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Asymptotic Consensus of Multi-Agent Systems with Unknown Nonlinear Dynamics via Smooth Barrier Integral Control

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Abstract— **We consider the consensus problem for 2nd-order MIMO multi-agent systems with unknown nonlinear terms. We propose a novel control algorithm based on the Barrier Integral Control (BRIC) that combines reciprocal barrier functions with integral terms of the multi-agent disagreement errors and guarantees *asymptotic* consensus despite the unknown dynamics. The control algorithm is distributed, in the sense that each agent calculates its own control signal based on local information from its neighbouring agents, and does not use any a priori information from the agents' dynamics. Furthermore and unlike previous works, it does not rely on boundedness assumptions or approximation of the dynamic terms and constitutes smooth feedback of the multi-agent states. Finally, simulation results verify the theoretical findings.**

I. INTRODUCTION

Distributed control of networked multi-agent systems has emerged as a prominent and widely studied topic in recent decades, owing to its diverse range of applications, including robotic systems, smart cities, or biological systems [1]. In such systems, each agent computes independently its control signal using only local information to achieve collaborative tasks with other agents. Such tasks usually consist of consensus, where the agents aim to synchronize their states, or special kinds of geometric formations, e.g., distance- or bearing-based formations [2]. At the same time, collaborative tasks often include maintaining certain transient properties, such as collision avoidance or connectivity maintenance [3].

When it comes to controlling multi-agent systems, a significant challenge that has not been adequately addressed in the related literature is dealing with uncertain dynamics. A large variety of engineering systems cannot be accurately modelled, contain plenty of geometric and dynamic parameters that cannot reliably identified, and suffer from unknown exogenous disturbances. Although many works in the related literature take into account such uncertainties, they tend to adopt a series of limiting assumptions. First, many works using traditional adaptive control [4] unrealistically assume linear parametrizations of the dynamic terms, limiting the uncertainty to constant parameters [5]–[7]. Another class of works approximates the unknown dynamics with single-layer neural networks, obtaining only local results and requiring large enough number of nodes [8]–[11]. Other works base their results on state-boundedness assumptions or growth conditions [3], [12] or assume a priori information on the underlying dynamics [13].

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Another issue associated with multi-agent control concerns the convergence of the underlying multi-agent errors. When the system dynamics suffer from uncertainties and disturbances, most related works establish convergence of these errors to sets around zero whose size is proportional to the uncertainties' bounds and approximation errors and inversely proportional to the algorithms' control gains [9], [10], [12]. Therefore, convergence of the errors close to zero requires prior tuning of the control gains to large enough values. This issue is overcome by employing prescribed performance control schemes [14]. In such schemes, the multi-agent errors evolve inside user-defined funnels, leading to pre-defined transient and steady-state performance, without requiring knowledge of the system dynamics or tuning of the control gains. Still, however, these works cannot provide *asymptotic* stabilization results, unless the considered dynamics are linear [15]; in the general nonlinear case, asymptotic stabilization is forced by funnels that become arbitrarily narrow around zero, which, however, can cause problematic behaviour for the control inputs [16]. The works [16]–[18] established asymptotic convergence for single- and multi-agent control-affine systems with entirely unknown dynamic terms at the cost, however, of using discontinuous feedback.

This paper considers the consensus problem for multi-agent systems that evolve subject to 2nd-order control-affine MIMO dynamics with unknown nonlinear terms. In particular, we consider that the multi-agent aims to synchronize its states to a pre-defined setpoint, whose coordinates are known only by a subset of agents. We introduce smooth Multi-Agent Barrier Integral Control (MAS - BRIC), which is a special case of adaptive control that integrates reciprocal barrier functions with integral adaptation terms and was introduced for single-agent systems in [19]. The proposed BRIC algorithm guarantees *asymptotic* convergence of the consensus errors to zero, without employing any a priori information from the agents' dynamic terms. These guarantees do not rely on global boundedness assumptions or growth conditions of such terms. Additionally, the proposed algorithm is distributed, in the sense that each agent calculates its own control signal based on local information from its neighbouring agents. Finally, unlike the previous version of the algorithm in [16], [17], the proposed BRIC scheme uses *smooth* feedback.

The rest of the paper is structured as follows. Section II describes the tackled problem and Section III provides the proposed control protocol and the stability analysis. Simulation results are given in Section IV and Section V concludes the paper.

Notation: The sets of real, positive real, and non-negative real numbers are denoted by $\mathbb{R}$, $\mathbb{R}_{>0}$, and $\mathbb{R}_{\geq 0}$, respectively; $\|\cdot\|$ denotes the vector 2-norm; $\lambda_{\min}(\cdot)$ and $\otimes$ are the minimum eigenvalue and Kronecker product, respectively.

II. PROBLEM FORMULATION

Consider a MIMO multi-agent team comprised of N agents evolving according to the 2nd-order dynamics

$$\dot{x}_{i,1} = x_{i,2} \tag{1a}$$

$$\dot{x}_{i,2} = f_i(x_{i,1}, x_{i,2}, z_i, t) + g_i(x_{i,1}, x_{i,2}, z_i, t)u_i \tag{1b}$$

$$\dot{z}_i = f_{z_i}(x_{i,1}, x_{i,2}, z_i, t) \tag{1c}$$

for $i \in \mathcal{N} := \{1, \dots, N\}$, where $z_i \in \mathbb{R}^{n_z}$, $x_i := [x_{i,1}^\top, x_{i,2}^\top]^\top \in \mathbb{R}^n \times \mathbb{R}^n$ are the states of agent $i \in \mathcal{N}$, $f_i : \mathbb{R}^{2n} \times \mathbb{R}^{n_z} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, $f_{z_i} : \mathbb{R}^{2n} \times \mathbb{R}^{n_z} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_z}$, and $g_i : \mathbb{R}^{2n} \times \mathbb{R}^{n_z} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{n \times n}$ are *unknown* vector fields, and $u_i \in \mathbb{R}^n$ is the control input of agent $i \in \mathcal{N}$. Eq. (1c) constitutes the zero dynamics of agent i; the signals z_i are considered to be agent i's internal variables and, unlike x_i, are *not* available for measurement, for all $i \in \mathcal{N}$. We make the following assumptions for $f_i(\cdot)$ and $g_i(\cdot)$:

Assumption 1: The maps $(x_{i,1}, x_{i,2}, z_i) \mapsto f_i(x_{i,1}, x_{i,2}, z_i, t) : \mathbb{R}^{2n+n_z} \to \mathbb{R}^n$ and $(x_{i,1}, x_{i,2}, z_i) \mapsto g_i(x_{i,1}, x_{i,2}, z_i, t) : \mathbb{R}^{2n+n_z} \to \mathbb{R}^{n \times n}$ are locally Lipschitz for each $t \in \mathbb{R}_{\geq 0}$ and the maps $t \mapsto f_i(x_{i,1}, x_{i,2}, z_i, t) : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ are uniformly bounded for each $(x_{i,1}, x_{i,2}, z_i) \in \mathbb{R}^{2n+n_z}$, for all $i \in \mathcal{N}$.

Assumption 2: The matrices $g_i(x_{i,1}, x_{i,2}, z_i, t)$ are positive definite, for all $(x_{i,1}, x_{i,2}, z_i, t) \in \mathbb{R}^{2n+n_z} \times \mathbb{R}_{\geq 0}$.

Assumption 3: There exist sufficiently smooth functions $U_{z_i} : \mathbb{R}^{n_z} \to \mathbb{R}_{\geq 0}$ and class $\mathcal{K}_\infty$ functions $\underline{\gamma}_{z_i}(\cdot)$, $\bar{\gamma}_{z_i}(\cdot)$, $\gamma_{z_i}(\cdot)$ such that $\underline{\gamma}_{z_i}(\|z_i\|) \leq U_{z_i}(z_i) \leq \bar{\gamma}_{z_i}(\|z_i\|)$, and

$$\left(\frac{\partial U_{z_i}}{\partial z_i}\right)^\top f_{z_i}(x_{i,1}, x_{i,2}, z_i, t) \leq -\gamma_{z_i}(\|z_i\|) + \pi_{z_i}(x_{i,1}, x_{i,2}, z_i, t),$$

for $i \in \mathcal{N}$, where $(x_{i,1}, x_{i,2}) \mapsto \pi_{z_i}(x_{i,1}, x_{i,2}, z_i, t) : \mathbb{R}^{2n} \to \mathbb{R}_{\geq 0}$ is continuous and class $\mathcal{K}_\infty$ for each $(z_i, t) \in \mathbb{R}^{n_z} \times \mathbb{R}_{\geq 0}$, and $(z_i, t) \mapsto \pi_{z_i}(x_{i,1}, x_{i,2}, z_i, t) : \mathbb{R}^{n_z} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is uniformly bounded for each $(x_{i,1}, x_{i,2}) \in \mathbb{R}^{2n}$, $i \in \mathcal{N}$.

Assumption 1 presents mild regularity conditions for the existence of solutions of (1). Assumption 2 is a sufficient controllability condition [3], [14], and Assumption 3 suggests that z_i are input-to-state practically stable implying stable zero (internal) dynamics.

The control objective is the *asymptotic* consensus of the multi-agent to a pre-defined configuration $x_\text{d} \in \mathbb{R}^n$, i.e., $\lim_{t\to\infty} x_{i,1}(t) = x_\text{d}$, for all $i \in \mathcal{N}$. However, we consider that not all agents have access to x_d, but rather a subset of them, as will be detailed later.

We use an undirected graph $\mathcal{G} := (\mathcal{N}, \mathcal{E})$ to model the communication among the agents, with $\mathcal{N}$ being the index set of the agents, and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ being the respective edge set, with $(i,i) \notin \mathcal{E}$ (i.e., simple graph). The adjacency matrix associated with the graph $\mathcal{G}$ is denoted by $\mathcal{A} := [a_{ij}] \in \mathbb{R}^{N \times N}$, with $a_{ij} \in \{0, 1\}$, $i, j \in \{1, \dots, N\}$. If $a_{ij} = 1$, then agent i obtains information regarding the state x_j of agent j (i.e., $(i,j) \in \mathcal{E}$), whereas if $a_{ij} = 0$ then there is no state-information flow from agent j to agent i (i.e., $(i,j) \notin \mathcal{E}$). Furthermore, the set of neighbors of agent i is denoted by $\mathcal{N}_i := \{j \in \mathcal{N} : (i,j) \in \mathcal{E}\}$, and the degree matrix is defined as $\mathcal{D} := \text{diag}\{|\mathcal{N}_1|, \dots, |\mathcal{N}_N|\}$. Since the graph is undirected, the adjacency is a mutual relation, i.e., $a_{ij} = a_{ji}$, rendering $\mathcal{A}$ symmetric. The *Laplacian* matrix of the graph is defined as $\mathcal{L} := \mathcal{D} - \mathcal{A}$ and is also symmetric. The graph is *connected* if there exists a path between any two agents. For a connected graph, it holds that $\mathcal{L}\bar{1} = 0$, where $\bar{1}$ is the vector of ones of appropriate dimension.

As mentioned before, we consider that only a subset of agents have access to the goal configuration x_d. We model such access using the matrix $\mathcal{B} := \{b_1, \dots, b_N\} \in \mathbb{R}^{N \times N}$; if $b_i = 1$, then agent i has access to x_d, whereas it does not if $b_i = 0$, for $i \in \mathcal{N}$. That is, x_d acts as a fixed "leader" agent, driving the multi-agent team [17]. We further denote $H := (\mathcal{L} + \mathcal{B}) \otimes I_n$.

By defining the disagreement vectors

$$\delta_i(t) := x_{i,1}(t) - x_\text{d}, \quad i \in \mathcal{N}, \tag{2}$$

the control objective becomes $\lim_{t\to\infty} \delta_i(t) = 0$, for all $i \in \mathcal{N}$. Since, however, δ_i is not accessible for the agents for which $b_i = 0$, we formulate the error variables

$$e_i := [e_{i,1}, \dots, e_{i,n}]^\top := \sum_{j \in \mathcal{N}_i} a_{ij}(x_{i,1} - x_{j,1}) + b_i(x_{i,1} - x_\text{d}), \tag{3}$$

for $i \in \mathcal{N}$, which will define the subsequent control design.

