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String stability for predecessor-following platoons over channels with heterogeneous reception probabilities*

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Abstract— **We study predecessor-following platoons in which each vehicle-to-vehicle (V2V) communication is affected by a different probability of successful transmission. We model the overall platoon as a stochastic hybrid system, and analyse its stochastic $\mathcal{L}_2$ string stability via a small-gain approach. We provide an explicit string stability condition that illustrates the interplay between the channel success probabilities, transmission rate, and time headway constant. We illustrate our findings through numerical simulations.**

I. Introduction

Vehicle platooning via cooperative adaptive cruise control (CACC) is crucial to mitigate the effects of rising road traffic [1], with support from advancements in wireless communication for automated cooperative driving [2]. Crucial aspects of these technologies include the growing complexity of the multi-agent system and the stochastic nature of wireless communications. Scalability issues like "string instability" [3] represent the former, where disturbances amplify as they propagate within interconnected systems. To address scalability in the presence of network-induced communication constraints, analysing platoons using networked control systems theory is beneficial [4]. This analysis must encompass platoon parameters (vehicle dynamics, topology, time headway) and their interaction with network parameters (packet losses, transmission rate, etc.).

While deterministic string stability has been extensively researched for over half a century [5]–[8], stochastic string stability, especially in the context of CACC schemes, remains sparsely explored [9], [10]. Existing work primarily relies on simulations for linear platoons under lossy channels [11]–[16], with limited availability of comprehensive string stability conditions. Notably, recent studies delve into stochastic string stability [17]–[19] in a more comprehensive manner. Furthermore, for linear systems with additive noise channels, conditions for "mean square string stability" are explored [20]. In the same context, recent conditions for the so-called *mean square string stability* and *$\mathcal{L}_p$-mean $\mathcal{L}_q$-variance string stability* are presented in [21].

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This work focuses on stochastic phenomena arising from packet losses in vehicle-to-vehicle (V2V) communications. Notable studies include [22], which explores leader-to-formation stability under random lead vehicle state dropouts. Simulations in [11], [13] reveal the adverse impact of packet losses on platooning string stability, while [12] highlights data-loss compensation strategies' varying effects on string stabilization. In [14], the use of Model Predictive Control (MPC) and buffer storage to handle packet dropouts is discussed. In [15], the authors explore the use of an observer to reduce the impact of losses on disturbance attenuation across the platoon string. In [17], CACC design with lossy communications using "average dynamics" was introduced, achieving string stability through $\mathcal{H}_\infty$ control methods. In [18], an $\mathcal{L}_2$ stochastic string stability definition is introduced and applied to an event-triggered platooning scheme with unreliable communications. Moreover, [19] employs Markov jump linear systems theory to formulate a minimisation problem, ensuring the feasibility of control designs for string stability in platooning schemes with random packet drops. Recently, [23] proposed a specific adaptive control strategy which ensures *almost surely $\mathcal{L}_\infty$ string stability* under packet dropouts. While other works like [24]–[26] investigate mean square stability, the analysis of string stability in the corresponding stochastic setting is limited. Additionally, in [16], an estimation scheme utilizing the intermittent Kalman filter is presented, with string stability analysis confined to simulations.

Apart from [17]–[19], [23], the majority of the cited works primarily study the effects of packet losses on stochastic string stability via simulations or internal stability (excluding string stability). In our previous work [27], we aimed to bridge this gap by introducing a framework for stochastic string stability and its relation to important vehicle and network parameters. However, this was carried out in a simpler network scenario where each V2V channel in the platoon had the same probability of successful transmission. In this paper, we aim to extend [27] to address a more general scenario with different probabilities of successful transmission in the underlying V2V links. While exploring related literature, we build on the concept of $\mathcal{L}_p$ string stability, proposed in works like [28]–[32], adapting it to handle stochastic conditions. Our contributions include employing a hybrid systems framework, encompassing both continuous and discrete time dynamics in wireless platoons. This approach distinguishes our work from others, like [17]–[19], which focus solely on discrete-time platoon models, enabling a more comprehensive analysis. Moreover, we provide explicit

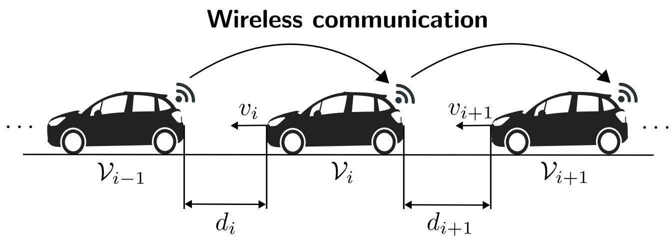

Fig. 1: Platoon configuration.

string stability conditions, establishing clear relationships between critical vehicle and network parameters, such as time headway, transmission rate, and the different channel success probabilities.

