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Safety-critical Stability of Switched Linear Autonomous Systems under Arbitrary Switching

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Abstract

Given a switched system and a safe region that is a subset of the state space, a state trajectory is safe if the whole trajectory is within the safe region. The safety-critical stability problem here is to determine the safe initial domain that is the set of initial states with safe trajectories under arbitrary switching. We prove that the safe initial domain is of full dimension iff the switched system is stable. Moreover, under the assumption that the switched system is exponentially stable and the safe region is regular, we present a computational procedure to numerically characterize the safe initial domain with the help of the newly introduced concept of 'cut-tail-points'. A numerical example is presented to verify the effectiveness of the proposed scheme.

1. Introduction

A switched linear autonomous system is composed by a finite set of linear forced-free subsystems and a switching signal that coordinates the switching among the subsystems. Though switched linear autonomous systems are the simplest hybrid dynamical systems, the analysis and design are shown to be extremely challenging. For stability analysis, while it had been established that stability under arbitrary switching is equivalent to the existence of a common Lyapunov function for all the subsystems, the construction of such a Lyapunov function is NP hard [3, 9]. Other approaches, for instance, the matrix measure approach [13, 18], the worst case phase portrait method [4, 8], have been developed for addressing stability of switched systems. The reader is referred to the monographs [12, 19] and the references therein.

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For complex dynamical systems, safety is an important pre-request in many practical scenarios. For example, when parking a car we have to drive carefully in a restricted physical space and with possible speed limit constraints. During the last two decades, safety-critical systems have been attracting much attention [11, 21]. The application fields include autonomous driving [17], aircraft control [23] and robotic control [6]. From the technical perspective, the barrier certificates approach, the control barrier functions method, and the invariance set approach have been developed [1, 2, 7]. For switched and hybrid systems, extensive theoretical investigation with practical applications has been a hot topic in recent years. For a class of hybrid systems, system safety is equivalent to the safety of each mode and the safety of the discrete transitions among the modes [20]. In [16], the forward invariance set approach was proposed for hybrid inclusions using barrier functions. For a switched system with unsafe subsystems, it is still possible to achieve safety via proper design of the switching signal [15]. In the recent work [22], we investigated the safety-critical stabilization of switched linear systems with designed switching signals, and presented a constructive procedure for estimating the safe initial domain, which is the set of initial states that admit safe trajectories under properly designed switching.

For switched systems, the safety-critical stability problem is challenging due to the fact that we have few handy tools for addressing the problem. Indeed, while the stability implies the existence of a common Lyapunov function, yet we do not have a constructive procedure for finding a common Lyapunov function, even for planar systems. This means that the invariance set approach is limited for stability and safety from a constructive viewpoint. An alternative approach is the spectral abscissa criterion for stability verification, which is based on the fact that the exponential stability of switched linear system is equivalent to the negative spectral abscissa of the set of subsystem matrices [13, 14]. To further address the safety-critical stability problem, we borrow from [22] the notion of safe initial domain, which is the set of initial states with safe trajec-

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tories. We argue that the introduction of the safe initial domain is more practically-oriented than the invariance set approach because we do not care about invariance in many practical scenarios like auto-parking. Our objective in this work is to provide a computational procedure to approximate the safe initial domain under mild assumptions.

The contribution of the work includes: (a) we establish that the safe initial domain is of full dimension iff the switched system is stable; (b) a computational procedure is developed for approximating the safe initial domain, and the 'cut-tail-points' method proposed recently in [10] is utilized to reduce the computational load; and (c) a motive example and an illustrative example are presented to show the motivation and effectiveness of the proposed approach.

