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Quantized Event-Based Sampled-Data Nonlinear Control for PEM Fuel Cell Air Supply

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Abstract— **The problem of oxygen starvation avoidance in air-feed proton exchange membrane fuel cells is widely known and studied in the literature because it crucially affects the performances of the overall system. To address this problem, many nonlinear control techniques have been provided which, however, do not take into account the effects on the performances induced by the digital hardware commonly used for the practical implementation of control strategies. In this paper, a methodology for the design of quantized sampled-data event-based stabilizers is proposed for a class of time-varying nonlinear systems and applied to the problem of oxygen starvation and maximum net power achievement in Proton Exchange Membrane Fuel Cells (PEMFCs). In particular, by exploiting a Lyapunov-based approach and the stabilization in the sample-and-hold sense theory, it is shown that there exist a suitably fast sampling and an accurate quantization of the input/output channels such that the digital event-triggered implementation of a proposed continuous-time stabilizer ensures the semi-global practical stability property of the related closed-loop system. In the theory developed here, time-varying sampling periods and non-uniform quantization of the input/output channels are allowed. Simulations confirm the effectiveness of the theoretical results.**

I. INTRODUCTION

Hydrogen fuel-cell powertrains are an active research area as an alternative energy conversion system due to their low impact on the environment (fuel cell emissions are practically zero), higher energy and power density than combustion engines, absence of moving mechanical parts [5]. In addition, they do not produce noise and their lifetime is longer [5]. The high efficiency of fuel cells and the fact that they scale with power [5], [3], contribute to the usage of fuel cell in several applications. Among these, PEMFCs are used mostly in automotive applications [3], mainly due to their low operative temperatures of around 60-80°C.

In a PEMFC, there are mainly three control loops [3]: heat management [15],[16], water management [17] and air/fuel supply control [18],[19]. In this paper, we refer to the latter control problem. The analyzed subsystem is composed of the fuel cell dynamic model, the air compressor system and the supply manifold dynamics, which represents the lumped volume associated with connections and pipes between compressor and fuel cell cathode. The objective is to regulate oxygen quantity inside the cathode avoiding oxygen starvation that happens when oxygen partial pressure drops below a critical value, damaging irreversibly the membrane. Moreover, this phenomenon is spatially variant, i.e. the oxygen partial pressure should not fall below a crucial level at all points of the cathode air stream [3].

Among the auxiliary components in a Fuel Cell, the most power-consuming device is the air compressor. Thus, in addition to the fuel cell performance, it is important to achieve a maximum net power (P_{net}), defined as the difference between the power dispensed by the fuel cell (P_{FC}) and the power consumed by the compressor ($P_{compressor}$). It was shown in [1], [2] that net power maximization can be obtained with oxygen ratio between 2 and 2.5, where the oxygen excess ratio λ_{O_2} is defined as the oxygen inlet flow injected to the cathode over the oxygen flux consumed during the reaction. Furthermore, it is known [3] that the aforementioned oxygen starvation problem can be avoided by regulating λ_{O_2}, so that the control objective is to achieve a desired $\lambda^*_{O_2}$ under load (disturbance) variations, thereby ensuring oxygen starvation avoidance and maximum Fuel Cell net power.

To deal with the problems above, several controllers have been designed in a continuous-time framework by exploiting approaches such as: Sliding Mode Control [7],[6], LPV Control [4], PID [21], Fuzzy Control [20]. On the other hand, although these approaches can simultaneously incorporate different control requirements in the design, the computed control laws are approximated and applied within digital hardware without formal performance guarantees, not taking into account crucial aspects such as (i) the presence of sampling and quantization in both input/output channels; (ii) possible event-based strategies used to optimize computational resources by updating the control law only when really necessary.

To fill this gap, in this work we apply to the problem of reactant supply control the theory of *stabilization in the sample-and-hold sense* [11], [10] which provides the theoretical background for the preservation of continuous-time guarantees in a semi-global practical sense when the controller is applied in the presence of sampling and quantization in both input/output channels [12] and updated through a triggering mechanism [13], [14]. As a first step, we will make use of a similar approach to the one proposed in [10]

