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Distributed Event-triggered Consensus Control from Noisy Data Using Matrix Polytopes

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***Abstract*— This paper presents a novel data-driven polytopic approach to event-triggered consensus control of unknown leader-following multi-agent systems (MASs). A distributed data-driven event-triggered consensus control protocol is proposed that utilizes noisy input-state data to enable all followers to track the leader while reducing communication and computational burden. Unlike previous research that relies on quadratic matrix inequalities to characterize system uncertainties, this paper devises a data-based polytopic representation for MASs, which enables addressing the consensus control problem without using explicit system matrices. Based on this representation, a data-based criterion is established, utilizing matrix polytopes to ensure the asymptotic stability of the closed-loop MAS. Moreover, a co-design method is presented for the distributed controller gain and the triggering matrix, using only data and expressed in terms of linear matrix inequalities. Finally, numerical simulations are conducted to demonstrate the validity and effectiveness of the proposed data-driven approach.**

I. INTRODUCTION

Consensus control of multi-agent systems (MASs) has been a popular research topic for the past two decades, particularly leader-following consensus control, where distributed control algorithms are developed through local information exchange between agents. Significant progress has been made in this area, as evidenced by numerous studies such as [1]–[3] and references therein.

Effective communication among agents is critical for achieving consensus control. However, continuous communication may not be feasible due to energy constraints and limited bandwidth shared by individual agents. To address this issue, researchers have investigated resource-efficient control strategies that reduce transmission frequency while maintaining consensus. Event-triggered control, in which measurements are taken, transmitted, and used to update the controller only when a certain event occurs, has been shown to be a promising approach to save communication resources [4]. This method has been extensively studied in MASs, as seen in [5]–[7].

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Most previous research on event-triggered control has been developed under the model-based control paradigm, which relies on explicit knowledge of system models or matrices. However, constructing an accurate system model is often challenging or even impossible in some engineering scenarios such as power and biological systems. Moreover, system identification-based control may result in unreliable performance due to uncertainties introduced by the identification process [8]. Consequently, data-driven methods that directly design control laws from measured data have received much attention in recent years [9], [10]. Indeed, various data-driven control approaches have been reported, including robust control [11], [12], state feedback control [13], [14], model predictive control [15], [16], quantized control [17], and control of complex networks [18]–[22].

One of the main challenges in data-driven control is building a data-based system representation, particularly when unexpected noise corrupts the offline data collection process. A general data-based representation, expressed as quadratic matrix inequalities (QMIs), was introduced in [23] and has been widely used to address theoretical and practical problems [24]. This paper proposes a data-driven polytopic approach for event-triggered consensus control of unknown leader-following MASs, taking into account more general forms of noise during the data acquisition phase. The proposed approach introduces a data-based polytopic representation for MASs, enabling the consensus control problem to be addressed without using explicit system matrices. Using this representation, a data-based criterion is established for the closed-loop system through the use of matrix polytopes, ensuring that all followers asymptotically track the leader. Moreover, a co-design method for the distributed controller gain and the triggering matrix is presented, expressed in terms of linear matrix inequalities, using only data. The proposed data-driven method provides a more effective approach compared to the system identification-based method, which is illustrated through a comparison study.

Our main contributions can be summarized as follows.

1) We propose a data-based polytopic representation of leader-following MASs, which is based on locally collected offline input-state data;
2) We develop a data-based stability condition using matrix polytopes, which guarantees asymptotic consensus while reducing computation frequency;
3) We offer a data-driven polytopic approach to co-designing the distributed controller gain and the triggering matrix using only input-state data.

Notation. Let $\mathbb{N}$ ($\mathbb{R}$) denote the set of all non-negative integers (real numbers). For integers $a < b$, let $\mathbb{N}_{[a,b]} := \mathbb{N} \cap [a, b]$. For vector $x \in \mathbb{R}^n$, $x > 0$ is understood entrywise. Symbols $(\cdot)^\top$ and $\otimes$ represent the transpose and the Kronecker product. For symmetric matrix P, $P \succ 0$ ($P \succeq 0$) means that P is positive (semi-)definite. $\mathrm{Sym}\{P\}$ takes the sum of $P^\top$ and P. Finally, we use I ($\mathbf{0}$) to denote the identity (zero) matrix of appropriate dimensions.

