2023 62nd IEEE Conference on Decision and Control (CDC)
December 13-15, 2023. Marina Bay Sands, Singapore

About a possible High Poles Observer design instead of High Gain for triangular nonlinear systems

Gildas Besançon

Abstract— **This paper is about observer design for a class of nonlinear systems admitting a possible solution via the well-known high-gain technique. Here instead, a design based on full pole placement for the linear part of the observer is discussed. In particular it is shown that exponential convergence of the observation error is guaranteed in spite of nonlinearities, for poles chosen large enough, with observer gains which can be lower than in a standard high gain approach. A focus is given on systems up to order three, and a discussion on measurement noise effect is provided. Simulation examples are included to illustrate the conclusions.**

Keywords—**Observer design, nonlinear systems, triangular form, high gain, pole placement.**

I. INTRODUCTION

Observer design for nonlinear systems is a problem still motivating a lot of research. Among the various available solutions so far [6], [7, ...], is the famous High Gain Observer (HGO) approach [10], allowing for arbitrarily fast convergence via a single tuning parameter, but for which the main drawback w.r.t. measurement noise is also well-known (see e.g. [12]). This has generated various refinements in the design so as to limit the adverse effect of this noise, such as gain adaptation (as in [1], [2], [9], [16] for instance), power gain limitation [4], [5, ...] or LMI-based techniques [17].

In the present paper, the primary motivation shares the same goal of minimizing the noise effect in this high-gain framework: noting indeed that the noise amplification problem is related to the design of the scalar parameter to be chosen large enough so as to dominate all nonlinear growth rates, the main idea here is that this amplification may be limited if one can distribute the domination of each nonlinearity into various gains, or can even take advantage of some nonlinearities which are in fact in favour of observer convergence. In that regard, the method here proposed can be related to so-called 'backstepping observers', for instance as in [14], or to dissipativity-based approaches (see e.g. [15], [3]) to some extent. The design then results in selecting an observer gain directly via observer poles (referring to the linear part of the system), to be still chosen large enough, but now each of them w.r.t. to each nonlinearity. In that sense, the design is turned into some 'High Poles' approach, instead of High Gain.

Gildas Besançon is with Univ. Grenoble Alpes, CNRS, Grenoble INP*, GIPSA-lab, Grenoble, France - email address: gildas.besancon@grenoble- inp.fr
* Graduate Schools of Engineering and Management

The paper continues as follows: section II first presents the considered class of systems and the proposed *High Poles* design, and section III discusses it, in particular regarding noise effect. Section IV then provides some illustrative examples, and section V finally concludes the paper.

II. HIGH POLES OBSERVER DESIGN

Let us consider systems under some canonical form as follows (typical of uniformly observable systems [11] admitting high gain observers):

$$\begin{array}{rcl} \dot{x}(t) & = & A_0x(t)+B(x(t),u(t)) \\ y(t) & = & C_0x(t) \\ \text{with} & & \\ A_0 & = & \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 1 & \ddots & \vdots \\ & & \ddots & \ddots & 0 \\ \vdots & & & 0 & 1 \\ 0 & \cdots & & \cdots & 0 \end{pmatrix} \\ C_0 & = & \begin{pmatrix} 1 & 0 & \cdots & & 0 \end{pmatrix} \end{array} \quad (1)$$

and each component B_i of B depending on components x_j's of x as:

$$B_i(x,u) = B_i(x_1,\cdots,x_i,u) \quad (2)$$

where $x \in I\!R^n,\ u \in I\!R^m,\ y \in I\!R$.
Let us recall that under a Lipschitz condition on B, system (1)-(2) admits a High Gain Observer as:

$$\begin{array}{rcl} \dot{\hat{x}}(t) & = & A_0\hat{x}(t)+B(\hat{x}(t),u(t))-\Lambda(\lambda)K_0(C_0\hat{x}(t)-y(t)) \\ \text{with} & & \\ \Lambda(\lambda) & = & \begin{pmatrix} \lambda & & 0 \\ & \ddots & \\ 0 & & \lambda^n \end{pmatrix} \\ K_0 & : & A_0-K_0C_0 \text{ is Hurwitz} \end{array} \quad (3)$$

and λ large enough (in particular w.r.t. Lipschitz constant of B).
In the present paper, the discussion is mostly restricted to systems of dimension up to 3 ($x \in I\!R^3$) for the sake of clarity, but we claim that the methodology extends to any n.
Let us start with second order systems:

Theorem 1: Given a system (1)-(2) with $x \in I\!R^2$, $u \in U \subset I\!R^m$, and B made of Lipschitz components B_1, B_2 with constants γ_1, γ_2 respectively, uniform w.r.t. u
(i.e. for $i=1,2$, $\forall \xi,\zeta, |B_i(\xi,u)-B_i(\zeta,u)| \leq \gamma_i|\xi-\zeta|$ for any $u \in U$),