2. Preliminaries

2.1. System Description

Let $\mathbf{R}$ be the set of real numbers. Given a positive integer l, let $\bar{l}$ be the set $\{1,\ldots,l\}$. **Co** denotes the convex hull of a set in $\mathbf{R}^n$. For two sets $\mathscr{G}_1$ and $\mathscr{G}_2$, define

$$\mathscr{G}_1 - \mathscr{G}_2 = \{g : g \in \mathscr{G}_1, g \notin \mathscr{G}_2\}.$$

A continuous-time switched linear autonomous system is described by

$$\dot{x}(t) = A_{\sigma(t)}x(t), \ \ t \geq t_0, \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the system state, $\sigma(t) \in \bar{m}$ is the switching signal, and $A_i \in \mathbf{R}^{n\times n}$, $i \in \bar{m}$ are known real constant matrices. Without loss of generality, we assume that $t_0 = 0$. Denote $\mathscr{A} = \{A_1,\ldots,A_m\}$.

Let $\phi(t;x_0,\sigma)$ denote the motion of system (1) at time t starting from $x(0) = x_0$ via switching signal σ. It is clear that

$$\phi(t;x_0,\sigma) = \Phi(t,\sigma)x_0, \tag{2}$$

where $\Phi(t,\sigma)$ is the state transfer matrix along switching signal σ. Let $\mathscr{S}$ be the set of well-posed switching signals.

A switching path θ is a time-driven switching signal defined over $[0,s)$, $s \in [0,+\infty)$, and its length is defined to be $|\theta| = s$. Let θ_1 and θ_2 be two switching paths. The concatenation of θ_1 and θ_2 is defined to be

$$(\theta_1 \wedge \theta_2)(t) = \begin{cases} \theta_1(t) & t \in [0,|\theta_1|) \\ \theta_2(t-|\theta_1|) & t \in [|\theta_1|,|\theta_1|+|\theta_2|). \end{cases}$$

The concatenation of multiple paths could be defined in a same way. Suppose that $\mathscr{P}_1$ and $\mathscr{P}_2$ are two sets of switching paths. Define their multiplication by

$$\mathscr{P}_1 \odot \mathscr{P}_2 = \{p_1 \wedge p_2 : p_1 \in \mathscr{P}_1, p_2 \in \mathscr{P}_2\}.$$

2.2. A Motive Example

Let us examine the two-form planar switched system (1) with

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0 & 1 \\ -10 & -4 \end{bmatrix},$$

where both A_1 and A_2 are Hurwitz stable. By the phase portrait analysis (Cf. [4, 5]), we could compute the worst-trajectory in the phase plane, which shows that the switched system is exponentially stable under arbitrary switching. On the other hand, it could be verified that the switched system does not admit a common quadratic Lyapunov function. In fact, we do not have a systematic approach for constructing a common Lyapunov function for planar switched systems.

Suppose that the unit ball is the safe region, and we need to determine the set of initial states with safe trajectories under arbitrary switching. Once again, by the computation of worse-case trajectory, we could characterize the safe initial domain, as depicted in Figure 1. Note that the construction of a common Lyapunov function is not needed here, and the estimation of the safe initial domain is exact. This shows the advantage of safe initial domain analysis against the set invariance approach, which is not available for the example.

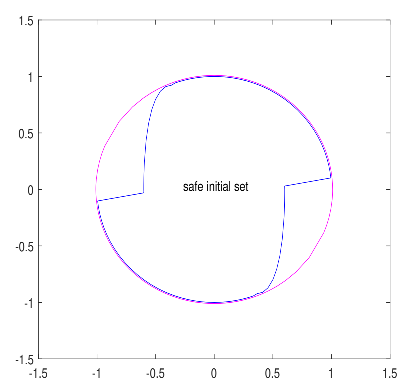

Figure 1. The safe initial domain vs safe region (unit ball)

2.3. Problem Formulation

Let $\mathscr{X} \subset \mathbf{R}^n$ be a compact region. The region is of full dimension (around the initial state) in the sense that it contains the origin as an interior point. Denote $\mathscr{X}_{bd}$ to be the set of boundary points of $\mathscr{X}$, and $\mathscr{X}_{int}$ to be the set of interior points of $\mathscr{X}$. We call $\mathscr{X}$ the safe region. A system trajectory is said to be safe, if the whole trajectory is inside the set region. The safe initial domain is defined by

$$\mathscr{X}^0 = \{x : \phi(t;x,\sigma) \in \mathscr{X},\ \forall\, \sigma \in \mathscr{S},\ t \geq 0\}. \quad (3)$$

It is clear that the safe initial domain $\mathscr{X}^0$ is a subset of the safe region $\mathscr{X}$, and is compact.