II. Preliminaries and Problem Setup

A. Graph theory

A weighted graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ defines over a nonempty node set $\bar{\mathcal{V}} = \{v_0, v_1, \ldots, v_N\}$ and an edge set $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ describes the interaction between agents. If node v_j can obtain information from node v_i, then the link $(v_i, v_j) \in \bar{\mathcal{E}}$. The induced subgraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ obtained from $\bar{\mathcal{G}}$ represents the information interaction relationship between followers, where $V = \{v_1, \ldots, v_N\}$ and $\mathcal{E} \subseteq (\mathcal{V} \times \mathcal{V})$. $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the adjacency matrix with $a_{ii} = 0$, $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$; and $a_{ij} = 0$, otherwise. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ associated with the subgraph $\mathcal{G}$ has $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$. Let the diagonal matrix $\mathcal{P} = \mathrm{diag}\{a_{10}, \ldots, a_{N0}\} \in \mathbb{R}^{N\times N}$ describe the accessibility of the leader to followers. Concretely, $a_{0i} > 0$ if follower i has access to the information of the leader; and $a_{0i} = 0$ otherwise. In addition, define the matrix $\mathcal{H} := \mathcal{L} + \mathcal{P}$. A directed graph contains a directed spanning tree if there exists a root node that has directed paths to all other nodes. To proceed, we need the following assumption.

Assumption 1 (Communication graph): The graph $\bar{\mathcal{G}}$ contains a directed spanning tree with the leader as the root node. Besides, only a subset of followers have direct access to the leader's information.

B. Distributed event-triggered control of MASs

Consider a discrete-time leader-following MAS with a leader indexed by 0 and N followers by $1, 2, \ldots, N$. For $t \in \mathbb{N}$, the dynamics of each agent are described by

$$\begin{cases} x_i(t+1) = A_{\mathrm{tr}} x_i(t) + B_{\mathrm{tr}} u_i(t), \quad i = 1, 2, \ldots, N, \\ x_0(t+1) = A_{\mathrm{tr}} x_0(t), \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ and $x_0(t) \in \mathbb{R}^n$ denote the state of follower i and the leader, respectively, and $u_i(t) \in \mathbb{R}^p$ denotes the control input of follower i. In the following, we assume that the true system matrices $A_{\mathrm{tr}} \in \mathbb{R}^{n\times n}$ and $B_{\mathrm{tr}} \in \mathbb{R}^{n\times p}$ are *unknown*, but the dimensions n and p are known and the pair $(A_{\mathrm{tr}}, B_{\mathrm{tr}})$ is stabilizable.

The problem of interest is the leader-following consensus phrased as designing a distributed event-triggered protocol that renders all followers tracking the leader asymptotically over a fixed directed communication network $\bar{\mathcal{G}}$.

Regarding this problem, we commence by adopting the following distributed event-triggered controller for (1)

$$u_i(t) = K \sum_{j=1}^{N} a_{ij} \left(\bar{x}_i(t) - \bar{x}_j(t)\right) + a_{i0} \left(\bar{x}_i(t) - \bar{x}_0(t)\right) \tag{2}$$

where a_{ij} denotes the ijth entry of the adjacency matrix $\mathcal{A}$, $K \in \mathbb{R}^{p\times n}$ is the feedback gain matrix to be designed. Let t_k^i denotes the kth ($k \in \mathbb{N}$) triggering time of agent i, dictated by an event-triggering mechanism to be designed later. For $t \in \mathbb{N}_{[t_k^i, t_{k+1}^i - 1]}$, $\bar{x}_i(t) := x_i(t_k^i)$, where $x_i(t_k^i)$ is the last broadcast state at t_k^i. On the other hand, the leader transmits the state at every time, i.e., $\bar{x}_0(t) = x_0(t_k^0) = x_0(t)$.

Without loss of generality, assume that $t_1^i = 0$ for $i = 1, \ldots, N$. Letting t_k^i designate the most recent triggering time, the subsequent triggering time t_{k+1}^i is given by

$$t_{k+1}^i = t_k^i + \inf_{t \geq t_k^i} \left\{ t \,\middle|\, f_i(e_i(t), z_i(t), t) \geq 0 \right\} \tag{3}$$

with the following triggering function

$$f_i(e_i(t), z_i(t), t) = e_i^\top(t) \Phi e_i(t) - \sigma z_i^\top(t) \Phi z_i(t)$$

where $\Phi \in \mathbb{R}^{n\times n}$ is a positive definite matrix to be designed, σ is a positive constant balancing between the transmission frequency and the system performance, $e_i(t) := \bar{x}_i(t) - x_i(t)$ denotes the measurement error between the last broadcast state at t_k^i and the current state at t, and $z_i(t) := \sum_{j=1}^{N} a_{ij} \left(\bar{x}_i(t) - \bar{x}_j(t)\right) + a_{i0} \left(\bar{x}_i(t) - \bar{x}_0(t)\right)$.