979-8-3503-0123-6/23/$31.00 ©2023 IEEE

there exist $\lambda_1 > \gamma_1$, $\lambda_2 > \gamma_2$ large enough so that the following system is an exponential observer for (1)-(2) :

$$\begin{array}{rcl} \dot{\hat{x}}(t) & = & A_0\hat{x}(t) - K(C\hat{x}(t) - y(t)) + B(\bar{x}(t), u(t)) \\ K & = & \begin{pmatrix} -\lambda_1 - \lambda_2 \\ -\lambda_1\lambda_2 \end{pmatrix} \\ \bar{x}(t) & = & \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 - \lambda_2(\hat{x}_1 - y) \end{pmatrix} \end{array} \tag{4}$$

where $\hat{x}_i$ stands for component i of $\hat{x}$. •

Here exponential observer means that $\|\hat{x}(t) - x(t)\|$ exponentially decays to 0 when t grows to infinity (where $\|.\|$ refers to euclidean norm). The result is global if the Lipshitz property of B holds globally (otherwise, one just has to consider some global Lipschitz extension of B as usual in high gain observer design [10]).

Notice that in this design, $-\lambda_1$, $-\lambda_2$ are the eigenvalues (or poles) of the linear part in the observer error system (i.e. eigenvalues of matrix $A_0 - KC_0$). Notice also that when $\lambda_1 = \lambda_2 = \lambda$ (and $\bar{x}$ is set to $\hat{x}$) we clearly recover the classical High Gain Observer (for $K_0 = [2\ 1]^T$).

Picking different values for λ_1 and λ_2 allows to split the domination of an overall Lipschitz constant of nonlinear vector function B by a single high gain, into 2 gains chosen to dominate the 2 Lipschitz components of this nonlinear function.

More precisely, if we define by γ_{ij} the Lipschitz constant of B_i w.r.t. its argument x_j uniformly in its other arguments (γ_{11} for B_1, γ_{21}, γ_{22} for B_2), a sufficient condition for choosing gains λ_1, λ_2 is that they should satisfy:

$$\begin{array}{l} M(\lambda_1, \lambda_2) > 0, \\ \text{with } M(\lambda_1, \lambda_2) := \\ \begin{pmatrix} \lambda_1 - \gamma_{11} & -(\lambda_2\gamma_{11} + \gamma_{21} + 1)/2 \\ -(\lambda_2\gamma_{11} + \gamma_{21} + 1)/2 & \lambda_2 - \gamma_{22} \end{pmatrix} \end{array} \tag{5}$$

This obviously needs $\lambda_i > \gamma_{ii}$, and can be achieved by λ_1 large enough (in particular such that $\lambda_1 > \lambda_2$).

Notice that when B_i does not depend on x_j, γ_{ij} reduces to 0. In particular, if $B_i(x, u) = B_i(x_i, u)$, then one just needs:

$$4(\lambda_1 - \gamma_{11})(\lambda_2 - \gamma_{22}) > (1 + \lambda_2\gamma_{11})^2.$$

Similarly, if $B_1(x, u) = B_1(u)$, then the condition reduces to:

$$4\lambda_1(\lambda_2 - \gamma_{22}) > (1 + \gamma_{21})^2.$$

Proof: The result of theorem 1 can be obtained as follows:

$$\begin{array}{rcl} \text{Set : e}_1 & := & \hat{x}_1 - x_1 \\ e_2 & := & \hat{x}_2 - x_2 - \lambda_2 e_1 \end{array} \tag{6}$$

$$\begin{array}{rcl} \text{Then : } \dot{e}_1 & = & -\lambda_1 e_1 + e_2 + \Delta B_1 \\ \dot{e}_2 & = & -\lambda_2 e_2 - \lambda_2\Delta B_1 + \Delta B_2 \end{array} \tag{7}$$

where $\Delta B_i = B_i(\bar{x}, u) - B_i(x, u)$.

By simply choosing as a candidate Lyapunov function for this error system

$$V(e) = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 \tag{8}$$

we easily get:

$$\dot{V} \leq -e^T M(\lambda_1, \lambda_2) e \tag{9}$$

with M as in (5), and by the 2nd method of Lyapunov we get the conclusion on exponential stability for $e = (\,e_1 \quad e_2\,)^T$, hence also for $(\,\hat{x}_1 - x_1 \quad \hat{x}_2 - x_2\,)^T$. ■

It can be noticed that from error equations (7), each observer pole λ_i is to be chosen depending on ΔB_i: in particular, if ΔB_i is in favour of stability, there is no need to dominate it, and λ_i can be chosen lower. If for instance $\Delta B_1 = \Delta\bar{B}_1 + \Delta\tilde{B}_1$, with $e_1\Delta\tilde{B}_1 \leq 0$, λ_1 just needs to dominate the Lipschitz constant of $\bar{B}_1$. This allows to reduce this gain λ_1. This feature is similar to the one used in the famous backstepping approach, and it turns out that the step-by-step construction of (6) can be extended to systems of higher dimension, in this backstepping spirit. For the sake of illustration, let us provide similarly the result for dimension 3 as follows:

Theorem 2: Given a system (1)-(2) with $x \in I\!R^3$, $u \in U \subset I\!R^m$, and B made of Lipschitz components B_1, B_2, B_3 with constants γ_1, γ_2, γ_3 respectively, uniform w.r.t. u there exist $\lambda_i > \gamma_i$ (for $i = 1, 2, 3$) large enough so that the following system is an exponential observer for (1)-(2) :

$$\begin{array}{rcl} \dot{\hat{x}}(t) & = & A_0\hat{x}(t) - K(C\hat{x}(t) - y(t)) + B(\bar{x}(t), u(t)) \\ K & = & \begin{pmatrix} -\lambda_1 - \lambda_2 - \lambda_3 \\ -\lambda_1\lambda_2 - \lambda_2\lambda_3 - \lambda_1\lambda_3 \\ -\lambda_1\lambda_2\lambda_3 \end{pmatrix} \\ \bar{x}(t) & = & \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 - \lambda_2(\hat{x}_1 - y) \\ \hat{x}_3 - \lambda_2\lambda_3(\hat{x}_1 - y) \end{pmatrix} \end{array} \tag{10}$$

where $\hat{x}_i$ stands for component i of $\hat{x}$. •

Practically, a sufficient condition for the choice of coefficients λ_i's (absolute values of poles of the error system) is that they satisfy (with same notations γ_{ij} as above):

$$\begin{array}{rcl} M(\lambda_1, \lambda_2, \lambda_3) & > & 0, \quad \text{with} \\ M(\lambda_1, \lambda_2, \lambda_3) & := & \begin{pmatrix} \lambda_1 - \gamma_{11} & m_{12} & m_{13} \\ m_{12} & \lambda_2 - \gamma_{22} & m_{23} \\ m_{13} & m_{23} & \lambda_3 - \gamma_{33} \end{pmatrix} \\ m_{12} & = & -((\lambda_2 + \lambda_3)\gamma_{11} + \lambda_3\gamma_{22} + \gamma_{21} + 1)/2; \\ m_{13} & = & -(\lambda_3(2\lambda_2 + \lambda_3)\gamma_{11} + \lambda_3^2\gamma_{22} \\ & & +\lambda_3(\gamma_{21} + \gamma_{32}) + \lambda_{31})/2; \\ m_{23} & = & -(\lambda_3(\gamma_{22} + \gamma_{33}) + \gamma_{32} + 1)/2. \end{array} \tag{11}$$

Once again, condition (11) clearly needs $\lambda_i > \gamma_{ii}$, and in addition, for the general case of nonzero γ_{ij}, $\lambda_1 > \lambda_2 > \lambda_3$ all large enough, by considering principal minors and Sylvester criterion for positive definite matrices.

Proof: The proof of theorem 2 goes similarly to that of theorem 1, as follows:

$$\begin{array}{rcl} \text{Set : e}_1 & := & \hat{x}_1 - x_1 \\ e_2 & := & \hat{x}_2 - x_2 - (\lambda_2 + \lambda_3)e_1 \\ e_3 & := & \hat{x}_3 - x_3 - \lambda_3 e_2 - \lambda_2\lambda_3 e_1 \end{array} \tag{12}$$

$$\begin{array}{rcl} \text{Then : } \dot{e}_1 &=& -\lambda_1 e_1 + e_2 + \Delta B_1 \\ \dot{e}_2 &=& -\lambda_2 e_2 + e_3 - (\lambda_2+\lambda_3)\Delta B_1 + \Delta B_2 \\ \dot{e}_3 &=& -\lambda_3 e_3 - \lambda_3(2\lambda_2+\lambda_3)\Delta B_1 - \lambda_3 \Delta B_2 \\ && +\Delta B_3 \end{array} \quad (13)$$

Choosing again as an immediate candidate Lyapunov function

$$V(e) = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 \quad (14)$$

and using Lipschitz properties of B_i's with definitions (12) to get:

$$\begin{array}{rcl} \|\Delta B_1\| &\leq& \gamma_{11}|e_1| \\ \|\Delta B_2\| &\leq& \gamma_{22}|e_2| + (\gamma_{21}+\lambda_3\gamma_{22})|e_1| \\ \|\Delta B_3\| &\leq& \gamma_{33}|e_3| + (\gamma_{32}+\lambda_3\gamma_{33})|e_2| \\ && +(\gamma_{31}+\lambda_3\gamma_{32})|e_1| \end{array} \quad (15)$$

we can obtain:

$$\dot{V} \leq -e^T M(\lambda_1, \lambda_2, \lambda_3) e \quad (16)$$

with M as in (11), and the conclusion follows. ∎

Our conjecture at this point is that the result extends to any dimension n, with $K = (\, k_1 \quad k_2 \quad \cdots \quad k_n \,)^T$ where k_i's are coefficients of a polynome $P(s) = \Pi_{i=1}^n (s+\lambda_i)$ with roots $-\lambda_i$'s such that, in the general case $\lambda_1 > \lambda_2 > \cdots > \lambda_1 > 0$ to be chosen large enough (each of them in particular larger than each Lipschitz constant γ_{ii} of component B_i of B w.r.t. x_i).

