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Predictive Norm Optimal Iterative Learning Control for High-performance Formation Control Problem

Yueqing Zhang[1] and Bin Chen[2]

Abstract— **This paper develops a predictive optimisation-based iterative learning control (ILC) strategy for the high-performance formation control problem in networked dynamical systems working repetitively. It avoids the need for exact model information in traditional methods and achieves high performance via a predictive framework incorporating a unique performance index that integrates both immediate and future performance. The proposed framework guarantees geometric convergence of the formation error norm to zero and is capable of handling both heterogeneous and non-minimum phase systems. A distributed implementation of the framework is developed using the Alternating Direction Method of Multipliers to guarantee the framework's scalability for large-scale networks. Rigorous convergence analysis and numerical examples are provided to confirm its effectiveness.**

I. INTRODUCTION

In the last decade, formation control of networked dynamical systems requiring all subsystems to form a predefined formation repetitively with high precision has attained increasingly more attention in wide research areas, such as satellites, search and rescue robots, and unmanned aerial vehicles (UAVs) [1], [2]. For example, UAVs in precision agriculture require repeated high-accuracy formation control to optimise resource use, prevent coverage gaps or overlaps, and improve crop health monitoring and disease detection. To achieve high-performance formation, traditional control methods require highly accurate models of each subsystem, which can be very difficult or expensive to know.

To address this limitation, recent designs apply a powerful control approach, namely iterative learning control (ILC), to the high-performance formation control problem. ILC is especially suitable for tasks working repetitively and requiring high precision, since it can learn the information (in particular input and error) of previous attempts on the same task to improve its control strategy without knowing accurate model information [3]–[5].

Due to the suitability of ILC for the high-performance formation control problem, many ILC-based methods have been proposed: [6], [7] propose Proportional-type ILC for linear networked dynamical systems with fixed and switching topology, respectively; [8]–[11] design Proportional-Integral-Derivative type ILC approaches for nonlinear networked dynamical systems; [12], [13] propose a high-order internal model-based ILC method for nonlinear networked dynamical systems; [14] designs a distributed norm optimal ILC (NOILC) algorithm with great scalability and convergence.

However, except for [14], the existing papers have limited convergence performance (e.g., monotonic convergence cannot be guaranteed without tunning any control parameters) and scalability to large-scale networks. Although the method proposed in [14] can achieve monotonic convergence of the formation error norm (without requiring parameter tuning) even on large-scale networks, its convergence could be slow if only the performance of the next step is considered.

To overcome the 'short-sighted' problem in the traditional NOILC for networked systems, a predictive NOILC has been proposed for consensus tracking of networked dynamical systems in our recent publication [15]. By incorporating predictions on future performance, [15] has shown a faster convergence speed with the other performances (e.g., monotonic convergence of the tracking error norm, certain robustness against model uncertainty) remaining unchanged as the distributed NOILC algorithm. However, the high-performance consensus tracking problem considered in [15] has only one unique input solution because there exists a reference signal, that is much easier for design when compared with the high-performance formation control problem which has an infinite number of solutions to achieve the desired formation. Furthermore, paper [15] only considers single-input-single-output (SISO) networked dynamical systems, while multi-input-multi-output (MIMO) networked dynamical system that is often seen in practice, have not be considered.

Motivated by the above ideas, this paper further extends [15] and proposes a predictive NOILC framework for high-performance formation control problems with MIMO dynamics. The main contributions are summarised as follows:

- we consider a general system formulation allowing the consideration of MIMO networked dynamical systems, which has a high degree of industrial applicability;
- we design a novel predictive norm optimal performance index for high-performance formation control problem and incorporate it into the optimal-based ILC framework. The resulting predictive NOILC framework can greatly enhance the convergence speed without losing the monotonic convergence properties;
- we prove that for the high-performance formation control problem which exists an infinite number of input selection, the proposed predictive NOILC framework can obtain the minimum energy solution by selecting the initial input to be zero, which is appealing in practice;

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[1]Yueqing Zhang is with the Faculty of Engineering and Physical Sciences, University of Southampton, SO17 1BJ Southampton, U.K. Yueqing.Zhang@soton.ac.uk

[2]Bin Chen is with the Department of Automatic Control and Systems Engineering, University of Sheffield, Sheffield S10 2TN, U.K. bin.chen@sheffield.ac.uk

- we design a distributed implementation for the predictive NOILC framework. The resulting distributed ILC algorithm has great scalability in dealing with large-scale heterogeneous networks.