II. Platoon setting

Consider a platoon of $N \geq 2$ identical vehicles, as depicted in Fig. 1, where d_i denotes the distance between vehicle $\mathcal{V}_i$ and its preceding vehicle $\mathcal{V}_{i-1}$, and v_i the velocity of vehicle $\mathcal{V}_i$. Each vehicle's primary objective is to follow its preceding vehicle while preserving a designated gap, denoted as r_i. We adopt a constant time-headway policy, whose objective is to maintain a consistent spacing between vehicles based on their velocities. This is expressed as $r_i(t) = \varepsilon_i + hv_i(t)$ for $i \in \mathcal{N}$, with $\mathcal{N} := \{1, \ldots, N\}$. Here, $h \geq 0$ is the time headway constant, and ε_i denotes the standstill distance. This policy is designed to reduce collision risk by increasing the "time gap" between vehicles [33], with known benefits for both string stability [34] and safety [35] when relying solely on nearest neighbor data. We consider a homogeneous platoon where all vehicles employ the same controller, maintain identical spacing policies, and share the same model. However, unlike [27], we here consider heterogeneous (different) success probabilities for the wireless links between vehicles.

Let the spacing error be defined by $\xi_i(t) := d_i(t) - r_i(t) = [s_{i-1}(t) - s_i(t) - L_i] - [\varepsilon_i + hv_i(t)]$, $i \in \mathcal{N}$, where s_i and L_i denote the position and length of vehicle $\mathcal{V}_i$, respectively. The first vehicle in the platoon, denoted by $\mathcal{V}_1$, follows a *virtual reference vehicle* denoted by $\mathcal{V}_0$. Then, ξ_1 corresponds to the spacing error between the leader and this virtual reference.

The control design is based in the following vehicle model, as seen in works such as [18], [28], [36].

$$\mathcal{V}_i : \begin{bmatrix} \dot{s}_i(t) \\ \dot{v}_i(t) \\ \dot{a}_i(t) \end{bmatrix} = \begin{bmatrix} v_i(t) \\ a_i(t) \\ -\frac{1}{\tau}a_i(t) + \frac{1}{\tau}u_i(t) \end{bmatrix}, \ i \in \mathcal{N} \cup \{0\}, \quad (1)$$

where a_i denotes the acceleration of vehicle i, u_i the desired acceleration, and τ the characteristic time constant representing drive-line dynamics.

In CACC schemes, the control law is typically designed based on the spacing error ξ_i and a feedforward component being the direct feedthrough of the predecessor's desired acceleration u_{i-1}. Inspired by [28], [37], we consider the CACC scheme in Fig. 2, where the control law is denoted by q_i, and it is filtered by $\dot{u}_i = -\frac{1}{h}u_i + \frac{1}{h}q_i$ before going into the vehicle drive-line in (1). This filter is given by $H(s) = hs + 1$. Formally, the control law takes the form

$$q_i(t) = \underbrace{k_p\xi_i(t) + k_d\dot{\xi}_i(t)}_{\mathcal{C}_i \text{ (feedback)}} + \underbrace{\hat{u}_{i-1}(t)}_{\text{feedforward}}, \ i \in \mathcal{N}, \quad (2)$$

with controller gains k_p and k_d to be designed. The feedback part of the controller frequently relies on data obtained from a forward-facing radar sensor. Consequently, we assume that each vehicle possesses the capability to measure the relative distance and relative velocity concerning the leading vehicle via the radar sensor. Additionally, it can measure its own absolute speed and acceleration. Nevertheless, acquiring relative accelerations using onboard sensors presents challenges, and, as a result, they are often acquired through wireless communications, as evidenced in [17], [36]. We thus use $\hat{u}_{i-1}$ to denote the desired acceleration u_{i-1} that is sent over the wireless channel from vehicle $i-1$. Because of the packet-based nature of the communication channel and the existence of packet dropouts, it is generally the case that $\hat{u}_{i-1}(t) \neq u_{i-1}(t)$ for $t \in \mathbb{R}_{\geq 0}$. We note that $\hat{u}_0(t) = u_0(t)$ for all $t \in \mathbb{R}_{\geq 0}$ as the first vehicle follows a virtual reference vehicle (i.e. no network imperfections). We will describe the dynamics of the communication channel, and thus $\hat{u}_{i-1}$, in detail in the following section.