In that general case, $\bar{x}$ keeps the same structure as above, with each component i of the form:

$$\bar{x}_i = \hat{x}_i - \lambda_i \lambda_{i+1} ... \lambda_n (\hat{x}_1 - y)$$

and by the fundamental theorem of symmetric polynomials,

$$k_{n-i} = \sum_{1 \leq j_1 < j_2 < ... < j_i \leq n} \lambda_{j_1}\lambda_{j_2}\cdots\lambda_{j_i}, \quad i = 1, ..n$$

III. OBSERVER DISCUSSION

As a first remark, it is clear that the approach extends to more general triangular forms, e.g. with non constant coefficients in A_0 (as in [14] for instance).

It is also clear, from the Lyapunov-based results establishing theorems 1 and 2, that the convergence rate of the proposed High Poles Observer (HPO) is directly tuned via the chosen poles λ_i (the larger they are, the faster the convergence can be).

But more importantly, it appears that in the proposed design, by selecting the observer gains via observer poles, we can adjust them according to each nonlinearity on the one hand, and take advantage of terms in favour of observer convergence on the other hand: both of those features result in possibly reducing the gains suitably. This in turn should be of interest for the observer behaviour w.r.t. measurement noise.

In order to study this aspect, let us focus on the case of second order systems, that is on result of theorem 1, when the output reads:

$$y = Cx + w \quad (17)$$

for some measurement noise w, assumed to admit an upper bound W.

Then defining the observer error as $e := \hat{x} - x$, the error system becomes:

$$\dot{e} = (A_0 - KC)e + \Delta B + \begin{pmatrix} \lambda_1 + \lambda_2 \\ \lambda_1 \lambda_2 \end{pmatrix} w \quad (18)$$

It is known that in the case of High Gain design (here $\lambda_1 = \lambda_2 = \lambda$), the observer error is in norm additively affected by w with a magnitude of order λW. More precisely, the additive effect of w on $|\hat{x}_1 - x_1|$ is proportional to W, and on $|\hat{x}_2 - x_2|$, proportional to λW (see e.g. [5]): this explains the increasing sensitivity to the noise, when the high gain is increased.

Let us check what happens when $\lambda_1 = \gamma\lambda_2$ in the HPO.

In that case we obtain an error similar to that of a High Gain Observer (HGO) of the form:

$$\begin{array}{rcl} \dot{e} &=& (A_0 - \Lambda(\lambda_2)K_\gamma C)e + \Delta B + \Lambda(\lambda_2)K_\gamma w \\ \text{with } \Lambda(\lambda_2) &=& \begin{pmatrix} \lambda_2 & 0 \\ 0 & \lambda_2^2 \end{pmatrix} \\ \text{and } K_\gamma &=& \begin{pmatrix} 1+\gamma \\ \gamma \end{pmatrix} \end{array} \quad (19)$$

Studying only the effect of noise w, let us omit ΔB for the error analysis, and consider P_γ such that

$$(A_0 - K_\gamma C)^T P_\gamma + P_\gamma(A_0 - K_\gamma C) = -I.$$

Then it can be checked that:

$$P = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & (2+\gamma)/(2\gamma) \end{pmatrix} \quad (20)$$

Considering the 'HGO Lyapunov function' $V(e) = e^T \Lambda(\lambda_2)^{-1} P \Lambda(\lambda_2)^{-1} e$, and using the property that

$$\Lambda(\lambda_2)^{-1}(A_0 - \Lambda(\lambda_2)K_\gamma C)\Lambda(\lambda_2) = \lambda_2(A_0 - K_\gamma C)$$

we can get (with similar computations as in HGO analysis):

$$\begin{array}{rcl} \dot{V} &\leq& -\lambda_2\|\Lambda(\lambda_2)^{-1}e\|^2 + 2e^T\Lambda(\lambda_2)^{-1}P_\gamma K_\gamma w \\ &=& -\lambda_2\|\Lambda(\lambda_2)^{-1}e\|^2 + e^T\Lambda(\lambda_2)^{-1}\begin{pmatrix} 1 \\ 1 \end{pmatrix} w \\ &\leq& -\|\Lambda(\lambda_2)^{-1}e\|(\lambda_2\|\Lambda(\lambda_2)^{-1}e\| - \sqrt{2}W) \end{array}$$

From this, we obtain that $|e_1|$ is ultimately bounded by $\sqrt{2}W$, and $|e_2|$ by $\lambda_2\sqrt{2}W$.