The rest of this paper is summarised as follows: Section II provides the problem formulation; Section III proposes a centralised predictive NOILC to achieve the desired mission with convergence properties analysed rigorously; Section IV develops a distributed implementation for the proposed predictive NOILC framework; Section V provides numerical simulations to verify the algorithm's effectiveness; Section VI concludes this paper and provides the future direction.

II. PROBLEM FORMULATION

A. System Dynamics

Considers a networked system contains p agents, where the i^{th} subsystem's dynamics is represented using a discrete-time, MIMO, linear time-invariant system as follows:

$$\begin{aligned} x_{i,k}(t+1) &= A_i x_{i,k}(t) + B_i u_{i,k}(t), \; x_{i,k}(0) = x_{i,0} \\ y_{i,k}(t) &= C_i x_{i,k}(t), \qquad\qquad 1 \le i \le p \end{aligned} \tag{1}$$

where $t \in [0, N]$ is the time index and N is the trial length; k is the trial index; $x_{i,k} \in \mathbb{R}^{n_i}$, $u_{i,k}(\cdot) \in \mathbb{R}^m$ and $y_{i,k}(\cdot) \in \mathbb{R}^l$ are the state, input and output of the system i at trial k respectively, in which n_i is the subsystem's order; A_i, B_i and C_i are system matrices of the system i with appropriate dimensions. The mission of the networked dynamical system is to achieve formation control with high precision repetitively over a finite time interval $[0, N]$, i.e., at $t = N + 1$, the time is reset to 0 and the state of each subsystem is reset to initial values $x_{i,0}$, and the networked system then perform the same formation task again.

A 'lifted matrix form' representation [4] is introduced for System (1) to facilitate later design:

$$y_{i,k} = G_i u_{i,k} + d_i \tag{2}$$

where u_k and y_k are the super-vectors to represent the control input and output of the i^{th} subsystem on trial k, i.e.,

$$\begin{aligned} u_{i,k} &= [u_{i,k}^T(0) \quad u_{i,k}^T(1) \quad \cdots \quad u_{i,k}^T(N-1)]^T \\ y_{i,k} &= [y_{i,k}^T(1) \quad y_{i,k}^T(2) \quad \cdots \quad y_{i,k}^T(N)]^T \end{aligned}$$

and the input-output mapping matrix G and the response of the initial condition d of the i^{th} subsystem are

$$G_i = \begin{bmatrix} C_iB_i & 0 & \cdots & 0 \\ C_iA_iB_i & C_iB_i & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_iA_i^{N-1}B_i & C_iA_i^{N-2}B_i & \cdots & C_iB_i \end{bmatrix},$$

$$d_i = [(C_iA_ix_{i,0})^T \; (C_iA_i^2x_{i,0})^T \; \cdots \; (C_iA_i^Nx_{i,0})^T]^T.$$

Denote the input, output, and initial response for all subsystems on the trial k respectively as

$$\begin{aligned} \hat{u}_k &= \begin{bmatrix} u_{1,k}^T & u_{2,k}^T & \cdots & u_{p,k}^T \end{bmatrix}^T \\ \hat{y}_k &= \begin{bmatrix} y_{1,k}^T & y_{2,k}^T & \cdots & y_{p,k}^T \end{bmatrix}^T \\ \hat{d} &= \begin{bmatrix} d_1^T & d_2^T & \cdots & d_p^T \end{bmatrix}^T. \end{aligned} \tag{3}$$

The 'lifted matrix form' of the networked dynamical system can then be written as follows

$$\hat{y}_k = \mathbb{G}\hat{u}_k + \hat{d} \tag{4}$$

where the global system matrix $\mathbb{G} = \text{diag}(G_1, G_2, \cdots, G_p)$.

B. Network Topology

An undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is used to describe the networked dynamical system's topology, where $\mathcal{V} = \{1, 2, ..., p\}$ is the set of nodes (corresponding to each subsystem), and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of edges. An edge between two subsystems suggests they are neighbours to each other, and $\mathcal{N}_i = \{j : (i, j) \in \mathcal{E}\}$ is the neighbour set of i, containing all of its neighbours.