Usually, a proficient CACC system should meet two primary aims. The first objective pertains to vehicle following, focusing on the regulation of spacing errors, commonly referred to as "individual vehicle stability." The second objective is the mitigation of disturbances within the vehicle platoon, known as "string stability," as introduced earlier. In order to pursue these objectives within a stochastic context featuring packet-based lossy communications among vehicles, we will adopt an "emulation approach" [38]. That is, we first design the controller (2) when network-induced imperfections are disregarded, i.e., when $\hat{u}_{i-1} = u_{i-1}$ in (2). A benefit of this approach is the compatibility with established design methods for the platoon when the network is disregarded. In fact, the design that ensures both individual vehicle stability and string stability, without network considerations, has been extensively explored in existing literature. Specifically, as demonstrated in [28], the desired stability properties of the platoon can be achieved for any positive values of h, k_d, k_p as long as $k_d > k_p\tau$. In the second stage of emulation, we put the network-free controller into operation within the wireless platoon and investigate how network imperfections impact string stability. Our specific objective is to establish conditions on both network and vehicle parameters that guarantee the preservation, to some extent, of the desired stability characteristics of the network-free system, even when dealing with packet-based communication and potential packet losses.

III. Wireless channels

We describe the wireless communication between vehicles, which governs the dynamics of the transmitted signals $\hat{u}_{i-1}$ in (2).

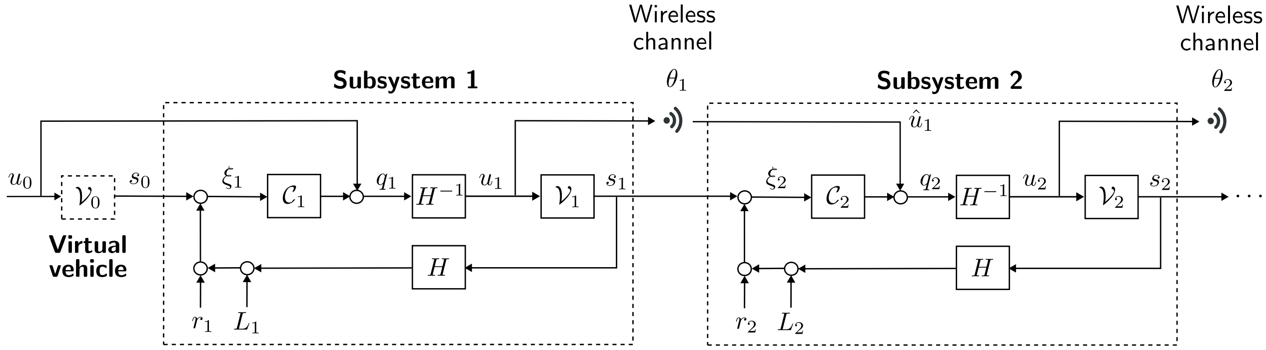

Fig. 2: Block diagram of the CACC scheme.

A. Transmission instants

We define $\mathcal{T} := \{t_0, t_1, t_2, \dots\}$ as the unbounded set of times at which the predecessor's information u_{i-1}, $i \in \mathcal{N} \setminus \{1\}$, is transmitted. Because of synchronisation times, acknowledgements, waiting times, etc., transmission times are typically neither equidistant nor deterministic, but rather exhibit randomness. For example, for networks with carrier sense multiple access, transmissions occur randomly as devices wait for channel clearance under *random back-off mechanisms*. Consequently, we assume the following.

Standing Assumption 1: Consider a Poisson point process $r(t)$ with rate $\boldsymbol{\lambda} \in \mathbb{R}_{>0}$ that satisfies $r(t) = 0$ for $t \in [0, t_0)$ and $r(t) = k$ for $t \in [t_{k-1}, t_k)$, where $t_k \in \mathcal{T}$, $k \in \mathbb{N}_0$, are defined inductively by: $t_0 = \tau_0$ with $\tau_0 \sim \text{Exp}(\boldsymbol{\lambda})$, and for each $k \in \mathbb{N}$, $t_k = t_{k-1} + \tau_k$, with $\tau_k \sim \text{Exp}(\boldsymbol{\lambda})$, where the sequence $\{\tau_k\}_{k \in \mathbb{N}_0}$ is i.i.d. □

The times $\{t_k\}_{k \in \mathbb{N}_0}$ are also called *arrival times* in the literature [39], $\{\tau_k\}_{k \in \mathbb{N}_0}$ are called *inter-transmission times* (or *inter-arrival times*), $\bar{\tau} := 1/\boldsymbol{\lambda}$ represents the *average inter-transmission time*, and $\boldsymbol{\lambda}$ is the *arrival rate*. Throughout this paper, we will use the terms arrival rate and transmission rate interchangeably. The exponential distribution that governs each τ_k describes the time between transmissions.