The safety-critical stability problem is to characterize the safe initial domain for the switched system.

Remark 1 *The problem is stability analysis with safety constraints. The formulation itself is more general than the forward invariance framework in the literature. Compared with the invariance set approach, the problem here is more physically and practically oriented.*

2.4. Technical Preparations

Definition 1 *Switched system (1) is said to be*

a) stable, if there is a class $\mathscr{K}$ function φ such that

$$\|\phi(t;x,\sigma)\| \leq \varphi(\|x\|),\ \ \forall\, t \geq 0,\ x \in \mathbf{R}^n,\ \sigma \in \mathscr{S};$$

b) exponentially stable, if there are positive real numbers α and β, such that

$$\|\phi(t;x,\sigma)\| \leq \beta e^{-\alpha t}\|x\|, \forall\, t \geq 0, x \in \mathbf{R}^n, \sigma \in \mathscr{S};$$

and

c) marginally stable if it is stable but is not exponentially stable.

For any norm ν in $\mathbf{R}^n$, there is an induced matrix measure

$$\mu_\nu(A) = \max_{\nu(x)=1} \lim_{s\searrow 0} \frac{\nu(x+sAx)-\nu(x)}{s},\ \forall\, A \in \mathbf{R}^{n\times n}.$$

Furthermore, define

$$\mu_\nu(\mathscr{A}) = \max_{i\in\bar{m}} \mu_\nu(A_i)$$

and

$$\mu_*(\mathscr{A}) = \inf_{\nu\in\Gamma} \mu_\nu(\mathscr{A}),$$

where Γ is the set of norms over $\mathbf{R}^n$. When there is a matrix measure μ_0 with $\mu_0(\mathscr{A}) = \mu_*(\mathscr{A})$, we call it an extreme measure.

Lemma 1 *[19, Corollary 2.29] For switched system (1), we have the following statements:*

1) The system is exponentially stable iff $\mu_*(\mathscr{A}) < 0$.

2) The system is marginally stable iff $\mu_*(\mathscr{A}) = 0$ and it admits an extreme measure.

Suppose A is Hurwitz and $x \in \mathbf{R}^n$. Define the convex hull

$$\mathscr{G}_x^A = \mathbf{Co}\{e^{At}x,\ -e^{At}x,\ \forall\, t \geq 0\}.$$

Definition 2 *Time $T > 0$ is called a cut tail time for x if for any $t \geq T$, $e^{At}x$ belongs to the relative interior of $\mathscr{G}_x^A$.*

Define $T_{cut}(x)$ be the smallest cut tail time for x.

Lemma 2 *[10] For all generic points of a Hurwitz matrix A, the value $T_{cut}(x)$ is the same and is the maximal one among all values $T_{cut}(x)$, $x \in \mathbf{R}^n$.*

According this lemma, for the convex hull of any state trajectory of a stable linear system, the trajectory within $[0, T_{cut}]$ is on the boundary and the trajectory within $(T_{cut}, +\infty)$ is in the interior.

3. Main Results

3.1. Basic Analysis

Proposition 1 *$0 \in \mathscr{X}_{int}^0$ iff the switched system is stable.*
Proof: Suppose that the switched system is stable. It follows from Lemma 1 that there is a norm ν with $\mu_\nu(\mathscr{A}) \leq 0$. This implies that the state trajectory is always norm contractive *w.r.t.* the norm, *i.e.*,

$$\nu(\phi(t;x,\sigma)) \leq \nu(x),\ \ \forall\, \sigma \in \mathscr{S},\ t \geq 0. \quad (4)$$