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concerning the *stabilization in the sample-and-hold sense* theory applied in the context of the attitude control problem of ground vehicles, in order to provide a methodology for the design of quantized event-based sampled-data stabilizers for a class of time-varying nonlinear systems. In particular, the *stabilization in the sample-and-hold sense* theory is here used as a tool to prove the existence of a suitably fast sampling and of an accurate quantization of the input/output channels, such that the digital event-based implementation of a proposed continuous-time controller ensures the semi-global practical stability property of the related closed-loop system. Then, the proposed methodology is exploited to design a quantized event-based sampled-data controller for the oxygen starvation avoidance in air-feed PEMFCs. In the provided results, aperiodic samplings and non-uniform quantization are allowed. To our best knowledge, it is the first time in the literature that: (i) for the considered class of time-varying nonlinear systems, a methodology for the design of quantized event-based sampled-data stabilizers is provided; (ii) theoretical results concerning the design and the application of quantized event-based sampled-data controllers for the oxygen starvation avoidance in air-feed PEMFCs are derived.

The paper is organized as follows. In Section II, the problem is formulated, i.e. the fuel cell cathode model is introduced as well as air compressor and supply manifold dynamics. In Section III, by exploiting the *stabilization in the sample-and-hold sense* theory, a methodology for the design of quantized event-based sampled-data stabilizers is proposed for a class of time-varying nonlinear systems. In Section IV, the proposed control strategy is applied to the fuel cell air-supply system. In Section V, simulation results are shown and in the last section the most important developments are highlighted as well as possible future improvements.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, the symbol $|\cdot|$ represents the L_2 norm of a real vector. Given a positive integer n and a positive real h, the symbol $\mathscr{B}^n_h$ is the set of points belonging to $\{x \in \mathbb{R}^n : |x| \leq h\}$.

A continuous function $\gamma : \mathbb{R}^+ \to \mathbb{R}^+$ is: of class $\mathscr{P}_0$ if $\gamma(0) = 0$; of class $\mathscr{P}$ if it is of class $\mathscr{P}_0$ and $\gamma(s) > 0$, $s > 0$; of class $\mathscr{K}$ if it is of class $\mathscr{P}$ and strictly increasing; of class $\mathscr{K}_\infty$ if it is of class $\mathscr{K}$ and unbounded.

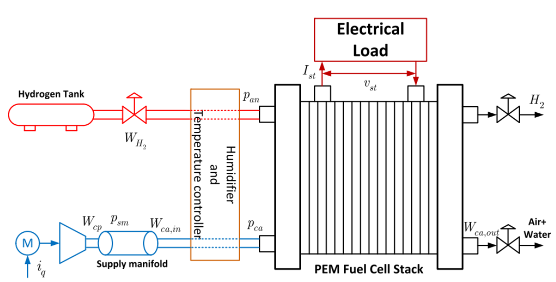

Fig. 1. Fuel Cell System.

II. PEMFC Air supply system and Control Problem

In this section, the PEMFC air supply system we consider is introduced and the control problem is stated. In particular, for the purpose of this study, we consider an air supply system for a PEMFC as the one depicted in Fig.1: an electric motor drives the air compressor which injects air to the cathode. Before entering the stack, humidifier and temperature controller manage air relative humidity and stack temperature to guarantee a proper thermal operative condition for the stack and cathode air inlet flow humidity. Connections and pipes between compressor and cathode are considered a lumped volume and are referred to as a supply manifold.

Our analysis is based on the fifth-order model that has been simulated and validated in HIL test bench in [1], where the following assumptions are made:

- Stack temperature and Air relative humidity are controlled;
- Anode hydrogen inlet flow is controlled so as to minimize anode and cathode pressure difference;
- All state variables are available for the controller;
- Current dynamics of the motor are neglected due to the faster time response with respect to the mechanical dynamics.

In the following, the mathematical model of the components characterizing the considered air supply PEMFC system (see Fig.1) [1], [3] is presented. In what follows, all the involved model parameters c_i, $i = 1, \cdots, 23$, are positive reals. The reader is referred to [1], [3] for more details. According to the classical mathematical models exploited, for instance in [1], [3], [8] for control design, the considered air supply PEMFC system is described by the following equations:

$$
\begin{aligned}
\dot{\omega}_{cp}(t) &= -c_1\omega_{cp}(t) - \frac{c_2}{\omega_{cp}(t)}\left[\left(\frac{p_{sm}(t)}{c_3}\right)^{c_4} - 1\right]h_2(t) + c_5 u(t) \\
\dot{p}_{sm}(t) &= c_6\left(1 + c_7\left[\left(\frac{p_{sm}(t)}{c_3}\right)^{c_4} - 1\right]\right)\left[h_2(t) - c_8(p_{sm}(t) - \chi(t))\right] \\
\dot{p}_{O_2}(t) &= c_9(p_{sm}(t) - \chi(t)) - c_{14}I_{st}(t) \\
&\quad - \frac{c_{10}\phi(\chi(t))p_{O_2}(t)}{c_{11}p_{O_2}(t) + c_{12}p_{N_2}(t) + c_{13}p_v(t)} \\
\dot{p}_{N_2}(t) &= c_{15}(p_{sm}(t) - \chi(t)) - \frac{c_{17}\phi(\chi(t))p_{N_2}(t)}{c_{11}p_{O_2}(t) + c_{12}p_{N_2}(t) + c_{13}p_v(t)} \\
\dot{p}_v(t) &= c_{16}(p_{sm}(t) - \chi(t)) + c_{20}I_{st}(t) + c_{22}W_{membr}(t) \\
&\quad - \frac{c_{23}\phi(\chi(t))p_v(t)}{c_{11}p_{O_2}(t) + c_{12}p_{N_2}(t) + c_{13}p_v(t)}.
\end{aligned}
\tag{1}
$$

where:

- $\omega_{cp}(t)$ is the angular velocity of the electric motor which drives the air compressor;
- $h_2(t)$ is the air flow generated by the compressor, which is proportional to the angular velocity $\omega_{cp}(t)$ according to the relation $h_2(t) = c_{21}\omega_{cp}(t)$;
- $p_{sm}(t)$, $p_{O_2}(t)$, $p_{N_2}(t)$ and $p_v(t)$ are the supply manifold pressure and oxygen, nitrogen and water vapor partial pressures, respectively;

- $\chi(t)$ is the cathode pressure defined as $\chi(t) = p_{O_2}(t) + p_{N_2}(t) + p_v(t)$;
- $I_{st}(t)$ is the stack current, considered as an external disturbance;
- $W_{membr}(t)$ is the net water flux across fuel cell membrane;
- $u(t) = i_q(t)$ is the control input characterized by the current (along q-axis) for the electric motor.

The oxygen excess ratio [1] and the net power delivered by the stack can be expressed as:

$$\lambda_{O_2}(t) = \frac{W_{O_2,in}(t)}{W_{O_2,react}(t)} = \frac{c_{18}}{c_{19} I_{st}(t)}(p_{sm}(t) - \chi(t)), \quad (2)$$

$$P_{net}(t) = P_{FC}(t) - P_{compressor}(t). \quad (3)$$

From (1)-(3) the oxygen excess ratio and air compressor flow rate dynamics follow:

$$\begin{aligned} \dot{\lambda}_{O_2}(t) &= g_{20}(t) h_2(t) + \phi_1(t) \\ \dot{h}_2(t) &= c_{21}\left\{ -c_1 \omega_{cp}(t) - \frac{c_2}{\omega_{cp}(t)}\left[\left(\frac{p_{sm}(t)}{c_3}\right)^{c_4} - 1\right] h_2(t) \right. \\ &\quad \left. + c_5 u(t) \right\} \end{aligned} \quad (4)$$

where (see [1])

$$\phi_1(t) = \frac{c_{18} c_6}{c_{19} I_{st}(t)} \left[c_8 \left(1 + c_7 \left[\left(\frac{p_{sm}(t)}{c_3} \right)^{c_4} - 1 \right] \right) (p_{sm}(t) - \chi(t)) - \dot{\chi}(t) \right],$$

$$g_{20}(t) = \frac{c_{18} c_6}{c_{19} I_{st}(t)} \left(1 + c_7 \left[\left(\frac{p_{sm}(t)}{c_3} \right)^{c_4} - 1 \right] \right).$$

Notice that $g_{20}(t)$ is a bounded function in the operative range of ω_{cp} satisfying $g_{20}(t) > 0$, $\forall t \geq 0$ [1].

The problem in this paper is to design a quantized sampled-data event-based controller for the system described by (1) in order to track a desired smooth signal $\lambda^*_{O_2}(t) \in [2, 2.5]$, for the maximization of the net power P_{net}, when the system is subject to load (I_{st}) variations. To deal with this control problem, according to (4), the air compressor flow rate $h_2(t)$ can be exploited to drive λ_{O_2} to $\lambda^*_{O_2}$ by tracking a desired flow $h^*_2(t)$ appropriately designed. The convergence of $h_2 \to h^*_2$ is obtained by means of the control input $u(t)$. Hence, there are two control loops: the first one to track $\lambda_{O_2} \to \lambda^*_{O_2}$ by means of h^*_2, the second one for the convergence $h_2 \to h^*_2$ through $u(t)$ (see, for instance, the reasoning in [1] used in the context of the continuous-time sliding mode control design). To this purpose, we introduce the tracking error variables:

$$e_1(t) = \lambda_{O_2}(t) - \lambda^*_{O_2}(t) = x_1(t) - x^*_1(t), \quad (5)$$

$$e_2(t) = h_2(t) - h^*_2(t) = x_2(t) - x^*_2(t), \quad (6)$$

where $x(t) = [x_1(t), x_2(t)]^T = [\lambda_{O_2}(t), h_2(t)]^T \in \mathbb{R}^2$ and $x^*(t) = [x^*_1(t), x^*_2(t)]^T = [\lambda^*_{O_2}(t), h^*_2(t)]^T \in \mathbb{R}^2$. Taking into account (5), (6), from (4), we obtain the following tracking error system:

$$\dot{e}_1(t) = g_{20}(t) h^*_2(t) + g_{20}(t) e_2(t) + \phi_1(t) - \dot{\lambda}^*_{O_2}(t) \quad (7)$$

$$\dot{e}_2(t) = \phi_2(t) - \dot{h}^*_2(t) + g_2 u(t), \quad (8)$$

where [1]

$$g_2 = c_5 c_{21},$$

$$\phi_2(t) = c_{21}\left\{ -c_1 \omega_{cp}(t) - \frac{c_2}{\omega_{cp}(t)}\left[\left(\frac{p_{sm}(t)}{c_3}\right)^{c_4} - 1\right] h_2(t) \right\}.$$

Let

$$h^*_2(t) = \frac{1}{g_{20}(t)}[-\phi_1(t) - k_1 e_1(t) + \dot{\lambda}^*_{O_2}(t)] \quad (9)$$

be the desired compressor air flow, where $k_1 > 0$ is a positive control tuning parameter. From (7)-(9), we obtain

$$\dot{e}_1(t) = -k_1 e_1(t) + g_{20} e_2(t) \quad (10)$$

$$\dot{e}_2(t) = \phi_2(t) - \dot{h}^*_2(t) + g_2 u(t). \quad (11)$$

In the following section, a methodology for the design of quantized event-triggered sampled-data controller is proposed for a class of time-varying nonlinear systems that includes the one corresponding to the error dynamics (10)-(11).

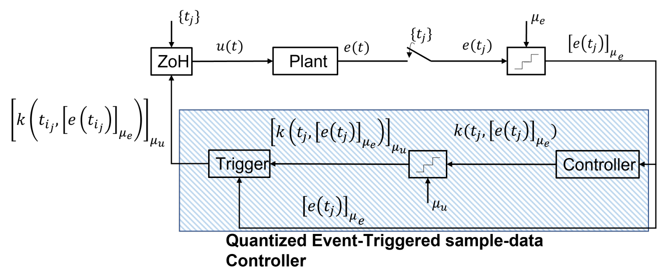

Fig. 2. Quantized Event-Triggered Sampled-Data Closed-Loop System.

III. Quantized Event-Based Sampled-Data Control Design

Let us consider a time-varying nonlinear system described by the following equations:

$$\begin{aligned} \dot{e}_1(t) &= f_1(t, e(t)) \\ \dot{e}_2(t) &= f_2(t, e(t)) + g(t, e(t)) u(t), \end{aligned} \quad (12)$$

where: $e(t) = [e_1(t)^T, e_2(t)]^T \in \mathbb{R}^n$, $e_1(t) \in \mathbb{R}^{n-1}$, $e_2(t) \in \mathbb{R}$ is the state; $u(t) \in \mathbb{R}$ is the control input; $f_1 : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^{n-1}$, $f_2 : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}$ are known functions. It is assumed that, for any given positive real E, there exist positive reals L_{f_1}, L_{f_2}, L_g, g_{min} and g_{max}, such that the following conditions hold

$$|f_i(t_1, e_x) - f_i(t_2, e_y)| \leq L_{f_i}(|t_1 - t_2| + |e_x - e_y|), \, i = 1, 2,$$
$$|g(t_1, e_x) - g(t_2, e_y)| \leq L_g(|t_1 - t_2| + |e_x - e_y|),$$
$$g_{min} \leq |g(t_1, e_x)| \leq g_{max}, \, \forall t_1, t_2 \in \mathbb{R}^+, \forall e_x, e_y \in \mathscr{B}^n_E,$$
$$g(t, e) \neq 0, \, \forall t \in \mathbb{R}^+, \forall e \in \mathbb{R}^n.$$