In order to solve the asymptotic consensus problem, we further need the following assumptions:

Assumption 4: The graph $\mathcal{G}$ is connected and there exists at least one $i \in \mathcal{N}$ such that $b_i = 1$.

Assumption 5: It holds that $f_i(x_\text{d}, x_{i,2}, z_i, t) = 0$, for all $x_{i,2} \in \mathbb{R}^n$, $z_i \in \mathbb{R}^{n_z}$, $t \in \mathbb{R}_{\geq 0}$, $i \in \mathcal{N}$.

Assumption 4 implies that $H = (\mathcal{L} + \mathcal{B}) \otimes I_n$ is an irreducibly diagonally dominant M-matrix [20]. An M-matrix is a square matrix having its off-diagonal entries non-positive and all principal minors nonnegative; thus H is positive definite. Assumption 5 guarantees the existence of an open-loop equilibrium point at the goal configuration x_d. Such an equilibrium is necessary since the drift term $f_i(\cdot)$ is state-dependent and time-varying and cannot be accurately cancelled or compensated. At the same time, each agent's control input u_i is smooth and it is therefore expected to vanish at x_d.

By stacking all e_i and using (3) and (2), one obtains

$$e := [e_1^\top, \dots, e_N^\top]^\top = H\delta \tag{4}$$

where $\delta := [\delta_1^\top, \dots, \delta_N^\top]^\top \in \mathbb{R}^{Nn}$. Therefore, since $H = (\mathcal{L} + \mathcal{B}) \otimes I_n$ and $\mathcal{L} + \mathcal{B}$ is positive definite owing to Assumption 4, we conclude that

$$\|\delta\| \leq \frac{\|e\|}{\lambda_{\min}(\mathcal{L} + \mathcal{B})}. \tag{5}$$

Therefore, the objective $\lim_{t\to\infty} \delta_i(t) = 0$ can be implicitly achieved by guaranteeing $\lim_{t\to\infty} e_i(t) = 0$, for al $i \in \mathcal{N}$.

III. MAIN RESULTS

We introduce a smooth version of Multi-Agent Barrier Integral Control (MAS-BRIC), which is an adaptation of BRIC developed for single-agent systems in [19]. BRIC consists of two main components: a reciprocal barrier term with respect to a set boundary, which establishes the boundedness of the multi-agent states and errors $e_i(t)$, and an integral term that guarantees asymptotic convergence of these errors to zero. BRIC was originally developed in our previous works [16], [17] for single and multi-agent systems, respectively, using discontinuous feedback. The scheme developed in this paper consists of smooth feedback.

We now describe the proposed BRIC algorithm. Let positive constants $R_{i,k} > 0$ such that $|e_{i,k}(0)| < R_{i,k}$ for all $i \in \mathcal{N}$, $k \in \{1, \dots, n\}$, and consider the transformation $\chi_i(e_i) := [\chi_{i,1}(e_{i,1}), \dots, \chi_{i,n}(e_{i,n})]^\top$, with

$$\chi_{i,k}(e_{i,k}) := \ln\left(\frac{1+\frac{e_{i,k}}{R_{i,k}}}{1-\frac{e_{i,k}}{R_{i,k}}}\right) \tag{6}$$

as well as its gradient $J_i(e_i) := \text{diag}\{[J_{i,k}(e_{i,k})]_{i\in\mathcal{N}}\}$, with

$$J_{i,k}(e_{i,k}) := \frac{2R_{i,k}^2}{R_{i,k}^2 - e_{i,k}^2} \tag{7}$$

for all $i \in \mathcal{N}$, $k \in \{1, \dots, n\}$. Note that $\chi_{i,k}$ is an increasing function of $e_{i,k}$ and it satisfies $\chi_{i,k}(0) = 0$ and $\lim_{e_{i,k}\to\pm R_{i,k}} \chi_{i,k}(e_{v_{i,k}}) = \pm\infty$, for all $i \in \mathcal{N}$, $k \in \{1, \dots, n\}$. Further note that $J_{i,k}(e_{i,k}) \geq 2$ for all $|e_{i,k}| < R_{i,k}$, $i \in \mathcal{N}$, $k \in \{1, \dots, n\}$.

We next design the reference signals

$$v_i := -k_{i,1} R_i^{-1} J_i(e_i)\chi_i(e_i) \tag{8}$$

where $R_i = \text{diag}\{[R_{i,k}]_{k\in\{1,\dots,n\}}\}$, and $k_{i,1}$ are constant positive gains, and define the associated errors

$$e_{v_i} := [e_{v_{i,1}}, \dots, e_{v_{i,n}}]^\top := x_{i,2} - v_i, \tag{9}$$

for all $i \in \mathcal{N}$. Let then positive constants $R_{v_{i,k}} > 0$ such that $|e_{v_{i,k}}(0)| < R_{v_{i,k}}$ for all $i \in \mathcal{N}$, $k \in \{1, \dots, n\}$, and consider the transformation $\chi_{v_i}(e_{v_i}) := [\chi_{v_{i,1}}(e_{v_{i,1}}), \dots, \chi_{v_{i,n}}(e_{v_{i,n}})]^\top$, with

$$\chi_{v_{i,k}}(e_{v_{i,k}}) := \ln\left(\frac{1+\frac{e_{v_{i,k}}}{R_{v_{i,k}}}}{1-\frac{e_{v_{i,k}}}{R_{v_{i,k}}}}\right) \tag{10}$$

and its gradient $J_{v_i}(e_{v_i}) := \text{diag}\{[J_{v_{i,k}}(e_{v_{i,k}})]_{i\in\mathcal{N}}\}$, with

$$J_{v_{i,k}}(e_{v_{i,k}}) := \frac{2R_{v_{i,k}}^2}{R_{v_{i,k}}^2 - e_{v_{i,k}}^2} \tag{11}$$

for $i \in \mathcal{N}$, $k \in \{1, \dots, n\}$. Similar to (6), (7), $\chi_{v_{i,k}}$ is an increasing function of $e_{v_{i,k}}$ and it satisfies $\chi_{v_{i,k}}(0) = 0$ and $\lim_{e_{v_{i,k}}\to\pm R_{v_{i,k}}} \chi_{v_{i,k}}(e_{v_{i,k}}) = \pm\infty$, while $J_{v_{i,k}}(e_{v_{i,k}}) \geq 2$ for all $|e_{v_{i,k}}| < R_{v_{i,k}}$, $i \in \mathcal{N}$, $k \in \{1, \dots, n\}$.