To indicate the relationship between two agents, a Laplacian matrix $\mathcal{L} = \{\mathcal{L}_{ij}\}$ is defined. It is a real positive semi-definite matrix (the proof can be found in [1]) with element:

$$\mathcal{L}_{ij} = \begin{cases} -W_{ij} & \text{if} \quad j \in \mathcal{N}_i \\ \sum_{l \in \mathcal{N}_i} W_{il} & \text{if} \quad j = i \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where $W_{ij} > 0$ is a weight that denotes the connection strength of the edge.

In literature, to achieve the formation control of networked dynamical systems, the following assumption is standard:

Assumption 1: There is at least one Euclidean path from one node to other nodes, i.e., the graph $\mathcal{G}$ is connected.

C. ILC Design Problem

The formation is defined by some desired relative positions between each pair of neighbouring subsystems, which is denoted as $r_{ij}(t)$ at time t. Consequently, its 'lifted matrix form' can be obtained in the following:

$$r_{ij} = [r_{ij}^T(1) \quad r_{ij}^T(2) \quad \cdots \quad r_{ij}^T(N)]^T \tag{6}$$

Now, by defining the relative position between subsystems i and j to be $y_{ij} = y_i - y_j$, the high-performance formation control problem can then be stated as designing inputs $u_i (i = 1, 2, \ldots, p)$ to achieve the desired formation such that

$$r_{ij} - y_{ij} = r_{ij} - y_i + y_j = 0 \tag{7}$$

Here, we can introduce a 'virtual' reference r_i for the subsystem i, and $r_{ij} = r_i - r_j$. The purpose of defining the 'virtual' reference is to facilitate the problem statement and it is not supposed to be known and tracked by the subsystem.

By defining the global virtual reference $\hat{r}$ as:

$$\hat{r} = \begin{bmatrix} r_1^T & r_2^T & \cdots & r_p^T \end{bmatrix}^T, \tag{8}$$

the following assumption is important to achieve the desired formation in the MIMO system:

Assumption 2: The global virtual reference $\hat{r}$ is in the range space of $\mathbf{L}\mathbb{G}$, where $\mathbf{L} = \mathcal{L} \otimes I_{Nl}$; $\otimes$ is the Kronecker product and I_{Nl} is the identity matrix with order $Nl \times Nl$.

Following from Assumptions 1 and 2, at least one solution exists to the formation control problem. Then, the ILC design problem can be stated as finding an input updating law

$$\hat{u}_{k+1} = f(\hat{u}_k, \hat{y}_k), \tag{9}$$

such that the output of the whole networked system can establish the desired formation in high precision, i.e.,

$$\lim_{k\to\infty} y_{i,k} - y_{j,k} = G_i u_{i,k} + d_i - (G_j u_{j,k} + d_j) = r_{ij}. \quad (10)$$

For this mission, there exists an infinite number of solutions to achieve the desired formation since no fixed reference signal in the control problem. However, out of these infinite solutions, there exists a solution that minimises the total input energy, which is appealing in practice. Hence, in the following section, we will design an ILC method that not only achieves the desired formation but also converges to the minimum input energy solution.

III. CENTRALISED PREDICTIVE NOILC ALGORITHM AND CONVERGENCE ANALYSIS

In this section, a novel predictive NOILC framework is proposed for high-performance formation control problems, and its convergence properties are given.

A. Centralised Predictive NOILC Framework

Predictive optimal ILC decides the control action by minimising a cost function evaluating the predicted future performance over upcoming trials. For the formation control problem, the performance index is expected to minimise some formation errors. According to (10), the formation error for the subsystem i is defined as

$$\hat{e}_{i,k} = \sum\nolimits_{j\in\mathcal{N}_i} W_{ij}(r_{ij} - y_{ij,k}), \quad (11)$$

and the formation error for the networked dynamical system is $\hat{e}_k = [\hat{e}_{1,k}^T \quad \hat{e}_{2,k}^T \quad \cdots \quad \hat{e}_{p,k}^T]^T$.