Assumption 1: There exist a symmetric positive definite matrix $P_1 \in \mathbb{R}^{n-1\times n-1}$, a locally Lipschitz function $\tilde{f}_1 : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}$ and a positive real γ_1 such that, for any $e = [e_1^T, e_2]^T \in \mathbb{R}^n$, $e_1 \in \mathbb{R}^{n-1}$, $e_2 \in \mathbb{R}$ and for any $t \in \mathbb{R}^+$, the following condition holds

$$2e_1^T P_1 f_1(t,e) \leq -\gamma_1 |e_1|^2 + e_2 \tilde{f}_1(t,e). \tag{13}$$

In the following, a quantized event-based sampled-data controller is proposed for the system (12) and, by exploiting the reasoning in [10] concerning the *stabilization in the sample-and-hold sense* theory applied in the context of the attitude control problem of ground vehicles, the semi-global practical stability property of the related closed-loop system is proved. Then, the proposed method is applied to system (10)-(11) in order to design a quantized event-based sampled-data controller for the oxygen starvation avoidance in air-feed PEMFCs.

Let $k : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}$ be the function defined for any $t \in \mathbb{R}^+$ and $e = [e_1^T, e_2]^T \in \mathbb{R}^n$ as:

$$k(t,e) = \frac{-f_2(t,e) - \tilde{f}_1(t,e) - \gamma_2 e_2}{g(t,e)}, \tag{14}$$

where γ_2 is a positive real.

In order to introduce the proposed quantized event-based sampled-data controller, the notions of partition of $[0,+\infty)$ [11], [10] and of quantizers [12] are now recalled.

Definition 1: A partition $\pi = \{t_i\}_{i\in\mathbb{Z}^+}$ of $[0,+\infty)$ is a countable, strictly increasing sequence t_i, with $t_0 = 0$, such that $\lim_{i\to\infty} t_i = +\infty$. The diameter of π, $diam(\pi)$, is defined as $\sup_{i\geq 0} t_{i+1} - t_i$.The dwell time of π, $dwell(\pi)$, is defined as $\inf_{i\geq 0} t_{i+1} - t_i$. For any real $a \in (0,1]$, $\delta > 0$, $\pi_{a,\delta}$ is any partition π with $a\delta \leq dwell(\pi) \leq diam(\pi) \leq \delta$.

In the following, a state quantizer and an input quantizer are piece–wise constant functions $[\cdot]_{\mu_e} : \mathbb{R}^n \to \mathscr{Q}_e$ and $[\cdot]_{\mu_u} : \mathbb{R} \to \mathscr{Q}_u$, with $\mathscr{Q}_e$, $\mathscr{Q}_u$ suitable finite subsets of $\mathbb{R}^n$ and $\mathbb{R}$, characterized, for some given positive reals E, U (range of the quantizers) and μ_e, μ_u (quantization errors), by the following implications [9]

$$|u| \leq U \implies |u - [u]_{\mu_u}| \leq \mu_u, \tag{15}$$

$$|e| \leq E \implies |e - [e]_{\mu_e}| \leq \mu_e. \tag{16}$$

In order to simplify the notation, let $F : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ be the function defined, for any $t \in \mathbb{R}^+$, $e \in \mathbb{R}^n$ and $u \in \mathbb{R}$, as follows

$$F(t,e,u) = \begin{bmatrix} f_1(t,e) \\ f_2(t,e) + g(t,e)u \end{bmatrix}, \tag{17}$$

where f_1, f_2 and g are the functions in (12). In the following, the proposed quantized event-triggered sampled-data controller for the system (12) is proposed. Under Assumption 1, for a given positive real $\sigma \in (0,1)$, for a given partition $\pi_{a,\delta}$, for a given state quantizer $[\cdot]_{\mu_e} : \mathbb{R}^n \to \mathscr{Q}_e$ and for a given input quantizer $[\cdot]_{\mu_u} : \mathbb{R} \to \mathscr{Q}_u$, the proposed quantized event-based sampled-data stabilizer for system (12) is described by (see Fig. 2) [10].