We finally design the MAS-BRIC law as

$$u_i = -\left(k_{i,2} + k_{i,3}\int_0^t h_i(e_{v_i}(\tau))\mathrm{d}\tau\right) R_{v_i}^{-1} J_{v_i}(e_{v_i})\chi_{v_i}(e_{v_i}) \tag{12}$$

where $h_i(e_{v_i}) := \|J_{v_i}(e_{v_i})\chi_{v_i}(e_{v_i})\|^2$, $R_{v_i} := \text{diag}\{[R_{v_{i,k}}]_{k\in\{1,\dots,n\}}\}$, and $k_{i,2}$, $k_{i,3}$ are positive constant gains, for all $i \in \mathcal{N}$.

Remark 1: The proposed control scheme does not use any information from the system's dynamics $f_i(\cdot)$ and $g_i(\cdot)$ and *does not* employ any approximation schemes, such as neural networks. Intuitively, the proposed MAS-BRIC framework employs the reciprocal barrier-like terms in (6) and (10) to guarantee the boundedness of the errors $e_{i,k}$ and $e_{v_{i,k}}$ in the domains defined by $R_{i,k}$ and $R_{v_{i,k}}$, respectively, $i \in \mathcal{N}$, $k \in \{1, \dots, n\}$, similar to funnel-based control [14], [17]. Consequently, the unknown dynamic terms $f_i(\cdot)$ and $g_i(\cdot)$ are also bounded due to their continuity properties. The integral term in (12) guarantees then their implicit compensation, leading to asymptotic convergence of $e_i(t)$ and $e_{v_i}(t)$ to zero, for all $i \in \mathcal{N}$. Further note that, unlike our previous works [16], [17], the control protocol (6)-(12) consists of smooth functions of the multi-agent states. Additionally, as will be shown in the sequel, the asymptotic consensus guarantees of the proposed BRIC protocol hold from all initial conditions satisfying $|e_{i,k}(0)| < R_{i,k}$ and $|e_{v_{i,k}}(0)| < R_{v_{i,k}}$, $i \in \mathcal{N}$, $k \in \{1, \dots, n\}$, resulting in a semi-global protocol. Nevertheless, the constants $R_{i,k}$ and $R_{v_{i,k}}$ can be always chosen to satisfy the aforementioned conditions, given the initial conditions, resulting in a *practically global* control scheme. Finally, note that each agent uses only local information from its neighbours to compute (8) and (12), illustrating the distributed nature of the algorithm.

The theoretical guarantees of the proposed MAS BRIC protocol are given in the next theorem.

Theorem 1: Consider a multi-agent system with unknown dynamics (1). Under Assumptions 1-5, the distributed control algorithm (6)-(12) guarantees robust asymptotic multi-agent consensus, i.e., $\lim_{t\to\infty}(x_{i,1}(t) - x_{j,1}(t)) = 0$ for all $i, j \in \mathcal{N}$ with $i \neq j$, and $\lim_{t\to\infty} x_{i,1}(t) = x_\mathrm{d}$, for all $i \in \mathcal{N}$ as well as the boundedness of all closed-loop signals, for $t \geq 0$.

Proof: We first introduce the stacked-vector notation that is necessary for the subsequent analysis:

$$\begin{aligned}
&\bar{x}_1 := [x_{1,1}^\top, \dots, x_{N,1}]^\top,\ \bar{x}_2 := [x_{1,2}^\top, \dots, x_{N,2}]^\top \\
&x := [x_1^\top, \dots, x_N]^\top,\ z := [z_1^\top, \dots, z_N^\top]^\top \\
&v := [v_1^\top, \dots, v_N^\top]^\top, e := [e_1^\top, \dots, e_N^\top]^\top \\
&e_v := [e_{v_1}^\top, \dots, e_{v_N}^\top]^\top,\ u := [u_1^\top, \dots, u_N^\top]^\top \\
&f := [f_1^\top, \dots, f_N^\top]^\top,\ f_z := [f_{z_1}^\top, \dots, f_{z_N}^\top]^\top \\
&\chi := [\chi_1^\top, \dots, \chi_N^\top]^\top,\ \chi_v := [\chi_{v_1}^\top, \dots, \chi_{v_N}^\top]^\top \\
&g := \text{diag}\{g_1, \dots, g_N\}, R := \text{diag}\{R_1, \dots, R_N\} \\
&R_v := \text{diag}\{R_{v_1}, \dots, R_{v_1}\}, J := \text{diag}\{J_1, \dots, J_N\} \\
&J_v := \text{diag}\{J_{v_1}, \dots, J_{v_N}\},\ K_1 := \text{diag}\{[k_{i,1}]_{i\in\mathcal{N}}\} \otimes I_n
\end{aligned}$$