Therefore, the cost function for the formation control by predictive NOILC can be defined as

$$J_{k+1,K}(\hat{u}_{k+1}) = \sum_{\kappa=1}^{K} \gamma^{\kappa-1} \left(\|\hat{e}_{k+\kappa}\|_{\hat{Q}}^2 + \|\Delta\hat{u}_{k+\kappa}\|_{\hat{R}}^2 \right), \quad (12)$$

where $\Delta\hat{u}_{k+\kappa} = \hat{u}_{k+\kappa} - \hat{u}_{k+\kappa-1}$; K is the number of trials to look-ahead; $\gamma > 0$ weights the importance of future costs; the norms are defined as

$$\|\hat{e}\|_{\hat{Q}} = \sqrt{\hat{e}^T\hat{Q}\hat{e}}, \quad \|\Delta\hat{u}\|_{\hat{R}} = \sqrt{\Delta\hat{u}^T\hat{R}\Delta\hat{u}},$$

where $\hat{Q} = \mathrm{diag}(Q_1, \ldots, Q_p)$, $\hat{R} = \mathrm{diag}(R_1, \ldots, R_p)$; Q_i and R_i (where $i = 1, 2, \cdots, p$) are positive-definite matrices.

With the above definitions, we have the following predictive NOILC algorithm:

Algorithm 1: For any initial input $\hat{u}_0$, the input sequence is calculated by minimising (12) as follows

$$\hat{u}_{k+1} = \arg\min \; J_{k+1,K}\left(\hat{u}_{k+1}\right). \quad (13)$$

This solves the formation control problem iteratively, i.e.,

$$\lim_{k\to\infty} y_{i,k} - y_{j,k} = r_{ij}, \quad (14)$$

Note that, when the number of subsystems is small, Algorithm 1 can be executed in a centralised manner by directly finding the stationary point of the performance index (12) according to the following input updating law

$$\hat{u}_{k+1} = \hat{u}_k + L_K\hat{e}_k, \quad (15)$$

where L_K is the learning gain matrix for $J_{k+1,K}$ and can be determined recursively

$$L_\kappa = \left(\hat{R} + \hat{\mathbb{G}}^T(\hat{Q} + \gamma P_{\kappa-1})\hat{\mathbb{G}}\right)^{-1}\hat{\mathbb{G}}^T(\hat{Q} + \gamma P_{\kappa-1}), \quad (16)$$

$$P_\kappa = (\hat{Q} + \gamma P_{\kappa-1})(I - \hat{\mathbb{G}}L_\kappa), \quad \forall\kappa = 1, 2, 3, \ldots, K \quad (17)$$

in which $\hat{\mathbb{G}} = \mathbf{L}\mathbb{G}$, and $P_{\kappa-1}$ is a positive definite matrix satisfying the above recursive relationship with $P_0 = 0$. Note that, the formation error used in (15) can be obtained by (11), with everything known or can be calculated from the system.

B. Convergence Analysis

Algorithm 1 has good convergence properties as shown in the following theorem:

Theorem 1: Given any initial input $\hat{u}_0$, the formation error norm $\|\hat{e}_k\|_{\hat{Q}}$ converges monotonically to zero, i.e.,

$$\|\hat{e}_{k+1}\|_{\hat{Q}} \le \|\hat{e}_k\|_{\hat{Q}}, \quad \lim_{k\to\infty} \hat{e}_k = 0. \quad (18)$$

The desired formation is then ultimately achieved, i.e.

$$\lim_{k\to\infty} (y_{i,k} - y_{j,k}) = r_{ij}, \quad \forall i, j = 1, 2, \cdots, p \quad (19)$$

Proof: The proof is omitted here for brevity. ■

Theorem 1 indicates that Algorithm 1 is capable of successfully achieving formation control for networked dynamic systems, with a monotonic decrease in the formation error norm. Additionally, the resulting control input converges to the one that achieves the target formation while minimising deviation from the initial control, as shown in the following:

Theorem 2: For any initial input choice $\hat{u}_0$, the input generated by Algorithm 1 converges as follows

$$\lim_{k\to\infty} \hat{u}_k = \hat{u}^*$$

where $\hat{u}^*$ is the optimal solution of the following problem

$$\begin{aligned} \min \quad & \|\hat{u} - \hat{u}_0\|_{\hat{R}}^2 \\ \text{s.t.} \quad & \mathbf{L}(\mathbb{G}\hat{u} + \hat{d}) = \mathbf{L}\hat{r} \end{aligned} \quad (20)$$

Consequently, when initialised with a zero input, Algorithm 1 converges to the control solution with the minimum energy.

Proof: The proof is omitted here for brevity. ■

Theorems 1 and 2 show that Algorithm 1 ensures a monotonic converging of formation error norm to zero and minimises the control energy without a need for parameter tuning, which is favouring in practical applications.