B. Packet losses

The next element that characterizes the dynamics of the transmitted signals $\hat{u}_{i-1}$ are packet dropouts. Consider Fig. 2, where a wireless channel is present between vehicles $\mathcal{V}_{i-1}$ and $\mathcal{V}_i$, for $i = 2, \dots, N$. This leads to a set of $\mathscr{L} := \{1, \dots, N-1\}$ wireless channels (or links). Two events may occur at transmissions, either the vehicle i receives the packet from its predecessor $i-1$ successfully (i.e. $\hat{u}_{i-1} = u_{i-1}$), or it gets dropped with some probability. To model this behaviour, we introduce a Bernoulli process $\{\theta_{j,k}\}_{k \in \mathbb{N}}$ for each wireless channel $j \in \mathscr{L}$ such that $\theta_{j,k} = 1$ with probability α_j (*probability of successful transmission for channel* j), and $\theta_{j,k} = 0$ with probability $1 - \alpha_j$.

Standing Assumption 2: The packet loss processes $\{\theta_{m,k}\}_{k \in \mathbb{N}}$ and $\{\theta_{n,k}\}_{k \in \mathbb{N}}$ are independent for all $m \neq n$, with $m, n \in \mathscr{L}$, and, for each $j \in \mathscr{L}$, $\{\theta_{j,k}\}_{k \in \mathbb{N}}$ is independent of $\{\tau_k\}_{k \in \mathbb{N}}$. □

This is a standard assumption in NCS literature that considers Bernoulli packet losses and multiple links, see e.g., [40].

Due to packet losses, it is useful to define the so-called *network-induced error*, which represents the error present in the information $\hat{u}_{i-1}$ available at vehicle $\mathcal{V}_i$, with respect to the sent information u_{i-1} from vehicle $\mathcal{V}_{i-1}$. We define it as $e_{u_{i-1}} := \hat{u}_{i-1} - u_{i-1}$, $i \in \mathcal{N} \setminus \{1\}$, noting that e_{u_0} (which is zero) is excluded as the leader follows a virtual reference without network imperfections.

We now use the network-induced error to model the dynamics of the V2V communications. Specifically, for every transmission instant t_k we assume that $e_{u_{i-1}}(t_k^+) = 0$ for every $i \in \mathcal{N} \setminus \{1\}$, i.e. the received signal is equal to the transmitted one $\hat{u}_{i-1}(t_k^+) = u_{i-1}(t_k)$, only if the transmission is successful in the corresponding wireless link ($\theta_{i-1,k} = 1$). Here, $e_{u_{i-1}}(t_k^+)$ denotes the right limit of $e_{u_{i-1}}(\cdot)$ at time t_k. On the other hand, whenever a packet loss occurs, then we assume the corresponding error components remain unchanged since the signal was not updated. That is, $e_{u_{i-1}}(t_k^+) = e_{u_{i-1}}(t_k)$ when $\theta_{i-1,k} = 0$ for every t_k and $i \in \mathcal{N} \setminus \{1\}$. We note that compensation strategies to cope with packet loss in vehicle platooning could be potentially used, see e.g. [12], but it is outside of the scope of this paper. Let $\mathbf{e} := (e_{u_1}, \dots, e_{u_{N-1}}) \in \mathbb{R}^{n_e}$, with $n_e := N - 1$ and $\Theta_k := \text{diag}\{\theta_{1,k}, \dots, \theta_{N-1,k}\}$. The above description is captured by

$$\mathbf{e}(t_k^+) = (I - \Theta_k)\mathbf{e}(t_k). \tag{3}$$

Lastly, in between two transmission events, the value of $\hat{u}_{i-1}$ is kept constant in a zero-order hold fashion. As such, we assume that $\dot{\hat{u}}_{i-1} = 0$, $i \in \mathcal{N} \setminus \{1\}$, for $t \in [t_{k-1}, t_k]$, and any $t_k \in \mathcal{T}$. This is common in multi-hop networks where communicating devices behave like routers/buffers to receive and transmit packets.