Next, we show that $\|e\|$ and $\|e_v\|$ are bounded by $\|J\chi\|$ and $\|J_v\chi_v\|$, respectively, which will be useful in the following. Towards that end, consider the function $w : \mathbb{R} \to \mathbb{R}$ with $w(\mathrm{y}) = \mathrm{y} - \frac{\exp(\mathrm{y})-1}{\exp(\mathrm{y})+1}$. It is easy to prove that $w(\mathrm{y})$ is increasing, for all y $\in \mathbb{R}$. Indeed, its derivative is

$$w(\mathrm{y})' = 1 - \frac{2\exp(\mathrm{y})}{(\exp(\mathrm{y})+1)^2} = \frac{\exp(\mathrm{y})^2+1}{(\exp(\mathrm{y})+1)^2},$$

which is positive for all y. Therefore, it holds that $|w(\mathrm{y})| \geq |w(0)| = 0$, for all $y \in \mathbb{R}$. In view of (6), (10), we obtain

$$e_{i,k} = R_{i,k}\frac{\exp(\chi_{i,k})-1}{\exp(\chi_{i,k})+1}$$
$$e_{v_{i,k}} = R_{v_{i,k}}\frac{\exp(\chi_{v_{i,k}})-1}{\exp(\chi_{v_{i,k}})+1},$$

for all $i \in \mathcal{N}$, $k \in \{1,\ldots,n\}$. Therefore, it holds that $|e_{i,k}| \leq R_{i,k}|\chi_{i,k}|$, $|e_{v_{i,k}}| \leq R_{v_{i,k}}|\chi_{v_{i,k}}|$ and by noting that $J_{i,k}(e_{i,k}) \geq 2$, $J_{v_{i,k}}(e_{v_{i,k}}) \geq 2$ for all $|e_{i,k}| < R_{i,k}$, $|e_{v_{i,k}}| < R_{v_{i,k}}$, we conclude that $|e_{i,k}| \leq R_{i,k}|J_{i,k}\chi_{i,k}|$, $|e_{v_{i,k}}| \leq R_{v_{i,k}}|J_{v_{i,k}}\chi_{v_{i,k}}|$ for all $i \in \mathcal{N}$, $k \in \{1,\ldots,n\}$. Hence, it holds that

$$\|e\| \leq \bar{R}\|J\chi\| \tag{13a}$$
$$\|e_v\| \leq \bar{R}_v\|J_v\chi_v\| \tag{13b}$$

for all e and e_v satisfying $|e_{i,k}| < R_{i,k}$, $|e_{v_{i,k}}| < R_{v_{i,k}}$, $i \in \mathcal{N}$, $k \in \{1,\ldots,n\}$, and we use $\bar{R} := \max_{i,k}\{R_{i,k}\}$, $\bar{R}_v := \max_{i,k}\{R_{v_{i,k}}\}$.

The multi-agent dynamics (1) can be compactly written as

$$\dot{\bar{x}}_1 = \bar{x}_2 \tag{14a}$$
$$\dot{\bar{x}}_2 = f(\bar{x}_1,\bar{x}_2,z,t) + g(\bar{x}_1,\bar{x}_2,z,t)u(x,t) \tag{14b}$$
$$\dot{z} = f_z(\bar{x}_1,\bar{x}_2,z,t) \tag{14c}$$

By further denoting $\zeta := [\bar{x}_1^\top, \bar{x}_2^\top, z^\top]^\top \in \mathbb{R}^{2Nn+Nn_z}$, we can write (14) as $\dot{\zeta} = f_\zeta(\zeta,t)$. Additionally, consider the nonempty and open set

$$\Omega := \{\zeta \in \mathbb{R}^{2Nn+Nn_z} : |e_{i,k}| < R_{i,k}, |e_{v_{i,k}}| < R_{v_{i,k}}, \forall i \in \mathcal{N}, k \in \{1,\ldots,n\}\} \tag{15}$$

and note that $\zeta(0) \in \Omega$. The closed-loop dynamic function $f_\zeta(\zeta,t)$ is locally Lipschitz in ζ in the set $\{\zeta \in \mathbb{R}^{2Nn+n_z} : (\zeta,t) \in \Omega\}$ for every fixed $t \geq 0$, and continuous in t in the set $\{t \geq 0 : (\zeta,t) \in \Omega\}$ for every fixed $\zeta \in \mathbb{R}^{2Nn+Nn_z}$. Therefore, according to [21, Theorem 2.1.1], there exists a positive time instant $t_{\max}$ and a solution $\zeta(t)$ to (14) satisfying $\zeta(t) \in \Omega$, for all $t \in [0,t_{\max})$, which further implies that $|e_{i,k}(t)| < R_{i,k}$, $|e_{v_{i,k}}(t)| < R_{v_{i,k}}$ for all $t \in [0,t_{\max})$. From (5), we further conclude that $\|\bar{x}_1(t)\| < \lambda_{\min}(H)^{-1}\sqrt{Nn}\bar{R} + \sqrt{N}\|x_{\mathrm{d}}\|$, for all $t \in [0,t_{\max})$.

Let now the function $V_1 = \frac{1}{2}\chi^\top K_1\chi$, which is well defined for $t \in [0,t_{\max})$. By differentiating V_1 along the solutions of (14), and using (4), $\bar{x}_2 = e_v + v$, and $v = -K_1R^{-1}J\chi$ from (8), $\dot{V}$ becomes

$$\dot{V}_1 = -\chi^\top K_1JR^{-1}HK_1R^{-1}J\chi + \chi^\top K_1JR^{-1}He_v \leq -\widetilde{\lambda}\|J\chi\|^2 + \|J\chi\|F_1 \tag{16}$$

where $\widetilde{\lambda} := \lambda_{\min}(R^{-1}K_1HK_1R^{-1})$, and F_1 is a constant independent of $t_{\max}$ satisfying $F_1 \geq \|K_1JR^{-1}He_v\|$ for all $t \in [0,t_{\max})$. Note that $\widetilde{\lambda}$ is positive due to Assumption 4. Therefore, we conclude that $\dot{V}_1 < 0$ when $\|J\chi\| > \frac{F_1}{\widetilde{\lambda}}$. Since $J_{i,k}(e_{i,k}) \geq 2$ for all $t \in [0,t_{\max})$, we infer [22, Theorem 4.18] to conclude the boundedness of $\chi(e(t))$ as $\|\chi(e(t))\| \leq \bar{\chi}$ for all $t \in [0,t_{\max})$, where $\bar{\chi}$ is a positive constant. Therefore, by inverting (6), we conclude that $|e_{i,k}(t)| = R_{i,k}\left|\frac{\exp(\chi_{i,k})-1}{\exp(\chi_{i,k})+1}\right| \leq R_{i,k}\widetilde{\chi} := R_{i,k}\left|\frac{\exp(\bar{\chi})-1}{\exp(\bar{\chi})+1}\right| < R_{i,k}$ for all $t \in [0,t_{\max})$, $i \in \mathcal{N}$, $k \in \{1,\ldots,n\}$. Similarly, we conclude from (7) that $2 \leq J_{i,k}(e_{i,k}(t)) \leq \bar{J}_{i,k} := \frac{2}{1-\widetilde{\chi}^2}$ for $t \in [0,t_{\max})$, $i \in \mathcal{N}$, $k \in \{1,\ldots,n\}$. From (8), we further conclude the boundedness of the reference signals $v(t)$ as $\|v(t)\| \leq \bar{v}$ as well as of $\|\bar{x}_2\| = \|v(t)+e_v(t)\| \leq \bar{v} + \sqrt{Nn}\bar{R}_v$ for all $t \in [0,t_{\max})$.