IV. DISTRIBUTED PREDICTIVE NOILC ALGORITHM AND CONVERGENCE ANALYSIS

A distributed implementation is developed in this section leveraging the Alternating Direction Multiplication Method (ADMM) to overcome the potential computational challenges posed by centralised algorithms in large-scale networked systems. Here, the control task is allocated to each subsystem and they achieve the desired formation together with only local information, enabling parallel processing to enhance the computational efficiency.

A. Distributed Predictive NOILC for Formation Control

Note that, the ILC problem (12) can be equivalent to finding the input increment in each trial, i.e.,

$$J_{k+1,K}\left(\Delta\hat{u}_{k+1}\right)=\sum\nolimits_{\kappa=1}^{K}\gamma^{\kappa-1}\left(\|\hat{e}_{k+\kappa}\|_{\hat{Q}}^{2}+\|\Delta\hat{u}_{k+\kappa}\|_{\hat{R}}^{2}\right),$$

which can be written into the following form

$$J_{k+1,K}\left(\Delta\hat{u}_{k+1}\right)=\sum\nolimits_{i=1}^{p}J_{i,k+1,K}\left(\Delta\hat{u}_{i,k+1}\right) \tag{21}$$

where $J_{i,k+1,K}\left(\Delta\hat{u}_{i,k+1}\right)$ denotes the local cost function as:

$$J_{i,k+1,K}\left(\Delta\hat{u}_{i,k+1}\right)=\sum_{\kappa=1}^{K}\gamma^{\kappa-1}(\|\hat{e}_{i,k+\kappa}\|_{Q_i}^{2}+\|\Delta\hat{u}_{i,k+\kappa}\|_{\mathbf{R}_i}^{2})$$

$$=\sum_{\kappa=1}^{K}\gamma^{\kappa-1}(\|\hat{e}_{i,k+\kappa-1}-\mathbf{L}_i\mathbb{G}_i\Delta\hat{u}_{i,k+\kappa}\|_{Q_i}^{2}+\|\Delta\hat{u}_{i,k+\kappa}\|_{\mathbf{R}_i}^{2}),$$

$\Delta\hat{u}_{i,k+1}=\hat{u}_{i,k+1}-\hat{u}_{i,k}$ is the local input increment; $\hat{u}_{i,k}$ is the local input; $\mathbf{R}_i$ is a local weight matrix; $\mathbf{L}_i$ is the local Laplacian matrix. As an example, if $\mathcal{N}_i=\{l,m\}$, then

$$\begin{aligned}
&\mathbb{G}_i=\text{diag}(G_i,G_l,G_m),\\
&\mathbf{L}_i=\begin{bmatrix}\sum_{j\in\mathcal{N}_i}W_{ij} & -W_{il} & -W_{im}\end{bmatrix}\otimes I_N,\\
&\mathbf{R}_i=\text{diag}(\frac{1}{1+|\mathcal{N}_i|}R_i,\frac{1}{1+|\mathcal{N}_l|}R_l,\frac{1}{1+|\mathcal{N}_m|}R_m),\\
&\hat{u}_{i,k}=\begin{bmatrix}u_{i,k}^T & u_{l,k}^T & u_{m,k}^T\end{bmatrix}^T,\quad \hat{d}_i=\begin{bmatrix}d_i^T & d_l^T & d_m^T\end{bmatrix}^T.
\end{aligned}$$

The formation problem can be formulated as the follows:

$$\begin{aligned}
\min_{\hat{u}_{i,k+1}}\quad & \sum\nolimits_{i=1}^{p}J_{i,k+1,K}\left(\Delta\hat{u}_{i,k+1}\right)\\
\text{s.t.}\quad & \Delta\hat{u}_{i,k+\kappa}-\tilde{E}_i z_{k+\kappa}=0,\\
& i=1,\cdots,p,\quad \kappa=1,\cdots,K.
\end{aligned} \tag{30}$$

Correspondingly, the Lagrangian is defined as

$$L_{\text{aug},i,k+1,K}\left(\Delta\hat{u}_{i,k+1},\lambda,\rho\right)=\frac{\rho}{2}\sum_{\kappa=1}^{K}\|\Delta\hat{u}_{i,k+\kappa}-\tilde{E}_i z_{k+\kappa}\|^2$$

$$+\sum_{\kappa=1}^{K}\lambda_{i,k+\kappa}^{T}(\Delta\hat{u}_{i,k+\kappa}-\tilde{E}_i z_{k+\kappa})+\frac{1}{2}J_{i,k+1,K}(\Delta\hat{u}_{i,k+1}).$$

Applying ADMM (more details can be found in [16]) to Algorithm 1, we have the following distributed implementation.