Let $d = \min_{x\in\mathscr{X}_{bd}} \nu(x)$. It is clear that $d > 0$, and $\mathscr{B}^\nu(d) \subset \mathscr{X}$, where $\mathscr{B}^\nu(d)$ is the ball centered at the origin with radius d *w.r.t.* norm ν. As a result, we have

$$0 \in (\mathscr{B}^\nu(d))_{int} \subset \mathscr{X}_{int}^0.$$

Conversely, suppose that $0 \in \mathscr{X}_{int}^0$. Let $\|\cdot\|$ be any norm on $\mathbf{R}^n$. Define

$$e = \min\{\|x\| : x \in \mathscr{X}_{bd}^0\}.$$

It is clear that $e > 0$. Let $L = \frac{d}{e}$. By the definition of safe initial domain, we have

$$\|\phi(t;x,\sigma))\| \leq L\|x\|,\ \ \forall\, \sigma \in \mathscr{S},\ t \geq 0, \quad (5)$$

which implies that the switched system is stable. ■

3.2. Computational Procedure

From a computational viewpoint, the case of marginal stability is not verifiable arithmetically. To address the safety-critical problem in a computational manner, it is natural to impose the following assumption.

Assumption 1 *The switched system is exponentially stable.*

To induce a norm from the safe region, we further impose the regularity as follows.

Assumption 2 *$\mathscr{X}$ is convex, simply connected, and origin symmetric.*

Examples of such regions includes ellipsoids and polytopes. Under Assumption 2, the safe region induces a Minkowski function

$$\|x\|_{\mathscr{X}} = \inf\{\alpha > 0 : x \in \alpha\mathscr{X}\}, \;\; x \in \mathbf{R}^n,$$

which is in fact a norm on $\mathbf{R}^n$. Under this norm, we have

$$x \in \mathscr{X} \Leftrightarrow \|x\|_{\mathscr{X}} \leq 1.$$

That is, the safe region $\mathscr{X}$ is the unit ball with respect to the norm, and the boundary set $\mathscr{X}_{bd}$ is the unit sphere.

Next, we discuss how to characterize the safe initial domain in a more concise manner. Suppose that we have verified that the switched system is exponentially stable with

$$\begin{aligned} \|\phi(t;x,\sigma)\|_{\mathscr{X}} \quad &\leq \quad \beta_0 e^{-\alpha t}\|x\|_{\mathscr{X}}, \\ &\forall\, t \geq 0,\; x \in \mathbf{R}^n,\; \sigma \in \mathscr{S}, \end{aligned} \quad (6)$$

where $\beta_0 \geq 1$ and $\alpha_0 > 0$ are known.

We fix $\|\cdot\|_{\mathscr{X}}$ to be the vector norm and induced matrix norm. For notational convenience, we drop the subscript of the symbol. Denote

$$\mathscr{H}_1^+ = \{x : \|x\| = 1, x_n \geq 0\},$$

which is the upper half unit sphere.

Define

$$r(x) = \sup_{\sigma\in\mathscr{S}} \sup_{t\geq 0} \|\phi(t;x,\sigma)\|/\|x\|, \;\; x \neq 0,$$

which is the state norm overshoot for all state trajectories initially from x. In addition, let $r(0) = 1$. It is clear that $r(x)$ is continuous (except at the origin) and homogeneous. By the definition of the safe initial domain, we have

$$\mathscr{X}^0 = \{x : r(x)x \in \mathscr{X}\}.$$

This implies that, if we could compute the function r, then we could determine the safe initial domain. Utilizing the homogeneity, we only need to compute r on the upper unit sphere, $\mathscr{H}_1^+$.

Denote T_i the cut-tail time of A_i, and $T = \frac{\ln\beta_0}{\alpha_0}$. Let $\mathscr{S}_T$ be the set of well-posed switching paths defined over $[0,T]$. Any switching path p in this set admits time/index pair sequence

$$p_{\varpi} = \{t_0, j_0), (t_1, j_1), \ldots, (t_s, j_s)\}$$

with $t_0 = 0$ and $t_s = T$. The i-th dwell time, $d_i(p)$, is

$$d_i(p) = \max\{t_{k+1} - t_k : j_k = i,\; k = 0,1,\ldots,s-1\}.$$

Define

$$\mathscr{S}_T^* = \{p \in \mathscr{S}_T : d_i(p) \leq T_i,\; i \in \bar{m}\}.$$