Next, we show that $\dot{v}$ can be bounded by terms containing $\|J\chi\|$ and $\|J_v\chi_v\|$. From (8), we obtain that

$$\begin{aligned}\dot{v} =& -K_1R^{-1}\dot{J}\chi - K_1R^{-1}J\dot{\chi}\\ \leq& \|K_1R^{-1}\|\|\dot{J}\|\|\chi\| - K_1R^{-1}J^2R^{-1}H\bar{x}_2\\ \leq& \|K_1R^{-1}\|\|\dot{J}\|\|\chi\| + K_1R^{-1}J^2R^{-1}HK_1R^{-1}J\chi\\ &- K_1R^{-1}J^2R^{-1}He_v\end{aligned}$$

By differentiating $J_{i,k}$ and using the aforementioned bounds for $e_{i,k}$ and $J_{i,k}$, we can obtain bounds for $\dot{J}_{i,k}$, for $i \in \mathcal{N}$, $k \in \{1,\ldots,n\}$. By further using the fact that $J_{i,k} \geq 2$ and $J_{v_{i,k}} \geq 2$ for all $t \in [0,t_{\max})$, $i \in \mathcal{N}$, $k \in \{1,\ldots,n\}$ as well as (13), we can bound $\dot{v}$ as

$$\dot{v} \leq \beta\|J\chi\| + \beta_v\|J_v\chi_v\| \tag{17}$$

for all $t \in [0,t_{\max})$ and positive constants β, β_v.

Next, we show the boundedness of χ_v and the asymptotic convergence of $\chi(e(t))$ and $\chi_v(e_v(t))$ to zero. Let

$$\hat{d}_i := k_{i,3}\int_0^t \|J_{v_i}(e_{v_i}(\tau))\chi_{v_i}(e_{v_i}(\tau))\|^2\mathrm{d}\tau$$

and the difference vectors $\widetilde{d}_i := \hat{d}_i - D$ for all $i \in \mathcal{N}$, where D is a constant to be defined later.

Consider now the candidate Lyapunov function

$$V = V_1 + \frac{1}{2\underline{g}}\|\chi_v\|^2 + \sum_{i\in\mathcal{N}}\frac{1}{2k_{i,3}}\widetilde{d}_i^2, \tag{18}$$

where $\underline{g} := \min_{i\in\mathcal{N}}\{\lambda_{\min}\{R_{v_i}^{-1}g_i(x_i,z_i,t)R_{v_i}^{-1}\}\}$. Note that $\underline{g}$ is positive due to Assumption 2. By differentiating V and using (16), we obtain

$$\begin{aligned}\dot{V} \leq& -\widetilde{\lambda}\|J\chi\|^2 + \chi^\top K_1JR^{-1}He_v + \sum_{i\in\mathcal{N}}\frac{1}{k_{i,3}}\widetilde{d}_i\dot{\hat{d}}_i\\ &+ \frac{1}{\underline{g}}\chi_v^\top J_vR_v^{-1}\left(f(x,z,t) + g(x,z,t)u - \dot{v}\right)\end{aligned}$$

Let now $\mu = \|K_1R^{-1}H\|$. By completing the squares and using (13), we obtain for the second term:

$$\chi^\top K_1JR^{-1}He_v \leq \mu\|J\chi\|\|e_v\| \leq \frac{\mu\alpha}{2}\|J\chi\|^2 + \frac{\mu\bar{R}}{2\alpha}\|J_v\chi_v\|^2$$

for a positive constant α.

Next, Assumption 3 and the boundedness of $x(t)$ imply that $z(t)$ is also bounded for all for $[0,t_{\max})$. Hence, the continuity of $f(\cdot)$ (see Assumption 1) implies the boundedness of $f(\bar{x}_1(t), \bar{x}_2(t), z(t), t)$ for $[0, t_{\max})$ by a bound independent of $t_{\max}$. Additionally, in view of Assumption 5, it holds that $f(\bar{x}_\mathrm{d}, \bar{x}_2, z, t) = 0$, where we use $\bar{x}_\mathrm{d} := [x_\mathrm{d}^\top, \ldots, x_\mathrm{d}^\top]^\top \in \mathbb{R}^{Nn}$. By further using (13), (4), and the positive definiteness of H, one obtains that

$$\begin{aligned}\chi_v^\top J_v R_v^{-1} f(\bar{x}_1, \bar{x}_2, z, t) &\le L_f \|R_v^{-1}\| \|\bar{x}_1 - \bar{x}_\mathrm{d}\| \|J_v \chi_v\| \\ &\le L_f \|R_v^{-1}\| \|H^{-1}\| \|e\| \|J_v \chi_v\| \\ &\le \mu_v \bar{R}_v \|J\chi\| \|J_v \chi_v\|\end{aligned}$$

where $\mu_v := L_f \|R_v^{-1}\| \|H^{-1}\|$, and L_f is the Lipschitz constant of $f(\cdot)$ in Ω. By completing the squares, we obtain

$$\mu_v \bar{R}_v \|J\chi\| \|J_v \chi_v\| \le \frac{\mu_v \alpha_v \bar{R}_v}{2} \|J\chi\|^2 + \frac{\mu_v \bar{R}_v}{2\alpha_v} \|J_v \chi_v\|^2$$