IV. Main results

We now present the main results of this paper, namely a stochastic hybrid model for the platoon, and the corresponding string stability conditions.

A. Stochastic hybrid model

Let $x_i = (\xi_i, v_i, a_i, u_i)$, $\mathbf{x} = (x_1, \dots, x_N)$ and $w := (v_0, u_0)$. Then, based on the descriptions in Sections II and

III, we can write the following state-space representation of the i-th subsystem in Fig. 2 during flows (i.e. $\forall t \in [t_{k-1}, t_k]$),

$$\begin{aligned}
\dot{x}_1(t) &= Ax_1(t) + B_w w(t) \\
\dot{x}_i(t) &= Ax_i(t) + Bx_{i-1}(t) + B_e e_{u_{i-1}}(t) \\
\dot{e}_{u_1}(t) &= -C_u A x_1(t) - C_u B_w w(t) \\
\dot{e}_{u_{i-1}}(t) &= -C_u A x_{i-1}(t) + C_u B x_{i-2}(t) + C_u B_e e_{u_{i-2}}(t)
\end{aligned}$$

for all $i \in \mathcal{N} \setminus \{1\}$, where

$$A = \begin{bmatrix} 0 & -1 & -h & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau} & \frac{1}{\tau} \\ \frac{k_p}{h} & -\frac{k_d}{h} & -k_d & -\frac{1}{h} \end{bmatrix},\ B_w = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{k_d}{h} & \frac{1}{h} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{k_d}{h} & 0 & \frac{1}{h} \end{bmatrix},\ B_e = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{h} \end{bmatrix},\ C_u = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}^\top .$$

Next, the dynamics of the platoon at jumps, i.e. for every $t = t_k^+$, are given as follows. Since the considered vehicle dynamics are in continuous time, we have that $\mathbf{x}(t_k^+) = \mathbf{x}(t_k)$. On the other hand, the dynamics of the network-induced error $\mathbf{e}$ at jumps are given by (3). Therefore, we can write the following stochastic hybrid model for the platoon

$$\mathcal{H}_1 : \begin{cases} \dot{\mathbf{x}}(t) = A_{11}\mathbf{x}(t) + A_{12}\mathbf{e}(t) + B_1 w,\ t \in [t_{k-1}, t_k], \\ \dot{\mathbf{e}}(t) = A_{21}\mathbf{x}(t) + A_{22}\mathbf{e}(t) + B_2 w,\ t \in [t_{k-1}, t_k], \\ \mathbf{x}(t_k^+) = \mathbf{x}(t_k), \\ \mathbf{e}(t_k^+) = (I - \Theta_k)\mathbf{e}(t_k), \end{cases} \tag{4}$$

with

$$A_{11} = \begin{bmatrix} A & & & & 0 \\ B & A & & & \\ & \ddots & \ddots & & \\ & & B & A & \\ 0 & & & B & A \end{bmatrix}, A_{12} = \begin{bmatrix} 0 & & & & 0 \\ B_e & 0 & & & \\ & \ddots & \ddots & & \\ & & & B_e & 0 \\ 0 & & & & B_e \end{bmatrix},$$

$$\begin{aligned}
B_1 &= \begin{bmatrix} B_w^\top & 0 & \cdots & 0 & 0 \end{bmatrix}^\top, \\
A_{21} &= \tilde{C}_u A_{11},\ A_{22} = \tilde{C}_u A_{12},\ B_2 = \tilde{C}_u B_1,
\end{aligned} \tag{5}$$

where

$$\tilde{C}_u := \begin{bmatrix} -C_u & 0 & & & 0 \\ 0 & -C_u & & & \\ & & \ddots & \ddots & \\ 0 & & & 0 & -C_u & 0 \end{bmatrix}.$$

System $\mathcal{H}_1$ captures the continuous dynamics given by the vehicles in the platoon (local controller and plant dynamics), and also the discrete—and stochastic—dynamics (jumps) given by the network-induced effects which in our case are random packet losses and stochastic transmission instants.

Moreover, in the context of our particular platooning configuration, the variable $w = (v_0, u_0)$ in (4) denotes changes in the leader's velocity and acceleration. Hence, our main objective in this paper is to explore how these changes impact the stochastic string stability of the underlying platoon.

B. String stability

We now establish explicit string stability conditions for system $\mathcal{H}_1$ in (4). Specifically, we focus on the following string stability property, which is highly motivated by the (deterministic) $\mathcal{L}_p$ *string stability* property proposed in [28], and extensively used in the literature, see e.g. [30], [31].