This looks similar to the HGO result, except that λ_2 may be chosen smaller than the unique gain λ of HGO, and thus the effect of noise may be attenuated.

This is illustrated in next section.

IV. ILLUSTRATIVE EXAMPLES

A. Pendulum example

For the sake of illustration, let us first consider the simple case of pendulum equations as follows:

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & -a_1 sin(x_1) - a_2 x_2 + bu \\ y & = & x_1 + w \end{array} \qquad (21)$$

where x_1 corresponds to the angular position, x_2 to the angular velocity, u to some external torque, w some measurement noise, and a_1, a_2, b, some positive known constants characterizing the pendulum.
It is obvious that for this simple case, when neglecting noise w, the error linearization method can be used by direct output injection, as in [13] for instance.
But owing to the fact that the system is clearly of the form (1), with B clearly globally Lipschitz (and with obvious constant $\rho = max(a_1, a_2)$), let us instead consider an option without output injection, and compare HGO with HPO design.
In the HGO approach, the gain must be chosen roughly larger than ρ, while in the HPO approach, one can take advantage of the fact that $-a_2 x_2$ is obviously here in favour of an observer error, and just rely on Lipschitz constant $\gamma_{11} = a_1$. If $a_1 < a_2$, then this second approach will be with a power lower than that of HGO.
For the comparison, let us consider a theoretical condition on High Gain selection for HGO design, for instance as in [5]: it should typically be larger than $2\|P\|\rho$, with P e.g. as in (20) (with $\gamma = 1$ for this HGO case). This means $2\|P\| = 2 + \sqrt{2}$.
For numerical illustration, let us consider here the following values:

$$a_1 = 2, \quad a_2 = 3, \quad b = 1, \quad u(t) = 2sin(2t)$$

and a noise simulated as a band-limited white noise with intensity equal to 0.5 (see Figure 1).

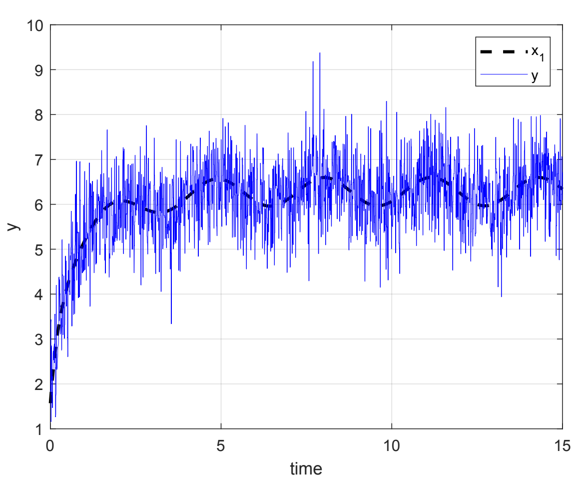

Fig. 1. x_1 vs noisy measurement.

Since in this case Lipschitz constant $\rho = 3$, we chose here $\lambda = 11$ in the HGO to satisfy the theoretical condition. The corresponding estimation results are displayed in Figure 2, where a strong noise effect can be seen.

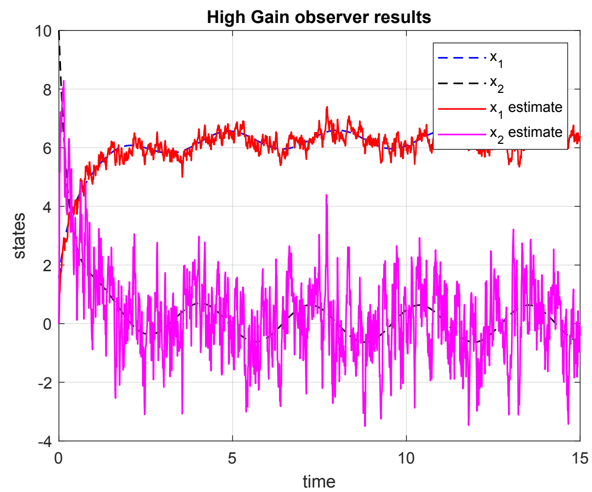

Fig. 2. HGO estimation results with $\lambda = 11$.

On the other hand, in the HPO approach, we just need here

$$4\lambda_1(\lambda_2 + a_2) > (1 + a_1)^2$$

hence $\lambda_1(\lambda_2 + 3) > 2.25$.
For instance $\lambda_2 = 0$ and $\lambda_1 = 1$ can do the job, and the corresponding estimation results are shown in Figure 3: in that case, the noise is significantly filtered out.
Of course, the convergence speed is slower than with the HGO, but it can be increased by increasing λ_1, λ_2, as shown in Figure 4, corresponding to $\lambda_1 = 10$, $\lambda_2 = 2$, and where the convergence rate is indeed significantly increased, while the noise effect is still pretty low.