for a positive constant α_v. Finally, in view of (17), we obtain

$$\tfrac{1}{\underline{g}} \chi_v^\top J_v R_v^{-1} \dot{v} \le \tfrac{\beta}{\underline{g}} \|R_v^{-1}\| \|J\chi\| \|J_v \chi_v\| + \tfrac{\beta_v}{\underline{g}} \|R_v^{-1}\| \|J_v \chi_v\|^2$$

and by completing the squares:

$$\begin{aligned}\tfrac{1}{\underline{g}} \chi_v^\top J_v R_v^{-1} \dot{v} \le& \frac{\beta \gamma \|R_v^{-1}\|}{2\underline{g}} \|J\chi\|^2 \\ &+ \|R_v^{-1}\| \left(\frac{\beta_v}{\underline{g}} + \frac{\beta}{2\gamma \underline{g}} \right) \|J_v \chi_v\|^2\end{aligned}$$

for a positive constant γ. Consequently, $\dot{V}$ becomes

$$\begin{aligned}\dot{V} \le& -\Lambda_v \|J\chi\|^2 + D \|J_v \chi_v\|^2 + \sum_{i \in \mathcal{N}} \frac{1}{k_{i,3}} \widetilde{d}_i \dot{\hat{d}}_i \\ &- \sum_{i \in \mathcal{N}} \frac{1}{\underline{g}} (k_{i,2} + \hat{d}_i) \chi_{v_i}^\top J_{v_i} R_{v_i}^{-1} g_i R_{v_i}^{-1} J_{v_i} \chi_{v_i}\end{aligned}$$

for $[0, t_{\max})$, where

$$\begin{aligned}\Lambda_v :=& \widetilde{\lambda} - \frac{\mu \alpha}{2} - \frac{\mu_v \alpha_v \bar{R}_v}{2} - \frac{\beta \gamma \|R_v^{-1}\|}{2\underline{g}} \\ D :=& \frac{1}{\underline{g}} \left[\frac{\mu \bar{R} \underline{g}}{2\alpha} + \frac{\mu_v \bar{R}_v}{2\alpha_v} + \|R_v^{-1}\| \left(\frac{\beta_v}{\underline{g}} + \frac{\beta}{2\gamma} \right) \right]\end{aligned}$$

We choose the constants α, α_v, and γ small enough so that $\Lambda_v > 0$. Note that these constants are independent form from the gains $k_{i,1}$, $k_{i,2}$, and $k_{i,3}$ of the control scheme and are introduced for analysis purposes. Next, we observe that $\hat{d}_i(t)$ is non-negative, for all $i \in \mathcal{N}$ and $[0, t_{\max})$, and since $\underline{g} = \min_{i \in \mathcal{N}} \{\lambda_{\min}\{R_{v_i}^{-1} g_i(x_i, z_i, t) R_{v_i}^{-1}\}\}$, we obtain

$$\begin{aligned}\dot{V} \le& -\Lambda_v \|J\chi\|^2 + D \|J_v \chi_v\|^2 - \sum_{i \in \mathcal{N}} (k_{i,2} + \hat{d}_i) \|J_{v_i} \chi_{v_i}\|^2 \\ &+ \sum_{i \in \mathcal{N}} \widetilde{d}_i \|J_{v_i} \chi_{v_i}\|^2\end{aligned}$$

for all $[0, t_{\max})$. By further observing that $D\|J_v \chi_v\|^2 = D \sum_{i \in \mathcal{N}} \|J_{v_i} \chi_{v_i}\|^2$ and using the fact that $\widetilde{d}_i = \hat{d}_i - D$, for all $i \in \mathcal{N}$, we finally obtain

$$\dot{V} \le -\Lambda_v \|J\chi\|^2 - \sum_{i \in \mathcal{N}} k_{i,2} \|J_{v_i} \chi_{v_i}\|^2 \le 0$$

TABLE I

INITIAL CONDITIONS AND PARAMETERS.

Agent	1	2	3	4	5
$x_{i,1}(0)$	(-10.4,-7.4)	(-3.8,-4.8)	(-7.6,0.8)	(-13.2,0)	(-10.3,-0.5)
m_i	0.5	0.5	0.1	0.9	0.5
$A_{d,i}$	0.1	0.4	0.6	0.6	0.4
ω_i	0.5	0.1	0.9	0.9	0.8
ϕ_i	0.6	0.7	0.9	0.8	0.1

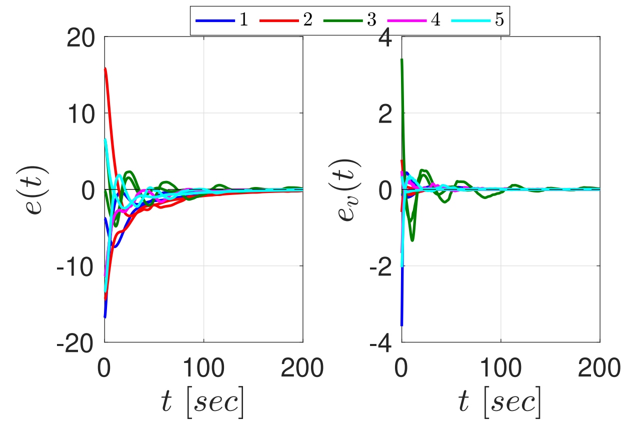

Fig. 1. The evolution of $e_i(t)$ (left) and e_{v_i} (right) of the simulations.