Definition 1: We say that $\mathcal{H}_1$ in (4) is $\mathcal{L}_2$ *string stable in expectation* if there exist non-negative constants K and γ such that, for any $\mathbf{x}(0) \in \mathbb{R}^{n_x}$ and $w \in \mathcal{L}_2$,

$$\mathbf{E}\left\{\|x_i\|_{\mathcal{L}_2[0,t]}\right\} \leq K|\mathbf{x}(0)| + \gamma\|w\|_{\mathcal{L}_2[0,t]}, \tag{6}$$

for all $i \in \mathcal{N}$ and all platoon lengths $N \geq 2$, with $t \geq 0$. □

In contrast to standard $\mathcal{L}_2$ stability, we emphasise that (6) must hold for any platoon length N to guarantee the *string* stability aspect. That is, the constants K and γ in the definition are independent of the number of vehicles. A similar notion to Definition 1 was adopted in [18], and called *stochastic* $\mathcal{L}_2$ *string stability*. Furthermore, in stochastic NCSs, similar definitions in terms of expectations have been used, albeit without considering the scalability (or "string") aspect. For instance, refer to [41] for the $\mathcal{L}_p$ *stability in expectation* property. Additionally, our focus is directed towards velocity/acceleration profiles $w = (v_0, u_0)$ characterised by a bounded 2-norm, as outlined in [30].

Formally, sufficient conditions for $\mathcal{L}_2$ string stability in expectation of $\mathcal{H}_1$ are stated in the following theorem, whose proof is omitted due to space constraints.

Theorem 1: Consider the wireless platoon $\mathcal{H}_1$ in (4), and let $P(s) := A_{21}(sI - A_{11})^{-1}[A_{12}\ B_1]$ be the transfer function of the $\mathbf{x}$–subsystem in (4), with (A_{11}, A_{21}) detectable. If the following holds

- (i) The time-headway h is such that there exists $\overline{\gamma}_x, \overline{K}_x \geq 0$ satisfying $\|P(j\omega)\|_{\mathcal{H}_\infty} \leq \overline{\gamma}_x$ and $|A_{21}| \leq \overline{K}_x$ for any platoon length $N \in \mathbb{N}$.
- (ii) $(\alpha_j, \boldsymbol{\lambda}, h)$ are such that

$$\prod_{j=1}^{N-1} \alpha_j > \frac{1}{\boldsymbol{\lambda}}\left(\overline{\gamma}_x + \frac{1}{h}\right). \tag{7}$$

Then, $\mathcal{H}_1$ is $\mathcal{L}_2$ string stable in expectation. □

Condition (i) pertains to the network-free design of h. Note that $P(s)$ corresponds to the transfer function of the $\mathbf{x}$-system, and thus condition (i) essentially states that the network-free platoon should at least be $\mathcal{L}_2$ string stability in absence of network imperfections. Then, condition (ii) establishes a direct relationship between crucial parameters of the networked platoon so that $\mathcal{L}_2$ string stability in expectation is achieved, namely the probabilities of successful transmission α_j, $j \in \mathscr{L}$, the transmission rate $\boldsymbol{\lambda}$, and the time-headway h.

As observed from (7), in the case of low-quality channels, achieving string stability relies not only on increasing the time-headway h but also on having more frequent transmissions via $\boldsymbol{\lambda}$. Additionally, we observe that maintaining the same value for h as in the network-free case can still lead to achieving string stability under packet losses through

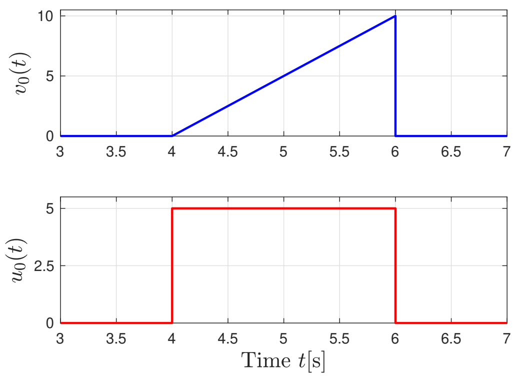

Fig. 3: External platoon input $w(t) = (v_0(t), u_0(t))$.