$$u(t) = u^*(t_{i_j}) = [k(t_{i_j}, [e(t_{i_j})]_{\mu_e}]_{\mu_u}, \tag{18}$$

$$t_j \leq t \leq t_{j+1}, \quad t_j \in \pi_{a,\delta}, \quad j = 0,1,\cdots, \tag{19}$$

where: k is the function in (14); the sequence $\{i_j\}$ is defined as $i_0 = 0$ and, for $j \geq 1$, $i_j = j$ if

$$\begin{gathered} 2[e(t_j)]_{\mu_e}^T S\Big[\sigma F\big(t_j, [e(t_j)]_{\mu_e}, u^*(t_j)\big) - \\ F\big(t_j, [e(t_j)]_{\mu_e}, u^*(t_{i_{j-1}})\big)\Big] \leq 0, \ S = \begin{bmatrix} P_1 & 0 \\ 0 & 0.5 \end{bmatrix} \end{gathered} \tag{20}$$

and $i_j = i_{j-1}$ otherwise. It is worth noting that the triggering condition (20) is checked at each sampling time t_j, $j = 0,1,\ldots,$ guaranteeing a minimum dwell-time between two consecutive sampling instants and, hence, avoiding Zeno phenomenon.

The main results are now provided. In particular, it is shown that there exist a suitably fast sampling δ and an accurate quantization of both input/output channels (i.e, ranges E, U and quantization errors μ_e, μ_u for the state and input quantizers) such that the digital event-triggered closed-loop system (12) with (18) is semi-globally practically stable with arbitrarily small final target ball of the origin.

Theorem 1: Let $\sigma \in (0,1)$, $a \in (0,1]$. Then, $\forall r$, $R \in \mathbb{R}^+$, $0 < r < R$, there exist positive reals δ, T, μ_e, μ_u, E and U, such that: for any partition $\pi_{a,\delta} = \{t_j, j = 0,1,\ldots,\}$ of $[0,+\infty)$, for any state and input quantizers $[\cdot]_{\mu_e}$ and $[\cdot]_{\mu_u}$ with error bounds μ_e, μ_u, and ranges E, U, respectively, for any initial state $e_0 \in \mathscr{B}_R^n$, the solution of the system (12) starting from e_0 with the quantized sampled-data event-triggered control law (18)-(20), exists $\forall t \geq 0$ and, furthermore, satisfies:

$$|e(t)| \leq E, \quad \forall t \geq 0, \qquad |e(t)| \leq r, \quad \forall t \geq T, \tag{21}$$

(i.e., the closed-loop system (12)-(18) is semi-globally practically stable).

Proof: In order to prove Theorem 1, we make use of the results provided in [10] concerning the stabilization in the sample-and-hold sense theory applied in the context of the attitude control problem of ground vehicles. In particular, by checking that Assumption 1 in [10] is satisfied for the continuous-time closed-loop system described by

$$\dot{e}(t) = F(t, e(t), k(t, e(t))), \tag{22}$$

where the functions F and k are defined in (17) (see also (12)) and (14), respectively, the same reasoning used in [10] can be here repeated for proving the results in Theorem 1. According to Assumption 1 and (25), (26) in [10], we have to prove that: there exist a continuous function $V : \mathbb{R}^n \to \mathbb{R}^+$ admitting locally Lipschitz partial derivatives and functions $\alpha_i \in \mathscr{K}_\infty$, $i = 1,2,3$, such that, for any $t \in \mathbb{R}^+$ and for any $e \in \mathbb{R}^n$, the following inequalities hold

$$\alpha_1(|e|) \leq V(e) \leq \alpha_2(|e|), \tag{23}$$

$$\frac{\partial V}{\partial e} F(t, e, k(t,e)) \leq -\alpha_3(|e|). \tag{24}$$

For this purpose, let $V : \mathbb{R}^n \to \mathbb{R}^+$ be the function defined for any $e \in \mathbb{R}^n$ as $V(e) = e^T S e$ where S is the matrix in (20). Notice that, the inequalities in (23) are here satisfied by choosing, for instance, functions $\alpha_i \in \mathscr{K}_\infty$, defined for $s \in \mathbb{R}^+$ as $\alpha_1(s) = \min\{\lambda_{min}(P_1), 0.5\}s^2$ and $\alpha_2(s) = \max\{\lambda_{max}(P_1), 0.5\}s^2$. Moreover, as far as (24) is concerned, for any $t \in \mathbb{R}^+$ and for any $e \in \mathbb{R}^n$, the following inequalities hold

$$\begin{aligned}\frac{\partial V}{\partial e}F(t,e,k(t,e)) &= 2e_1^T P_1 f_1(t,e) + e_2(-\gamma_2 e_2 - \tilde{f}_1(t,e)) \\ \le -\gamma_1|e_1|^2 + e_2\tilde{f}_1(t,e) - \gamma_2 e_2^2 - e_2\tilde{f}_1(t,e) &\le -\gamma_1|e_1|^2 - \gamma_2 e_2^2 \\ &\le -\min\{\gamma_1,\gamma_2\}|e|^2.\end{aligned} \tag{25}$$