Based on the settings above, define the tracking error by $\delta_i(t) := x_i(t) - x_0(t)$. It follows from (1) and (2) that the dynamics of $\delta_i(t)$ satisfies

$$\begin{aligned} \delta_i(t+1) = A_{\mathrm{tr}} \delta_i(t) + B_{\mathrm{tr}} K \\ \times \sum_{j=1}^{N} a_{ij} (\bar{\delta}_i(t) - \bar{\delta}_j(t)) - a_{i0} \bar{\delta}_i(t), \end{aligned} \tag{4}$$

where $\bar{\delta}_i(t) := \bar{x}_i(t) - x_0(t)$. By stacking the vectors in matrices $\delta(t) = [\delta_1^\top(t), \delta_2^\top(t), \ldots, \delta_N^\top(t)]^\top$ and $\bar{\delta}(t) = [\bar{\delta}_1^\top(t), \bar{\delta}_2^\top(t), \ldots, \bar{\delta}_N^\top(t)]^\top$, the closed-loop system (4) can be written as

$$\delta(t+1) = (I_N \otimes A_{\mathrm{tr}}) \delta(t) + (\mathcal{H} \otimes B_{\mathrm{tr}} K) \bar{\delta}(t). \tag{5}$$

It can be inferred that the consensus problem of the leader-following MAS (1) is converted to the stability problem of the closed-loop system (5) under the event-triggered consensus control protocol. Although existing solutions have demonstrated success in tackling this problem, they all require prior knowledge of the matrices, see, e.g. [3], [5], [6], [25]. To overcome this issue, we focus on preforming a distributed event-triggered control directly from data in this paper.

C. Pre-collected noisy data

To compensate for the lack of true system matrices, it is assumed that a T-long stream of input-state data $\{x_i(k)\}_{k=0}^{T}$, $\{u_i(k)\}_{k=0}^{T-1}$ of all followers from the following *purterbed* system can be collected through some offline experiments

$$x_i(k+1) = A_{\mathrm{tr}} x_i(k) + B_{\mathrm{tr}} u_i(k) + w_i(k), \tag{6}$$

where $k \in \{0, 1, \ldots, T\}$ and $w_i(k) \in \mathbb{R}^n$ is *unknown* noise satisfying the following assumption.

Assumption 2 (Polytopic noise): For every k and $i = 1, \cdots, N$, the noise $w_i(k) \in \mathcal{P}_{w_i}$, where $\mathcal{P}_{w_i}$ is a known polytope given by

$$\mathcal{P}_{w_i} = \Big\{ w_i | w_i = \sum_{\rho=1}^{\gamma_i} \beta_i^{(\rho)} \hat{w}_i^{(\rho)}, \beta_i^{(\rho)} \geq 0, \sum_{\rho=1}^{\gamma_i} \beta_i^{(\rho)} = 1 \Big\},$$

with γ_i and $\hat{w}_i^{(\rho)}$ the number of vertices, and the ρth vertex of polytope $\mathcal{P}_{w_i}$, respectively. Moreover, $0 \in \mathcal{P}_{w_i}$.

Imitating the definition of the tracking error $\delta_i(t)$, the system (6) can be transformed into the ensuing linear system

$$\delta_i(k+1) = A_{\mathrm{tr}} \delta_i(k) + B_{\mathrm{tr}} u_i(k) + w_i(k). \tag{7}$$

Based on the collected data $\{x_i(k)\}_{k=0}^{T}$, $\{u_i(k)\}_{k=0}^{T-1}$, we then compute and collect the tracking error measurements $\{\delta_i(k)\}_{k=0}^{T}$ per agent over time step $k \in \{0, 1, \ldots, T\}$. Store these data vectors into the following matrices

$$\begin{aligned} U_i &:= [u_i(0)\ u_i(1)\ \cdots\ u_i(T-1)], \\ \Delta_i &:= [\delta_i(0)\ \delta_i(1)\ \cdots\ \delta_i(T-1)], \\ \Delta_{i+} &:= [\delta_i(1)\ \delta_i(2)\ \cdots\ \delta_i(T)]. \end{aligned}$$

To guarantee that these matrices are sufficiently representative of the MAS [11], the following assumption is made.

Assumption 3 (Rank guarantee): $\mathrm{rank} \begin{bmatrix} \Delta_i \\ U_i \end{bmatrix} = n + p.$

Furthermore, we denote the sequence of unknown noise $\{w_i(k)\}_{k=0}^{T-1}$ by $W_i := [w_i(0)\ w_i(1)\ \cdots\ w_i(T-1)]$. Here, $W_i \in \mathcal{M}_{W_i}$, and $\mathcal{M}_{W_i}$ is a matrix polytope described by

$$\mathcal{M}_{W_i} = \Big\{ W_i | W_i = \sum_{\rho=1}^{\gamma_i T} \beta_{W,i}^{(\rho)} \hat{W}_i^{(\rho)}, \beta_{W,i}^{(\rho)} \geq 0, \sum_{\rho=1}^{\gamma_i T} \beta_{W,i}^{(\rho)} = 1 \Big\}$$

which results from the concatenation of multiple noise polytopes $\mathcal{P}_{w_i}$ as