Proposition 2 *For any $x \in \mathscr{H}_1^+$, we have*

$$r(x) = \sup_{\sigma\in\mathscr{S}_T^*} \max_{t\geq 0} \|\phi(t;x,\sigma)\|.$$

Proof: We proceed by contradiction. Suppose that there is an $x \in \mathscr{H}_1^+$ and a switching path $p \notin \mathscr{S}_T - \mathscr{S}_T^*$ with

$$\max_{t\geq 0} \|\phi(t;x,p)\| > \sup_{\sigma\in\mathscr{S}_T^*} \max_{t\geq 0} \|\phi(t;x,\sigma)\|. \quad (7)$$

This implies the existence of an $i \in \bar{m}$ such that $d_i(p) > T_i$. Clearly we have

$$\phi(t;x,p) = e^{A_{j_{s-1}}h_{s-1}} \ldots e^{A_i h_k} e^{A_{j_{k-1}}h_{k-1}} \ldots e^{A_{j_1}h_0}x,$$

with $h_l = t_l - t_{l-1}$, $l = 0,1,\ldots,s-1$, and $h \geq T_i$. Let

$$y = e^{A_{j_{k-1}}h_{k-1}} \ldots e^{A_{j_1}h_0}.$$

According the Lemma 2, $e^{A_i h}y$ is inside the convex hull

$$\mathbf{Co}\{e^{A_i t}y, -e^{A_i t}y : t \in [0,T_i]\}.$$

This implies that $\phi(t;x,p)$ is in the convex hull

$$\mathbf{Co}\{Cy, -Cy : t \in [0,T_i]\},$$

where $C = e^{A_{j_{s-1}}h_{s-1}} \ldots e^{A_i h_0}$. In this way, we have $\phi(t;x,p)$ is in the convex hull

$$\mathbf{Co}\{\phi(s;x,\sigma), \phi(s;-x,\sigma) : \sigma \in \mathscr{S}_T^*, s \leq t\}.$$

By definition, we obtain

$$\max_{t\in[0,T]} \|\phi(t;x,p)\| \leq \sup_{s\in[0,T]} \{\|\phi(t;x,\sigma)\| : \sigma \in \mathscr{S}_T^*\},$$

which contradicts (7). ■

Next, under Assumption 1, suppose that we have a norm ν, such that

$$\mu_0 = \mu_\nu(\mathscr{A}) < 0.$$

It follows that

$$\nu(\phi(t;x,\sigma)) \leq e^{-\mu_0 t}\nu(x), \;\; \forall\, t \geq 0, x \in \mathbf{R}^n, \sigma \in \mathscr{S}_T^*.$$

Define

$$\rho = \max\{\nu(x) : x \in \mathscr{H}_1^+\}.$$

For any $x \in \mathscr{H}_1^+$, define

$$\psi(x) = \rho/\nu(x).$$

It can be seen that

$$r(x) \le \psi(x), \;\; \forall\, x \in \mathscr{H}_1^+. \tag{8}$$

With the above preparations, we are to develop a computational procedure to estimate $r(x)$ in a more accurate manner. For this, fix an $x \in \mathscr{H}_1^+$, and let

$$T_x = \frac{\ln(\psi(x))}{\mu_0}.$$

Computational Procedure for Calculating $r(x)$

Initialization: Grid the time space $[0, T_i]$ by $0 < t_{i,1} < \ldots < t_{i,j_i} = T_i$, $i \in \bar{m}$. Denote p_k^i, $k = 1, \ldots, j_i$ the switching path over $[0, t_{i,k}]$ taking the value i constantly. Let $f = \psi(x)$ and $\mathscr{P} = \{p_k^i : i \in \bar{m}, k \in \bar{j}_i\}$. Let $\mathscr{P}_0 = P$ and $\mathscr{P}_1 = \emptyset$.