It follows that (24) is satisfied by choosing, for instance, the function $\alpha_3 \in \mathscr{K}_\infty$, defined for $s \in \mathbb{R}^+$ as $\alpha_3(s) = \min\{\gamma_1,\gamma_2\}s^2$. From here on, the same steps used in the proof of Theorem 1 in [10] can be repeated to show that the solution of the quantized event-based sampled-data closed-loop system (12)-(18) exists for any $t \in \mathbb{R}^+$ and (21) holds. The reader is referred to steps (32), (33), (36), (37) and (40)-(54) in [10], with the functions F and k defined in (17) (see also (12)) and (14), respectively. This concludes the proof. ■

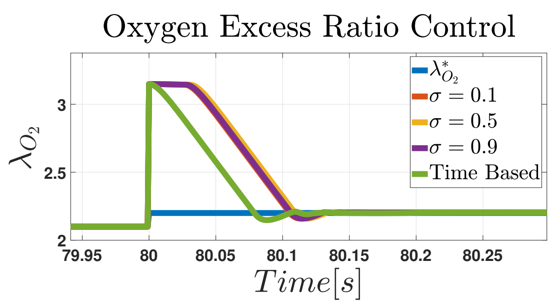

Fig. 3. λ_{O_2} Control - Transient response detail.

IV. A Digital Event-Based Controller for PEMFC Air-Supply System

In this section, the design methodology proposed in Section III is applied in the context of the oxygen starvation avoidance in air-feed PEMFCs. In particular, by exploiting the results provided in Section III, a quantized event-triggered sampled-data controller is designed for the system described by (10)-(11) ensuring the semi-global practical stability property of the related closed-loop tracking error system (see Theorem 1). Notice that Assumption 1 is satisfied for system (10)-(11) by choosing, for instance, $P_1 = 0.5$, $\gamma_1 = k_1$ and $\tilde{f}_1(t,e) = g_{20}(t)e_1$. Hence, we can apply the design methodology proposed in Section III to system (10)-(11). According to (14), $k : \mathbb{R}^+ \times \mathbb{R}^2 \to \mathbb{R}$ is the function defined, for any $t \in \mathbb{R}^+$ and $e = [e_1, e_2]^T \in \mathbb{R}^2$, by

$$k(t,e) = \frac{1}{g_2}[-\phi_2(t) + \dot{h}_2^*(t) - \gamma_2 e_2 - g_{20}(t)e_1]. \tag{26}$$

Moreover, according to (17) and taking into account (10)-(11), $F : \mathbb{R}^+ \times \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}^2$ is the function defined, for any $t \in \mathbb{R}^+$ and $e = [e_1, e_2]^T \in \mathbb{R}^2$, by

$$F(t,e,u) = \begin{bmatrix} -k_1 e_1 + g_{20}(t)e_2 \\ \phi_2(t) - \dot{h}_2^*(t) + g_2 u \end{bmatrix}. \tag{27}$$

The next proposition follows from the results in Theorem 1:

Proposition 1: The tracking error system described by (10)-(11) in closed-loop with the quantized sampled-data event-based controller (18)-(20) and (26) is semi-globally practically stable in the sense of Theorem 1.

Proof: The proof follows from Theorem 1. ■

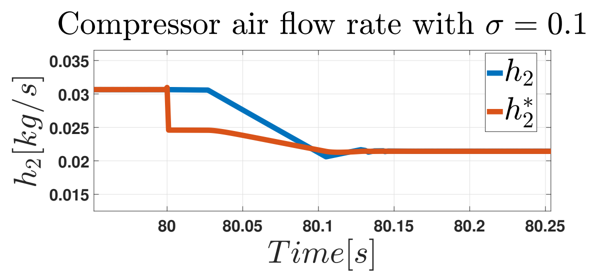

Fig. 4. h_2 Event-Based Control with $\sigma = 0.1$ - Transient response detail.