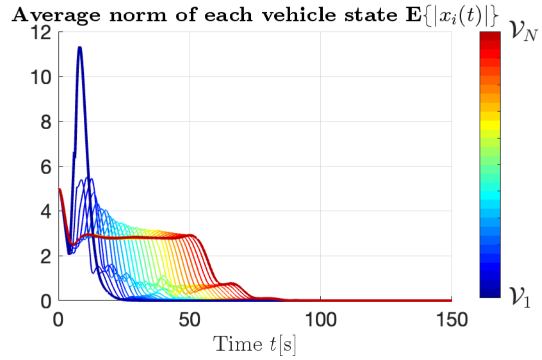

Fig. 4: Expected value of the norm of each vehicle state for a platoon of size $N = 30$, over 2000 realisations, with parameters $h = 1.8$, $1/\boldsymbol{\lambda} = 1$ and $\alpha_1 = \cdots = \alpha_{10} = 0.8$, $\alpha_{11} = \cdots = \alpha_{20} = 0.7$, $\alpha_{21} = \cdots = \alpha_{29} = 0.9$.

increasing the transmission rate, provided the network has enough capacity.

V. Numerical example

Consider the wireless platoon described by $\mathcal{H}_1$ with controller gains $k_p = 0.2$ and $k_d = 0.7$, alongside a drive-line constant of $\tau = 0.1$. As shown in [28], these parameters ensure (network-free) individual vehicle stability. The aim of this example is to showcase how the interplay among pivotal platoon parameters, including h, α_j's, and $\boldsymbol{\lambda}$, directly impacts string stability. We use the Hybrid Equations Toolbox (HyEQ) [42] to simulate the stochastic hybrid system $\mathcal{H}_1$ under different scenarios of interest. The velocity and acceleration profile that determines the external $\mathcal{L}_2$ input $w = (v_0, u_0)$ are given in Fig. 3. We consider a platoon size of $N = 30$ vehicles, and the initial condition for every vehicle is taken to be $x_i(0) = (5, 0, 0, 0)$ for all $i \in \{1, \ldots, 30\}$. In this example, we examine three noteworthy cases.

Case 1: We first study a scenario where the V2V channels have good quality (i.e. large success probabilities). Particularly, we consider probabilities $\alpha_1 = \cdots = \alpha_{10} = 0.8$, $\alpha_{11} = \cdots = \alpha_{20} = 0.7$, and $\alpha_{21} = \cdots = \alpha_{29} = 0.9$. The time headway is taken to be $h = 1.8$ and we consider an average transmission rate of $1/\boldsymbol{\lambda} = 1$ second. The expected value of the norm of each vehicle state over 2000 realisations is depicted in[1] Fig. 4. As per Definition 1, we can see this platoon is indeed $\mathcal{L}_2$ string stable in expectation. As shown by our explicit string stability condition in Theorem 1, there exists a direct relationship between h, $\boldsymbol{\lambda}$, and the α_j's. Consequently, in the subsequent case, we will manipulate these variables to investigate whether string stability can be maintained even when certain segments of the platoon experience significantly degraded channel quality.

Case 2: Here, we consider that the tail of the platoon has worse V2V communication quality, and thus take $\alpha_{21} = \cdots = \alpha_{29} = 0.1$, whilst keeping the other probabilities, headway and transmission rate as in Case 1. From Fig. 5, we can see that maintaining string stability, which was achieved in Case 1 with the same average transmission rate and time headway, becomes unfeasible when certain segments of the platoon encounter a higher dropout rate compared to others. We could anticipate this finding based on our string stability bound (7), given its dependence on the product of all probabilities. Hence, to guarantee $\mathcal{L}_2$ string stability on average when some vehicles in the platoon have low success probabilities, adjustments to either the transmission rate $\boldsymbol{\lambda}$ or the time headway h may be necessary. This leads to the remaining two cases below.

Case 3: In this scenario, we maintain the low-quality communication at the tail of the platoon, as in Case 2, with $\alpha_{21} = \cdots = \alpha_{29} = 0.1$, and retain the same headway of $h = 1.8$. However, we increase the transmission rate to achieve an average transmission interval of $1/\boldsymbol{\lambda} = 0.1$ seconds, contrasting with the one-second interval in Case 2. As depicted in Fig. 6, we managed to restore the string stability property that was compromised by the high dropout rate at the tail by adjusting $\boldsymbol{\lambda}$. In essence, boosting the transmission rate can help in managing losses within segments of the platoon.