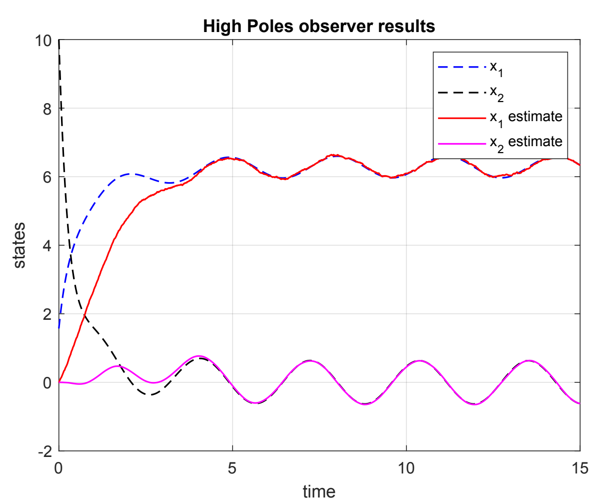

Fig. 3. HPO estimation results with $\lambda_1 = 1$, $\lambda_2 = 0$.

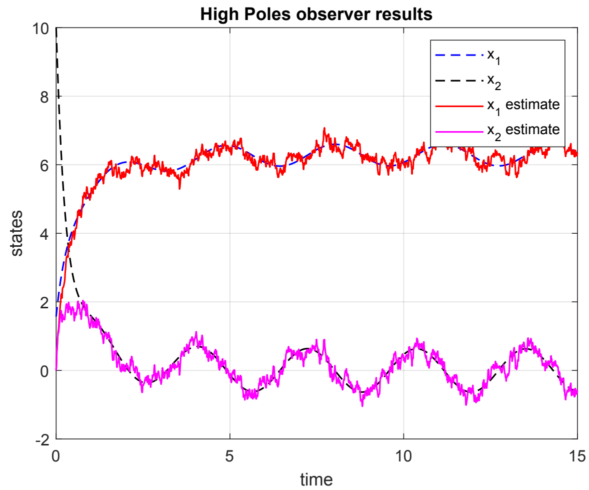

Fig. 4. HPO estimation results with $\lambda_1 = 10$, $\lambda_2 = 2$.

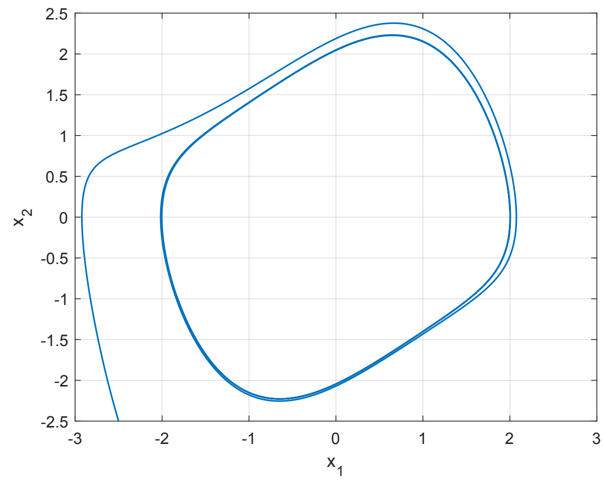

Fig. 5. Example of Van der Pol state trajectory.

B. Van der Pol oscillator

As a second example, let us consider the well-known model of Van der Pol oscillator, possibly driven by some input u, as follows:

$$\ddot{z}(t) = -\alpha^2 z(t) + \beta(1 - z^2(t))\dot{z}(t) + u(t) \tag{22}$$

for positive constant parameters α, β.
Considering that z is measured (with additive measurement noise w again), the model again reads as (1):

$$\begin{array}{rcl} \dot{x}_1 &=& x_2 \\ \dot{x}_2 &=& -\alpha^2 x_1 + \beta(1 - x_1^2)x_2 + u =: B_2(x) + u \\ y &=& x_1 + \nu \end{array} \tag{23}$$

Omitting again other possible designs (e.g. as in [8]), let us focus on HGO and HPO: clearly, here, the HGO needs a gain to dominate the Lipschitz constant of B_2, while the proposed High Poles based design can for instance take advantage of the fact that both $-\beta x_1^2 x_2$ and $-\alpha^2 x_1$ can be rather in favour of observer convergence, allowing in turn to reduce the required observer gain.
Let us consider first the autonomous behaviour ($u = 0$), with $\alpha = 1$, $\beta = 0.5$, and initial conditions so that state trajectories remain in $[-3, 3] \times [-2.75, 2.75]$, i.e. $|x_1(t)| \leq Y$ and $|x_2(t)| \leq Y_d$, with $Y = 3$, $Y_d = 2.75$ (see Figure 5).
Then, following the construction of HPO previously discussed, it can be checked that we just need to select λ_1, λ_2 here such that:

$$\alpha^2 \lambda_1(\lambda_2 - \beta) > (\beta Y Y_d)^2$$

while the HGO theoretically needs a gain

$$\lambda > (2 + \sqrt{2}) max(\alpha^2 + 2\beta Y Y_d, \beta(Y^2 + 1)).$$

Numerically, one can choose $\lambda = 32$, and $\lambda_1 = 5.5$, $\lambda_2 = 4$ for instance.
The corresponding estimation results, simulated with a band-limited white noise of intensity equal to 0.01, are shown in Figures 6-7 where, as obviously expected, the HPO is much less sensitive to noise than HGO.

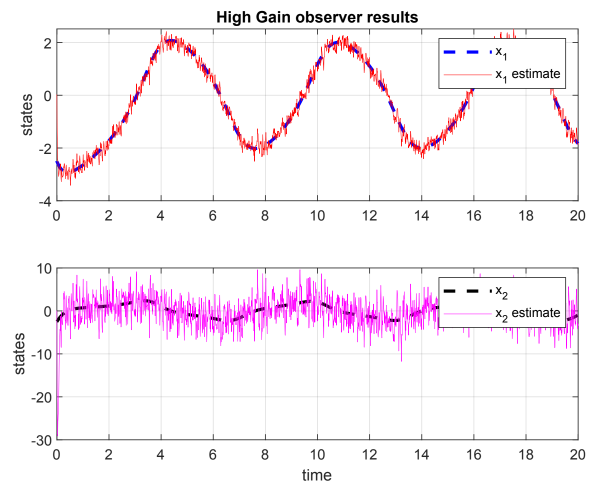

Fig. 6. HGO estimation results with $\lambda = 32$.

Finally, for the purpose of illustrating also the third order case, let us consider a scenario when u is now unknown, in the simplest case of a constant signal.
By setting $x_3 := u$, and $\dot{x}_3 = 0$, the model reads once more as (1), and the proposed High Poles Observer still applies, and can still be implemented with a smaller gain as compared to the standard HGO. Corresponding simulation results can be seen in Figure 8 for HGO case, and Figure 9 for HPO one, under a step variation of unknown input u at time 20 (Figure 10 shows that corresponding states remain within the considered bounds).

V. CONCLUSIONS

In this paper, a possible pole placement approach instead of single high gain design for nonlinear observer solution has been discussed, emphasizing its interest for gain reduction, and in turn noise limitation. Connections with backstepping-like design have also been commented, and extensions to

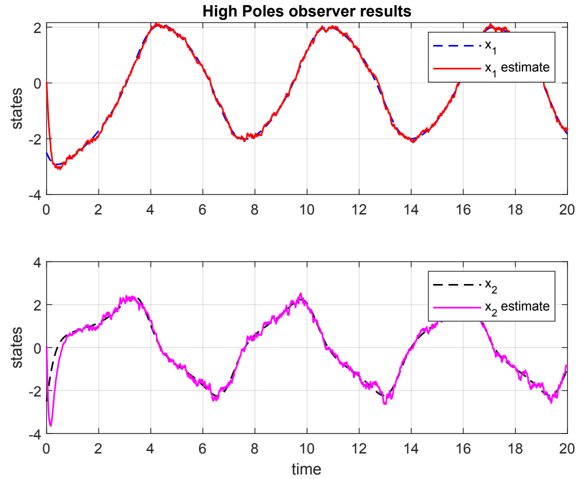

Fig. 7. HPO estimation results with $\lambda_1 = 5.5$, $\lambda_2 = 4$.

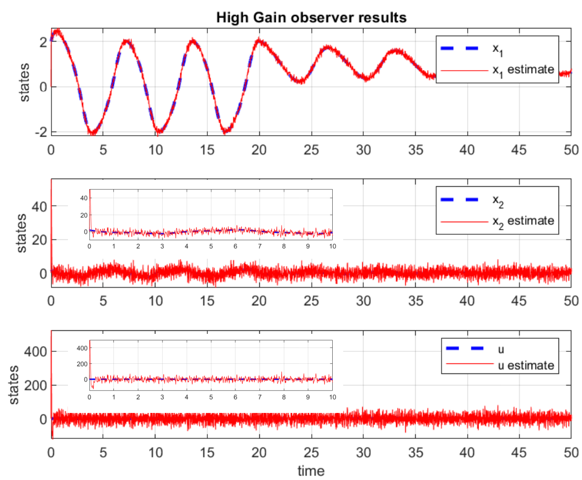

Fig. 8. HGO estimation results with $\lambda = 32$ for estimation of state variables *and u* (with zooms for x_2 and u).

more general cases will be part of future studies (including links to other techniques like adaptation, LMIs...).