Algorithm 2: At the $k+1^{\text{th}}$ trial, the input $\hat{u}_{k+1}$ is generated by the following ADMM steps (with $\kappa=1,2,\cdots,K$):

$$\Delta\hat{u}_{i,k+\kappa}^{q+1}=\arg\min\{L_{\text{aug},i,k+1,K}\} \tag{31}$$

$$z_{i,k+\kappa}^{q+1}=\frac{1}{1+|\mathcal{N}_i|}\sum\nolimits_{o\in(\mathcal{N}_i\cup i)}(\Delta\hat{u}_{o,k+\kappa}^{q+1})_i \tag{32}$$

$$\lambda_{i,k+\kappa}^{q+1}=\lambda_{i,k+\kappa}^{q}+\rho(\Delta\hat{u}_{i,k+\kappa}^{q+1}-\tilde{E}_i z_{k+\kappa}^{q+1}), \tag{33}$$

which solves the formation problem (12) distributedly, i.e.,

$$\lim_{q\to\infty}z_{k+1}^{q}=\arg\min\{J_{k+1,K}(\Delta\hat{u}_{k+1})\}, \tag{34}$$

where $z_{i,k+1}$ is the i^{th} component of the global vector z_{k+1} at the $k+1^{\text{th}}$ ILC trial; $(\Delta\hat{u}_{o,k+1})_i$ is the element in $\Delta\hat{u}_{o,k+1}$ that corresponds to the subsystem i; and $|\mathcal{N}_i|$ is the amount of the neighbours of the subsystem i.

Algorithm 3 Distributed predictive NOILC algorithm for the high-performance formation control problem

Input: Initial input u_0; reference signal r; number of ILC trials k_{max}; number of ADMM iterations q_{max}

Output: Input of the each subsystem $u_{i,max}$ at trial k_{max}

1: **for** $k=0$ to k_{max} **do**
2: Input $u_{i,k}$ to each plant to collect the output.
3: **for** $q=0$ to q_{max} **do**
4: **for** $i=1$ to p **do**
5: Receive information from neighbours
6: Update $\Delta\hat{u}_{i,k+1}^{q+1}$ according to (31)
7: Update $z_{i,k+1}^{q+1}$ according to (32)
8: Update $\lambda_{i,k+1}^{q+1}$ according to (33)
9: Send information to neighbours
10: **end for**
11: **end for**
12: Transform $\Delta\hat{u}_{i,k+1}^{q_{max}}$ into $\Delta u_{i,k+1}$
13: $u_{i,k+1}=u_{i,k}+\Delta u_{i,k+1}$.
14: **end for**

Combining Algorithms 1 and 2 leads to the distributed predictive NOILC Algorithm 3. This algorithm generates the optimal input for each subsystem. Note that, there are two indexes for iteration: k is the index for an ILC trial and q is the index for an ADMM iteration. Each ILC trial includes an ILC experiment (where the computed control inputs for each subsystem are implemented) and some ADMM iterations (to iteratively solve the Lagrangian and distributedly find the optimal input increment for the next ILC trial). Within one ADMM iteration, each subsystem first calculates an optimal input increment according to the local predicted performance for itself and for its neighbours in Step 6. In this step, the local optimal input increments can be computed in parallel. By averaging the corresponding local policy in Step 7, the new global value can be obtained. Step 8 updates the dual variable by dual ascent to gradually reduce the difference between the local plan and its global counterpart. Every subsystem achieves agreement on their respective plans, thereby collectively resolving the optimisation problem by repeating the above procedure. It also should be noted that, during the ADMM iterations, the optimal input increments for the future K ILC trials are predicted, but only the control input for the next ILC trial (rather than the future K ILC trials) is applied to the system.

Theoretically, ADMM is expected to converge to the optimal solution as the number of ADMM iterations approaches infinity. In practice, however, ADMM is typically efficient and a relatively small number of ADMM iterations is sufficient to closely approximate the optimal solution.