$$\begin{aligned} \hat{W}_i^{(1+(k-1)\rho)} &= \begin{bmatrix} \hat{w}_i^{(k)} & \mathbf{0}_{n_i \times (\rho-1)} \end{bmatrix}, \\ \hat{W}_i^{(m+(k-1)\rho)} &= \begin{bmatrix} \mathbf{0}_{n_i \times (m-1)} & \hat{w}_i^{(k)} & \mathbf{0}_{n_i \times (\rho-m)} \end{bmatrix}, \\ \hat{W}_i^{(\rho+(k-1)\rho)} &= \begin{bmatrix} \mathbf{0}_{n_i \times (\rho-1)} & \hat{w}_i^{(k)} \end{bmatrix}, \end{aligned}$$

for all $k = \{1, 2, \ldots, \gamma_i\}$, $m = \{2, 3, \ldots, T-1\}$, and $i = 1, 2, \ldots, N$.

With the preliminaries above, the problem to be addressed in this paper is formally stated as follows.

Problem 1: For the unknown closed-loop system (5), under Assumptions 1-3, design a distributed event-triggered consensus control protocol from noisy input-state data using matrix polytopes to save transmissions while ensuring $\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0$, $\forall i = 1, 2, \ldots, N$.

III. Distributed Data-driven Event-triggered Control of MASs

This section proposes a data-driven event-triggered consensus control protocol to address Problem 1 for unknown MASs (1). The main challenge can be attributed to three factors: c1) How to accurately characterize an MAS using pre-collected input-state data and the polytopic noise description? c2) How to establish theoretical consensus guarantees for the resulting data-based event-triggered MAS? c3) How to obtain a consensus controller and a triggering mechanism based solely on data instead of system matrices?

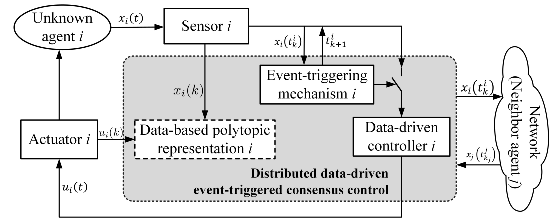

Fig. 1. Distributed data-driven event-triggered consensus control.

To overcome the first challenge, we construct a data-based polytopic representation for MASs. Subsequently, data-based stability conditions are derived using matrix polytopes to ensure the asymptotic consensus, thereby solving c2). Finally, we provide a data-driven co-design approach for the controller gain and the event-triggering matrix to address c3). See Fig. 1 for a pictorial description of the distributed data-driven event-triggered consensus controller.

Note that the correspondence between the trajectories $(U_i, \Delta_i, \Delta_{i+})$ and the true system model is not unique in general, particularly in light of the fact that the noise sequences are unknown. We denote this set by Σ_i as follows

$$\Sigma_i := \{[A,\ B] | \Delta_{i+} = A\Delta_i + BU_i + W_i, W_i \in \mathcal{M}_{W_i}\}.$$

To provide stability analysis guarantees of the MAS (1) with unknown A_{tr}, B_{tr}, we need to derive a stability criterion for all $[A,\ B]$ that are consistent with the input-state data and the given noise bound. For this purpose, inspired by previous works using zonotope [26], a data-based representation of system matrices $[A,\ B]$ is constructed via matrix polytopes in the following lemma.

Lemma 1 (Data-based polytopic representation of MASs): Suppose Assumptions 2-3 hold. Given input-state data $(U_i, \Delta_i, \Delta_{i+})$ of the MAS (7), then $\mathcal{M}_i \supseteq \Sigma_i$ for each $i = 1, 2, \ldots, N$, where $\mathcal{M}_i$ is a matrix polytope defined as

$$\mathcal{M}_i = (\Delta_{i+} - \mathcal{M}_{W_i}) \begin{bmatrix} U_i \\ \Delta_i \end{bmatrix}^{\dagger}. \tag{8}$$

Note that different with the general quadratic form in existing works [20], [21], [23], [24], Lemma 1 provides a more precise characterization of MASs, resulting in less conservative data-based stability conditions in the subsequent analysis.

Consider that the tracking error $\delta_i(t)$ belongs to a well-defined polytope $\mathcal{P}_{\delta_i,t}$, i.e., $\delta_i(t) \in \mathcal{P}_{\delta_i,t}$ for $i = 1, 2, \ldots, N$. Let $\mathcal{P}_{\delta_i,0} = \delta_i(0)$. It follows from (4) that the polytope $\mathcal{P}_{\delta_i,t}$ is described as

$$\mathcal{P}_{\delta_i,t} = A_{\mathrm{tr}} \mathcal{P}_{\delta_i,t-1} + B_{\mathrm{tr}} K z_i(t), \quad i = 1, 2, \ldots, N. \tag{9}$$

We are now ready to advocate a data-driven solution for achieving the consensus of (1) with unknown matrices A_{tr}, B_{tr} under the event-triggered consensus control protocol (2)-(3). The core idea is to guarantee the stability of the tracking error polytope $\mathcal{P}_{\delta_i,t}$ for $i = 1, 2, \ldots, N$ using input-state data. Following this line, a data-based stability condition is obtained based on Lemma 1.