Case 4: Finally, we illustrate that string stability can be restored not only by adjusting $\boldsymbol{\lambda}$ but also by modifying the time headway h. We consider the same string-unstable configuration from Case 2, where $1/\boldsymbol{\lambda} = 1$ and $\alpha_{21} = \cdots = \alpha_{29} = 0.1$, but we now increase the headway from $h = 1.8$ to $h = 3$. From Fig. 7, we can see that by modifying h, we are capable of achieving $\mathcal{L}_2$ string stability on average, even under a significant dropout rate at the platoon's tail.

Moreover, it is apparent that adjusting the transmission rate, at least in this example, results in improved platoon behaviour, despite both scenarios being considered string stable (cf. Fig. 6 and Fig. 7).

VI. Conclusions

We provided explicit conditions for stochastic string stability in an $\mathcal{L}_2$ sense for predecessor-following platoons subject

[1]We note that every figure in this example utilises colour coding to distinguish each vehicle in the platoon, transitioning from bluish hues for the leading vehicles to reddish tones towards the rear. Additionally, we have highlighted the first and last vehicles in the platoon by making them bold.

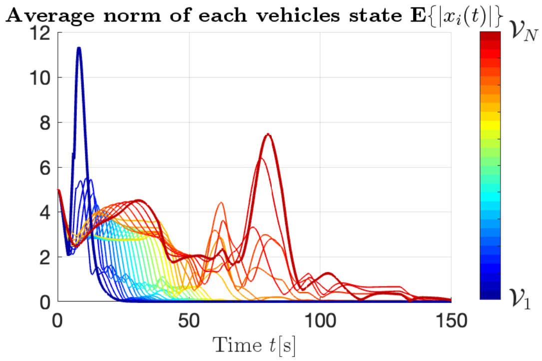

Fig. 5: Expected value of the norm of each vehicle state for a platoon of size $N = 30$, over 2000 realisations, with parameters $h = 1.8$, $1/\boldsymbol{\lambda} = 1$ and $\alpha_1 = \cdots = \alpha_{10} = 0.8$, $\alpha_{11} = \cdots = \alpha_{20} = 0.7$, $\alpha_{21} = \cdots = \alpha_{29} = 0.1$.

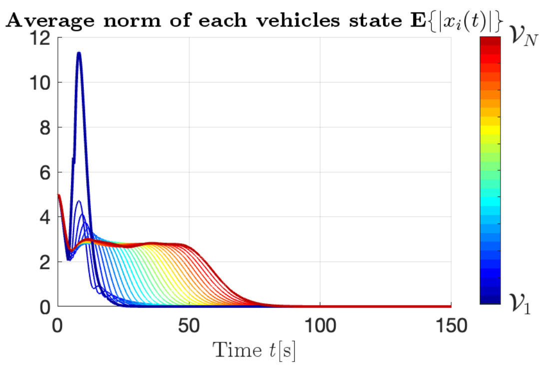

Fig. 6: Expected value of the norm of each vehicle state for a platoon of size $N = 30$, over 300 realisations, with parameters $h = 1.8$, $1/\boldsymbol{\lambda} = 0.1$ and $\alpha_1 = \cdots = \alpha_{10} = 0.8$, $\alpha_{11} = \cdots = \alpha_{20} = 0.7$, $\alpha_{21} = \cdots = \alpha_{29} = 0.1$.

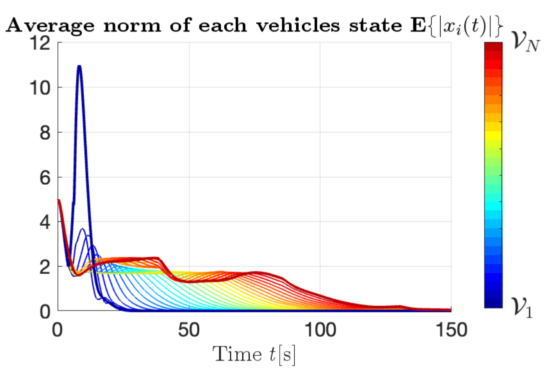

Fig. 7: Expected value of the norm of each vehicle state for a platoon of size $N = 30$, over 2000 realisations, with parameters $h = 3$, $1/\boldsymbol{\lambda} = 1$ and $\alpha_1 = \cdots = \alpha_{10} = 0.8$, $\alpha_{11} = \cdots = \alpha_{20} = 0.7$, $\alpha_{21} = \cdots = \alpha_{29} = 0.1$.

to (heterogeneous) packet losses. These conditions illustrate the interaction between crucial vehicle and network parameters. Through numerical examples, we have highlighted that, despite lower channel quality, adjusting the transmission rate or time headway proves effective in preserving string stability.

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