B. Convergence Analysis

Algorithm 3 has good convergence properties as shown in the following theorem:

Theorem 3: The distributed optimisation-based ILC algorithm with K trials look-ahead is given by Algorithm 3. Given any initial input $\hat{u}_0$, and any initialisation for the

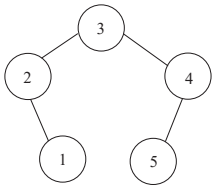

Fig. 1. The network topology of the example system

related parameters, the formation error norm $\|\tilde{e}_k\|_{\bar{Q}}$ convergences monotonically to zero, i.e.,

$$\|\tilde{e}_{k+1}\|_{\bar{Q}} \leq \|\tilde{e}_k\|_{\bar{Q}}, \quad \lim_{k\to\infty} \tilde{e}_k = 0. \tag{35}$$

Therefore, the desirable formation is ultimately achieved, i.e,

$$\lim_{k\to\infty} y_{ij,k} = r_{ij}, \quad i = 1, 2, \cdots, p. \tag{36}$$

The generated input converges to the one with the least deviation from the initial input selection

$$\lim_{k\to\infty} \begin{bmatrix} u_{1,k}^T & u_{2,k}^T & \cdots & u_{p,k}^T \end{bmatrix}^T = \hat{u}^*$$

where $\hat{u}^*$ is the solution of the following problem

$$\begin{aligned} \min \quad & \|\hat{u} - \hat{u}_0\|^2 \\ \text{s.t.} \quad & \mathbf{L}(\mathbb{G}\hat{u} + \hat{d}) = \mathbf{L}\hat{r}. \end{aligned}$$

Specially, if the zero initial input is applied, then the Algorithm 3 converges to the control solution with the minimum energy required to achieve the desired formation.

Proof: The proof is omitted here for brevity. ■

Remark 1: The proposed algorithms are applicable to any networked dynamical system, regardless of its size, trial length or topology, as long as Assumptions 1 and 2 are met.

Remark 2: A large K ensures fast convergence to the desired formation, with $K = 2$ is enough to achieve extremely rapid reduction in the formation error for a large enough γ.

V. Numerical Simulation

To illustrate the performance of the proposed method, an example application is shown in this section. Consider the system with $p = 5$ subsystems, and the transfer function of each is described as:

$$G_i(s) = \begin{bmatrix} \frac{s+1.2}{s+i+10} & 0 \\ 0 & \frac{s+0.1}{s+i+8} \end{bmatrix} \tag{37}$$

where $i = 1, 2, \cdots, p$ and sampled using a zero-order hold with the sampling time to be 0.3s. The trial length is 3s and zero initial conditions of all subsystems are assumed. Fig. 1 shows its topology with the weight of each edge to be one.

The system is required to achieve the following formation:

$$r_{i,i+1} = R(t) \begin{bmatrix} \cos(2i\pi/p) - \cos(2(i+1)\pi/p) \\ \sin(2i\pi/p) - \sin(2(i+1)\pi/p) \end{bmatrix} \tag{38}$$

where $i = 1, 2, \ldots, p-1$ and $R(t)$ is a rotation matrix

$$R(t) = \begin{bmatrix} \cos(2\pi t/p) & -\sin(2\pi t/p) \\ \sin(2\pi t/p) & \cos(2\pi t/p) \end{bmatrix}.$$