Theorem 1 (Data-based stability condition): Consider the MAS (1) under the event-triggered consensus control protocol (2)-(3) over graph $\bar{\mathcal{G}}$. Suppose Assumptions 1-3 hold. Given scalars $\sigma > 0$ and $\epsilon > 0$, the tracking error polytope $\mathcal{P}_{\delta_i,t}$ of follower i is asymptotically stable for any $\mathcal{P}_{\delta_i,0}$ with $t \in \mathbb{N}_{[t_k^i, t_{k+1}^i-1]}$, $i = 1, 2, \ldots, N$, if there exist matrices $P \succ 0$, $\Phi \succ 0$, F, K_G, such that for any $M \in \mathcal{M}_K^F$ the following LMI is satisfied

$$\Omega + \Psi + \mathrm{Sym}\{M\} \prec 0 \tag{10}$$

where

$$\begin{aligned}
&\mathcal{Q} := [(I_N \otimes L_1)^\top, (\mathcal{H} \otimes L_3)^\top]^\top, \mathcal{M}_i^F := (I_N \otimes F\mathcal{M}_i)\mathcal{Q},\\
&\Omega := L_2^\top (I_N \otimes P) L_2 - L_1^\top (I_N \otimes P) L_1,\\
&\Psi := \mathrm{Sym}\{-(I_N \otimes F L_2)\} - \sigma(L_3^\top(\mathcal{H} \otimes \Phi) L_3)\\
&\qquad + (L_3 - L_1)^\top (I_N \otimes \Phi)(L_3 - L_1),\\
&L_\kappa := \left[\mathbf{0}_{n\times(\kappa-1)n},\, I_n,\, \mathbf{0}_{n\times(3-\kappa)n}\right],\ \kappa = 1,\, 2,\, 3.
\end{aligned}$$

Proof: Consider the following Lyapunov function

$$V(\delta, t) = \delta^\top(t)(I_N \otimes P)\delta(t) \tag{11}$$

where $P \succ 0$. The forward difference of the function $V(t)$ along the closed-loop system (5) is given by

$$\Delta V(\delta, t) = \xi^\top(\delta, t)\big[L_2^\top (I_N \otimes P) L_2 \tag{12}$$

$$- L_1^\top (I_N \otimes P) L_1\big]\xi(\delta, t) \tag{13}$$

where $\xi(\delta, t) := [\delta^\top(t),\ \delta^\top(t+1),\ \bar{\delta}^\top(t)]^\top$.

Leveraging the descriptor method [27], for $t \in \mathbb{N}_{[t_k^i,\ t_{k+1}^i-1]}$, the system (5) can be expressed as follows

$$\begin{aligned}
&2(I_N \otimes F)\big[(I_N \otimes A_{\mathrm{tr}})\delta(t) + (\mathcal{H} \otimes B_{\mathrm{tr}}K)\bar{\delta}(t) - \delta(t+1)\big]\\
=&2\xi(\delta, t)^\top\big[(I_N \otimes F A_{\mathrm{tr}} L_1) + (\mathcal{H} \otimes F B_{\mathrm{tr}} L_3)\\
&- (I_N \otimes F L_2)\big]\xi(\delta, t) = 0
\end{aligned}$$

where $F \in \mathbb{R}^{3n\times n}$. Evidently, in light of the triggering mechanism (3), when an event has not been triggered, it yields

$$\begin{aligned}
\xi^\top(\delta, t)\Big[&(L_3 - L_1)^\top (I_N \otimes \Phi)(L_3 - L_1)\\
&- \sigma\left(L_3^\top (\mathcal{H} \otimes \Phi) L_3\right)\Big]\xi(\delta, t) \geq 0.
\end{aligned} \tag{14}$$

By summing up (12)-(14), $\Delta V(\delta, t)$ is bounded by

$$\Delta V(\delta, t) \leq \xi^\top(\delta, t)\Upsilon\xi(\delta, t) \tag{15}$$

where $\Upsilon := \Omega + \Psi + \mathrm{Sym}\{(I_N \otimes F A_{\mathrm{tr}},\, I_N \otimes F B_{\mathrm{tr}})\mathcal{Q}\}$.

It follows from the data-based polytopic representation of MASs proposed in Lemma 1 that $(I_N \otimes F A_{\mathrm{tr}}, I_N \otimes F B_{\mathrm{tr}})\mathcal{Q} \in \mathcal{M}_i^F$, where $\mathcal{M}_i^F := (I_N \otimes F\mathcal{M}_i)\mathcal{Q}$ is a well-defined matrix polytope.

