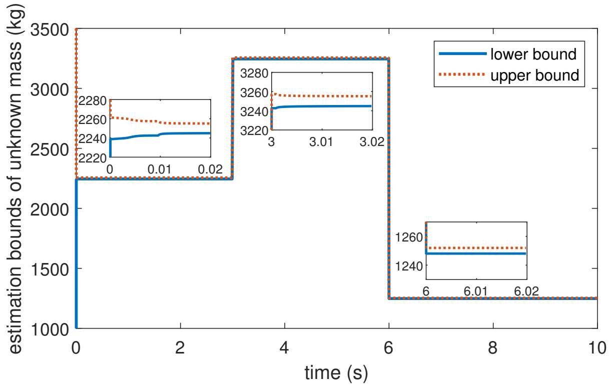

Fig. 5. Estimated boundaries of the system mass by set menbership identification.

an adaptive safe collaboration control strategy under system mass variation. In the beginning, a model for two coupled electromagnets has been formulated with practical constraints. To address adaptive collaboration, we have proposed a novel adaptive collaborative control by constructing a novel Lyapunov function and an adaptive law for the unknown mass. Then, a control barrier function based on adaptive safe backstepping is newly built to derive a safe set for control inputs thus ensuring the controller errors are within the permissible range. To achieve the goals of asymptotic stability and safety of the coulpled levitation system, a QP is formulated to synthesize the control. Comparative simulation results demonstrate that the adaptive safe collaboration outperforms the pure adaptive collaborative controller for the mutually coupled system.

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