implying that $V(t) \le V(0)$, for all $t \in [0, t_{\max})$. Therefore, we conclude that $\|\chi_v(e_v(t))\| \le \bar{\chi}_v := \sqrt{2\underline{g}V(0)}$, and $|\widetilde{d}_i| \le \bar{d}_i := \sqrt{2k_{i,3}V(0)}$ for all $i \in \mathcal{N}$ and $t \in [0, t_{\max})$, which proves the boundedness of $u(x(t), t)$. Further, inverting (10) leads to $e_{v_{i,k}}(t) \le \bar{e}_v := R_{v_{i,k}} \frac{\exp(\bar{\chi}_v)-1}{\exp(\bar{\chi}:v)+1} < R_{v_{i,k}}$ $t \in [0, t_{\max})$, $i \in \mathcal{N}$, $k \in \{1, \ldots, n\}$. Hence, it holds that $\zeta(t) \in \bar{\Omega}$, where $\bar{\Omega}$ is a compact subset of Ω, defined in (15), leading to $t_{\max} = \infty$ ([21, Theorem 2.1.4]). Therefore, V has a finite limit $\lim_{t \to \infty} V(t)$. By differentiating $\dot{V}$ and using (6), (10), and the boundedness of $\chi(e(t))$, $\chi_v(e_v(t))$, and $\widetilde{d}_i(t)$, $i \in \mathcal{N}$, we conclude that $\ddot{V}(t)$ is bounded for all $t \ge 0$, which implies the uniform continuity of $\dot{V}$. Therefore, Barbalat's lemma [22, Lemma 8.2] dictates that $\lim_{t \to \infty} \dot{V}(t) = 0$, i.e., $\lim_{t \to \infty} \chi_{i,k}(e_{i,k}(t)) = \lim_{t \to \infty} \chi_{v_{i,k}}(e_{v_{i,k}}(t)) = 0$. Since $\chi_{i,k}(e_{i,k})$ and $\chi_{v_{i,k}}(e_{v_{i,k}})$ are increasing functions of $e_{i,k}$ and $e_{v_{i,k}}$, respectively, and $\chi_{i,k}(0) = 0$ and $\chi_{v_{i,k}}(0) = 0$, we conclude that $\lim_{t \to \infty} e_{i,k}(t) = \lim_{t \to \infty} e_{v_{i,k}}(t) = 0$, for all $i \in \mathcal{N}$, $k \in \{1, \ldots, n\}$, concluding the proof. ■

Remark 2: Note that the correctness of Theorem 1 does not rely on any boundedness assumptions or growth conditions for the unknown dynamic terms f_i and g_i, $i \in \mathcal{N}$. These are proven bounded due to the barrier terms of (8)-(12) and hence appropriately compensated by the integral term of (12). Note further that no gain tuning is needed to obtain asymptotic convergence of the consensus errors. Nevertheless, such tuning might prove useful in real scenarios to limit the control inputs within the feasible range that can be produced from the respective actuators.

IV. SIMULATION RESULTS

We perform simulation studies to illustrate the proposed MAS-BRIC scheme. We consider the agent dynamics (1)

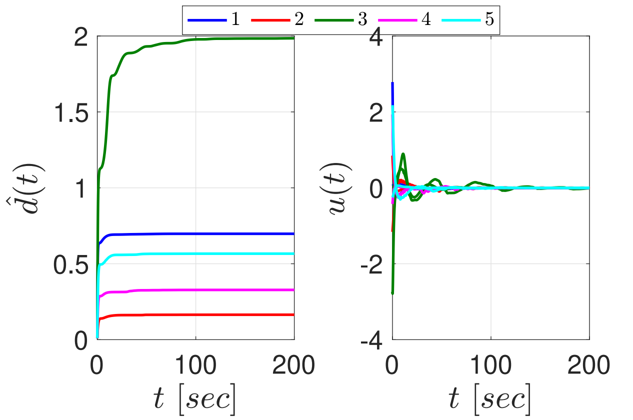

Fig. 2. The evolution of $\hat{d}_i(t)$ (left) and $u_i(t9$ (right) of the simulations.

with $n = 2$. We choose the dynamic terms as $f_i(\cdot) = \sin(\|x_{i,1} - x_{\rm d}\|t)F_i(\cdot)x_{i,2}$, where $F_i = [F_{i,j,\ell}]_{j,\ell\in\{1,2\}}$ with

$$\begin{aligned}
F_{i,1,1} =& - m_i x_{i,2,2} - A_{d,i}\sin(\omega_i t) \\
F_{i,1,2} =& m_i(x_{i,2,1} + x_{i,2,2}) \\
F_{i,2,1} =& x_{i,2,2} A_{d,i}\cos(\omega_i t + \phi_i) \\
F_{i,2,2} =& 0
\end{aligned}$$

and $g_i(\cdot) = m_i \text{diag}\{0.5, 1.4\}$, for all $i \in \mathcal{N}$. We first consider $N = 5$ agents with $x_{\rm d} = [-3, 2.24]^\top$, graph edge set $\mathcal{E} = \{(1,2),(1,4),(1,5),(2,3),(2,5),(3,4),(3,5)\}$, and $b_1 = b_4 = 0$, $b_2 = b_3 = b_5 = 1$. The initial conditions $x_{i,1}(0)$ and parameters of $f_i(\cdot)$ can be found in Table I, while we set $x_{i,2}(0) = [0,0]^\top$, for all $i \in \mathcal{N}$. For the execution of the BRIC scheme (8)-(12), we choose $R_{i,k} = \|e_i(0)\| + 2$ and $R_{v_{i,k}} = \|e_{v_i}(0)\| + 2$, for all $k \in \{1,2\}$, $i \in \mathcal{N}$, and the control gains as $k_{i,1} = k_{i,2} = 3$, $k_{i,3} = 0.1$, for all $i \in \mathcal{N}$.

The results are depicted in Figs. 1 and 2 for $t = 200$ seconds. In particular, Fig. 1 depicts the errors $e_i(t)$ and $e_{v_i}(t)$, respectively, which are shown to converge to zero, for all $i \in \{1,\dots,5\}$; Fig. 2 shows the evolution of the integral terms $\hat{d}_i = k_{i,3}\int_0^t \|J_{v_i}\chi_{v_i}\|^2 \mathrm{d}\tau$ and control inputs $u_i(t)$ for $i \in \{1,\dots,5\}$. It can be concluded that the results verify the theoretical analysis regarding asymptotic convergence of the consensus errors to zero.

V. CONCLUSION AND FUTURE WORK

This paper presents smooth multi-agent Barrier Integral Control (BRIC), a distributed control algorithm that guarantees asymptotic consensus for a class of 2nd-order multi-agent systems with unknown, nonlinear dynamics. The algorithm relies on the novel integration of reciprocal barrier functions and adaptive control in order to drive the consensus errors to zero, without using any information from the agents' dynamic terms. Future efforts will be devoted towards extending the proposed scheme to more general systems, directed graphs, and controllability relaxations.

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