Finally, for any $M \in \mathcal{M}_i^F$, inequality $\Omega+\Psi+\mathrm{Sym}\{M\} \prec 0$ implies that $\Delta V(\delta, t) < 0$ holds for all $[A,\, B] \in \mathcal{M}_i$. It is immediate that the LMI (10) guarantees the stability of the tracking error polytope $\mathcal{P}_{\delta_i,t}$, i.e., $\delta_i(t) \in \mathcal{P}_{\delta_i,t}$ is asymptotically stable for any $\delta_i(0)$. Therefore, we conclude that the MAS (1) achieves leader-following consensus asymptotically for any $[A,\, B] \in \mathcal{M}_i$. This ends the proof. ■

Theorem 1 allows us to analyze stability properties of the closed-loop system under event-triggered consensus control protocol, without any model knowledge. Based on this result, a data-driven method for co-designing the distributed controller gain and the triggering matrix will be derived in the subsequent. To this end, we proceed by defining $\delta_i(t) = G s_i(t)$, where $G \in \mathbb{R}^{n\times n}$ is assumed nonsingular. Then, for $t \in \mathbb{N}_{[t_k^i,\ t_{k+1}^i-1]}$, an algebraically equivalent system to (5) is established by

$$s(t+1) = (I_N \otimes G^{-1} A_{\mathrm{tr}} G)s(t) + (\mathcal{H} \otimes G^{-1} B_{\mathrm{tr}} K_G)\bar{s}(t) \tag{16}$$

where $K_G = KG$, $s(t) = [s_1^\top(t),\, s_2^\top(t), \ldots,\, s_N^\top(t)]^\top$, and $\bar{s}(t) = [\bar{s}_1^\top(t),\, \bar{s}_2^\top(t), \ldots,\, \bar{s}_N^\top(t)]^\top$ with $\bar{s}_i(t) = s_i(t_k^i)$. System (16) exhibits the same characteristics as (5) in terms of stability and performance. According to Theorem 1 and the equivalent system expression (16), the following theoretical result is proposed.

Theorem 2 (Data-based co-design): Consider the MAS (1) under the event-triggered consensus control protocol (2)-(3) over graph $\bar{\mathcal{G}}$. Suppose Assumptions 1-3 hold. Given the same scalars σ, ϵ as in Theorem 1, the leader-following consensus is achieved asymptotically for any initial states, if the following LMIs are feasible for any $[A,\, B] \in \mathcal{M}_i$, $i = 1, 2, \ldots, N$, and returns the solutions $\bar{\Phi}$, G, and K_G

$$\Omega + \bar{\Psi} + \mathrm{Sym}\left\{\left[I_N \otimes \mathcal{R}\Theta_i \begin{bmatrix}\Delta_i\\ U_i\end{bmatrix}^\dagger\right]\mathcal{T}\right\} \prec 0 \tag{17}$$

where

$$\begin{aligned}
&\mathcal{R} := (L_1 + \epsilon L_2)^\top, \mathcal{T} := [(I_N \otimes G L_1)^\top,\ (\mathcal{H} \otimes K_G L_3)^\top]^\top,\\
&\bar{\Psi} := \mathrm{Sym}\{-(I_N \otimes \mathcal{R} G L_2)\} - \sigma(L_3^\top(\mathcal{H} \otimes \bar{\Phi}) L_3)\\
&\qquad + (L_3 - L_1)^\top (I_N \otimes \bar{\Phi})(L_3 - L_1),\\
&\Theta_i := \Delta_{i+} - \sum_{\rho=1}^{\gamma_{w_i}T} \beta_{W,i}^{(\rho)} \hat{W}_i^{(\rho)}.
\end{aligned}$$

Moreover, the controller gain is given by $K = K_G G^{-1}$ and the triggering matrix is co-designed as $\Phi = (G^{-1})^\top \bar{\Phi} G^{-1}$.

Proof: We construct a Lyapunov function $V(s, t) = s^\top(t)(I_N \otimes P)s(t)$ for system (16) by substituting δ of the function $V(\delta, t)$ in (11) with s. Similar to the proof of Theorem 1, it can be obtained that

$$\begin{aligned}
\Delta V(s, t) = \zeta^\top(s, t)\big[&L_2^\top (I_N \otimes P) L_2\\
&- L_1^\top (I_N \otimes P) L_1\big]\zeta(s, t)
\end{aligned}$$

with $\zeta(s, t) := [s^\top(t),\ s^\top(t+1),\ \bar{s}^\top(t)]^\top$.