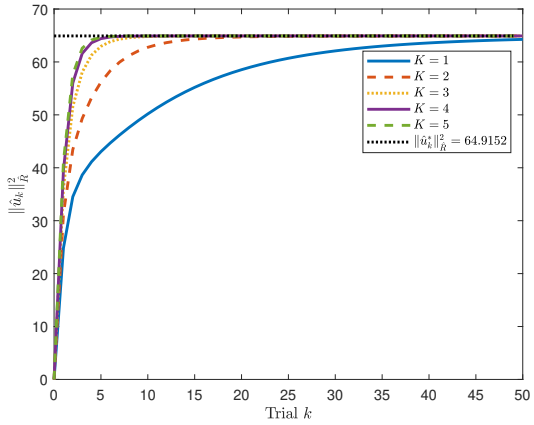

Fig. 2. Convergence of $\|\hat{u}_k^*\|_{\hat{R}}^2$ for different K

To quantitatively explore the power of 'looking forward' on the improvement of the performance in-depth, two experiments are designed and conducted. The first one compares the performance for different formulation of performance index (varying look-ahead horizon K) while maintaining consistency in the parameter setting; the second one uses an same formulation of performance index (i.e., a same K) and examines the impact of varying the weights assigned to future performance. For the first experiment, Fig. 3 compares the results for centralised (solid lines) and distributed (dashed lines) implementations. Here, the parameter setting is: $Q = I$, $R = 0.1I$ and $\gamma = 0.2$. For each K, the formation error norms converge monotonically to zero along with the increasing ILC trials, and the two lines are almost on top of each other, which suggests the equivalence of two implementations. For a large K, it requires more ADMM iterations to reproduce exactly the same performance as centralised implementation. Besides, it can be seen that looking further enables a huge improvement in the convergence speed, which holds for both centralised and distributed implementations. Also, Fig. 2 shows the convergence of the energy consumption to the optimal solution (obtained from (20) and $\|\hat{u}_k^*\|_{\hat{R}}^2 = 64.9152$) for varying K. It can be seen that, a larger look-ahead horizon enables a more rapid reduction in the energy consumption, which again supports the theory.

For the second experiment, $K = 2$, $Q = I$ and $R = 0.1I$ remain unchanged and we just focus on the impact of varying γ on the formation results. The weighting parameter γ is selected from 0.01 to 100 and the convergence in the formation error norm square is compared in Fig. 4. It can be seen that, even with the same formulation, a large focus on the (predicted) future performance makes the algorithm less shortsighted and enables a more efficient reduction in the formation error. By comparing Fig. 3 and 4, it can be found that, with a large enough weight on the predicted future performance, a smaller looking-ahead ($K = 2, \gamma = 100$) can achieve or even suppress the performance of the design with more steps to look-ahead but less attention on the future information ($K = 5, \gamma = 0.2$). This observation suggests

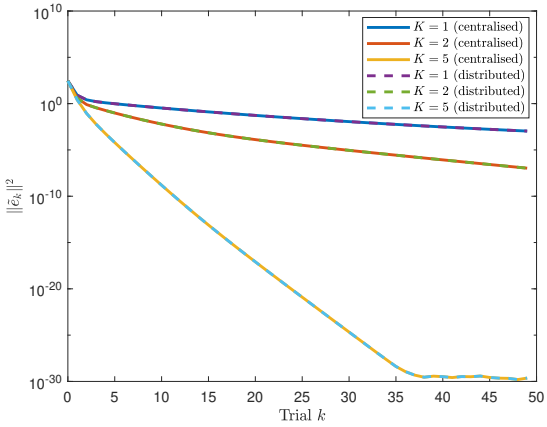

Fig. 3. Convergence of $\|\tilde{e}_k\|^2$ for different K

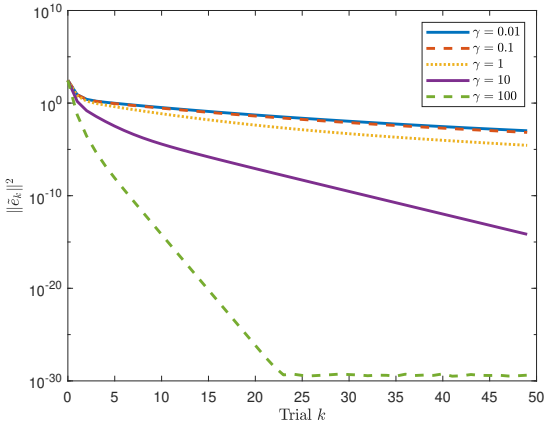

Fig. 4. Convergence of the formation error norm for different γ

that, 'looking forward' is important and too many steps looking-ahead can be replaced by more focusing on the predicted future performance in the more recent future. A simpler performance index formulation can achieve desirable formation rapidly and requires less computational complexity, which is desirable in practice.

Simulations with different network typologies have been conducted. The algorithm also works for different network typologies, hence they are omitted here for brevity.

VI. CONCLUSIONS

This paper addresses high-performance formation control problems in MIMO networked dynamical systems. A novel predictive norm optimal performance index is designed for high-performance formation control problem, which enables a great enhancement in the convergence speed while maintaining the monotonic convergence properties, even for non-minimum phase networked dynamical systems, which is important but quite challenging for traditional designs. Moreover, though there are an infinite number of input selections for high-performance formation control problems, our proposed algorithm can be guaranteed to converge to the one with the minimum energy consumption, which is appealing in practice. A distributed implementation for the proposed predictive framework is also designed, which shows great scalability in the large-scale networks. Simulation results are provided to support the effectiveness of the proposed design. Future work includes investigating the convergence of the proposed algorithm under uncertainty or unknown dynamics. Some results in [17] and [18] may answer this question, and we will investigate them in the future.

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