References

[1] J. H. Ahrens and H. K. Khalil. High-gain observers in the presence of measurement noise: A switched-gain approach. *Automatica*, 45(4):946–943, 2009.

[2] V. Andrieu, L. Praly, and A. Astolfi. High gain observers with updated gain and homogeneus correction term. *Automatica*, 45(2):422–428, 2009.

[3] M. Arcak and P. Kokotović. Ononlinear observers: a circle criterion design and robustness analysis. *Automatica*, 37:1923–1930, 2001.

[4] D. Astolfi and L. Marconi. A high-gain nonlinear observer with limited gain power. *IEEE Transactions on Automatic Control*, 60(11):3059–3064, 2015.

[5] D. Astolfi, L. Marconi, L. Praly, and A.R. Teel. Low-power peaking-free high-gain observers. *Automatica*, 98:169–179, 2018.

[6] P. Bernard. *Observer design for nonlinear systems*. Springer, Vol 479, 2019.

[7] G. Besançon. *Nonlinear Observers and applications*. Springer, vol.363, 2007.

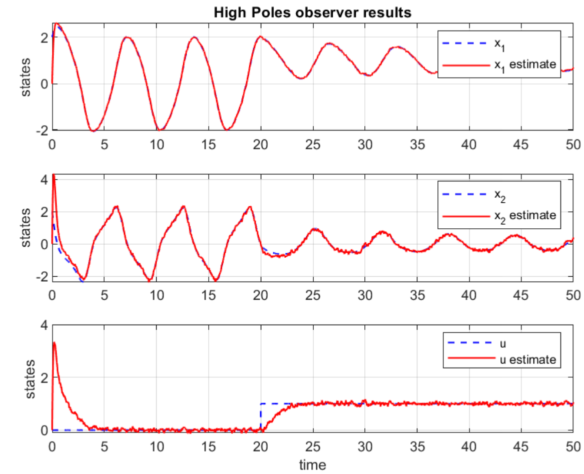

Fig. 9. HPO estimation results with $\lambda_1 = 5.5$, $\lambda_2 = 4$ for estimation of state variables *and u*.

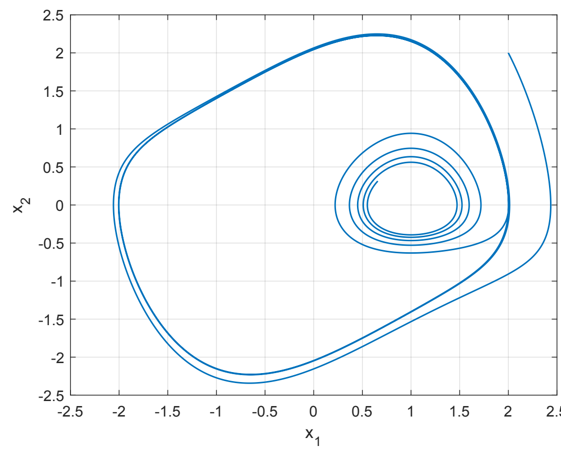

Fig. 10. Van der Pol state trajectory under step input.

[8] G. Besançon, A. Voda, and G. Jouffroy. A note on state and parameter estimation in a Van der Pol oscillator. *Automatica*, 46, 2010.

[9] N. Boizot, E. Busvelle, and J. P. Gauthier. An adaptive high-gain observer for nonlinear systems. *Automatica*, 46(9):1483–1488, 2010.

[10] J. P. Gauthier, H. Hammouri, and S. Othman. A simple observer for nonlinear systems-applications to bioreactors. *IEEE Transactions on Automatic Control*, 37(6):875–880, 1992.

[11] J.P. Gauthier and G. Bornard. Observability for any u(t) of a class of nonlinear systems. *IEEE Trans. Autom. Cont.*, 26(5):922–926, 1981.

[12] H.K. Khalil and L. Praly. High-gain observers in nonlinear feedback control. *Int. J. Robust and Nonlinear Control*, 24:993–1015, 2014.

[13] A.J. Krener and A. Isidori. Linearization by output injection and nonlinear observers. *Systems & Control Letters*, 3:47–52, 1983.

[14] J. De Leon, I. Souleiman, A. Glumineau, and G. Schreier. On nonlinear equivalence and backstepping observer. *Kybernetika*, 37(5):521–6, 2001.

[15] J. Moreno. Observer design for nonlinear systems: A dissipative approach. In *Proc. IFAC Systems, Structure & Control, Oaxaca, Mexico*, 2004.

[16] R.G. Sanfelice and L. Praly. On the performance of high-gain observers with gain adaptation under measurement noise. *Automatica*, 47:2165–2176, 2011.

[17] A. Zemouche, F. Zhang, F. Mazenc, and R. Rajamani. High gain nonlinear observer with lower tuning parameter. *IEEE Trans. Automatic Control*, 64(8):3194–3209, 2019.