Using the descriptor method again, (16) is represented as

$$\begin{aligned}
0 = 2\zeta(s, t)^\top (I_N \otimes \mathcal{R})\Big[&(I_N \otimes A_{\mathrm{tr}} G L_1) + (\mathcal{H} \otimes B_{\mathrm{tr}} K_G L_3)\\
&- (I_N \otimes G L_2)\Big]\zeta(s, t).
\end{aligned}$$

In addition, it can be deduced by imitating (15) that $\Delta V(s, t) \leq \zeta^\top(s, t)\bar{\Upsilon}\zeta(s, t)$, where $\bar{\Upsilon} := \Omega + \bar{\Psi} +$

$\mathrm{Sym}\{(I_N \otimes \mathcal{R}A_{\mathrm{tr}}) + (I_N \otimes \mathcal{R}B_{\mathrm{tr}})\mathcal{T}\}$. Recalling the data-based polytopic representation in Lemma 1 again, one gets

$$\mathcal{M}_i = \left(\Delta_{i+} - \sum_{\rho=1}^{\gamma_{w_i}T} \beta_{W,i}^{(\rho)} \hat{W}_i^{(\rho)}\right) \begin{bmatrix}\Delta_i \\ U_i\end{bmatrix}^{\dagger}.$$

Thus, it is easy to verify that we have $\bar{\Upsilon} \prec 0$ for any $[A\ B] \in \mathcal{M}_i$, if the following LMI holds

$$\Omega + \bar{\Psi} + I_N \otimes \left[\mathcal{R}\Big(\Delta_{i+} - \sum_{\rho=1}^{\gamma_{w_i}T} \beta_{W,i}^{(\rho)} \hat{W}_i^{(\rho)}\Big)\right] \begin{bmatrix}\Delta_i \\ U_i\end{bmatrix}^{\dagger} \mathcal{T} \prec 0.$$

Observing that the above LMIs are equivalent to LMIs (17), this provides a sufficient guarantee similar as that in Theorem 1, and hence ensuring asymptotic consensus of the MAS (1) under the event-triggered consensus control protocol (2)-(3). Moreover, this guarantee is established with the desired $K = K_G G^{-1}$ and $\Phi = (G^{-1})^{\top} \bar{\Phi} G^{-1}$. The proof is completed by observing that system (16) exhibits the same stability behavior as (1). ■

Note that Theorem 2 provides a data-driven tool to co-design the distributed controller gain and triggering matrix, while guaranteeing the asymptotic consensus and therefore tackling Problem 1. In order to solve the LMIs (17), one only needs to take the vertices of $\mathcal{M}_i$ into account. This is due to the fact that the polytope is convex. Hence, if the LMIs are addressed at all the vertices of the polytope, then it covers the inside of polytope [28]. As a result, our approach eliminates the need on system models and can potentially be adopted in a wider range of practical applications.

IV. Simulation

To demonstrate the correctness and numerical effectiveness of the proposed data-driven event-triggered control method, a comparative experiment is provided in this section.

Consider an MAS composed of six followers indexed by $1, 2, \ldots, 6$ and the leader indexed by 0; see Fig. 2. Each agent is modeled by the linear dynamics adapted from [14].

$$\dot{x}_i(t) = \begin{bmatrix}0 & 0 \\ -1 & -2\end{bmatrix} x_i(t) + \begin{bmatrix}1 \\ 0\end{bmatrix} u_i(t), \quad i = 1, 2, \ldots, 6.$$

Here, we consider the discrete-time version of this system with a sampling period of $0.02s$. The proposed data-driven event-triggered consensus control protocol (2)-(3) are then applied to this MAS. The co-designed results are displayed in the following part.

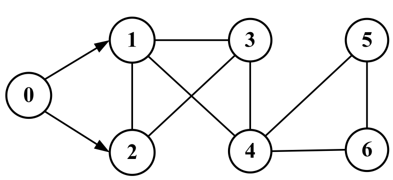

Fig. 2. Communication topology $\bar{\mathcal{G}}$ between agents.

(Testing the proposed data-driven method) Since the true system matrices A_{tr} and B_{tr} are unknown, we collect the input-state measurements $\{x_i(k)\}_{k=0}^{T}$, $\{u_i(k)\}_{k=0}^{T-1}$ for each agent from the perturbed system (6) with $T = 20$. Besides, the data-generating input is taken uniformly from a polytope $\mathcal{P}_{u_i} = \{u_i | u_i = \sum_{\rho=1}^{\gamma_{u_i}} \beta_{u,i}^{(\rho)} \hat{u}_i^{(\rho)}, \beta_{u,i}^{(\rho)} \geq 0, \sum_{\rho=1}^{\gamma_{u_i}} \beta_{u,i}^{(\rho)} = 1\}$. Set $\gamma_{u_i} = 2$, and the two vertices are $\hat{u}_i^{(1)} = 1$ and $\hat{u}_i^{(2)} = -1$ for $i = 1, 2, \ldots, 6$. As stated in Assumption 2, the noise $w_i(k)$ is sampled from the polytope $\mathcal{P}_{w_i}$ with four vertices $\hat{w}_i^{(1)} = [1, 1]^{\top}$, $\hat{w}_i^{(2)} = [1, -1]^{\top}$, $\hat{w}_i^{(3)} = [-1, 1]^{\top}$, and $\hat{w}_i^{(4)} = [-1, -1]^{\top}$. Select $\epsilon = 2$ and $\sigma = 0.2$. By solving the data-based LMIs (17) in Theorem 2, the controller gain K and the triggering gain matrix Φ were co-designed as

$$K = [-0.2302\ 0.2950],\ \Phi = \begin{bmatrix}0.2463 & 0.1232 \\ 0.1232 & 0.0616\end{bmatrix}.$$