Iteration:

1. For any $p \in \mathscr{P}_0$, compute

   $$ftmp = \max_{t \in [0,|p|]} \{\|\phi(t; x_0, p)\|\}.$$

   If $f < ftmp$, then let $f = ftmp$. If $f = \psi(x)$, output $r(x) = \psi(x)$ and stop. Otherwise, for any $p \in P_0$, compute

   $$s_1 = \arg \min_{t \in [0,|p|]} \{\|\phi(t; x_0, p)\|\}.$$

   If

   $$\psi(\phi(s_1; x_0, p))\|\phi(s_1; x_0, p)\| > f, \tag{9}$$

   then let $\mathscr{P}_1 = \mathscr{P}_1 \cup p$.

2. If $\mathscr{P}_1 = \emptyset$, then output $r(x) = f_2$ and stop. Otherwise, let $\mathscr{P}_0 = \mathscr{P}_0 \bigodot \mathscr{P}_1$. Remove any p with $|p| > T_x$ from $\mathscr{P}_0$, and continue the iteration.

Remark 2 *Note that the procedure terminates in a finite steps as the stopping condition $|p| > T_x$ is satisfied in a finite steps. Moreover, condition (9) could prune the switching paths with relatively small overshoots. The computational load depends on T_x and the time grids. When the load is affordable, we could dense the grids adaptively to produce better estimation of $r(x)$.*

3.3. Illustrative Example

Let us examine the two-form third-order switched linear system with

$$A_1 = \begin{bmatrix} 5.8841 & 7.9921 & 5.2029 \\ -7.8556 & -10.0016 & -5.7216 \\ 3.9653 & 4.6037 & 2.0073 \end{bmatrix}$$

and

$$A_2 = \begin{bmatrix} 4.0249 & 5.3079 & 2.3262 \\ -5.9310 & -7.1902 & -2.5825 \\ 3.2662 & 3.4672 & 0.5499 \end{bmatrix}.$$

By applying the state transform technique [13], we could compute that $\mu_*\{A_1, A_2\} \le -0.3137$, which shows that the switched system is exponentially stable.

First, suppose that the safe region is the unit ball, and the norm $\|\cdot\|_{\mathscr{X}}$ is the standard ℓ_2 norm. It can computed that $\rho = 11.75$. Take state grids with 2400 points equally distributed over the upper unit sphere, and apply the Computational Procedure for Calculating $r(x)$ for each state. The approximated safe initial domain is depicted in Figure 2.

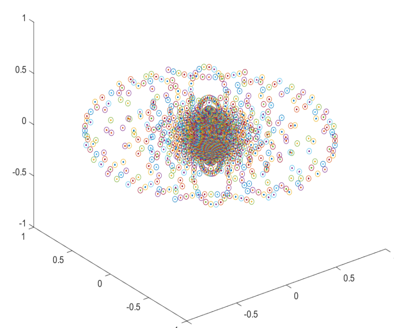

Figure 2. The safe initial domain for unit ball safe region

Next, suppose that safe region is the polytope $\mathscr{X} = \{x : \max_{j=1}^3 |x_j| \le 1\}$, which corresponds to ℓ_1 norm. Take state grids with 4066 points equally distributed over the upper unit tube sphere. The approximated safe initial domain is depicted in Figure 3.

4. Concluding Remarks

In this work, the safety-critical stability problem has been investigated for continuous-time switched linear autonomous systems under arbitrary switching. We

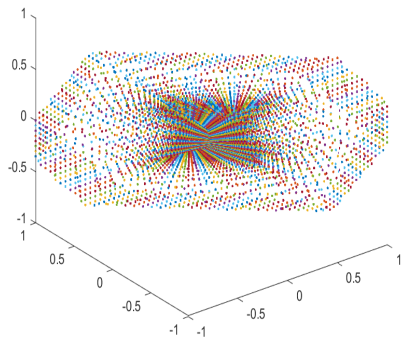

Figure 3. The safe initial domain for unit cube safe region

proved that the safe initial domain is of full dimensional iff the switched system is stable. A computational procedure was developed to numerically estimate the domain. The effectiveness of the proposed method was verified by a numerical example.

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