V. Simulations

In this section, numerical simulations of the proposed quantized event-based sampled-data controller for the oxygen starvation avoidance in air-feed PEMFCs are illustrated. This approach was explored in different system operating conditions. In the simulations, (i) the control parameters are chosen as $\gamma_1 = 100$, $\gamma_2 = 50$; (ii) a uniform (i.e. $a = 1$) sampling $\delta = 10^{-3}$ [s] is chosen; (iii) quantizers based on the round-to-nearest approach are considered with

$$\begin{aligned}\mathscr{Q}_e = \{e \in \mathbb{R}^2 | e_1 = \pm 10^{-2}\bar{k}, \bar{k} = 0,1,\dots,2500; \\ e_2 = \pm 10^{-4}\tilde{k}, \tilde{k} = 0,1,\dots,5000\} \\ \mathscr{Q}_u = \{u \in \mathbb{R} | u = 10^{-2}\bar{k}, \bar{k} = 0,1,\dots,5000\}\end{aligned}$$

Since $u(t)$ is the current (along q-axis) for the inverter,

TABLE I

Frequency of Control Updates as a Function of σ.

Trigger Percentage with $\sigma = 0.1$	62.85%
Trigger Percentage with $\sigma = 0.5$	63.62%
Trigger Percentage with $\sigma = 0.9$	64.46%

values of γ_1, γ_2 were chosen in order to have an acceptable control input behavior in terms of amplitude oscillations during system responses.

The subsequent Fig.3 and Fig.4 focus on transient response of the system to better show controller's performances with different control input update methods.

In the event-based approach three values for σ are considered, namely $\sigma = \{0.1, 0.5, 0.9\}$ and $\lambda_{O_2}^* \in [2.1, 2.5]$, while I_{st} varies from $100A$ to $300A$. Both time-based and event-based controller track $\lambda_{O_2}^*$ and h_2^* : Fig.3 shows a sudden overshoot of λ_{O_2} when I_{st} decreases from $300A$ to $150A$ (according to (2)), settling time for time-based and event-based approach are approximately $0.15s$ and $0.2s$, respectively. Air compressor flow rate, h_2, is controlled with a settling time of almost $0.15s$ in all four cases; Fig.4 illustrates a transient behaviour detail with $\sigma = 0.1$. Following an event-based approach, control input is not updated at each sample, but only when the triggering condition is satisfied. Since the triggering condition is checked at each sampling instant, the trigger percentage is defined as the ratio of the times at which the condition is satisfied over the total number of samples. Results show that higher σ translates into a higher trigger percentage, as reported in Table I. Fig. 5 illustrates a detail of control input comparison with $\sigma = 0.1$ and time-based approach: it can be clearly seen how control is kept constant for several time samples, translating into less switches.

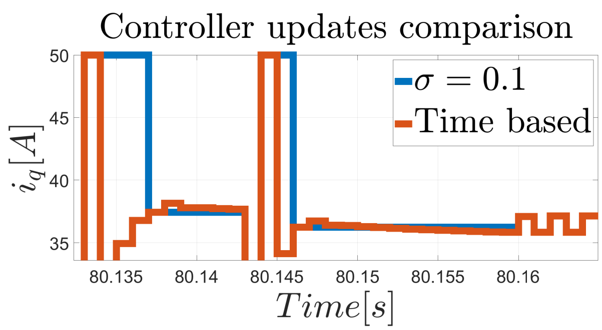

Fig. 5. Control updates - a comparison detail between the time-based approach and the event-based approach with $\sigma = 0.1$.

VI. CONCLUSIONS

In this paper, the problem of oxygen starvation avoidance in air-feed proton exchange membrane fuel cells via quantized event-triggered sampled-data controllers has been investigated. In particular, a methodology for the design of quantized sampled-data event-based stabilizers was provided for a class of time-varying nonlinear systems, which was then applied to the oxygen starvation and maximum net power achievement in Proton Exchange Membrane Fuel Cells (PEMFCs). A Lyapunov-based approach and the stabilization in the sample-and-hold sense theory were used as tools to show that there exist a suitably fast sampling and an accurate quantization of the input/output channels such that the digital event-triggered implementation of a proposed continuous-time stabilizer ensures semi-global practical stability of the corresponding closed-loop system. Numerical simulations confirm the effectiveness of the theoretical results and highlight the better behaviour of the event-based controller, that shows less oscillations with respect to the time-based stabilizer. Future investigations will concern the extensive validation of the proposed quantized event-triggered sampled-data controller and the design of digital event-based output feedback controllers for the oxygen starvation avoidance in air-feed PEMFCs.

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