Let the initial state of each pendulum be $x_0(0) = [1,\ 1]^{\top}$, $x_1(0) = [2,\ -1]^{\top}$, $x_2(0) = [1,\ -4]^{\top}$, $x_3(0) = [-4,\ 2]^{\top}$, $x_4(0) = [4,\ 2]^{\top}$, $x_5(0) = [5,\ 0]^{\top}$, and $x_6(0) = [3,\ -1]^{\top}$. The tracking error trajectories between the leader and each follower are depicted in Fig. 3. Obviously, leader-following consensus is achieved, which validates the correctness of the data-driven control design.

(Comparing with the identification-based method.) We tested the identification-based STC on this multi-pendulum system. The system identification step was proceeded by using e.g., the subspace space system identification (n4sid) technique [29], followed by the model-based STC design. Specifically, we first estimated a discrete-time state-space model for the system using the n4sid toolbox in MATLAB based on the same set of data. The controller gain K in (2) and the triggering matrix Φ in (3) were designed as

$$K = [-2.5565\ \ -1.9286],\ \Phi = \begin{bmatrix}0.4880 & 0.0010 \\ 0.0010 & 0.4827\end{bmatrix}.$$

We used the same parameters and initial states for both data-driven and identification-based settings. Fig. 4 shows the tracing error trajectories of each agent. Moreover, to evaluate communication efficiency quantitatively, we counted the numbers of triggering events across the entire MAS within $10s$ for these two approaches, and reported them in Fig. 5. By analyzing Figs. 3-5, it is evident that the identification-based approaches require fewer settling steps ($t = 4s$) to stabilize the system compared to the data-driven method ($t = 6s$), while necessitating considerably more frequent communication. It can be concluded that the proposed data-driven method strikes a good trade-off between control performance and communication efficiency, particularly when dealing with finite and noisy data.

V. Conclusions

In this work, we have investigated the event-triggered consensus control problem of unknown leader-following MASs. By developing a novel data-based polytopic representation of MASs, a distributed data-driven approach using matrix polytopes was proposed to design an event-triggered control protocol directly from pre-collected noisy data. Our data-based stability criterion has enabled us to co-design the controller gain and the triggering matrix while ensuring the stability of the closed-loop system.

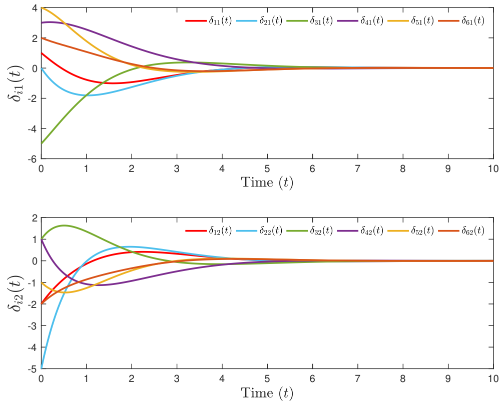

Fig. 3. Tracking error trajectories under the data-driven method.

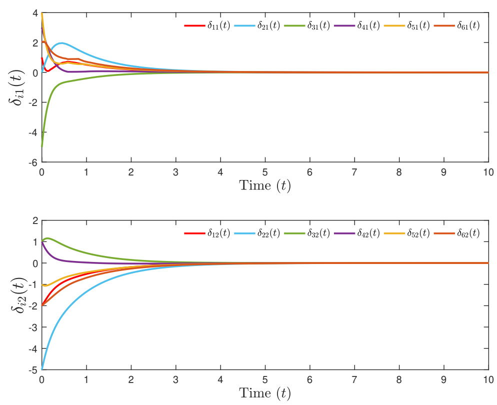

Fig. 4. Tracking error trajectories under the identification-based method.

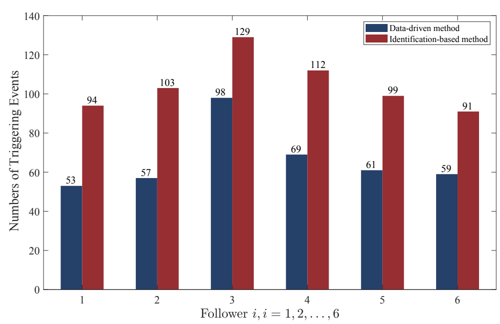

Fig. 5. Triggering times for different methods.

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